Some Useful Truth-Table Equivalences

Because of the interdefinability of the connectives, each truth function can be expressed by a number of different FOL sentences. It is useful to be aware of the variant, but equivalent, ways of expressing familiar truth-functions, such as conjunction, disjunction, etc. Each of the following five groups contains five **equivalent sentences**, all expressing the same truth-function. Use truth-tables, if necessary, to confirm any equivalances you find surprising.

Group 1: Conjunction

$A \wedge B$	$B \wedge A$	$\neg(\neg A \vee \neg B)$
$\neg(A \rightarrow \neg B)$	$\neg(B \rightarrow \neg A)$	

Group 2: Disjunction

$A \vee B$	$B \vee A$	$\neg(\neg A \wedge \neg B)$
$\neg A \rightarrow B$	$\neg B \rightarrow A$	

Group 3: Conditional

$A \rightarrow B$	$\neg(A \wedge \neg B)$	$\neg A \vee B$
$B \vee \neg A$	$\neg B \rightarrow \neg A$	

Group 4: Biconditional

$A \leftrightarrow B$	$B \leftrightarrow A$	$\neg A \leftrightarrow \neg B$
$(A \rightarrow B) \wedge (B \rightarrow A)$		$(A \wedge B) \vee (\neg A \wedge \neg B)$

Group 5: Negation of biconditional

$\neg(A \leftrightarrow B)$	$\neg A \leftrightarrow B$	$A \leftrightarrow \neg B$
$(A \wedge \neg B) \vee (B \wedge \neg A)$		$(A \vee B) \wedge \neg\ (A \wedge B)$

Note the last group, negation of biconditional, expresses **exclusive disjunction** (A xor B) – either A or B, but not both.