Midterm Exam Translation Practice

For practice, **translate** these sentences (English into FOL and FOL into English). Where there is more than one obviously correct answer, alternative (correct) translations are given.

1. If *a* is a cube but *b* isn't, then neither *c* nor *d* is large.
2. *c* is a large tetrahedron if either *b* or *a* adjoins *d*.
3. If *d* is a dodecahedron, then *c* isn't; but *b* is small only if *a* is a cube.
4. Either *a* or *b* is not small; however, *c* is a cube just in case *d* is, too.
5. Not both *a* and *b* adjoin *c* unless *d* is a dodecahedron.
6. $(\text{Cube(c)} \wedge \neg\text{Small(c)}) \rightarrow (\text{Adjoins(c, d)} \vee \text{FrontOf(c, d)})$
7. $\text{Adjoins(d, c)} \leftrightarrow \neg(\text{Large(b)} \wedge \text{Tet(b)})$
8. If either one of *b* or *c* is left of the other, then at least one of them is large.
9. In the event that *b* is a cube that is in the same column as *d*, *b* and *d* are the same block.
10. *b* is a cube, and it is in the same column as *d* only if *b* and *d* are the same block.

Now try translating this argument into FOL.

> *d* is large if it either is a dodecahedron that is not in front of *c* or is a cube. If *d* is a dodecahedron and is in front of *c*, then *b* is a tetrahedron. *d* is a dodecahedron, but *b* is not a tetrahedron. Therefore, *d* is large.

Just for fun, can you figure out whether it is valid? Solutions for all the problems can be found on the next page.

Solutions

Sentences

1. (Cube(a) ∧¬Cube(b)) → ¬(Large(c) ∨ Large(d))
   (Cube(a) ∧¬Cube(b)) → (¬Large(c) ∧ ¬Large(d))
2. (Adjoins(b, d) ∨ Adjoins(a, d)) → (Large(c) ∧ Tet(c))
3. (Dodec(d) → ¬Dodec(c)) ∧ (Small(b) → Cube(a))
4. (¬Small(a) ∨ ¬Small(b)) ∧ (Cube(c) ↔ Cube(d))
5. ¬(Adjoins(a, c) ∧ Adjoins(b, c)) ∨ Dodec(d)
   ¬Dodec(d) → ¬(Adjoins(a, c) ∧ Adjoins(b, c))
   (Adjoins(a, c) ∧ Adjoins(b, c)) → Dodec(d)
   ¬Dodec(d) → (¬Adjoins(a, c) ∨ ¬Adjoins(b, c))
6. If *c* is cube that isn't small, it either adjoins *d* or is in front of it.
   c is a non-small cube only if it either adjoins or is in front of *d*.
7. *d* adjoins *c* if and only if *b* is not a large tetrahedron.
   d adjoins *c* just in case *b* is not a large tetrahedron.
8. (LeftOf(b, c) ∨ LeftOf(c, b)) →(Large(b) ∨ Large(c))
9. (Cube(b) ∧ SameCol(b, d)) → b = d
10. Cube(b) ∧ (SameCol(b, d) → b = d)

Argument:

((Dodec(d) ∧ ¬FrontOf(d, c)) ∨ Cube(d)) → Large(d)

(Dodec(d) ∧ FrontOf(d, c)) → Tet(b)

Dodec(d) ∧ ¬Tet(b)

Large(d)

The argument is valid. To see whether you can prove it, open MidtermPrep1.prf on the Supplementary Exercises page, under Chapter 8. The best strategy is proof by cases, with the two cases being FrontOf(d, c) and ¬FrontOf(d, c). You may use **TautCon** to introduce the disjunction of these two cases, which is an instance of Excluded Middle.

If you get stuck and want to see a completed proof, open ProofMidtermPrep1.prf