Aristotle's Syllogistic

1. **Propositions** consist of two terms (a subject and a predicate) and an indicator of quantity/quality: 'every', 'no', 'some', 'not every'.

TYPE	TAG	IDIOMATIC	TECHNICAL	ABBREVIATION
Universal Affirmative	***a***	Every *F* is *G*	*G* belongs to every *F* *G* is predicated of every *F* *F* is in *G* as in a whole	G***a***F
Universal Negative	***e***	No *F* is *G*	*G* belongs to no *F* *G* is predicated of no *F*	G***e***F
Particular Affirmative	***i***	Some *F* is *G*	*G* belongs to some *F* *G* is predicated of some *F* *G* is related to *F* in part	G***i***F
Particular Negative	***o***	Some *F* is not *G*	*G* does not belong to some *F* *G* belongs to not every *F*	G***o***F

2. **Contraries and Contradictories** (cf. *De Int*. $17^{b}16$-28)

   a. *p* and *q* are **contradictories** iff

      i. *p* and *q* cannot both be true, and — ***a*** and ***o*** are contradictories
      ii. necessarily, either *p* or *q* is true. — ***e*** and ***i*** are contradictories

   b. *p* and *q* are **contraries** iff

      i. *p* and *q* cannot both be true, and
      ii. *p* and *q* can both be false. — ***a*** and ***e*** are contraries

   c. *p* and *q* are **subcontraries** iff they are the contradictories of a pair of contraries.
   That is, *p* and *q* are subcontraries iff

      i. *p* and *q* cannot both be false, and
      ii. *p* and *q* can both be true — ***i*** and ***o*** are subcontraries

3. **Syllogistic Figures**

- '*P*' represents the predicate of the conclusion (major term), and '*S*' represents the subject of the conclusion (minor term). Aristotle calls these terms the "extremes."
- '*M*' represents the term that is common to the premises but absent from the conclusion. Aristotle calls this the "middle term."
- Aristotle calls the premise containing '*P*' the "major premise," and the premise containing '*S*' the "minor premise."
- '*' represents any of the four quantity/quality indicators ('***a***', '***e***', '***i***', '***o***').

First Figure	**Second Figure**	**Third Figure**
P*M	M*P	P*M
M*S	M*S	S*M
P*S	P*S	P*S

4. Syllogistic Moods

When we replace each ‘*’ with a quantity/quality indicator we get a triple of propositions, <major premise, minor premise, conclusion>. Each distinct triple (e.g., <***a***,***a***,***a***>, <***e***,***i***,***o***>, etc.) constitutes a **mood**. (Aristotle, however, did not call these ‘moods’; this expression is due to later commentators.)

There are 64 such moods ($4^3 = 64$). Each mood can occur in each of the three figures. Hence, there are 3 x 64 = 192 candidate syllogisms among the various mood/figures.

Each candidate syllogism can be labeled as to mood and figure as in the following example:

G***a***F
G***e***H
F***e***H

The mood is <***a***,***e***,***e***>, and the figure is the second; we can abbreviate this as <***a***,***e***,***e***>-2

Aristotle attempts to identify and prove the validity of the syllogisms (= valid mood/figures) and to identify and show the invalidity of the others. [More precisely, he attempts to show which premise-pairs do, and which do not, yield syllogistic conclusions; to prove the validity of those that do; and to show (by counter-example) the invalidity of those that do not.]

5. The Valid Moods

Aristotle recognizes the following as syllogisms (i.e., as valid moods):

First Figure		**Second Figure**		**Third Figure**	
<***a***,***a***,***a***>	(*Barbara*)	<***e***,***a***,***e***>	(*Cesare*)	<***a***,***a***,***i***>	(*Darapti*)
<***e***,***a***,***e***>	(*Celarent*)	<***a***,***e***,***e***>	(*Camestres*)	<***e***,***a***,***o***>	(*Felapton*)
<***a***,***i***,***i***>	(*Darii*)	<***e***,***i***,***o***>	(*Festino*)	<***i***,***a***,***i***>	(*Disamis*)
<***e***,***i***,***o***>	(*Ferio*)	<***a***,***o***,***o***>	(*Baroco*)	<***a***,***i***,***i***>	(*Datisi*)
				<***o***,***a***,***o***>	(*Bocardo*)
				<***e***,***i***,***o***>	(*Ferison*)

The standard predicate-logic representations of all but two of these are valid; but *Darapti* and *Felapton* require existential import (of the middle term). If the minor term also has existential import, we get four additional “weakened” syllogisms with valid predicate-logic representations:

First Figure		**Second Figure**	
<***a***,***a***,***i***>	(*Barbari*)	<***e***,***a***,***o***>	(*Cesaro*)
<***e***,***a***,***o***>	(*Celaront*)	<***a***,***e***,***o***>	(*Camestros*)

Aristotle excludes these from among the valid syllogisms not because they require existential import, but because their premises support stronger (universal) conclusions than the (particular) ones claimed here.

6. **Disproof (rejection) procedure by counterexample**

Example at 26a36-38:

> "Suppose that B belongs to no C, and that A either belongs or does not belong to some B or does not belong to every B; there will be no deduction in this case either. Let the terms be white—horse—swan, and white—horse—raven."

We have been given two premise-pairs of the first figure, <***i***,***e***> and <***o***,***e***>:

	Case 1	**Case 2**
Premises	A***i***B B***e***C	A***o***B B***e***C

There are four candidate syllogisms in each case:

Positive Conclusions	A***a***C A***i***C	A***a***C A***i***C
Negative Conclusions	A***e***C A***o***C	A***e***C A***o***C

Aristotle gives two sets of values for the variables. The first set has a **positive** relation between A and C; the second set has a **negative** relation between A and C.

Positive relation:	**Negative relation**:
A = white B = horse C = swan	A = white B = horse C = raven

The swan example (*positive* relation) disproves the four candidates with *negative* conclusions.

The raven example (*negative* relation) disproves the four candidates with *positive* conclusions. So Aristotle in effect gives us counter-examples to all eight candidates. In every case, the premises are true and the conclusion is false, so none of the candidates is a syllogism:

	Case 1	**Case 2**
Major premise	Some horse is white	Some horse is not white
Minor premise	No swan is a horse No raven is a horse	No swan is a horse No raven is a horse
Positive Conclusions	Every raven is white Some raven is white	Every raven is white Some raven is white
Negative Conclusions	No swan is white Some swan is not white	No swan is white Some swan is not white

For another example see 27a18-21.

7. Syllogistic as a Natural Deduction System

RULES:

Premise-introduction: Any premise-pair of any of the three figures may be assumed as premises.

Barbara:	From A***a***B and B***a***C, infer A***a***C
Celarent:	From A***e***B and B***a***C, infer A***e***C
Darii:	From A***a***B and B***i***C, infer A***i***C
Ferio:	From A***e***B and B***i***C, infer A***o***C

Simple Conversion

e*-Conversion*	From A***e***B infer B***e***A
i*-Conversion*	From A***i***B infer B***i***A

Accidental Conversion

a*-Conversion*	From A***a***B infer B***i***A

Reduction *per impossibile*:

r may be inferred from the premise-pair {*p*, *q*}, if either the contradictory or the contrary of *q* has been inferred from the premise-pair {*p*, the contradictory of *r*}.

That is, to prove a syllogist*ic* conclusion, show that the assumption of its contradictory (along with one of the premises) leads to the contradictory or the contrary of the other premise.

[Examples of reduction *per impossibile*: For the contradictory of *q* inferred from {*p*, the contradictory of *r*}, see $28^{b}17$-18. For the contrary of *q* inferred from {*p*, the contradictory of *r*}, see $29^{a}37$-39.]

> <*p*, *q*, *r*> is a syllogism iff it is a mood of one of the three figures and *r* is deducible from {*p*, *q*} by the rules above.

Aristotle eventually shows how *Darii* and *Ferio* can be obtained as derived rules and hence dropped from the set of primitive rules (cf. $29^{b}1$-25).

8. Some Proofs of Syllogisms (texts **omitted** in Fine/Irwin)

a. *Cesare* ($27^{a}5$-9)

"Let M be predicated of no N but of every X. Then, since the privative converts, N will belong to no M. But M was assumed to belong to every X, so that N belongs to no X (for this has been proved earlier)."

1.	M***e***N	Premise
2.	M***a***X	Premise
3.	N***e***M	1 ***e****-Conversion*
4.	N***e***X	2,3 *Celarent*

b. *Camestres* (27^a9-13)

"Next, if M belongs to every N but to no X, then neither will N belong to any X. For if M belongs to no X, neither does X belong to any M; but M belonged to every N; therefore, X will belong to no N (for the first figure has again come about). And since the privative converts, neither will N belong to any X, so that there will be the same deduction. (It is also possible to prove these results by leading to an impossibility.)"

1.	M***a***N	Premise
2.	M***e***X	Premise
3.	X***e***M	2 ***e***-*Conversion*
4.	X***e***N	1,3 *Celarent*
5.	N***e***X	4 ***e***-*Conversion*

c. *Baroco* (27^a36-^{b}1)

"Next, if M belongs to every N but does not belong to some X, it is necessary for N not to belong to some X. For if it belongs to every X and M is also predicated of every N, then it is necessary for M to belong to every X: but it was assumed not to belong to some. And if M belongs to every N but not to every X, then there will be a deduction that N does not belong to every X. (The demonstration is the same.)"

1.	M***a***N	Premise
2.	M***o***X	Premise
→ 3.	N***a***X	Assumption for *RPI*
4.	M***a***X	1,3 *Barbara*
5.	N***o***X	1-4 *RPI*

From 1 and the contradictory of 5 (= 3) we have derived the contradictory of 2 (= 4). Hence we may (by rule *RPI*) derive 5 from 1 and 2.

d. *Darapti* (28^a18-25)

"When both P and R belong to every S, it results of necessity that P will belong to some R. For since the positive premise converts, S will belong to some R; consequently, since P belongs to every S and S to some R, it is necessary for P to belong to some R (for a deduction through the first figure comes about). It is also possible to carry out the demonstration through an impossibility or through the setting-out (*ekthesis*). For if both terms belong to every S, then if some one of the Ss is chosen (for instance N), then both P and R will belong to this; consequently, P will belong to some R."

i. By conversion:

1.	P***a***S	Premise
2.	R***a***S	Premise
3.	S***i***R	2 ***a***-*Conversion*
4.	P***i***R	1,3 *Darii*

ii. By “*ekthesis*”:

1.	P***a***S	Premise
2.	R***a***S	Premise
→ 3.	Let N be one of the S’s	Assumption
4.	P(N)	1,3 *Barbara* ?
5.	R(N)	2,3 *Barbara* ?
6.	P***i***R	? (*ekthesis*)

What is the syntactical status of ‘N’? It may be an implicitly quantified class term (so that (4) is, in effect, P***a***N), or a name of an individual (so that (4) is, in effect, *n is P*). If it is the former, Aristotle seems to have a problem. For if the implicit quantifier is universal the rule of “*ekthesis*” would then be: from P***a***N and R***a***N, infer P***i***R. But that has exactly the logical form of *Darapti*, and so this proof of *Darapti* would be circular.

So, if ‘N’ is an individual rather than a class term, what is the rule justifying line 6? It appears to be something like Existential Generalization: if there is an individual, *x*, such that both *Fx* and *Gx*, infer F***i***G).

e. Proofs of Two “Derived Rules”

i. *Darii* (29b9-11)

“The particular deductions in the first figure are brought to completion through themselves, but it is also possible to prove them through the second figure, leading away to an impossibility. For instance, if A belongs to every B and B to some C, we can prove that A belongs to some C. For if it belongs to no C and to every B, then B will not belong to any C (for we know this through the second figure).”

1.	A***a***B	Premise
2.	B***i***C	Premise
→ 3.	A***e***C	Assumption for *RPI*
<4.	C***e***B>	1,3 *Cesare* (derived rule)
5.	B***e***C	<4 ***e***-*Conversion*>[1]
6.	A***i***C	1-5 *RPI*

ii. *Ferio* (29b12-15)

“If A belongs to no B and B to some C, then A will not belong to some C. For if it belongs to every C but to no B, then neither will B belong to any C (this was the middle figure).”

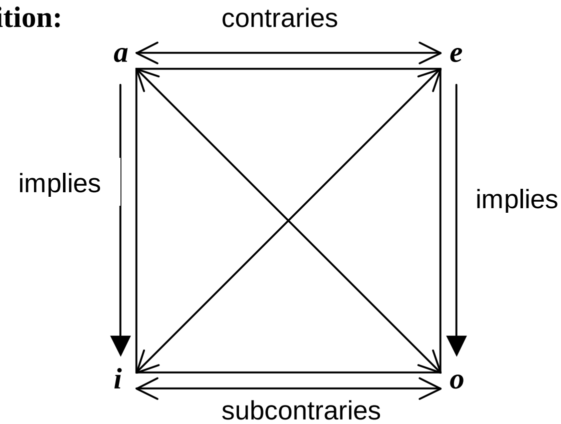
1.	A***e***B	Premise
2.	B***i***C	Premise
→ 3.	A***a***C	Assumption for *RPI*
4.	B***e***C	1,3 *Camestres* (derived rule)[2]
5.	A***o***C	1-4 *RPI*

[1] Aristotle omits the material in angle brackets, neglecting to mention that he is using the conversion rule as well as the “second figure” syllogism (*Cesare*).

[2] Aristotle is appealing to only the first four lines of his proof of *Camestres*.

9. Aristotle's Square and the Modern Square

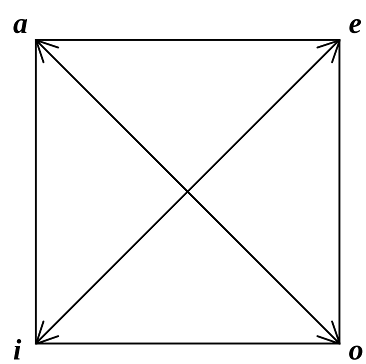
a. Aristotle's square of opposition:



Notice all the logical relationships contained in this diagram: ***a*** and ***o*** are contradictories (always have opposite truth-values), as are ***e*** and ***i***. ***a*** logically implies ***i***, and ***e*** logically implies ***o***. ***a*** and ***e*** are contraries (can't both be true, but may both be false), while ***i*** and ***o*** are subcontraries (can't both be false, but may both be true).

Aristotle's square of opposition is notoriously out of step with the "modern" square. That is, when Aristotle's ***a***, ***e***, ***i***, and ***o*** propositions are "translated" in a standard way into the notation of contemporary first-order logic, these traditional relationships of entailment and contrariety do not survive.

b. The modern square:

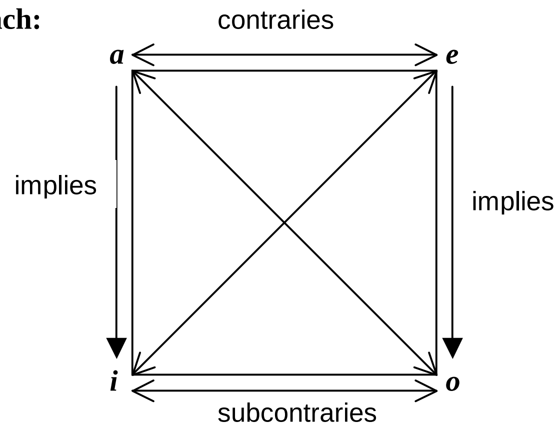


All that remains of the traditional square is that ***a*** and ***o*** are contradictories, and that ***e*** and ***i*** are contradictories. That is because the "modern" versions of these propositions look like this:

a	All *F*s are *G*s	$\forall x\ (Fx \rightarrow Gx)$
e	No *F*s are *G*s	$\forall x\ (Fx \rightarrow \neg Gx)$
i	Some *F*s are *G*s	$\exists x\ (Fx \wedge Gx)$
o	Some *F*s are not *G*s	$\exists x\ (Fx \wedge \neg Gx)$

The cause of the discrepancy is that the FOL versions of ***a*** and ***e*** propositions lack "existential import." That is, $\forall x\ (Fx \rightarrow Gx)$ and $\forall x\ (Fx \rightarrow \neg Gx)$ are both **true** when $\neg\exists x\ Fx$ is true, that is, when there are no *F*s. (These are the so-called "vacuously true" universal generalizations.) So 'All *F*s are *G*s', on the modern reading, does not imply that there are *F*s, and so does not imply that some *F*s are *G*s.

c. **The Presupposition Approach:**



The simplest way to bring the ancient and modern squares into line is to assume that Aristotelian syllogistic is intended to be used only with **non-empty predicates**. If we assume that '*F*' and '*G*' are non-empty, we can keep the standard FOL translations of the four Aristotelian forms:

a	All *F*s are *G*s	$\forall x\ (Fx \rightarrow Gx)$
e	No *F*s are *G*s	$\forall x\ (Fx \rightarrow \neg Gx)$
i	Some *F*s are *G*s	$\exists x\ (Fx \wedge Gx)$
o	Some *F*s are not *G*s	$\exists x\ (Fx \wedge \neg Gx)$

Each of the four propositions is now taken to **presuppose** that the letters *F*, *G*, etc. are assigned **non-empty sets**.

Presupposition vs. Entailment

P entails *Q*	If *Q* is false, *P* is false.
P presupposes *Q*	If *Q* is false, *P* is neither true nor false.

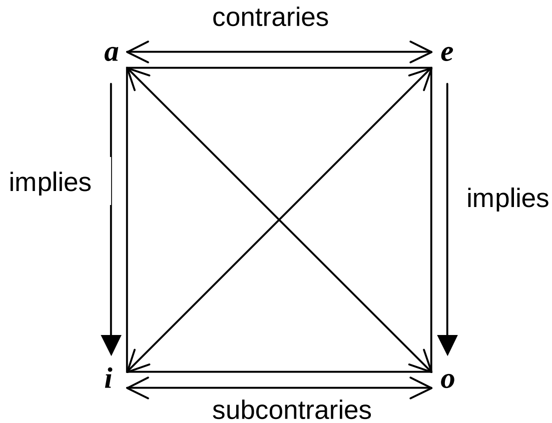
The presupposition assumption, at a single stroke, disables all the counterexamples to the traditional entailment and contrariety relations. For the only way in which the FOL version of ***a*** (or ***e***) could be true while ***i*** (or ***o***) is false is if there are no *F*s. Likewise, the only way in which ***a*** and ***e*** could both be true (or ***i*** and ***o*** both false) is if there are no *F*s. So if the syllogistic propositions presuppose that there are *F*s, there are no counterexamples.

This simple solution reinstates all the traditional relationships of contrariety, subcontrariety, and implication. But there are shortcomings with this approach, as well. For it is not enough to say that each simple predicate we use must be non-empty; we will also have to require that any logical operation we perform on predicates will also yield a non-empty result.

For example, 'swan' is non-empty, and so is 'black'. But suppose that, in fact, all swans are white. Still, we can conjoin the predicates 'swan' and 'black' to get 'black swan'. This compound predicate is **as a matter of fact** empty. So we cannot allow it to be the subject of an ***a*** proposition, or else we will get embarrassing results. For if "all black swans have long necks" is true, then (since ***a*** implies ***i***) so is "some black swans have long necks" and hence we are committed to the existence of black swans. But if "all black swans have long necks" is false, then (since ***a*** and ***o*** are contradictories) "some black swans don't have long necks" is true, and once again we are committed to the existence of black swans.

Thus we will have to say that syllogistic simply does not apply to such sentences. But the emptiness of 'black swan' seems to be a factual matter. And should the applicability of an inferential system depend on such things? This seems to bring too much of the world into the logical system. We may only discover the emptiness of the compound predicates we create by means of logical operations on non-empty predicates by reasoning with them. And the presupposition approach prohibits us from doing that.

d. **A Revised Modern Square:**



One way to reconcile Aristotle's square of opposition with the modern square. is to revise the modern versions of some of the four propositions to make the existential import explicit. That is, instead of **presupposing** that there are *F*s, we can actually **assert** it, by conjoining $\exists x\, Fx$ to the Aristotelian propositions.

But one cannot simply conjoin $\exists x$ Fx to each of ***a*** and ***e***. That would salvage two of the entailments (***a*** $\Rightarrow$ ***i*** and ***e*** $\Rightarrow$ ***o***) but wreck the contradictories (since, e.g., ***a*** and ***o*** would both be false when there are no *F*s). The only plausible way to revise the FOL versions of the Aristotelian propositions so as to preserve the traditional relationships would look like this:

a	All *F*s are *G*s	$\forall x\, (Fx \rightarrow Gx) \wedge \exists x\, Fx$
e	No *F*s are *G*s	$\forall x\, (Fx \rightarrow \neg Gx)$
i	Some *F*s are *G*s	$\exists x\, (Fx \wedge Gx)$
o	Some *F*s are not *G*s	$\exists x\, (Fx \wedge \neg Gx) \vee \neg\exists x\, Fx$

Notice that all the traditional relationships of contrariety, subcontrariety, and implication have returned. But there are some oddities. Now it is only the positive (***a*** and ***i***) propositions that have existential import; the negative ones lack it. And the ***o*** proposition no longer says "There are *F*s that are not *G*s". Rather, it says that either there are *F*s that are not *G*s or there are no *F*s at all. That is, it is precisely the contradictory of ***a*** when the existential import of ***a*** is made explicit as a conjunct of ***a***.

Still, there are some textual reasons that favor this approach. First, there is the fact that Aristotle frequently paraphrases ***o*** as '*G* belongs to not every *F*', i.e. , as the explicit contradictory of ***a***. Second, there are these passages:

> "If Socrates does not exist, 'Socrates is ill' is false" (*Categories* 13b18).

> "If Socrates does not exist, 'Socrates is not ill' is true" (*Categories* 13b26).

These examples show that Aristotle is willing to assign a truth value to a sentence with a non-denoting singular term as subject. This suggests that he would be willing to do the same for an ***a***-proposition with an empty subject term. This amounts to a rejection of the presupposition approach, and hence favors the Revised Modern Square.

Revised 12/28/07