Some Final Exam Practice Questions

Here are some additional practice problems to help you prepare for the final exam. Be sure to attempt as many questions as you can before consulting the answer sheet. If you get a problem wrong, be sure to read the explanation on the answer sheet; it should help you figure out why your answer was incorrect.

1. Which of the following are logical truths, but not FO-validities?
   a. ∀x ¬(Square(x) ∧ Circle(x))
   b. Cube(a) ∨ ¬Cube(a)
   c. Dodec(d) ∧ d = c ∧ Cube(c)
   d. ∀x SameRow(x, x)
   e. (a) and (d)

2. Which of the following are FO-validities, but not tautologies?
   a. c = c → c = c
   b. ∀x ¬Larger(x, x)
   c. ¬(Large(a) ∧ Adjoins(a, b))
   d. ∃x ¬Cube(x) → ¬∀x Cube(x)
   e. (b) and (d)

3. Which of the following is true?
   a. All TW-necessities are logical truths.
   b. All tautologies are logical truths.
   c. Some FO-validities are tautologies.
   d. All FO-validities are tautologies.
   e. (b) and (c)

4. Which of the following are tautologies?
   a. ((∀x Cube(x) → ∃y Large(y)) ∧ ¬∃y Large(y)) → ¬∀x Cube(x)
   b. ∀y (y = y)
   c. ¬(Medium(a) ∧ Small(a))
   d. ∃x Cube(x) ∧ ¬∃x Cube(x)
   e. ∀x (Cube(x) ∧ Small(x)) → (∀x Cube(x) ∧ ∀x Small(x))

5. Which of the following is a TT-contradiction?
   a. Cube(a) ∧ ¬Cube(a)
   b. (Tet(a) ∧ a = b) ∧ Dodec(b)
   c. Tet(a) ∧ Tet(b) ∧ ¬(Tet(a) ∨ Tet(b))
   d. ∃x ¬Cube(x) ∧ ¬∀x Cube(x)
   e. (a) and (c)

6. $\exists x\ (S(x) \land C(x))$ is equivalent to which of the following?
    a. $\exists x\ (C(x) \land S(x))$
    b. $\exists x\ S(x) \land \exists x\ C(x)$
    c. $\neg\forall x\ (S(x) \to \neg C(x))$
    d. Both (a) and (c)
    e. All of the above.

7. How would you say in the blocks language that all the dodecahedra are between two particular blocks?
    a. $\forall x\ (Dodec(x) \to \exists y \exists z\ Between(x, y, z))$
    b. $\exists y \forall x\ (Dodec(x) \to \exists z\ Between\ (x, y, z))$
    c. $\exists y \exists z \forall x\ (Dodec(x) \to Between(x, y, z))$
    d. $\exists y \exists z \forall x\ (Dodec(x) \land Between\ (x, y, z))$
    e. (a) and (c)

8. How could you translate *There are at most two apples* into FOL?
    a. $\forall x \forall y \forall z\ ((Apple(x) \land Apply(y) \land Apple(z)) \to (x = y \lor x = z \lor y = z))$
    b. $\forall x \forall y\ (Apple(x) \land Apple(y)) \to (x = y \lor y = x))$
    c. $\exists x \exists y\ (Apple(x) \land Apple(y) \land x \neq y \land \forall z\ (Apple(z) \to (z = x \lor z = y))$
    d. (a) and (c)
    e. None of the above

9. How could you translate *There are exactly two apples* into FOL?
    a. $\exists x \exists y\ (Apple(x) \land Apple(y) \land \forall z\ (Apple(z) \to (z = x \lor z = y))$
    b. $\exists x \exists y\ (Apple(x) \land Apple(y) \land x \neq y \land \forall z\ (Apple(z) \to (z = x \lor z = y))$
    c. $\exists x \exists y\ (x \neq y \land \forall z\ (Apple(z) \leftrightarrow (z = x \lor z = y)))$
    d. $\exists x \exists y\ (Apple(x) \land Apple(y) \land x \neq y) \land \forall x \forall y \forall z\ ((Apple(x) \land Apple(y) \land Apple(z)) \to (x = y \lor x = z \lor y = z))$
    e. (b), (c), and (d)
    f. All of the above

10. What is the truth-functional form of the following sentence?:

    $(\exists x \forall y\ Larger(x, y) \land \forall x \forall y\ ((Tet(x) \land \neg\ Tet(y)) \to Adjoins(y, x))) \to \neg\forall x\ \neg\forall y\ Larger(x, y)$

    a. $(A \land (B \to C\ )) \to E$
    b. $(A \land B) \to C$
    c. $(A \land B) \to \neg C$
    d. $(A \land ((B \land \neg C) \to D)) \to \neg E$
    e. None of the above

11. Which of the following means that R is a symmetrical relation?
    a. $\forall x \forall y \forall z\ ((R(x, y) \land R(y, z)) \rightarrow R(x, z))$
    b. $\forall x \forall y\ (R(x, y) \rightarrow \neg R(y, x))$
    c. $\forall x\ R(x, x)$
    d. $\forall x\ \neg R(x, x)$
    e. None of the above

12. Which of the following are symmetric relations?
    a. The *sibling of* relation
    b. The *parent of* relation
    c. The *same height as* relation
    d. The *larger than* relation
    e. (a) and (c)

13. How might the sentence "The small tetrahedron adjoins *b*" be translated into FOL according to Bertrand Russell's Theory of Descriptions?
    a. $\exists x\ (Small(x) \land Tet(x) \land \forall y\ ((Small(y) \land Tet(y)) \rightarrow y = x) \land Adjoins(x, b))$
    b. $\exists x \forall y\ (((Small(y) \land Tet(y)) \leftrightarrow y = x) \land Adjoins(x, b))$
    c. $\neg \forall x\ ((Small(x) \land Tet(x)) \rightarrow \neg\ Adjoins(x, b))$
    d. All of the above are correct.
    e. (a) and (b) are correct, but (c) is not.

14. Which of the following is a correct translation of *Every dodecahedron is in front of a small tetrahedron*?
    a. $\forall x\ (Dodec(x) \lor \forall y\ \neg(Small(y) \land Tet(y) \land FrontOf(x, y)))$
    b. $\forall x\ (Dodec(x) \rightarrow \exists y\ (Small(y) \land FrontOf(x, y) \land Tet(y)))$
    c. $\forall x\ (Dodec(x) \rightarrow \exists y\ (Tet(y) \land Small(y) \land FrontOf(y, x)))$
    d. $\forall x\ \neg(Dodec(x) \land \exists y\ (Tet(y) \land Small(y) \land FrontOf(x, y)))$
    e. (b) and (d)

15. Which of the following is true?
    a. Strawson's theory of descriptions withholds truth-values from sentences with non-denoting definite descriptions.
    b. Russell believes that *The golden mountain is golden* makes these three claims: (1) there is at least one golden mountain, (2) there is at most one golden mountain, and every golden mountain is golden.
    c. Russell's Theory of Descriptions does not have "truth-value gaps".
    d. All of the above
    e. (a) and (c)

16. ∃x (Dodec(x) → ∃y Cube(y)) is equivalent to which of the following?
    a. ¬∀x ¬ (Dodec(x) → ∃y Cube(y))
    b. ∃x (¬Dodec(x) ∨ ∃y Cube(y))
    c. ∃x (¬∃yCube(y) → ¬Dodec(x))
    d. ¬∀x (Dodec(x) ∧ ¬∃y Cube(y))
    e. (a) and (b)
    f. (b) and (c)
    g. (a) and (c)
    h. (a), (b), and (c).
    i. All of the above.

17. Fill in the missing sentences, inference rules, and support steps in lines 6 through 12 in this proof (essentially identical to exercise 13.50):

1. ∃x(Tet(x) ∧ ∀y (Tet(y) → y = x))	
2. [a b] Tet(a) ∧ Tet(b)	
3. Tet(a)	✓ ∧ Elim: 2
4. Tet(b)	✓ ∧ Elim: 2
5. [c] Tet(c) ∧ ∀y (Tet(y) → y = c)	
6. ∀y (Tet(y) → y = c)	Rule?:
7.	∀ Elim: 6
8. a = c	Rule?: 3,7
9. Tet(b) → b = c	Rule?:
10. b = c	Rule?:
11.	= Elim: 8,10
12. a = b	Rule?:
13. ∀x ∀y ((Tet(x) ∧ Tet(y)) → x = y)	✓ ∀ Intro: 2-12

Goals

▷ ∀x ∀y ((Tet(x) ∧ Tet(y)) → x = y)

18. Which techniques are used in this proof, and where are they used?
    a. Proof by cases
    b. General conditional proof
    c. Existential instantiation
    d. Proof by contradiction
    e. Both (a) and (b)
    f. Both (b) and (c)