Answers to Final Exam Practice Questions

Answers to multiple choice questions are listed here. For more detailed explanations, see the next section; it should help you figure out why any of your answers was incorrect.

1. e
2. d
3. e
4. a
5. e
6. d
7. c
8. a
9. e
10. c
11. e
12. e
13. e
14. b
15. d
16. i
17. 6. ∧ **Elim:** 5
    7. Tet(a) → a = c
    8. → **Elim**
    9. ∀ **Elim:** 6
    10. → **Elim:** 4, 9
    11. a = b
    12. ∃ **Elim:** 1, 5-11
18. f

Explanations

1. Which of the following are logical truths, but not FO-validities?

a.	∀x ¬(Square(x) ∧ Circle(x))	
b.	Cube(a) ∨ ¬Cube(a)	Tautology, hence FO-valid.
c.	Dodec(d) ∧ d = c ∧ Cube(c)	Not a logical truth.
d.	∀x SameRow(x, x)	
e.	**(a) and (d)**	**This is the correct answer.**

2. Which of the following are FO-validities, but not tautologies?

a.	c = c → c = c	Tautology.
b.	∀x ¬Larger(x, x)	Logical truth, but not FO-valid.
c.	¬(Large(a) ∧ Adjoins(a, b))	TW-true, but not FO-valid.
d.	**∃x ¬Cube(x) → ¬∀x Cube(x)**	**This is the correct answer.**
e.	(b) and (d)	

3. Which of the following is true?

a.	All TW-necessities are logical truths.	No. Cf. 2c above.
b.	All tautologies are logical truths.	Yes.
c.	Some FO-validities are tautologies.	Yes.
d.	All FO-validities are tautologies.	No. Cf. 2d above.
e.	**(b) and (c)**	**This is the correct answer.**

4. Which of the following are tautologies?

   a. ((∀x Cube(x) → ∃y Large(y)) ∧ ¬∃y Large(y)) → ¬∀x Cube(x)

   b. ∀y (y = y) No.

   c. ¬(Medium(a) ∧ Small(a)) No.

   d. ∃x Cube(x) ∧ ¬∃x Cube(x) No.

   e. ∀x (Cube(x) ∧ Small(x)) → (∀x Cube(x) ∧ ∀x Small(x)) No.

   The correct answer is (a), whose truth-functional form is ((P → Q) ∧ ¬Q) → ¬P. Since this is a tautology, so is (a). (b) and (e) are FO-validities, but not tautologies. The logical truth of (b) depends on the meanings of the identity predicate and the quantifier. The logical truth of (e) depends on the distribution principle for quantifiers (which is not part of propositional logic). The logical truth of (c) depends on the meanings of the predicates. The truth-value of (d) can be settled by truth tables, but it is a contradiction, hence logically false, not a tautology.

5. Which of the following is a TT-contradiction?

   a. Cube(a) ∧ ¬Cube(a)

   b. (Tet(a) ∧ a = b) ∧ Dodec(b) Logically false, but TT-possible.

   c. Tet(a) ∧ Tet(b) ∧ ¬(Tet(a) ∨ Tet(b))

   d. ∃x ¬Cube(x) ∧ ¬∀x Cube(x) FO-impossible, but TT-possible.

   e. (a) and (c) This is the correct answer.

6. ∃x (S(x) ∧ C(x)) is equivalent to which of the following?

   a. ∃x (C(x) ∧ S(x)) Yes, simply commute the conjunct wffs.

   b. ∃x S(x) ∧ ∃x C(x) Does not require any object satisfy both S(x) and C(x).

   c. ¬∀x (S(x) → ¬C(x)) Simple chain shows equivalence.

   d. Both (a) and (c) This is the correct answer.

   e. All of the above. No, (b) is not equivalent.

7. How would you say in the blocks language that all the dodecahedra are between two particular blocks?

   a. ∀x (Dodec(x) → ∃y∃z Between(x, y, z))

   b. ∃y∀x (Dodec(x) → ∃z Between (x, y, z))

   c. ∃y∃z∀x (Dodec(x) →Between(x, y, z))

   d. ∃y∃z∀x (Dodec(x) ∧ Between (x, y, z))

   e. (a) and (c)

   The correct answer is (c) The trouble with (a) is that it says that every dodecahedron is between **some** pair of blocks, but not necessarily the **same** pair of blocks. The 'particular blocks' means that you pick a pair of blocks **first**, and then check to be sure that all the dodecahedra are between **them**. This means the existential quantifiers must have wide scope. (b) gets part of the way there, but not quite all the way. It says that there is a particular block, *y*, that has each dodecahedron between it and some block or other. But (b) allows for the possibility that one dodecahedron is between *y* and a certain cube, while another dodecahedron is between *y* and a certain tetrahedron. In this case, not all the dodecahedra would be between two **particular** blocks. (d) says that there is a pair of dodecahedra which are such that **everything** is both a dodecahedron and between them. This is much too strong—in fact, it's logically false—since it asserts that each of our particular blocks *y* and *z* is **itself** a dodecahedron that is between *y* and *z*. But nothing can be between itself and something else.

8. How could you translate *There are at most two apples* into FOL?

   a. ∀x∀y∀z ((Apple(x) ∧ Apply(y) ∧ Apple(z)) → (x = y ∨ x = z ∨ y = z))

   b. ∀x∀y (Apple(x) ∧ Apple(y)) → (x = y ∨ y = x))

   c. ∃x∃y (Apple(x) ∧ Apple(y) ∧ x ≠ y ∧ ∀z (Apple(z) → (z = x ∨ z = y))

   d. (a) and (c)

   e. None of the above

   (a) is correct. It says that if you make three selections and come up with an apple each time, you've selected the same apple (at least) twice. (That is, either your first selection = your second selection, or your first selection = your third selection, or your second selection = your third selection.) Hence, the number of apples must be either 0 or 1 or 2. So (a) is a correct translation. (b) says that if you make two selections and come up with an apple each time, you've selected the same apple twice. (That is, either your first selection = your second selection, or [redundantly] your second selection = your first selection.) Hence, the number of apples must be either 0 or 1, which means that there is at most one apple. So (b) is wrong. (c) says that there are two distinct apples and every apple is identical to one or the other of them, that is, that there are **exactly** two apples. So (c) is wrong.

9. How could you translate *There are exactly two apples* into FOL?

   a. ∃x∃y (Apple(x) ∧ Apple(y) ∧ ∀z (Apple(z) → (z = x ∨ z = y))

   b. ∃x∃y (Apple(x) ∧ Apple(y) ∧ x ≠ y ∧ ∀z (Apple(z) → (z = x ∨ z = y))

   c. ∃x∃y (x ≠ y ∧ ∀z (Apple(z) ↔ (z = x ∨ z = y)))

   d. ∃x∃y (Apple(x) ∧ Apple(y) ∧ x ≠ y) ∧ ∀x∀y∀z ((Apple(x) ∧ Apple(y) ∧ Apple(z)) → (x = y ∨ x = z ∨ y = z))

   e. (b), (c), and (d)

   f. All of the above

   (a) says that there is an apple *x* and an apple *y* such that every apple is identical to either *x* or *y*. But it does not guarantee that *x* and *y* are two **distinct** apples. Since (a) allows that *x* = *y*, (a) comes out true even if there is only one apple. So (a) is incorrect. But (b), (c), and (d) are all adequate translations. (b) is like (a) except that it adds the non-identity clause that (a) lacks. (c) says that there are distinct objects such that **anything** is an apple if and only if it is identical to one or the other of them. (d) is a conjunction of *There are at least two apples* and *There are at most two apples*. Some simple math shows that (d) means that there are **exactly** two apples. Therefore (e) is the correct answer.

10. What is the truth-functional form of the following sentence?:

    (∃x∀y Larger(x, y) ∧ ∀x∀y ((Tet(x) ∧ ¬ Tet(y)) → Adjoins(y, x))) → ¬∀x ¬∀y Larger(x, y)

a.	(A ∧ (B → C))→ E	No. The wff corresponding to B → C is not a sentence.
b.	(A ∧ B) → C	No. The negation in ¬∀x has been omitted.
c.	**(A ∧ B) → ¬C**	**This is correct.**
d.	(A ∧ ((B ∧ ¬C)→D)) → ¬E	No. The wff corresponding to (B ∧ ¬C)→D) is not a sentence.
e.	None of the above	

11. Which of the following means that R is a symmetrical relation?

a.	∀x∀y∀z ((R(x, y) ∧ R(y, z)) → R(x, z))	No. This means *transitivity*.
b.	∀x∀y (R(x, y) → ¬R(y, x))	No. This means *asymmetry*.
c.	∀x R(x, x)	No. This means *reflexivity*.
d.	∀x ¬R(x, x)	No. This means *irreflexivity*.
e.	**None of the above**	**This is the correct answer.**

12. Which of the following are symmetric relations?

a.	The *sibling of* relation	Yes.
b.	The *parent of* relation	No. This is *asymmetric*.
c.	The *same height as* relation	Yes.
d.	The *larger than* relation	No. This is *asymmetric*.
e.	**(a) and (c)**	**This is the correct answer.**

13. How might the sentence “The small tetrahedron adjoins *b*” be translated into FOL according to Bertrand Russell’s Theory of Descriptions?

    a. ∃x (Small(x) ∧ Tet(x) ∧ ∀y ((Small(y) ∧ Tet(y))→ y = x) ∧ Adjoins(x, b))
    b. ∃x∀y (((Small(y) ∧ Tet(y)) ↔ y = x) ∧ Adjoins(x, b))
    c. ¬∀x ((Small(x) ∧ Tet(x)) → ¬ Adjoins(x, b))
    d. All of the above are correct.
    e. (a) and (b) are correct, but (c) is not.

    (a) says that there is a small tetrahedron, *x*, which every small tetrahedron, *y*, is identical to, and that *x* adjoins *b*. That is, (a) says that there is exactly one small tet, and it adjoins *b*. So it is an adequate translation. (b) says that there is something, *x*, such that it alone is a small tetrahedron, and that *x* adjoins *b*. So (b) is also an adequate translation. (c) denies that no small tet adjoins *b*, so it asserts, in effect, that **some** small tet adjoins *b*. But it does not say that there is no more than one small tet, and so (c) is incorrect. Therefore (e) is the correct answer.

14. Which of the following is a correct translation of *Every dodecahedron is in front of a small tetrahedron*?

    a. ∀x (Dodec(x) ∨ ∀y ¬(Small(y) ∧ Tet(y) ∧ FrontOf(x, y)))

    b. ∀x (Dodec(x) → ∃y (Small(y) ∧ FrontOf(x, y) ∧ Tet(y)))

    c. ∀x (Dodec(x) → ∃y (Tet(y) ∧ Small(y) ∧ FrontOf(y, x)))

    d. ∀x ¬(Dodec(x) ∧ ∃y (Tet(y) ∧ Small(y) ∧ FrontOf(x, y)))

    e. (b) and (d)

    (a) says "everything is either a dodecahedron or …" and you can stop right there—it must be wrong. For this sentence (however it continues) will be true in any world in which every block is a dodec (even if it's not in front of anything), and so it cannot be correct. (b) says that for any dodec, *x*, there's a small tet, *y*, that *x* is in front of. So (b) is correct. (c) looks like (b), but what it says is that every dodec has a small tet in front of **it**. So (c) is incorrect. (d) says that nothing is both a dodec and in front of a small tet. In other words, it says that **no** dodec is in front of a small tet. So (d) is wrong. (b) is therefore the only correct translation in the group.

15. Which of the following is true?

    a. Strawson's theory of descriptions withholds truth-values from sentences with non-denoting definite descriptions.

    b. Russell believes that *The golden mountain is golden* makes these three claims: (1) there is at least one golden mountain, (2) there is at most one golden mountain, and every golden mountain is golden.

    c. Russell's Theory of Descriptions does not have "truth-value gaps".

    d. All of the above

    e. (a) and (c)

    Strawson's theory says that a sentence with a non-denoting definite description fails to make any true-or-false statement, so (a) is true. (b) is also true—Russell does propose this three-part analysis. (3) is also true—Russell's theory provides a truth-value for every sentence with a description, even if the description is non-denoting. There are no truth-value gaps. So the correct answer is (d).

16. ∃x (Dodec(x) → ∃y Cube(y)) is equivalent to which of the following?

    a. ¬∀x ¬ (Dodec(x) → ∃y Cube(y))

    b. ∃x (¬Dodec(x) ∨ ∃y Cube(y))

    c. ∃x (¬∃yCube(y) → ¬Dodec(x))

    d. ¬∀x (Dodec(x) ∧ ¬∃y Cube(y))

    e. (a) and (b)

    f. (b) and (c)

    g. (a) and (c)

    h. (a), (b), and (c).

    i. All of the above.

    (a) is equivalent—just replace ∃x in (16)with ¬∀x ¬ to obtain (a). (b) is also equivalent—just replace Dodec(x) → in (16) with ¬Dodec(x) ∨ to obtain (b). (c) is also equivalent—

just replace Dodec(x) → ∃y Cube(y) in (16) with ¬∃yCube(y) → ¬Dodec(x) to obtain (c). (d) is also equivalent, but it may take a two-step process to see this. First, replace Dodec(x) → ∃y Cube(y) in (16) with ¬(Dodec(x) ∧ ¬∃y Cube(y) to obtain ∃x ¬(Dodec(x) ∧ ¬∃y Cube(y). Then replace ∃x ¬ with ¬∀x to obtain (d).

17. Fill in the missing sentences, inference rules, and support steps in lines 6 through 12 in this proof (essentially identical to exercise 13.50):

1. ∃x(Tet(x) ∧ ∀y (Tet(y) → y = x))	
2. [a b] Tet(a) ∧ Tet(b)	
3. Tet(a)	∧ Elim: 2
4. Tet(b)	∧ Elim: 2
5. [c] Tet(c) ∧ ∀y (Tet(y) → y = c)	
6. ∀y (Tet(y) → y = c)	Rule?:
7.	∀ Elim: 6
8. a = c	Rule?: 3,7
9. Tet(b) → b = c	Rule?:
10. b = c	Rule?:
11.	= Elim: 8,10
12. a = b	Rule?:
13. ∀x ∀y ((Tet(x) ∧ Tet(y)) → x = y)	∀ Intro: 2-12

Goals

∀x ∀y ((Tet(x) ∧ Tet(y)) → x = y)

18. Which techniques are used in this proof, and where are they used?

a. Proof by cases
b. General conditional proof
c. Existential instantiation
d. Proof by contradiction
e. Both (a) and (b)
f. Both (b) and (c)

General conditional proof is used in the subproof 2-12, culminating in step 13 (justified by ∀ **Intro**). Existential instantiation is used in the subproof 5-11 (based on step 1), culminating in step 12 (justified by ∃ **Elim**). Neither proof by cases nor indirect proof is used (note the absence of the rules ∨ **Elim** and ¬ **Intro**). So (f) is the correct answer.