

Analyzing Plato's Arguments: Plato and Platonism

S. Marc Cohen and David Keyt

1. Introduction

Does Plato have a philosophy? If so, what is it and how does he argue for it? Simple questions. But there are well-known obstacles standing in the way of their answer. First of all, Plato writes dialogues; and it is often unclear which character, if any, in a given dialogue speaks for Plato. Secondly, when a character in a dialogue advances a thesis, it is often unclear what the thesis is. And, finally, when a thesis is backed up by an argument, crucial premises are often missing.

In this paper we wish to focus on this last obstacle and consider some of the issues it raises, though the other two obstacles will not go unnoticed. Suppose, then, that one has at least surmounted the first obstacle and is dealing with an argument that can reasonably be attributed to Plato himself and not just to a character in a dialogue. Suppose, further, that the argument is missing a crucial premise. The basic issue that we shall address is the proper goal of an interpretation that supplies the missing premise. Is the goal to divine Plato's thought or to extend it? When an interpreter supplies the missing premise, what is he really doing? Is he expounding Plato or platonizing? This issue arises because of the paradoxical consequences of a major principle of interpretation.

In attempting to understand a passage from a major philosopher such as Plato, Aristotle, Kant, or Wittgenstein, interpreters often seem to be guided by a principle of charity that directs them to put as favorable a construction as possible on the passage under consideration. And this is wise strategy. For an interpreter who ignores this principle risks missing a good, perhaps even a profound, point. If the passage under consideration contains an argument, the principle of charity says that, other things being equal, one interpretation is better than another just to the extent that the one produces a better argument than the other. Suppose, now, that the principle is applied to a philosophical text that contains an argument whose conclusion follows from its explicit premises only by the addition of a "tacit" or "suppressed" premise¹—the sort of argument that traditional logic calls an *enthymeme*. Paradoxical consequences follow in six stages.

2. A Paradox of Interpretation

First Stage

Following the principle of charity, an interpreter, when faced with a passage from a major philosopher that contains an enthymeme, will search for a suppressed premise rather than charge the philosopher with a *non sequitur*. Ordinarily an interpreter will have an indefinite number of possible premises to choose among. Still pursuing the principle of charity, the interpreter tries to make the most sympathetic choice among the possible premises. Often it will be possible for an interpreter to judge that one premise is a better choice than a second by appealing to other passages in the same work or, failing this, in other writings of the same philosopher. If there is a passage in which the one premise is asserted by the philosopher but no passage in which the second is asserted, then in selecting a tacit premise the one is a better choice than the other. In this case the interpreter fills the hole in the argument by extending the context of the argument. We shall call such an argument an *apparent* enthymeme. An argument

with a suppressed premise is a *real* enthymeme, on the other hand, if, and only if, the suppressed premise cannot be supplied by extending the context.

In the case of a real enthymeme how does an interpreter choose among the various possibilities? He continues to invoke the principle of charity. If the proposition expressed by one possible premise seems to the interpreter to be more reasonable than the proposition expressed by another, the interpreter, led by the principle of charity, will supply the one rather than the other. But reasonable in what sense, and to whom? One possibility is that it is reasonable for the interpreter to believe that the author of the text believed the proposition, or would have believed it had he entertained it. Such a proposition is reasonable for the interpreter *to attribute* to the author. Another possibility is that it is reasonable for the interpreter himself to believe the proposition according to his own lights. Such a proposition is reasonable for the interpreter *to hold*. Thus the proposition that the earth is shaped like a drum is a reasonable one for us to attribute to Anaximander, but not, of course, a reasonable one for us to hold.

Which sort of reasonableness is demanded in the interpretation of a real enthymeme? At first sight, it may appear that it is only the first sort—reasonableness to attribute—that is at issue. For historians are expected, aren't they, to ferret out the beliefs of the historical figures they study, not necessarily to share them? But we contend that a tacit premise for a real enthymeme must be reasonable for the interpreter to hold. For any proposition that it is reasonable to attribute to an author is reasonable on the basis of the text or the context. But the gap in a *real* enthymeme is precisely the sort of gap that cannot be filled by an appeal to the text or context however much the context is broadened. Consequently, an interpreter here has no basis for supplying the missing premise beyond his own sense of what it is reasonable for a rational person to hold.

Suppose two interpreters differ on the question of reasonableness. They propose competing and inconsistent interpretations of a real enthymeme.² Now, if the correctness of an interpretation were an objective fact, at most one of the interpreters could be right. But since we are dealing with a *real* enthymeme, there is no principle beyond the principle of charity to adjudicate between the two interpretations. And by hypothesis the principle of charity is unable to decide between them; each interpreter applies the principle correctly, and each prefers his own interpretation. Of course one interpreter may be an idiot and the other a Princeton professor, but the situation envisaged often arises when both interpreters are good philosophers and equally rational. This brings us to our first conclusion:

Either there is no objectively correct interpretation of any real enthymeme found in the text of a major philosopher or else it is inaccessible to us.

Second Stage

The situation envisaged so far is this. An interpreter is analyzing an argument with a suppressed premise and considering which premise among various competing candidates to supply. In choosing among the candidates, the principle of charity directs him to choose the most reasonable. But the interpreter may regard none of the possible premises as reasonable to hold. This is an unstable situation in scholarship and leads to the second stage. An interpreter who is guided by the principle of charity always seeks a premise that is reasonable to hold, and he will not regard the enthymeme as adequately interpreted until such a premise has been found. Hence our second conclusion:

No adequately interpreted real enthymeme of a major philosopher has a *conclusion* that an interpreter will judge to be false unless it also has at least one explicit premise that he also judges to be false.

Third Stage

In attempting to understand a passage from a major philosopher that contains a real enthymeme, it is usually necessary not only to supply a suppressed premise but also to interpret the enthymeme's conclusion and its explicit premises. By the principle of charity an interpreter puts as favorable a construction on a given sentence as possible. He tries to find an interpretation under which it expresses a proposition that in his judgment is close to the truth. Indeed, if a given sentence appears to express a false proposition, he will try to find an interpretation under which it expresses a true proposition.³ Consequently, an interpreter strives for an interpretation under which *all* the premises of a real enthymeme, explicit as well as implicit, express propositions that are close to the truth. Thus we arrive at our third conclusion:

An interpreter will not regard a real enthymeme of a major philosopher as adequately interpreted until he has found a way of reading it that makes it into a good argument, that is to say, an argument that is valid and all of whose premises, explicit and implicit, express propositions that in his judgment are close to the truth.

Fourth Stage

An interpreter's judgments about truth and reasonableness are affected, if not determined, by the philosophy that is current in his day. Thus an interpreter will always strive for a reading of a passage from a major philosopher by which the passage expresses something that would be reasonable for a contemporary philosopher to hold. Hence our fourth conclusion:

All philosophy, including that written 2400 years ago, is contemporary philosophy. Or, phrased another way, all interpretation is anachronistic.

Fifth Stage

The foregoing conclusion can be strengthened. A charitable interpreter will undoubtedly rank Plato higher than any contemporary philosopher. Thus in interpreting Plato he will set his sights higher than the philosophy of his own day. His standard of reasonableness will be perfect reasonableness rather than that of contemporary philosophy. But such reasonableness belongs only to a god. Hence our fifth conclusion:

To a charitable interpreter every classical text is a sacred text and every classical philosopher infallible and omniscient.

Sixth Stage

Furthermore, there can be no discord in heaven. Charitable interpretation cannot allow for the possibility that two major philosophers might disagree. For the contrary hypothesis leads to a contradiction. Suppose that Plato and Aristotle disagree over some issue. Suppose, for example, that Aristotle claims that an idea of Plato's is false and that Plato's argument for it is invalid. Since Plato and Aristotle are both major philosophers, both must be interpreted charitably. On a charitable interpretation of Aristotle, Aristotle reads Plato correctly, and his criticism of Plato is well-taken. On the

other hand, on a charitable interpretation of Plato, Plato's idea is reasonable and his argument for it valid. Thus an interpreter who is interpreting Aristotle interpreting Plato and who is charitable to both Plato and Aristotle must find the very same argument both valid and invalid. This is impossible. So the original hypothesis, that it is possible for two major philosophers to disagree, must be false. Hence our sixth and final inference from the principle of charity:

Major philosophers never disagree.

This conclusion seems preposterous. But the venerable philosophical tradition of syncretism reflects the propensity of interpreters to reach the sixth stage.

3. An Illustration: The *TMA*

Modern Platonic scholarship might seem far removed from syncretism. But it is not difficult to find in recent scholarship a series of discussions of a single Platonic text guided by the principle of charity that maps most of the stages of the paradox of interpretation. The recent history of the interpretation of the Third Man Argument (*TMA*) in Plato's *Parmenides* is one example.

The *TMA*, as all students of the *Parmenides* will recall, threatens the Theory of Forms with an infinite regress:⁴

This, I suppose, is what leads you to believe that each form is one. Whenever many things seem to you to be large, some one form probably seems to you to be the same when you look at them all. So you think that largeness is one. . . . But what about largeness itself and the other large things? If you look at them all in your mind in the same way, won't some one largeness appear once again, by virtue of which they all appear large? . . . So another form of largeness will have made an appearance, besides largeness itself and its participants. And there will be yet another over all these, by virtue of which they will all be large. So each of your forms will no longer be one, but an infinite multitude.

The argument is plainly an enthymeme. Its only explicit premise is a One-Over-Many assumption:⁵

(*OM*) If a number of things are all *F*, it follows that there is a Form in virtue of which they are all *F*.

The reasoning proceeds as follows. *OM* is applied to an initial collection of *F* things and generates a Form of *F*-ness. The Form is then added to the initial collection, and *OM* is applied to this new collection, generating another Form. The new Form is added to the previous collection, and *OM* is applied again. The process can be repeated, ad infinitum.

In Gregory Vlastos's seminal article,⁶ *OM* appears in the following form:

(*OM_v*) If a number of things . . . are all *F*, there must be a single Form *F*-ness, in virtue of which we apprehend [them] as all *F*.

And as Vlastos observes, *OM_v* alone does not entail an infinite regress of Forms. For (a) no provision has been made for the second application of *OM_v*, and (b) nothing explicitly stated entitles us to infer that the second application of *OM_v* generates a second Form, distinct from the one generated by the first application of *OM_v*.

Did Vlastos dismiss the *TMA* as invalid? Not at all. Declaring that “there must have been something more in Plato’s mind” (p. 236) than what appears in OM_v , he proposed two additional tacit premises to fill in the gaps in the reasoning. To justify the second application of OM_v , Vlastos proposed a Self-Predication assumption:

(SP_v) *F*-ness is *F*.

Now when *F*-ness is collected together with the *F* things that participate in it, SP_v guarantees that they are, all of them, *F*. Hence, the second application of OM_v is provided for.

To justify the inference to a new Form, Vlastos proposed a Non-Identity assumption:

(NI_v) If *x* is *F*, then *x* cannot be identical with *F*-ness.

The idea is simple enough. Something which is *F* by virtue of participating in a Form cannot be identical to that Form. Hence the Form generated by the first application of OM_v cannot be identical to the Form generated by the second application; otherwise, it would be identical to the Form in virtue of which it is *F*.

A striking feature of this reconstruction, as Vlastos himself noted, is that his two tacit premises are mutually inconsistent. (The contrapositive of NI_v says that if *x* is identical to *F*-ness, then *x* is not *F*. More simply put, this says that *F*-ness is not *F*, which is the contradictory of SP_v .) Still, he was convinced that these two assumptions had to be made for the argument to go through. So, believing that the premises necessary to generate the regress are inconsistent, Vlastos maintained that Plato must have been unaware of them. He concluded that there is a buried inconsistency in the Theory of Forms of which Plato was only dimly aware, and that the *TMA* is a “record of [Plato’s] honest perplexity” (p. 254).

Not surprisingly, Vlastos’s article unleashed a torrent of critical response. From the first, Vlastos’s critics have urged that there need be no inconsistency in the *TMA*’s premises. Wilfrid Sellars⁷ advanced a reconstruction of the *TMA* which not only reconciled its two tacit premises (SP and NI) but also resolved the inconsistency between them and OM . The first reconciliation was carried out by reformulating NI as a principle that denies self-participation rather than self-predication. To assert that nothing participates in itself (and, in particular, that *F*-ness does not participate in *F*-ness) does not contradict SP . It merely entails, when conjoined with SP , that at least one *F* thing—namely, *F*-ness itself—does not participate in *F*-ness.

But this last proposition conflicts with OM_v , which entails that every *F* thing (including *F*-ness itself) participates in *F*-ness. So further repairs were needed to remove the inconsistency from the entire premise-set. Sellars observed that Vlastos had used the expression “*F*-ness” as if it represented a proper name of a Form, and proposed instead that “*F*-ness” be taken to represent a quantifiable variable.⁸ This simple syntactic maneuver removes the remaining inconsistency. Here are the *TMA*’s premises as Sellars formulated them:

(OM_s) If a number of things are all *F*, it follows that there is an *F*-ness in virtue of which they are all *F*.

(SP_s) All *F*-nesses are *F*.

(NI_s) If *x* is *F*, then *x* is not identical with the *F*-ness by virtue of which it is *F*.⁹

These three premises are mutually consistent; they do not entail a contradiction. But they do entail that if there are any *F* things at all, there is an infinite regress of *F*-nesses.

In response,¹⁰ Vlastos conceded that his formulation of the Non-Identity assumption had been defective, and that Sellars had succeeded in “deriv[ing] the regress by an internally consistent argument” (p. 353). But Vlastos denied that this could have been the argument that Plato intended. For where Plato’s version of *OM* posits a *unique* Form (“whenever many things seem to you to be large, some *one* form probably seems to you to be the same when you look at them all,” 132a2-3), Sellars’ *OM_s* has an ordinary existential quantifier (“there is *an F*-ness”). But throughout this context, and in numerous others, Vlastos insists, it is clear that by ‘one Form’ Plato means ‘exactly one Form’. Concluding that *OM* cannot be correctly formulated without a uniqueness quantifier, Vlastos offered this revised version of the *TMA*’s premise-set:

(*OM_{v2}*) If any set of things share a given character, then there exists a unique Form corresponding to that character; and each of these things has that character by participating in that Form (p. 348).

(*SP_{v2}*) The Form corresponding to a given character itself has that character (p. 351).

(*NI_{v2}*) If anything has a given character by participating in a Form, it is not identical with that Form (p. 351).

On this revision *SP* and *NI* are compatible, but the three axioms together are not.¹¹ For suppose there are *F* things. Then there is a unique Form corresponding to the character *F* (*OM_{v2}*); but this Form is an *F* thing (*SP_{v2}*), and so shares the character *F* with its own participants, and so participates in the unique Form corresponding to *F* (*OM_{v2}*), that is, participates in itself; but this contradicts *NI_{v2}*. Vlastos thus continued to maintain that the *TMA*’s premise-set is inconsistent.

Cohen¹² agreed with Vlastos that the correct formulation of *OM* will have a uniqueness quantifier but did not concede that this must render the premise-set inconsistent. Just as a more careful formulation of *SP* and *NI* showed them to be consistent with one another, so a more sophisticated approach to the formulation of *OM* was required.

Cohen’s analysis of the *TMA* exploits an analogy with number theory: Plato’s infinite regress of Forms is analogous to the generation of the infinite sequence of natural numbers (NNs). Consider how the NNs are defined by Peano’s Postulates:¹³

1. 0 is a NN.
2. The successor of any NN is a NN.
3. No two NNs have the same successor.
4. 0 is not the successor of any NN.
5. Any property that belongs to 0, and also to the successor of every NN that has the property, belongs to all NNs.

In the usual (von Neumann) set-theoretic construction in which the NNs are represented by sets, the *successor* of a set is defined as the union of that set with its own unit set:

$$\alpha' = \alpha \cup \{ \alpha \}.$$

The NNs are thus represented as the members of the following infinite sequence of sets:

$$\begin{aligned}
0 &= \Lambda \\
1 &= \Lambda \cup \{0\} \\
2 &= \Lambda \cup \{0, 1\} \\
3 &= \Lambda \cup \{0, 1, 2\} \\
&\text{etc.}^{14}
\end{aligned}$$

Zero is identified with the empty set; the successor of zero (i.e., 1) is identified with the set whose only member is the empty set; etc. The sequence clearly satisfies the five Peano Postulates. Note that it also has the curious feature that every NN “belongs” to every one of its “descendants”:

$$0 \in 1 \in 2 \in 3 \in \dots$$

This is a harmless side-effect of the construction in number theory, but it captures the central idea in Plato’s regress.

The symbols of the foregoing sequence of equations are, of course, subject to reinterpretation. Suppose we take \in to represent *participation*, rather than set-membership, Λ to denote the set of F particulars, rather than the empty set, and $=$ to denote a one-one relation pairing a form with the set of its participants, rather than identity. So interpreted, our formulas represent a different (but structurally identical) sequence of objects:

$$\begin{aligned}
F_0 &= \Lambda \\
F_1 &= \Lambda \cup \{F_0\} \\
F_2 &= \Lambda \cup \{F_0, F_1\} \\
F_3 &= \Lambda \cup \{F_0, F_1, F_2\} \\
&\text{etc.}
\end{aligned}$$

F_0 is the Form which has all and only F particulars as its participants; F_1 is the Form whose participants are F_0 and all the participants in F_0 ; F_2 is the Form whose participants are F_1 and all the participants in F_1 ; etc. Suppose finally that ‘NN’ denotes the set $\{F_0, F_1, F_2, \dots\}$, instead of the set of natural numbers.

Each of the Peano Postulates now has a familiar Form-theoretic analogue: (1) asserts that there is a Form which has (all and only) the F particulars as its participants. (2) amounts to *SP* (each Form by virtue of which an F thing is F is itself F). (3) has *NI* as a consequence.¹⁵ (4) asserts that F_0 , the first Form in the sequence, has only particulars as participants. (5) guarantees (although this is not obvious) that, for every n , F_n is a member of the sequence.

The only premise of the *TMA* that has no counterpart among the Peano Postulates is *OM*. Since *OM*’s role is to generate a new Form at each stage of the regress, its number-theoretic counterpart is the successor function, which generates the members of the infinite sequence of NNs. If *OM* is to have a uniqueness quantifier, then, it will need to be based on something stronger than Plato’s *over* relation, which is not a function. But we can use Plato’s *over* relation to define an *immediately-over* function that corresponds to the *successor* function:¹⁶

y is immediately over x =_{df} y is over x , and there is no z such that y is over z and z is over x .

(That is, one Form is “immediately over” another if no third Form intervenes between the two.¹⁷) Cohen’s One-Immediately-Over-Many axiom thus guarantees that every Form has a unique “successor”:

(*IOM*-axiom) For any set of *F*s, there is exactly one Form immediately over that set.

This axiom blocks self-participation, since it entails that Forms do not belong to the sets they are over. *NI* is thus built into *IOM*-axiom. *SP* is presupposed as well, for the values of the variables in the definition of the *immediately-over* function have been restricted to things that are *F*.

The regress develops as we would expect. We are given a set of *F*s; *IOM*-axiom generates a Form they all participate in (and which is the unique Form immediately over *that set*). That Form is itself *F*, and we may thus obtain a new set of *F*s (in the usual way) by adding the Form to the previous set. *IOM*-axiom is applied to the new set, generating a new Form, and so on. Assuming that the Peano Postulates are consistent, the *TMA*’s premise-set is thus capable of a consistent formulation, even with a “uniqueness” quantifier in the One-Over-Many premise.

Vlastos was clearly correct in conceding that *SP* and *NI*, properly formulated, are not incompatible; but he was mistaken in supposing that any version of *OM* with a uniqueness quantifier would reintroduce the inconsistency. But notice what was required to show this. Vlastos had been working in first-order logic with quantifiers ranging over particulars and Forms, whereas Cohen’s reconstruction required quantifying over sets, as well.¹⁸ To demonstrate the consistency of the *TMA*’s premises it had been necessary to employ more sophisticated logical machinery.

Let us pause for a moment to review the history of the use of the principle of charity in interpreting the *TMA*. In his 1954 article, Vlastos used the principle of charity to convert a *non sequitur* into a valid argument, but two of the premises in his reconstruction contradicted one another. His 1969 revision again illustrates the same use of the principle of charity, but now at a second level: he replaced this blatant inconsistency with a more subtle one—an inconsistent dyad with an inconsistent triad. Sellars, Geach,¹⁹ Strang,²⁰ Cohen and others too numerous to mention illustrate a further use of the principle of charity; they sought to remove inconsistency altogether.

But consistency (though much preferable to inconsistency) is still a weak requirement. After all, the members of a consistent set might all be false. Our hypothesis about the use of the principle of charity would predict that scholars would next turn their attention to the truth or reasonableness of the various premises of the *TMA*, and this, in fact, is what we find. In the face of a long tradition of ridiculing the Self-Predication assumption as too absurd to attribute to Plato,²¹ Sandra Peterson published a paper entitled “A Reasonable Self-Predication Premise for the Third Man Argument.”²² In it she proposed a version of *SP* both plausible enough to be attributed to Plato and powerful enough to fill the gap in the *TMA*. The use of the principle of charity was raised to another level.

The key to Peterson’s interpretation is to treat instances of *SP* as being akin to what she calls “Pauline” predications. One of her standard examples of this kind of predication, appropriately enough, is the sentence ‘Charity is kind.’ People normally take it to express something true, she notes, even though its subject (‘Charity’) names an abstraction (what Plato would call a Form), and its predicate (‘. . . is kind’) seems inappropriate to such an entity. So we should not reject self-predications as absurd merely because their predicates seem inappropriate to their subjects.

What, then, do self-predications mean? Consider Pauline predications again. In saying that charity is kind, we may mean something like this: charity is a virtue that causes those who have it to be kind. Similarly, the sentence 'Justice is just' may be construed as asserting (plausibly enough) that Justice is a Form which causes its participants to be just. The Form may be just in a different way from the way in which its participants are just; but this does not mean that we equivocate on 'is just' when we say that each of them is just. Peterson puts the point as follows:²³

The difference between the way the *F* is *F* and the way many of its participants are *F* is best brought out by saying that the *F* is a form which is *F* perhaps solely by bringing about that its participants are *F*. Many of its participants may be *F* without being able to have participants. Such categorial differences, however, do not make a difference to what it is to be *F*. . . .

Hence not only can one say, without absurdity, that *F*-ness is *F*; one can also say that *F*-ness and its participants are all *F* in the same sense. That is, *SP* is both reasonable to hold and powerful enough to help generate the *TMA*.

There are, of course, degrees of reasonableness. The highest degree (at least among mortals) is that of a first-rate contemporary philosopher. So our hypothesis about the use of the principle of charity would now predict that some contemporary scholars would find such a high degree of plausibility in Plato's metaphysics. One scholar who makes a strong case for reading Plato's dialogues as a contemporary text is Terry Penner.²⁴ Indeed, Penner treats Plato himself as a participant in a dialogue with Frege over contemporary issues in philosophy of language.²⁵ Consequently, historians of philosophy must also be thoroughly versed in contemporary theorizing. Penner writes (p. 288):

You can't expect to do good exegesis of passages on Plato's Forms—or indeed on almost any other metaphysical topics in Plato—without making up your own mind on matters as fundamental as the nature and metaphysical presuppositions of logic, and the sources of the paradoxes of logic, semantics and set theory.

Since Plato is on the cutting edge of contemporary thought, so, too, must his interpreters be, if they are going to understand him. Penner's work thus carries us to the fourth stage of the paradox of interpretation where the distinction between historical and contemporary philosophizing breaks down altogether.

4. The Principle of Parsimony

An interpreter who is guided solely by the principle of charity is doomed, it seems, to commit the twin sins of overinterpretation and anachronism. A second principle seems demanded to curb the excesses of the first. Charity needs to be limited by some principle of economy. One plausible candidate is the traditional principle of parsimony or simplicity. In following such a principle an interpreter seeks the simplest explanation for the text before him. Other things being equal, the simplest explanation for a writer's use of a particular sentence is that it provided the most straightforward way of expressing what he wanted to say. The simplest explanation for a missing premise in an argument is that its author, being human, failed to notice that his argument is invalid without it. And the simplest explanation for an apparent inconsistency in a philosophical work is that the philosopher's thought actually is inconsistent.

Most interpretation involves balancing charity and parsimony. The stage is set for a use of the principle of charity whenever the evidence supporting an interpretative

hypothesis underdetermines the hypothesis—that is to say, whenever the evidence is compatible with the falsity of the hypothesis. The principle of charity is used at every step from establishing a text to supplying the tacit premises in an argument. A modern edition of the Greek text of a Platonic dialogue with its accentuation, punctuation, and separation of words is far removed from the unaccented, unpunctuated, unbroken string of capital letters actually written by Plato. Since there is often more than one way to divide an unbroken string of capitals into individual words and the words into sentences, the principle of charity must be invoked to choose among competing hypotheses. A grammatical sentence, for example, will be preferred to one that is ungrammatical; a grammatical sentence that makes sense to one that does not; and a true sentence to a false one. If the text contains an argument, the identification of its explicit premises and its conclusion is often a matter of dispute; and once identified, these sentences are still subject to conflicting interpretations. The stage is thus set for further uses of the principle of charity.

If the paradoxical results outlined earlier are to be avoided, the principle of charity must be reined in at some point by an opposing principle of parsimony. There is, as far as we can see, no third principle for determining the point where charity and parsimony are properly balanced. This is why one cannot hope for consensus among interpreters. The situation is similar to that which arises in an ethical system that tries to balance a maximizing principle (say, of utility) and a fairness principle. In both situations the balancing of the two principles involves judgment and common sense (nebulous as these are).

There is, however, a special problem when it comes to supplying the missing premise in a real enthymeme. The simplest explanation for the missing premise is that the author of the argument was unaware that it was needed. Once one moves beyond this explanation, the principle of parsimony loses its foothold and seems unable to provide any restraint on the principle of charity. For in interesting cases there will always be an indefinitely large number of interpretative possibilities; and since the enthymeme is real and not apparent, there is no broader context to provide a basis for choosing among them. Although an interpretation of a text is always underdetermined by the evidence, in normal cases an interpreter has at least some evidence he can appeal to in ranking one interpretative hypothesis ahead of another. In the case of a real enthymeme such evidence does not exist. The various ways of filling the gap in a real enthymeme are not simply underdetermined by the evidence—they are *hyper*-underdetermined.

5. An Example from Euclid

It may be helpful in dealing with this problem of hyper-underdetermination to contrast the interpretation of the Third Man Argument with the interpretation of a text from the closely related area of the history of mathematics. Euclid's *Elements* have been studied as closely as Plato's dialogues and present many of the same problems of interpretation. One of the major shortcomings of the *Elements* from a modern perspective is their constant use of tacit premises. Furthermore, the Euclidean scholar is likely to be as well versed in modern mathematics as the Platonic scholar in modern philosophy. So we might expect the one to be as prone to anachronism and overinterpretation as the other. But this turns out not to be the case. Thus it may be enlightening to consider how the tools of modern mathematics are used in the interpretation of an ancient mathematical text.

The very first proof in Book I, which shows how to construct an equilateral triangle on a given finite straight line, is a good example. The proof, divided into discrete steps, runs as follows:²⁶

1. Let AB be the given finite straight line.

2. With center A and distance AB , let the circle BCD be described. (Postulate 3)
3. With center B and distance BA , let the circle ACE be described. (Postulate 3)
4. The circles cut one another at point C .
5. From the point C to the points A and B let the straight lines CA and CB be joined. (Postulate 1)
6. Since the point A is the center of the circle CDB , AC is equal to AB . (Definition 15²⁷)
7. Since the point B is the center of the circle CAF , BC is equal to BA . (Definition 15)
8. Therefore, AC is equal to BC . (From 6 and 7 by axiom 1²⁸)
9. Hence the three straight lines AC , AB , and BC are equal to each other. (From 6, 7, and 8)
10. Therefore, the triangle ABC is equilateral. (Definition 20²⁹)

An obvious flaw in this proof and one that has often been pointed out³⁰ is that no justification is given for step (4). What guarantees that the two circles intersect at a point and do not slip through each other without touching? Step (4) tacitly assumes that Euclidean circles are continuous. Can this be proved? Is there anything in Euclid's postulates that ensures that Euclidean lines have no gaps?

Here are Euclid's five postulates:

- "1. Let it be postulated to draw a straight line from any point to any point,
2. and to produce a limited straight line in a straight line,
3. and to describe a circle with any center and distance,
4. and that all right angles are equal to one another,
5. and that, if one straight line falling on two straight lines makes the interior angles in the same direction less than two right angles, the two straight lines, if produced *ad infinitum*, meet one another in that direction in which the angles less than two right angles are."³¹

One way to settle the question of continuity is to see if there is an interpretation of the concepts that enter into these five postulates and into the above proof under which the postulates are true but under which step (4) is false. Such an interpretation can be found by exploiting the techniques of analytic geometry and set theory. The basic idea is to give the relevant terms an arithmetic interpretation in the domain of rational numbers. The domain of rational numbers is chosen since a line whose points correspond to rational numbers, though everywhere dense (between any two points there is a third), is not continuous. (There is a gap, for example, between all the points of such a line greater than the square root of two and all the points less than the square root of two.) Following this strategy each of the relevant terms is assigned an arithmetic meaning that corresponds by way of a Cartesian (or rectangular) coordinate system to the intended geometric meaning of the term.

Under this arithmetic interpretation the word 'point' means 'ordered pair of rational numbers'; 'straight line' means 'set of points that satisfy an equation of the form $ax + by + c = 0$ ', and 'circle' means 'set of points that satisfy an equation of the form $x^2 + y^2 + ax + by + c = 0$ '. (In these equations and in those that follow 'x' and 'y' are variables and all other letters are constants.) These two equations are chosen since the graph of the first, using a Cartesian coordinate system, is a geometric straight line and that of the second

is a geometric circle. Finally, 'line AB intersects line CD ' means 'the intersection of the set of points identified with line AB and the set of points identified with line CD is not the null set'.

Consider now Euclid's five postulates. Postulate Four is provable,³² so it can be set aside. The other four are all true under the foregoing arithmetic interpretation when it is elaborated in an obvious way.

If the points mentioned in Postulate One are the two ordered pairs $\langle h_1, k_1 \rangle$ and $\langle h_2, k_2 \rangle$, the following set is the straight line through these points (where Ra is the set of rational numbers):

$$\{ \langle x, y \rangle \mid x, y \in Ra \ \& \ (k_1 - k_2)x + (h_2 - h_1)y + h_1k_2 - h_2k_1 = 0 \}.$$

Under the arithmetic interpretation Postulate One makes the true assertion that this set has members. Postulate Two makes the true assertion that if $h_1 \neq h_2$, then for any given rational number, n , the set contains an ordered pair whose first element is larger than n and a second ordered pair whose first element is smaller than n , and it makes a similar true assertion for the second element if $k_1 \neq k_2$. (If $h_1 = h_2$, the line is parallel to the y -axis; and if $k_1 = k_2$, it is parallel to the x -axis.)

In order to interpret Postulate Three it is necessary to specify that by 'any center' is meant 'any point as center' and by 'distance' 'distance between two points'. If this is laid down, then under the arithmetic interpretation Postulate Three makes the true assertion that the following set, which is determined by the equation for a circle with the center $\langle h, k \rangle$ and the radius r , is not null:

$$\{ \langle x, y \rangle \mid x, y \in Ra \ \& \ (x - h)^2 + (y - k)^2 = r^2 \}$$

(In the equation determining this set r^2 must always be a rational number though r can be irrational.)

The fifth, or parallel, postulate is the most complex. Consider the two following sets:

$$A = \{ \langle x, y \rangle \mid x, y \in Ra \ \& \ a_1x + b_1y + c_1 = 0 \}$$

$$B = \{ \langle x, y \rangle \mid x, y \in Ra \ \& \ a_2x + b_2y + c_2 = 0 \}$$

Postulate Five asserts that if A and B each have at least two members and if $a_1/a_2 \neq b_1/b_2$, then $A \cap B \neq 0$. This amounts to the claim that the equations that determine A and B have a simultaneous solution in the domain of rational numbers. It is established by showing that a linear equation that is satisfied by two distinct ordered pairs of rational numbers has rational coefficients³³ and that the simultaneous solution of two linear equations with rational coefficients must be a pair of rational numbers.³⁴

All of Euclid's postulates are true under the proposed arithmetic interpretation, but under this interpretation the key step in Euclid's proof of his first theorem and the theorem itself are false. For suppose that $\langle -1, 0 \rangle$ and $\langle 1, 0 \rangle$ are the end points of the finite straight line upon which the equilateral triangle is to be constructed. Then under the arithmetic interpretation the two circles are:

$$\{ \langle x, y \rangle \mid x, y \in Ra \ \& \ (x - 1)^2 + y^2 = 2^2 \}$$

$$\{ \langle x, y \rangle \mid x, y \in Ra \ \& \ (x + 1)^2 + y^2 = 2^2 \}$$

But these sets do not intersect. (In the domain of real numbers the points of intersection are $\{ \langle 0, \sqrt{3} \rangle, \langle 0, -\sqrt{3} \rangle \}$.) Thus Euclid's first proof cannot be repaired, and

the reason is that his postulates do not guarantee that the lines of his geometry are continuous.

Euclid's geometry is in need of a continuity postulate. What form should it take? There are various possibilities of increasing levels of generality. The following postulate is sufficient to deal with the gap in Euclid's first proof:

1. If A is the center of one circle and B the center of another and if the straight line joining A and B is a radius of both circles, then the two circles cut one another at a point C.

The difficulty with this postulate is that it is too specific. Unlike Euclid's other postulates, it speaks to one case only: it guarantees that two circles have a point in common only when the circles are the same size and share a radius. It seems unlikely that Euclid would have considered it sufficiently general to deserve a place among his other postulates.

To increase generality one might try something like the following:

2. If one point of a circle lies inside and another point lies outside another circle, then the two circles have exactly two points in common.

This postulate may still be too specific. Does it, for example, entail that a straight line that joins a point inside a circle with a point outside the circle intersects the circle at a point? A further question is whether this postulate expresses the assumption about continuity that Euclid was tacitly assuming in his first proof. Although a charitable interpretation fuses these two questions, parsimony keeps them apart. A parsimonious interpreter will point out that proposition (2) is not sufficient by itself to bridge the gap in Euclid's first proof. Before Euclid can appeal to proposition (2) he will need to establish its antecedent, a process requiring several additional steps. A charitable interpreter will need to supply these steps as well as proposition (2). Charity is getting out of hand. A simpler, more parsimonious interpretation is clearly preferable: Euclid did not notice the hole in his first proof and did not realize that his postulates do not guarantee the continuity of his lines. Although a continuity postulate and a revised proof are needed to establish Euclid's first theorem, both of these belong, not to Euclid's geometry, but to Euclidean geometry.³⁵

By pursuing this example in such detail we have tried to show, among other things, how modern techniques can deepen one's understanding of an historical text. A scholar using the tools of modern mathematics can understand Euclid's *Elements* better than Euclid understood them himself. Thus Euclid could have discovered that his first proof is defective by noticing that no justification is offered for step (4). But an unproved proposition might still be provable. The tools of analytic geometry and model theory are needed to show that Euclid's first theorem is not provable from his postulates and why. Such insights were beyond Euclid's scope. The main point of the example, however, is that in the history of mathematics a sharp line is drawn between Euclid's geometry and Euclidean geometry, even though the latter rises on the basis of the former. The principle of parsimony seems to play a larger role in the interpretation of mathematical texts than it does in the interpretation of philosophical ones.

6. Two Models of Interpretation

When an interpreter of Plato supplies a suppressed premise in a real enthymeme, what is his aim? There are two models. One is retrospective; the other, prospective. On the retrospective model the interpreter supposes either that Plato consciously entertained the suppressed premise but, for one reason or another, did not write it down, or that there is a premise he would have supplied if he had been queried about the gap in his argument. The interpreter's job, on this model, is to discover the idea that

was in Plato's head or the answer he would have given if the question had been put to him. Thus his interpretation is correct or incorrect depending upon whether or not the premise he supplies corresponds to an actual or latent premise of Plato's. On the prospective model, on the other hand, the interpreter supposes that the gap in Plato's argument reflects a gap in Plato's thinking. When the interpreter fills the gap, he considers it a free act of creation on his part. His goal is not to recapture Plato's thought (since there is no thought to recapture) but to construct as good an argument as possible on the foundation that Plato lays.

An analogy may be helpful. On the retrospective model the interpreter is like a scholar who is attempting to establish a text from a sole surviving manuscript that happens to be worm-eaten. Due to the worm holes one of the important words in the manuscript is missing. The scholar makes a conjecture about this word. His conjecture is either true or false depending upon the word the author actually wrote. There may be a serious epistemic problem about recovering this word, but nevertheless one word is correct and all others wrong. On the prospective model, on the other hand, the interpreter is like a poet whose help is sought by a colleague having difficulty finding the right word to end a stanza of a poem he is writing. In this case the right word is the one that makes the best poem. There is no question of truth and falsity.

Both models have their place in the interpretation of Euclid. The historian of mathematics who is intent on recapturing Euclid's thought adopts the retrospective model, whereas the mathematician who repairs Euclid's proofs by supplying missing premises adopts the prospective model. (The same person might, and often will, wear both hats.) This does not mean that the historian eschews the principle of charity, and the mathematician, the principle of parsimony. As we have already pointed out, one cannot even establish a text without using the principle of charity. Thus a retrospective interpretation will be guided by charity as well as parsimony. Similarly, parsimony as well as charity has a role in prospective interpretation. The principle of charity exhorts the interpreter to maximize validity, truth, and content. Since there is always more than one way to do this—since there is always more than a single true (or reasonable) proposition that will restore the validity of a real enthymeme—a second principle is needed to guide the choice among the candidates nominated by the principle of charity. The principle of parsimony, in exhorting the interpreter to choose the simplest, is a principle of elegance. Although the principle of parsimony plays a role in both models of interpretation, it functions differently in the two. In the prospective interpretation of a real enthymeme, the principle of parsimony guides the interpreter to the simplest, most elegant *supplement* of the text; in the retrospective interpretation of such an enthymeme, it guides him to the simplest *explanation* for the text.

It is our contention that the interpretation of Euclid provides a guide for the interpretation of Plato. The missing premises in the real enthymemes in Plato's dialogues reflect gaps in Plato's thought just as the missing premises in Euclid's proofs reflect gaps in his thought. And in both cases, when an interpreter supplies a missing premise, he is extending his author's thought rather than expounding it. As the distinction between retrospective and prospective interpretation leads in the one case to the distinction between Euclid's geometry and Euclidean geometry, it leads in the other to that between Plato and Platonism.

7. Two Objections

One objection that might be made to the prospective model of interpretation is that it presupposes that there are real gaps in Plato's arguments, that is, gaps that cannot be filled by scouring the dialogues and by increasing one's background knowledge of Greek culture. But, the objection goes, there are no such gaps. Plato did not write in a vacuum, and research will always permit a diligent scholar to isolate the premise Plato

intended his reader to supply. According to this objection, all the enthymemes in the dialogues are apparent; none are real.

But how are we to understand the claim that there are no real enthymemes in Plato? Is it a maxim of interpretation or is it a factual generalization over all the arguments in the dialogues? If it is a maxim, the objector is begging the question. For to assert as a maxim that there are no real enthymemes in Plato is simply to use the principle of charity retrospectively, and the issue is whether there are cases where such retrospective use is inappropriate. If, on the other hand, the claim that there are no real enthymemes in Plato is a generalization, it will be difficult to establish, as negative existential propositions typically are. For in each case the evidence one brings forward to fill the gap is bound to be controversial.

Suppose, for example, one attempts to support the claim that the *TMA* is not a real enthymeme by producing external evidence that Plato indeed subscribed to literal Self-Predication, one of the argument's implicit premises. One might point to such a passage as *Symposium* 210e-211b where Plato clearly asserts that the Form of Beauty is perfectly beautiful. Or, appealing to Plato's characterization of the Forms as *paradeigmata* (*Rep.* 472c4, 484c8, 500e3, 540a9, *Parm.* 132d2, *Tim.* 29b4, 31a4, 39e7, 48e5, and elsewhere), an intrepid gap-filler might even refer beyond the dialogues to the nature of the *paradeigmata* used by Greek craftsmen. In Greek architecture they were three-dimensional full-scale models. This is how they are described in a recent book on Greek architects:

The *paradeigma* . . . was used for elements like triglyphs or capitals which required a design in three dimensions, and in cases where carved or painted decoration had to be shown. These specimens were often made of wood, stucco, or clay, even where they were to be copied in more permanent materials; but in at least one case the material was stone, for the specimen capital was to be set in place in the actual building, along with the others. In this case the specimen must obviously have been full size, and that is likely to have been normal, for the use of full-size specimens would be the easiest way of ensuring the uniformity which is so important to the effect of a Greek building.³⁶

This practice is noteworthy because it shows that the *paradeigma* of a capital could itself be used as a capital. In some cases, at least, *paradeigmata* were literally self-predicational.

Evidence of this sort is extremely important and should be avidly sought by anyone attempting to understand Plato's dialogues. The particular evidence just cited lends a good deal of support to the claim that Plato subscribed to literal Self-Predication. But a scholar who finds the idea of literal Self-Predication absurd will not find it compelling. Such a scholar can avoid the implications of the *Symposium* passage either by denying the relevance of an earlier dialogue in interpreting the *Parmenides* (Plato changed his mind in the interval between the two dialogues) or by denying the legitimacy of inferring that Plato held that *all* Forms are literally self-predicational from the fact that he held that one Form, the Form of Beauty, is literally self-predicational.³⁷ The external evidence from Greek architecture is even easier for an opponent of literal Self-Predication to discount; for one can hold that Plato's Forms, even though described as *paradeigmata*, are only analogous to the literally self-predicational *paradeigmata* used by Greek craftsmen. All analogies break down; this analogy breaks down, one can argue, on precisely the matter of literal self-predication. So the best external evidence that we have for ascribing this tacit premise to Plato falls short of being conclusive. Thus it will be virtually impossible to establish on the basis of either internal or external evidence that no enthymeme in Plato's dialogues is real.

A second objection that might be made to the prospective model of interpretation is that it presupposes that the arguments in Plato's dialogues are assertoric: it presupposes, that is, that Plato is attempting to establish certain theses by advancing arguments for them. But, the objection goes, this presupposition is obviously false since Plato never advances *any* thesis in his own name. Certain arguments are presented by the characters in his dialogues, but Plato never tells the reader how these arguments are to be taken. And there are various possibilities. Plato may intend the reader to take a given argument as an example of sophistry, or as an interesting intellectual experiment, or as a piece of rhetoric, or as raising an interesting problem, or as an *ad hominem*, or simply as a report of an argument current in fourth or fifth century Athens. The prospective model of interpretation supposes, so the objection goes, that Plato is a forerunner of Spinoza; but this is simply to misunderstand the point of writing dialogues.

The interpretation of the *TMA* advanced by R.E. Allen and by Harold Cherniss provides a good example of a maneuver that illustrates this last objection.³⁸ According to Allen and Cherniss, Plato deliberately presented an argument he knew to be invalid.³⁹ They hold that the *TMA* tacitly assumes that Largeness is large, but they also hold that this self-predicational language expresses nothing more than a trivial claim of self-identity. When Plato asserts that the Form of justice is just, for example, he is simply making the true (and trivial) assertion that justice is justice. Hence all that the tacit assumption that Largeness is large can mean for Plato (and all he is committed to) is that Largeness is Largeness. Since this premise is too weak to generate the regress, the *TMA* poses no threat to Plato's theory of Forms. According to this interpretation, the *TMA* is an *aporia*, or perplexity, to which Plato knew the answer, not an assertoric argument indicating a flaw in the theory of Forms.⁴⁰

Although strategies of this sort can be used to evade the putative conclusion of any given argument in the dialogues, it seems to us to be methodologically unsound to use them to evade the putative conclusion of *every* argument in the dialogues. To do so would be to sacrifice the idea that Plato is ever committed to any thesis that he is prepared to argue for. If Plato's philosophy is not to be thus cut off from his own expression of it, it must be conceded that at least some of Plato's arguments are assertoric. And among these it should not be too difficult to find a real enthymeme. So the prospective model is back in business.

8. Conclusion

Given that the prospective model of interpretation leads to Platonism rather than to Plato, the question arises whether prospective interpretation has a place in Platonic scholarship. We believe it does. For without it the interpretation of Plato is dry and barren, lacking in intelligence and imagination. Consider what a purely retrospective interpretation of the *TMA* would amount to. It could establish no more than that the argument is a *non sequitur*. The aridity of interpretation that is purely retrospective explains, no doubt, the need for both models of interpretation in accounting for the actual practice of historians of philosophy. And this need partially explains in turn why the study of the history of philosophy is such a peculiarly philosophical enterprise. To do good work in the history of philosophy, it has often been observed, one must be a good philosopher, not just a good historian. Why this should be so is not a simple question,⁴¹ but one that is much easier to answer if philosophical interpretation has a prospective as well as a retrospective component. For while he is engaged in prospective interpretation, the historian of philosophy is augmenting the philosophical work of his subject: that is, he is doing philosophy.

If a Platonic scholar needs to employ both models of interpretation, he also needs to maintain the distinction between them. Otherwise he will end up attributing his own

contribution to Plato. He will end up conflating Platonism and Plato. He will be tempted, for example, in searching for the true point, meaning, or moral of a text, to discover one bestowed on the text by his own augmentation of it. Interpreters of the *TMA* have often succumbed to this temptation. There is no such thing as *the* moral of the *TMA* if, as we contend, retrospective interpretation is unable to advance beyond the observation that the *TMA* is a *non sequitur*.⁴²

We suggest, finally, that Plato himself might find our view of interpretation congenial. For Plato's complaint about written words—that they do not respond to questions but repeat the same thing endlessly (*Protagoras* 329A, *Phaedrus* 275D)—corresponds to our complaint about the sterility of retrospective interpretation. On the other hand, since Plato devoted so much of his life to putting words on paper, he must have hoped that this defect of writing could sometimes be alleviated. He must have hoped that his words would occasionally kindle a philosophical dialogue in the mind of an attentive reader, albeit a dialogue that the reader would have to conduct on his own or with another prospective interpreter.⁴³

University of Washington

NOTES

¹ Since two premises can always be reduced to one by means of conjunction, no enthymeme stands in need of more than a single additional premise.

² The premise that the one supplies need not itself be inconsistent with the premise supplied by the other. But so long as the premises are distinct, they cannot both be the most reasonable one to supply.

³ One finds an instance of this in the famous Simonides episode of Plato's *Protagoras* (339a-347a). The text that Socrates is constrained to interpret contains these lines (345d):

But all who do no evil
Voluntarily I praise and love.

Charitably refusing to attribute a false belief to the poet, Socrates construes 'voluntarily' with the words that follow rather than with those that precede.

⁴ *Parm.* 132a1-b2. The translation is our own.

⁵ It is almost universally assumed that Plato intended the *TMA* to hold for any predicate for which there is a Form; hence the schematic letter 'F' is typically used in place of 'large' to express this generality. For an interesting alternative view, see William E. Mann, "The Third Man = The Man Who Never Was," *American Philosophical Quarterly* 16 (1979), pp. 167-76.

⁶ "The Third Man Argument in the *Parmenides*," *Philosophical Review* 63 (1954), pp. 319-49; reprinted with an addendum in *Studies in Plato's Metaphysics*, ed. by R.E. Allen (London, 1965), pp. 231-63. Subsequent references will be to the reprinted version.

⁷ "Vlastos and 'The Third Man,'" *Philosophical Review* 64 (1955), pp. 405-37; reprinted in *Philosophical Perspectives* (Springfield, IL, 1967), pp. 23-54. Subsequent references will be to the reprinted version.

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- 8 For further details and a more precise characterization of this revision, see S. Marc Cohen, "The Logic of the Third Man" (cited henceforth as LTM), *Philosophical Review* 80 (1971), pp. 452-53.
- 9 Sellars' formulation of *NI* is still not quite right, as was pointed out in LTM, p. 453, n. 14. The problem is that NI_S entails (or presupposes) that there is such a thing as the (unique) *F*-ness by virtue of participating in which a given *F* thing is *F*. This conflicts with Plato's idea that particular *F*s are *F* in virtue of participating in the first Form in the regress and also (along with that Form) in virtue of participating in the second Form in the regress, etc. Hence, for Plato, there will not be such a thing as the (unique) *F*-ness by virtue of which a given *F* thing is *F*. The correct Sellarsian formulation of *NI* would be this: If *x* is *F*, then *x* is not identical with any of the *F*-nesses by virtue of which it is *F*.
- 10 "Plato's 'Third Man' Argument [*Parm.* 132a1-b2]: Text and Logic," *Philosophical Quarterly* 19 (1969), pp. 289-301. Reprinted with revisions in his *Platonic Studies*, Second Edition (Princeton, 1981), pp. 342-65. Page references will be to the revised version.
- 11 Strictly speaking, the three axioms are consistent but entail that nothing has the given character. But clearly all participants in this dialogue agree that some things are large.
- 12 LTM, pp. 448-75. Cf. note 8.
- 13 Our formulation of the Peano Postulates is due to Bertrand Russell, *Introduction to Mathematical Philosophy* (London, 1919), pp. 5-6.
- 14 The occurrences of ' $\wedge \cup$ ' are redundant under the usual interpretation of \wedge as the empty set but not under the reinterpretation to follow.
- 15 Recall that on the von Neumann construction, each NN is a member of all its "descendants"; this, together with postulate 3, entails that no NN is its own successor. The Form-theoretic analogue of this is that no Form in the sequence participates in itself.
- 16 One might object that two distinct Forms may be immediately over the same object since Socrates, for example, participates in both the Form *man* and the Form *philosopher*. But these two Forms belong to different sequences, the Third Man sequence and the Third Philosopher sequence. The *immediately-over* relation is a function only with respect to a single sequence.
- 17 In LTM, "immediately over" was defined differently (but equivalently) in terms of the *over* relation and the notion of the *level* of an object: *x* is immediately over *y* iff the level of *x* is *one* greater than the level of *y* (whereas *x* is over *y* iff the level of *x* is greater than the level of *y*). The notion of *level* is defined recursively in terms of participation: *F* things in which nothing participates (i.e., *F* particulars) are on level 0; *F* things which have as participants all and only the things on level 0 are on level 1; *F* things which have as participants all and only the things on level *n* or lower are on level *n* + 1. The possibility of simplifying the definition of *immediate overhead* as we have in the text was also discovered (independently) by Richard Patterson. See his *Image and Reality in Plato's Metaphysics* (Indianapolis, 1985) p. 54.
- 18 Vlastos's 1981 version of the *TMA* (*Platonic Studies*, Second Edition, p. 363) contains a statement of *OM* which appears to quantify over sets of *F*s, but this appearance is misleading. His English version begins promisingly: "If any set of things share a given character, say, *large*, then there is a unique corresponding

Form, Largeness” But what is it to which this Form uniquely corresponds? His wording seems to suggest: corresponding to whatever set the initial quantifier picks out. That, we maintain, would be the right idea. But formalizing that idea requires a quantifier ranging over sets, and no such quantifier is to be found in Vlastos’s formalization. (In English, what his formalization says is this: if a , b , and c are all large, then this is in virtue of their participating in a Form, Largeness, the one and only Form in virtue of participating in which things are large.) And his glossary of logical symbols confirms that he intends a unique Form corresponding to a given character, as he did in OM_{V2} , where the quantification over sets was purely adventitious.

- 19 “The Third Man Again,” *Philosophical Review* 65 (1956), pp. 72-82.
- 20 “Plato and the Third Man,” *Proceedings of the Aristotelian Society*, supplementary volume 37 (1963), pp. 147-63.
- 21 R.E. Allen and Harold Cherniss are two prominent members of this tradition. Their view will be discussed below, p. 17.
- 22 *Philosophical Review* 82 (1973), pp. 451-70.
- 23 *Ibid.*, p. 470. We have made minor revisions in Peterson’s notation to make it conform to our own.
- 24 *The Ascent From Nominalism* (Dordrecht, 1987).
- 25 *Ibid.*, pp. 57 ff.
- 26 Thomas L. Heath, *The Thirteen Books of Euclid’s Elements*, 2nd ed. (Cambridge, England, 1926), vol. I, pp. 241-2.
- 27 “A circle is a plane figure contained by one line such that all the straight lines falling upon it from one particular point among those lying within the figure are equal.”
- 28 “Things which are equal to the same thing are also equal to one another.”
- 29 “Of the three trilateral figures, an *equilateral triangle* is the one which has its three sides equal”
- 30 Heath, *op. cit.*, pp. 235, 242; Felix Klein, *Elementary Mathematics from an Advanced Standpoint: Geometry* trans. E.R. Hedrick and C.A. Noble (New York, 1939), p. 197; Howard Eves, *A Survey of Geometry* revised ed. (Boston, 1972), p. 321; Ian Mueller, *Philosophy of Mathematics and Deductive Structure in Euclid’s Elements* (Cambridge, MA, 1981), p. 28.
- 31 Mueller’s translation, *op. cit.*, pp. 318-9.
- 32 The proof, which goes back at least to Proclus, is given in Mueller, *op. cit.*, p. 22.
- 33 The equation for a straight line through the two points $\langle h_1, k_1 \rangle$ and $\langle h_2, k_2 \rangle$ is:

$$(k_1 - k_2)x + (h_2 - h_1)y + h_1k_2 - h_2k_1 = 0$$

Since h_1 , h_2 , k_1 , and k_2 are all rational, the coefficients in the equation are rational.

- 34 A system of two linear equations can be solved by successive uses of addition and multiplication. Hence if all of the coefficients of each equation are rational, the solution must also be rational.

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- ³⁵ In modern Euclidean geometry the continuity postulate that is favored is Dedekind's Postulate: "If all the points of a straight line fall into two classes, such that every point of the first class lies to the left of every point of the second class, there exists one and only one point of the line which produces this division of all points into two classes, this division of the straight line into two parts." On the basis of this postulate the proof that the two circles of Euclid's first proof share a point C is quite elaborate. For details see Heath, *op. cit.*, vol I, pp. 234-40.
- ³⁶ J.J. Coulton, *Ancient Greek Architects at Work* (Ithaca, NY, 1977), p. 55.
- ³⁷ This second alternative is a relatively weak response since the paradox of the Third Man breaks out again with the Form of Beauty. The Third Man Argument will simply be replaced by the Third Beauty Argument. Thus this response would at most localize the paradox without eliminating it.
- ³⁸ R.E. Allen, "Participation and Predication in Plato's Middle Dialogues," in *Studies in Plato's Metaphysics* edited by R.E. Allen (London, 1965), pp. 46, 59. Harold Cherniss, "The Relation of the *Timaeus* to Plato's later Dialogues," in Allen, *op. cit.*, pp. 370-4. See also Cherniss, *Aristotle's Criticism of Plato and the Academy* (New York, 1944), pp. 294-8.
- ³⁹ Thus Cherniss writes: "I take it therefore as proved not only that both versions of the regress are invalid arguments but also that when Plato put them into Parmenides' mouth he believed them to be invalid and invalid for reasons which he felt himself to have indicated satisfactorily for anyone who would compare the assumptions of these arguments with what he had already said concerning the nature of the ideas" (in Allen, *op. cit.*, p. 374).
- ⁴⁰ See in this regard R.E. Allen, *Plato's Parmenides* (Minneapolis, 1983), pp. 96-8.
- ⁴¹ For a useful examination of this unjustly neglected question, see David M. Rosenthal, "Philosophy and its History," in *The Institution of Philosophy: a Discipline in Crisis?*, ed. Avner Cohen and Marcelo Dascal (LaSalle, IL, 1989).
- ⁴² Unless of course, as Cherniss and Allen maintain, the point is to present an invalid argument.
- ⁴³ Earlier versions of this paper were presented to the Philosophy Department at Rutgers University and at the March, 1989 meeting of the Pacific Division of the American Philosophical Association in Berkeley. We would like especially to thank our two commentators on the latter occasion, Gerasimos Santas and Martin Tweedale, for their helpful comments and criticisms. We also gratefully acknowledge the help of Christine Keyt, Gareth Matthews, Nicholas Smith, Charles Young, and the participants in the discussions at Rutgers and Berkeley.