Glossary

Antecedent: The antecedent of a conditional is its first component clause (the *if* clause). In $P \rightarrow Q$, P is the antecedent and Q is the consequent.

Antisymmetric: a binary relation R is antisymmetric iff no **two** things ever bear R to one another, i.e., R satisfies the condition that $\forall x \forall y [(R(x, y) \wedge R(y, x)) \rightarrow x = y]$.

Argument: "Argument" is used in two different senses in logic.

1. Arguments as pieces of reasoning: an argument is a sequence of statements in which one (the conclusion) is supposed to follow from or be supported by the others (the premises).

2. Arguments in the mathematical sense: an argument is an individual symbol (variable or constant) taken by a predicate in an atomic wff. In the atomic wff LeftOf(x, a), x and a are the arguments of the binary predicate LeftOf.

Aristotelian forms (A, E, I, O): The four main sentence forms treated in Aristotle's logic: the A form (universal affirmative) *All P's are Q's*, the E form (universal negative) *No P's are Q's*, the I form (particular affirmative) *Some P's are Q's*, and the O form (particular negative) *Some P's are not Q's*.

Arity: The arity of a predicate indicates the number of individual constants (names) it takes to combine with the predicate to form a complete sentence. A predicate with an arity of one is called unary. A predicate with an arity of two is called binary. It's possible for a predicate to have any arity, so we can talk about 6-ary or even 113-ary predicates.

Asymmetric: a binary relation R is asymmetric iff it is never reciprocated, i.e., R satisfies the condition that $\forall x \forall y (R(x, y) \rightarrow \neg R(y, x))$.

Atomic sentence: Atomic sentences are the most basic sentences of FOL, those formed by a predicate followed by the right number (see arity) of names. Atomic sentences of FOL correspond to the simplest sentences of English.

Biconditional: An *if and only if* statement. In FOL, the biconditional $P \leftrightarrow Q$ is comes out true whenever P and Q have the same truth value and false when they differ in truth value. $P \leftrightarrow Q$ is equivalent to $(P \rightarrow Q) \wedge (Q \rightarrow P)$; that is, a biconditional is equivalent to a conjunction of "one-way" conditionals.

Biconditional Elimination ($\leftrightarrow$ Elim): A rule of systems $\mathcal{F}$ and $\mathcal{F}_T$ that permits us, given a biconditional on one line and either of its components on another line, to infer the other component of the biconditional. Also known as *modus ponens* for the biconditional.

Biconditional Introduction ($\leftrightarrow$ Intro): A rule of systems $\mathcal{F}$ and $\mathcal{F}_T$ that permits us to infer a biconditional $P \leftrightarrow Q$ from a pair of closed subproofs, one of which assumes P and deduces Q, the other of which assumes Q and deduces P.

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Boolean connective: The logical connectives conjunction ($\wedge$), disjunction ($\vee$), and negation ($\neg$) allow us to form complex claims from simpler claims and are known as the Boolean connectives after the logician George Boole. Conjunction corresponds to the English word *and*, disjunction to *or*, and negation corresponds to the phrase *it is not the case that*. (See also Truth-functional connective.)

Bound variable: A bound occurrence of a variable is an instance of a variable occurring within the scope of a quantifier used with the same variable. For example, in $\forall x\, P(x, y)$ both occurrences of the variable x are bound, but y is free. (See also Free variable.)

Boxed constant: An individual constant placed inside a box when it is introduced in a subproof in system $\mathcal{F}$. Boxed constants are used in conjunction with rules $\exists$ **Elim** and $\forall$ **Intro** to indicate that those constants denote arbitrary objects. The rules of system $\mathcal{F}$ do not permit a boxed constant to appear outside the subproof in which it is introduced.

Completeness: A formal system is complete if every valid inference is provable by means of the rules of the system. (See also Soundness.)

Complex noun phrase: a noun phrase containing more than just a single semantically significant word, such as noun + adjective, or adverb + adjective + noun. Examples of complex noun phrases would be *small happy dog* or *politician who admires no Democrats*. In FOL, we normally use truth-functional connectives and quantifiers in translating complex noun phrases.

Conclusion: The statement in an argument that is meant to follow from the other statements (the premises).

Conditional: An *if ... then* sentence, i.e., a sentence that expresses some kind of conditional relationship between the two parts of the sentence. Not all conditionals in a natural language, such as English, are truth-functional. (See Material conditional, Truth-functional.)

Conditional Elimination ($\rightarrow$ Elim): A rule of systems $\mathcal{F}$ and $\mathcal{F}_{\text{T}}$ that permits us to infer the consequent of a conditional sentence whose antecedent occurs alone on a separate line. Also known as *modus ponens*.

Conditional Introduction ($\rightarrow$ Intro) : A rule of systems $\mathcal{F}$ and $\mathcal{F}_{\text{T}}$ that permits us to infer a conditional sentence from the fact that we have proved the conditional's consequent in a closed subproof that has the conditional's antecedent as its assumption. (See Conditional proof.)

Conditional proof: A method of proof in which one proves a conditional sentence by assuming its antecedent and then deducing its consequent. (See Conditional Introduction.)

Conjunct: One of the component sentences in a conjunction. For example, A and B are the conjuncts of $A \wedge B$.

Conjunction: The Boolean connective $\wedge$ corresponding to the English word *and*. An FOL sentence whose main connective is $\wedge$ is also called a conjunction. Such a sentence is true if and only if each conjunct is true.

Conjunction Elimination ($\wedge$ Elim): A rule of systems $\mathcal{F}$ and $\mathcal{F}_{\text{T}}$ that permits us to infer any of the conjuncts of a conjunction.

Conjunction Introduction (∧ Intro): A rule of systems $\mathcal{F}$ and $\mathcal{F}_T$ that permits us to infer a conjunction from the fact that we have proved all of its conjuncts separately.

Consequent: The consequent of a conditional is its second component clause (the *then* clause). In $P \rightarrow Q$, Q is the consequent and P is the antecedent.

Contradiction Elimination (⊥ Elim): A rule of systems $\mathcal{F}$ and $\mathcal{F}_T$ that permits us to infer any sentence we like from a contradiction.

Contradiction Introduction (⊥ Intro): A rule of systems $\mathcal{F}$ and $\mathcal{F}_T$ that permits us to enter a contradiction, ⊥, into a proof or subproof if we have already proved both P and $\neg P$ on separate lines in that proof or subproof.

Contradiction symbol, ⊥: The symbol ⊥ represents contradiction, i.e., something that cannot possibly be true in any set of circumstances. An example would be the conjunction of a sentence and its negation, $S \wedge \neg S$. (⊥ can be pronounced simply "contradiction.")

Conversational implicature: An implicature of a speaker's assertion of a sentence *S* is a conclusion that a hearer might draw from the speaker's assertion of *S*, but that is not strictly part of the meaning of *S*.

Counterexample: An individual case or instance that falsifies a universal generalization. A counterexample to an argument is a possible situation in which the premises of the argument are true and the conclusion is false. Such a situation is therefore a counterexample to the generalization that the conclusion comes out true in **all** situations in which the premises come out true.

Deductive vs. inductive: A deductive argument attempts to show that the conclusion is a logical consequence of the premises—that the conclusion **must** be true if the premises are true. An inductive argument does not attempt to show that the conclusion must be true, but only that its truth is made more **probable** by the truth of the premises.

Default and generous uses of rules (in Fitch): A default use of a rule is what Fitch does when you cite that rule and some previous line(s) as justification and, without entering any new sentence, ask Fitch to check out the step. For example, if you cite a conjunction and the rule ∧ **Elim** and ask Fitch to check out the step, Fitch will enter the **leftmost** conjunct on the new line. A **generous** use of a rule is one that is not is not strictly in accordance with the rule as stated in $\mathcal{F}$ (i.e., , $\mathcal{F}$ would not allow you to derive it in a single step), but is still a valid inference and will be approved of by Fitch.

Definite description: A phrase of the form *the so-and-so* that purports to refer to exactly one object, e.g., *the king of Norway* or *the sum of 3 and 5*.

DeMorgan's laws: There are two such laws. The first tells us that the negation of a conjunction, $\neg(P \wedge Q)$, is logically equivalent to the disjunction of the negations of the original conjuncts: $\neg P \vee \neg Q$. The second tells us that the negation of a disjunction, $\neg(P \vee Q)$, is logically equivalent to the conjunction of the negations of the original disjuncts: $\neg P \wedge \neg Q$.

DeMorgan laws for quantifiers: There are two such laws. The first tells us that the negation of a universal generalization, $\neg\forall x\, P(x)$, is logically equivalent to an existential generalization of a negation: $\exists x\, \neg P(x)$. The second tells us that the negation of an existential generalization, $\neg\exists x\, P(x)$, is logically equivalent to a universal generalization of a negation: $\forall x\, \neg P(x)$. These laws are also known as the quantifier/negation equivalences.

Disjunct: One of the component sentences in a disjunction. For example, A and B are the disjuncts of $A \vee B$.

Disjunction: The Boolean connective $\vee$ corresponding to the English word *or*. An FOL sentence whose main connective is $\vee$ is also called a disjunction. Such a sentence is true if and only if at least one disjunct is true.

Disjunction Elimination ($\vee$ Elim): A rule of systems $\mathcal{F}$ and $\mathcal{F}_T$ that permits us to infer a sentence from a disjunction if we have inferred it from each disjunct. (See Proof by cases.)

Disjunction Introduction ($\vee$ Intro): A rule of systems $\mathcal{F}$ and $\mathcal{F}_T$ that permits us to infer a disjunction from any of it disjuncts.

Distributing quantifiers: In some cases (but not all) an FOL sentence beginning with a quantifier is equivalent to the corresponding sentence with the quantifier "distributed through" the sentence. For example, given the sentence $\forall x\, (P(x) \wedge Q(x))$ we can distribute the universal quantifier through and obtain the equivalent sentence $\forall x\, P(x) \wedge \forall x\, Q(x)$, and vice versa. This is called distributing $\forall$ through $\wedge$. One may also distribute $\exists$ through $\vee$. But it is **not** legitimate to distribute $\forall$ through $\vee$ or $\exists$ through $\wedge$.

Domain of discourse: the set of objects under consideration when claims involving quantifiers are evaluated —that is, the entire collection of things that we take the quantifiers to "range over" or pick out. For example, the truth value of the claim "Every student received a passing grade" depends on whether our domain of discourse includes all the students in the world, in the university, or just in one particular class. Also called Domain of quantification, or Universe of discourse. (See Infinite domain.)

Donkey sentence: An English sentence whose grammatical subject contains what appears to be an existential noun phrase (e.g., "a donkey") that serves as the grammatical antecedent to a pronoun (e.g., "it") in the verb phrase. These sentences get their name from the classic example *every farmer who owns a donkey beats it.* Donkey sentences are especially tricky to translate into FOL, and always require paraphrasing first. (See Paraphrasing.)

Equivalence chain: A sequence of equivalent sentences designed to show the equivalence of the first and last sentences in the chain. In a typical equivalence chain, the equivalence between each sentence in the chain and the next is obvious, although the equivalence between the first sentence and the last is not.

Equivalence relation: An equivalence relation is a binary relation that is reflexive, symmetric, and transitive.

Exceptive: An *exceptive* is a claim that makes a universal generalization with an exception, such as *everything is a cube except c*. Exceptives go into FOL most conveniently as negative biconditionals, such as $\forall x\, (\text{Cube}(x) \leftrightarrow x \neq c))$.

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Existential Elimination (∃ Elim): A rule of system $\mathcal{F}$ that permits us to infer a sentence S from an existential generalization together with a closed subproof in which we have proved S from an instance of that generalization.

Existential Introduction (∃ Intro): A rule of system $\mathcal{F}$ that permits us to infer an existential generalization from any of its instances.

Existential quantifier (∃): In FOL, the existential quantifier is expressed by the symbol ∃ and is used to make claims asserting the existence of some object in the domain of discourse. In English, we express existentially quantified claims with the use of words like *something*, *at least one thing*, *a*, etc.

$\mathcal{F}$ vs. $\mathcal{F}_T$: $\mathcal{F}$ is the formal system of inference rules for FOL; $\mathcal{F}_T$ is a part of $\mathcal{F}$, and consists of just the rules for propositional logic. That is, $\mathcal{F}_T$ is the set of introduction and elimination rules for ¬, ∧, ∨, →, ↔, and ⊥. $\mathcal{F}$ contains all of these rules plus the introduction and elimination rules for =, ∀, and ∃.

First-order consequence: A sentence S is a first order consequence of some premises if S follows from the premises simply in virtue of the meanings of the truth-functional connectives, identity, and the quantifiers. More precisely, a sentence S is an FO consequence of sentences $P_1,\ldots, P_n$ iff there is no interpretation under which all of $P_1,\ldots, P_n$ come out true and S comes out false. (See Interpretation.)

First-order logic: A logical system in which quantifiers range over individuals, but not over properties or relations. A first-order logic thus contains individual variables, but not predicate variables.

First-order validity: A sentence S is a first order validity (FO validity, for short) if S is a logical truth simply in virtue of the meanings of the truth-functional connectives, identity, and the quantifiers. More precisely, a sentence S is an FO validity iff it comes out true on every interpretation. (See Interpretation, Validity.)

FO-contradiction: A sentence that comes out false simply in virtue of the meanings of the truth-functional connectives, identity, and the quantifiers. Every FO-contradiction is also a logical contradiction, but not conversely. (See also Logical contradiction, TT-contradiction, TW-contradiction.)

Formal proof: A step-by-step demonstration, given in a formal system of deduction, that a conclusion follows from its premises.

Free variable: A free occurrence of a variable is one that is not bound. (See Bound variable.)

General Conditional Proof (∀ Intro): A rule of system $\mathcal{F}$ that permits us to infer a generalized conditional sentence ∀x (P(x) → Q(x)) from a closed subproof in which we have proved the instance of the consequent, Q(c), from the assumption P(c) for an arbitrary object c. (See Universal Introduction, Generalized conditional sentence.)

Generalized conditional sentence: An FOL sentence of the form ∀x (P(x) → Q(x)), that is, a universal generalization of a conditional wff.

Inconsistent set of sentences: A set of sentences is called inconsistent if there is no possible circumstance in which they can all be true together. For an argument to be valid is for its premises together with the **negation** of its conclusion to form an inconsistent set of sentences.

Indirect proof: See Proof by contradiction.

Individual constant: The FOL version of a name—a symbol that stands for an object or individual. In FOL it is assumed that each individual constant names one and only one object.

Infinite domain: A domain of discourse is infinite if it contains infinitely many members. The domain of positive integers is an example of an infinite domain. There are some invalid FOL arguments whose counter-examples must have infinite domains. Similarly, there are some FOL sentences that are logically possible but come out true only in infinite domains. (See Domain of discourse, Logically possible.)

Interpretation: If we take a quantified FOL sentence and replace its names with individual constants and its predicates with predicate letters, we get an uninterpreted FOL sentence. An interpretation of such a sentence is an assignment of values to those individual constants and predicate letters. Basically, an interpretation takes each individual constant to be a name of some object in the domain of discourse, and each predicate letter to stand for some property of objects in the domain. For example, from the FOL sentence $\forall x$ (Cube(x) $\rightarrow$ Larger(x, b)) we obtain the uninterpreted sentence $\forall x$ (F(x) $\rightarrow$ R(x, b)). One interpretation of this sentence takes the domain to be the positive integers, and assigns the number 2 to b, the property of being an even number to F, and the relation *divisible by* to R. So interpreted, the sentence says that every even number is divisible by 2.

Irreflexive: a binary relation R is irreflexive iff nothing stands in the relation R to itself, i.e., R satisfies the condition that $\forall x \neg R(x, x)$.

Literal: A literal is a sentence that is either an atomic sentence or the negation of an atomic sentence.

Logical consequence: A sentence S is a logical consequence of a set of premises if it is impossible for the premises all to be true while the conclusion S is false.

Logical contradiction: A sentence that comes out false in every possible circumstance. Every logical contradiction is also a TW-contradiction, but not conversely. (See also TW-contradiction, FO-contradiction, TT-contradiction.)

Logical necessity: See Logical truth.

Logical truth: A sentence that is a logical consequence of any set of premises. That is, no matter what the premises may be, it is impossible for the conclusion to be false. A logical truth thus comes out true in every possible circumstance. This is also called logical necessity.

Logically equivalent sentences: Two sentences are logically equivalent if they have the same truth values in all possible circumstances.

Logically equivalent wffs: A pair of wffs with free variables are logically equivalent if, in any possible circumstance, they are satisfied by the same objects. (See Satisfaction.)

Logically possible: A sentence is logically possible if there is no logical reason it cannot be true, i.e., if there is a possible circumstance in which it is true.

Material conditional: A truth-functional version of the conditional *if ... then ...*. The material conditional $P \rightarrow Q$ is false if P is true and Q is false, but otherwise true.

Metatheory: The branch of logic that studies the properties of a system of logic. Proofs of the soundness and completeness of system $\mathcal{F}$, for example, belong to metatheory.

Mixed quantifiers: An FOL sentence that contains at least one universal quantifier and at least one existential quantifier is said to contain mixed quantifiers. When a sentence contains mixed quantifiers, the order in which the quantifiers occur is especially important semantically.

Necessary and sufficient conditions: A necessary condition for a statement S is a condition that must hold in order for S to obtain. $S \rightarrow P$ says that P is a necessary condition for S. A sufficient condition for a statement S is a condition that guarantees that S will obtain. $P \rightarrow S$ says that P is a sufficient condition for S.

Negation: An FOL sentence that begins with a negation sign $\neg$ is called a negation. The negation of a true sentence is false; the negation of a false sentence is true.

Negation Elimination ($\neg$ Elim): A rule of systems $\mathcal{F}$ and $\mathcal{F}_T$ that permits us to infer a sentence from the negation of its negation (e.g., to infer S from $\neg\neg S$).

Negation Introduction ($\neg$ Intro): A rule of systems $\mathcal{F}$ and $\mathcal{F}_T$ that permits us to prove S by showing that $\neg S$ leads to a contradiction. (See Proof by contradiction.)

Null quantification: A null quantification is an FOL sentence beginning with a quantifier that does not bind any variables. That is, the variable used in the quantifier does not occur free in the wff to which the quantifier is prefixed. For example, the sentences $\forall x\, \text{Cube}(b)$ and $\forall y\, \exists y\, \text{Small}(y)$ are null quantifications. A null quantification is equivalent to the same sentence with the null quantifier deleted.

Numerical quantifier: English contains numerical quantifiers such as *at least two*, *at most one*, *exactly one*, *at least three*, *at most two*, *exactly two*, etc. FOL does not have numerical quantifiers, but we can paraphrase English sentences containing these phrases to enable their translation into FOL. These FOL translations use universal and existential quantifiers together with identity or nonidentity clauses to capture the meaning of the English numerical quantifiers.

Paraphrasing: To paraphrase a sentence is to say the same thing in a different way. It is often convenient to paraphrase an English sentence before attempting to translate it into FOL. For example, before translating *if something is a cube, it is not a tetrahedron* into English, it is helpful to paraphrase it (e.g., as *everything is such that, if it is a cube, it is not a tetrahedron*) to make clear that the quantifier intended is universal.

Predicate symbol: A symbol of FOL that is used to express a property of objects or a relation between objects.

Prefix vs. infix notation: In prefix notation, the predicate or relation symbol precedes its arguments, e.g., Larger(a, b). In infix notation, the relation symbol appears between its two arguments, e.g., a = b.

Premise: A statement meant to support (lead us to accept) the conclusion of an argument.

Prenex form: An FOL sentence is in prenex form when all its quantifiers are "out in front." More precisely, an FOL sentence is in prenex form when it contains no quantifier that is preceded by any symbol other than a quantifier. Thus, ∀x ∃y (F(x) → R(x, y)) and Cube(a) are both in prenex form. (The second example may seem surprising, but since it contains no quantifiers at all, it contains no quantifier that is preceded by any other symbol, and so counts as being in prenex form.) On the other hand, ∀x (F(x) → ∃y R(x, y)) and ∀x ¬∃y R(x, y) are not in prenex form. Every FOL sentence is equivalent to some sentence in prenex form.

Presupposition: The presuppositions of a sentence *S* are those conditions that must be fulfilled in order for *S* to have a truth value, i.e., in order for *S* to make any claim at all. Whereas Russell's theory of descriptions holds that a sentence of the form *the F is G* logically implies that there exists an *F*, Strawson's theory contends that such sentences presuppose the existence of an *F*, rather than logically imply it. So whereas Russell says that if nothing is *F*, the sentence *the F is G* is false, Strawson maintains that in this case the sentence is neither true nor false.

Proof by cases: A proof strategy that consists in proving some statement S from a disjunction by proving S from each disjunct. (See Disjunction Elimination.)

Proof by contradiction (indirect proof): To prove ¬S by contradiction, we assume S and prove a contradiction. In other words, we assume the negation of what we wish to prove and show that this assumption leads to a contradiction. (See Negation Introduction.)

Proofs without premises A proof without premises, as the name implies, contains no premises. Such a proof typically begins with a subproof assumption and ends when all subproofs have been closed. The conclusion of a proof without premises is called a *theorem* of the system of proof. In a sound system, every theorem is a logical consequence of the empty set of premises, i.e., is a logical truth. (See Soundness, Theorem.)

Reflexive: a binary relation *R* is reflexive iff everything stands in the relation *R* to itself, i.e., *R* satisfies the condition that ∀*x R(x, x)*.

Satisfaction: An object named a satisfies an atomic wff S(x) if and only if S(a) is true, where S(a) is the result of replacing all free occurrences of x in S(x) with the name a. Satisfaction for wffs with more than one free variable is defined similarly. For example, an ordered pair of objects named a and b, respectively, satisfies an atomic wff S(x, y) if and only if S(a, b) is true, where S(a, b) is the result of replacing all free occurrences of x in S(x, y) with the name a and all free occurrences of y with the name b.

Scope of quantifier: The scope of a quantifier in a wff is that part of the wff that falls under the "influence" of the quantifier. Parentheses play an important role in determining the scope of quantifiers. When a quantifier is immediately followed by a left parenthesis, the scope of that quantifier is the wff that is contained between that parenthesis and the right parenthesis that is its mate.

Semantics of quantified FOL: The semantics of a language concerns the meaning of its sentences and the conditions under which they come out true. For quantified FOL, the semantics consists of the rules for assigning truth values to sentences of FOL that are built up out of predicates, individual variables and constants, connectives, and quantifiers. The semantics of quantified FOL is based on the notion of *satisfaction*. (See Satisfaction.)

Sentence: In propositional logic, atomic sentences are formed by combining names and predicates. Compound sentences are formed by combining atomic sentences by means of the truth-functional connectives. In FOL, the definition is a bit more complicated. A sentence of FOL is a wff with no free variables.

Soundness: "Sound" is used in two different senses in logic.

1. An argument is sound if it is both valid and all of its premises are true.

2. A formal system is sound if all the inferences that are permitted by the rules of the system are valid inferences, that is, if no invalid arguments are provable within the system. (See also Completeness.)

Step-by-step method: A method of translating a natural language sentence into FOL that begins with the outer or "gross" structure of the sentence, and then moves inward, one step at a time.

Subproof: A proof that occurs within the context of a larger proof. In system $\mathcal{F}$, subproofs are required by each of the following rules: ∨ **Elim**, ¬ **Intro**, → **Intro**, ↔ **Intro**, ∀ **Intro**, and ∃ **Elim**.

Superlative: An adjective ending in *–est*, such as *largest*, *oldest*, *strongest*, etc. These are often translated into FOL in terms of the corresponding relational predicates (*larger than*, *older than*, *stronger than*), quantifiers, and identity.

Symmetric: a binary relation R is symmetric iff it is always reciprocated, i.e., R satisfies the condition that $\forall x \forall y\, (R(x, y) \rightarrow R(y, x))$.

Syntax of quantified FOL: The syntax of a language is, roughly speaking, its grammar—the rules for constructing wffs and sentences of the language. For quantified FOL, the syntax consists of the rules for constructing wffs using predicates, individual variables and constants, connectives, and quantifiers.

⊢T vs. ⊢: These "turnstile" symbols stand for the relation of provability in a formal system. ⊢ indicates provability in system $\mathcal{F}$, while $\vdash_T$ indicates provability in system $\mathcal{F}_T$. We write $P_1,\ldots, P_n \vdash_T S$ to mean that there is a formal proof in $\mathcal{F}_T$ of S from premises $P_1,\ldots, P_n$, while we write $P_1,\ldots, P_n \vdash S$ to mean that there is a formal proof in $\mathcal{F}$ of S from premises $P_1,\ldots, P_n$. These symbols are sometimes written with no sentences to their left. When we write $\vdash_T S$ we mean that S can be proved without premises in system $\mathcal{F}_T$, and $\vdash S$ means that S can be proved without premises in system $\mathcal{F}$. (See $\mathcal{F}$ vs. $\mathcal{F}_T$.)

Tarski's World necessity: A sentence that comes out true in every world in Tarski's World. Such a sentence may not be logically necessary, since there might be a circumstance that cannot arise in Tarski's World in which it comes out false. An example would be ¬(Large(b) ∧ Larger(a, b)).

Tautological consequence: A sentence S is a tautological consequence of some premises if S follows from the premises simply in virtue of the meanings of the truth-functional connectives. We can check for tautological consequence by means of truth tables, since S is a tautological consequence of the premises if and only if every row of their joint truth table that assigns true to each of the premises also assigns true to S. All tautological consequences are logical consequences, but not all logical consequences are tautological consequences.

Tautological equivalence: Two sentences are tautologically equivalent if they are equivalent simply in virtue of the meanings of the truth-functional connectives. We can check for tautological equivalence by means of truth tables since two sentences Q and S are tautologically equivalent if and only if every row of their joint truth table assigns the same value to the main connectives of Q and S.

Tautology: A sentence that is logically true in virtue of its truth-functional structure. This can be checked using truth tables since S is a tautology if and only if every row of the truth table for S assigns true to the main connective.

Theorem: In a formal system such as $\mathcal{F}$ or $\mathcal{F}_T$, a theorem is any sentence that has been proved within the system from the empty set of premises. (See also Proofs without premises.)

Total relation: a binary relation R is total iff for each object in the domain there is something to which it stands in the relation R, i.e., R satisfies the condition that $\forall x \exists y \, R(x, y)$.

Transitive: a binary relation R is transitive just in case if one thing bears R to a second, and the second bears R to a third, then the first thing bears R to the third, i.e., R satisfies the condition that $\forall x \forall y \forall z \, [(R(x, y) \wedge R(y, z)) \rightarrow R(x, z)]$.

Truth table: Truth tables show the way in which the truth value of a sentence built up using truth-functional connectives depends on the truth values of the sentence's components.

Truth-functional connective: A sentence connective with the property that the truth value of the newly formed sentence is determined solely by the truth value(s) of the constituent sentence(s), nothing more. Examples are the Boolean connectives ($\neg, \wedge, \vee$) and the material conditional and biconditional ($\rightarrow, \leftrightarrow$).

Truth-functional form: The truth-functional form of a sentence is the structure that results from replacing all of its constituent quantified sentences with atomic sentence letters. Thus, the truth-functional form of $\forall x \, F(x) \rightarrow \forall x \, G(x)$ is $A \rightarrow B$, whereas the truth-functional form of $\forall x \, (F(x) \rightarrow G(x))$ is A.

TT-contradiction: A sentence that comes out false on every row of its truth table; that is, it comes out false simply in virtue of the meanings of the truth-functional connectives. Every TT-contradiction is also an FO-contradiction, but not conversely. (See also FO-contradiction, TW-contradiction.)

TT-possible ("truth table possible"): An FOL sentence is TT-possible if comes out true on at least one row of its truth table. Such a sentence may not be logically possible, but every sentence that is logically possible is also TT-possible.

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TW-contradiction: A sentence that comes out false in every word in Tarski's World. Not every TW-contradiction is a logical contradiction. For example, Large(b) ∧ Larger(a, b) is a TW-contradiction (since there is no Tarski world in which there is a block that is larger than a large block), but not a logical contradiction (since there are possible circumstances—not modeled by Tarski's World—in which one large object can be larger than another large object). (See also Logical contradiction, FO-contradiction, TT-contradiction.)

TW-possible ("Tarski's World possible"): A sentence that comes out true in at least one world in Tarski's World. Such a sentence is guaranteed to be both logically possible and truth table possible. But not all logically possible sentences are TW-possible (an example would be Large(a) ∧ Adjoins(a, b), which is logically possible but not TW-possible).

Universal Elimination (∀ Elim): A rule of system $\mathcal{F}$ that permits us to infer any instance of a universal generalization.

Universal Introduction (∀ Intro): A rule of system $\mathcal{F}$ that permits us to infer a universal generalization ∀x P(x) from a closed subproof in which we have proved the instance P(c) for an arbitrary object c. (See General Conditional Proof.)

Universal quantifier (∀): In FOL, the universal quantifier is expressed by the symbol ∀ and is used to express universal claims. It corresponds, roughly, to English expressions such as *everything*, *all things*, *each thing*, etc.

Validity: "Validity" is used in two ways in logic.

1. Validity as a property of arguments: An argument is valid if the conclusion must be true in any circumstance in which the premises are true. (See also Logical consequence.)

2. Validity as a property of sentences: A first-order sentence is said to be valid if it is logically true simply in virtue of the meanings of its connective, quantifiers, and identity. (See First-order validity.)

Variable: Variables are expressions of FOL that function somewhat like pronouns in English. They are like individual constants in that they may be the arguments of predicates, but unlike constants, they can be bound by quantifiers. Generally letters from the end of the alphabet, x, y, z, etc., are used for variables.

Well formed formula (wff): Wffs are the "grammatical" expressions of FOL. A wff is either atomic (an *n*-ary predicate followed by *n* individual symbols) or a complex wff constructed using connectives, quantifiers, and other wffs. Wffs may have free variables. Sentences of FOL are wffs with no free variables. (See Bound variable.)

Working backwards strategy: A basic proof strategy in which you begin by figuring out what intermediate conclusion you might reach that would enable you to obtain your ultimate conclusion, and then take that intermediate conclusion as your new goal. You then repeat the strategy with the intermediate conclusion, and so on.

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