

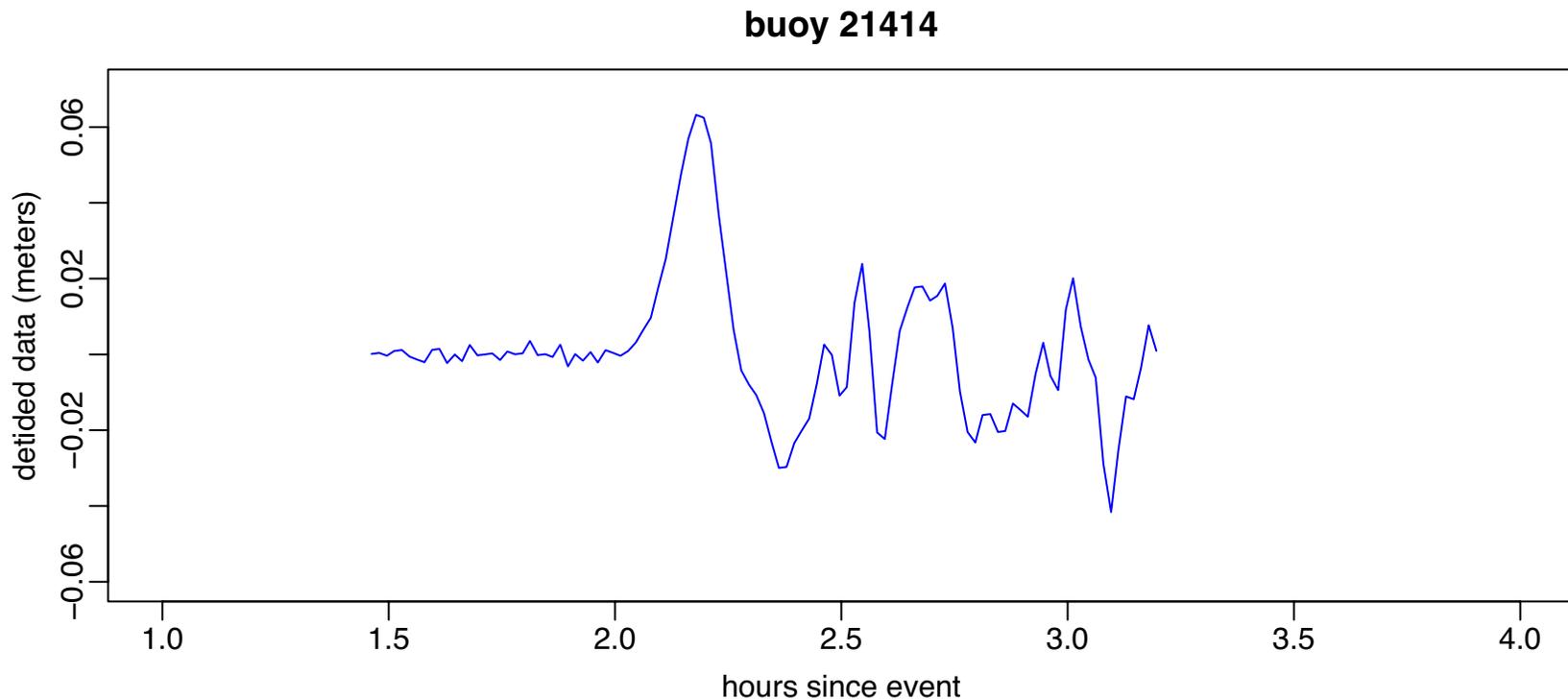
Simulation of DART Buoy Data

- goal: develop procedure for simulating detided DART buoy data that can be used to test inversion algorithm, train users etc.
- simulated data has three stochastic components:
 - background noise
 - seismic noise
 - distorted tsunami signal

(as explained later, these will be added to a deterministic component)

Background Noise: I

- background noise is what is observed at DART buoy when nothing is going on – properties can be deduced from data prior to arrival of tsunami wave (in absence of seismic noise)



- in the above, background noise observed up to time 2.0

Background Noise: II

- let X_t be a stochastic process that models the background noise
- for simplicity, will assume that X_t obeys a first order autoregressive (AR(1)) process with zero mean:

$$X_t = \phi_B X_{t-1} + \epsilon_t^{(B)}, \quad t = 0, 1, \dots, N - 1,$$

where $\epsilon_t^{(B)}$ is a Gaussian white noise process with zero mean and variance σ_B^2 (can estimate ϕ_B and σ_B^2 from data)

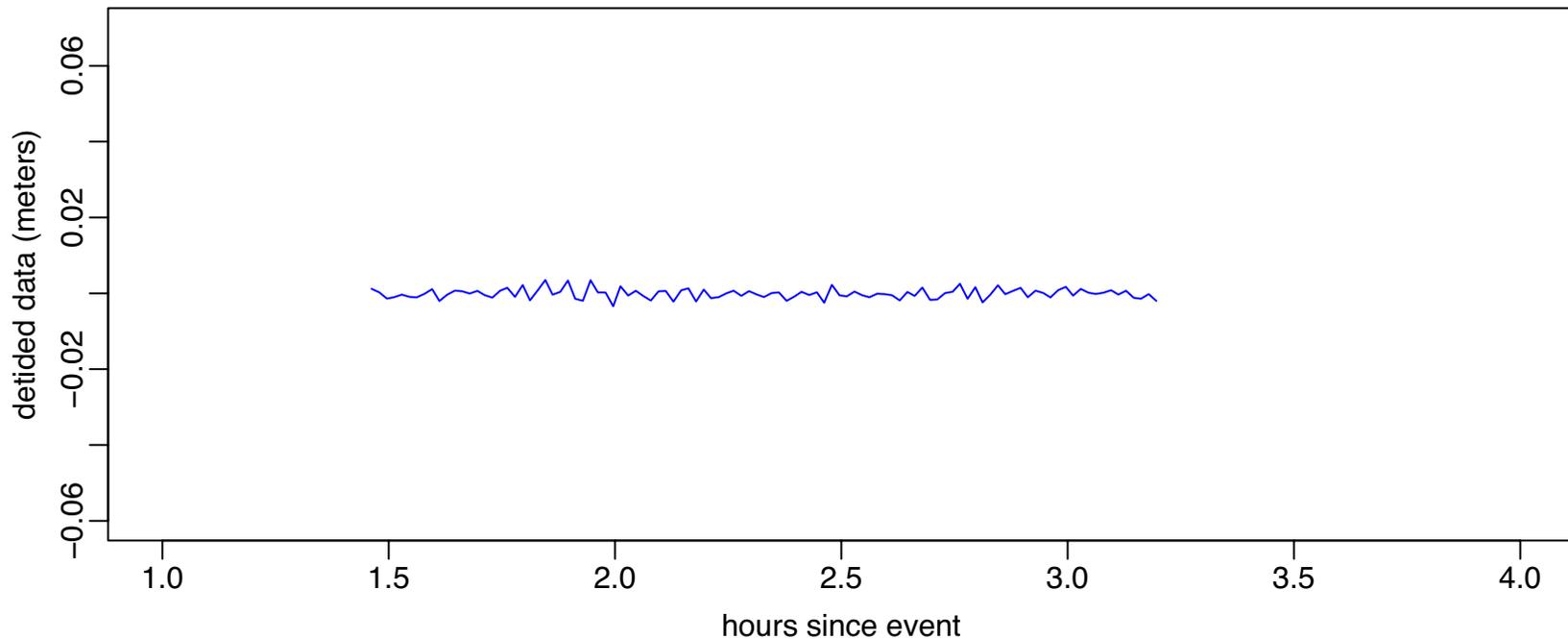
- will assume that ϕ_B and σ_B^2 are the same for each buoy, but noise processes at different buoys are uncorrelated
- can show that variance of X_t is $\sigma_B^2 / (1 - \phi_B^2)$
- can initialize simulations by letting

$$X_0 = \epsilon_0^{(B)} \times \sigma_B / \sqrt{(1 - \phi_B^2)}$$

Background Noise: III

- use first 33 data points (33 minutes) from buoy 21414 to get estimates $\hat{\phi} = -0.3$ and $\sigma_B^2 = 1.94 \times 10^{-6}$

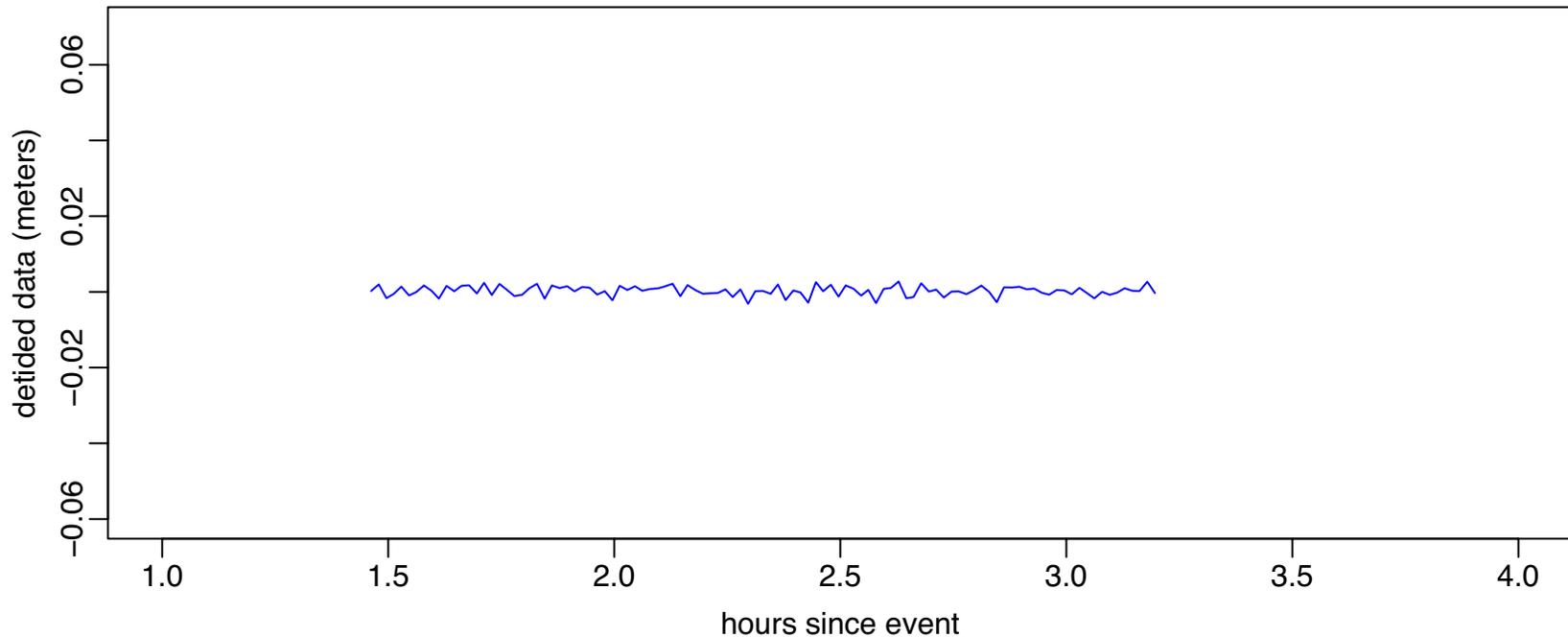
simulated background noise for buoy 21414



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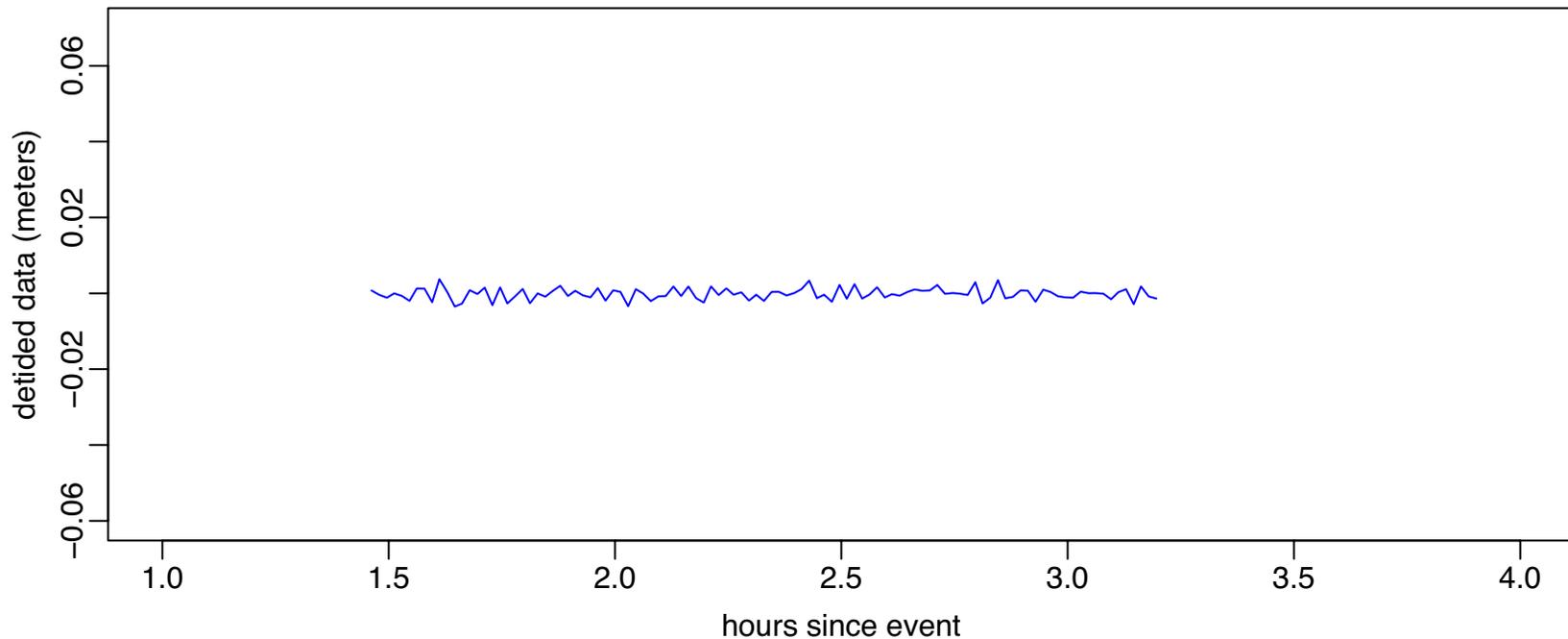
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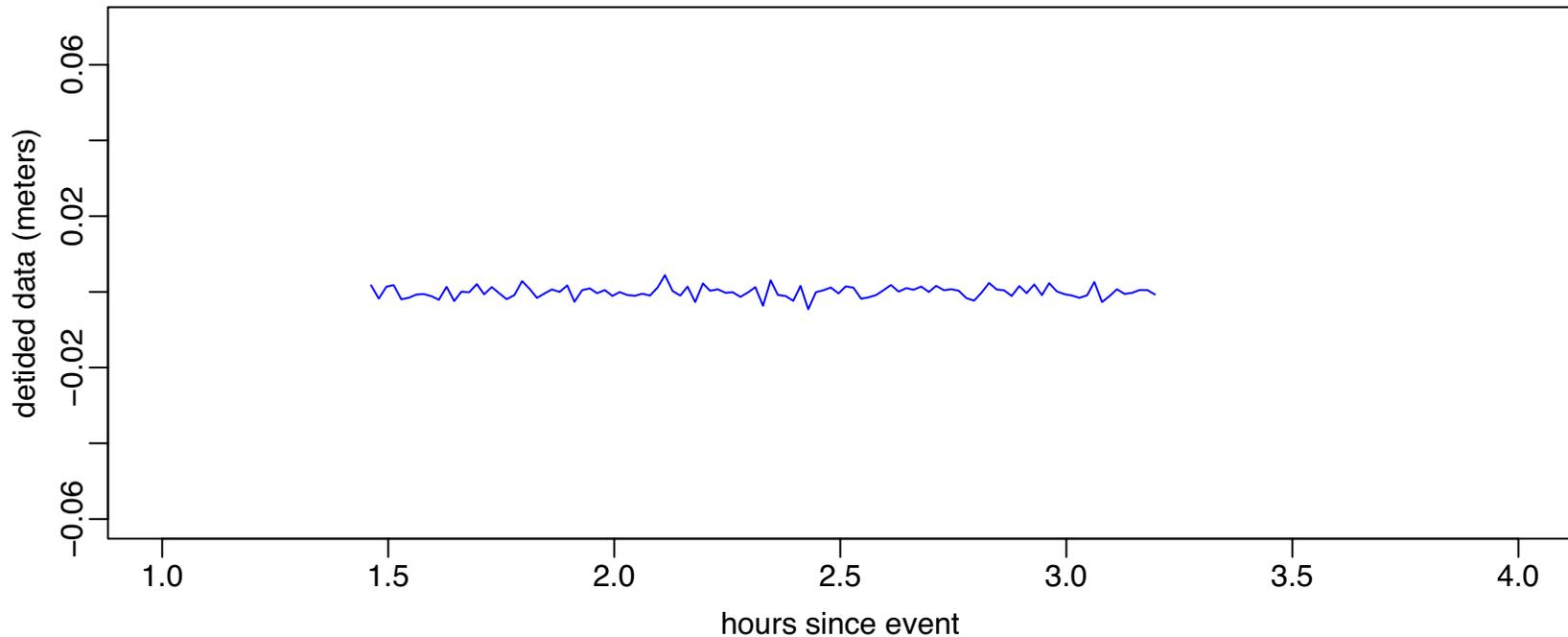
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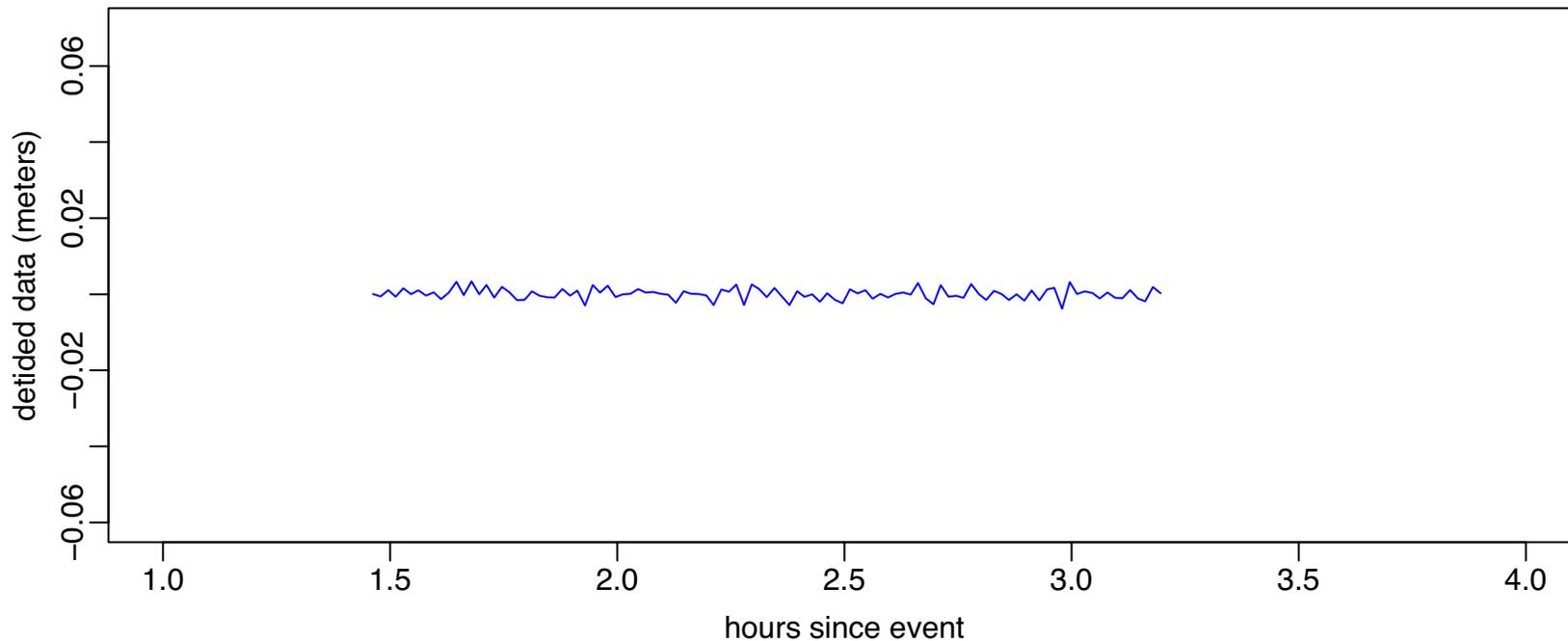
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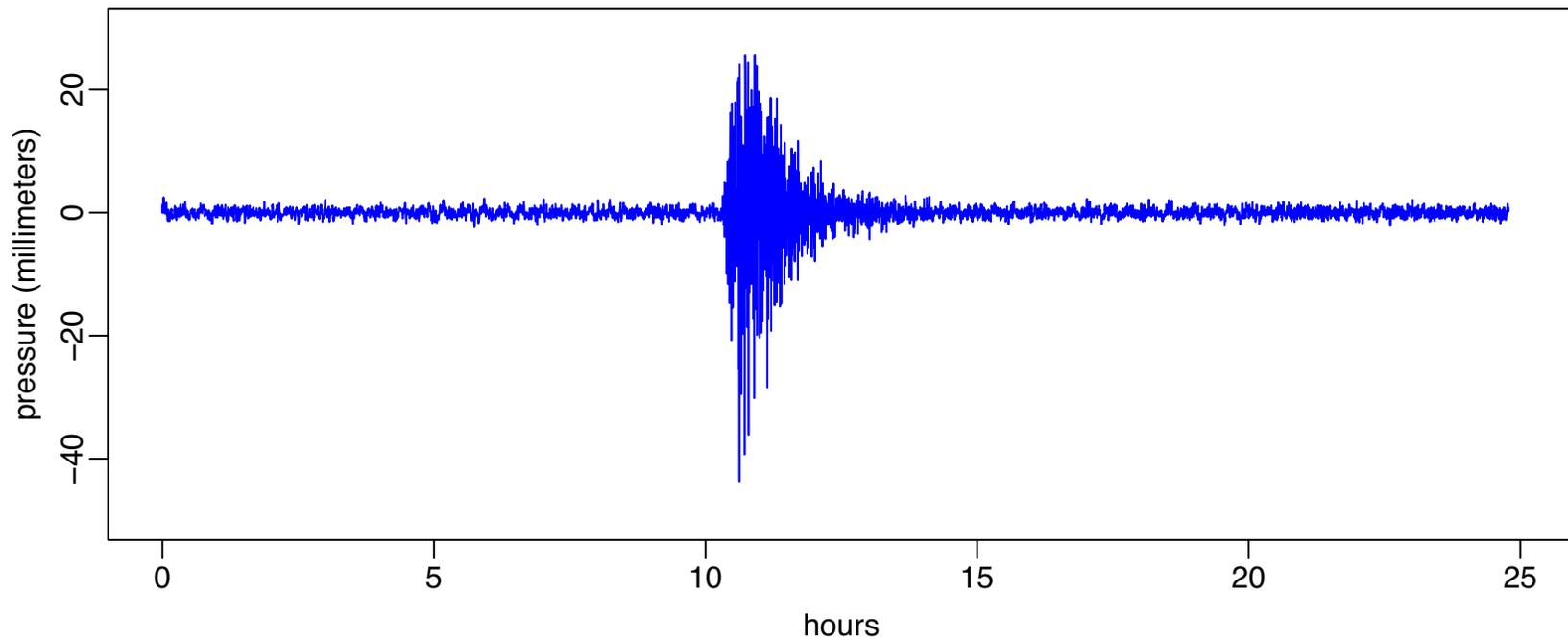
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Seismic Noise

- example of seismic noise (after detiding)

detided seismic data (sampled every 15 seconds)

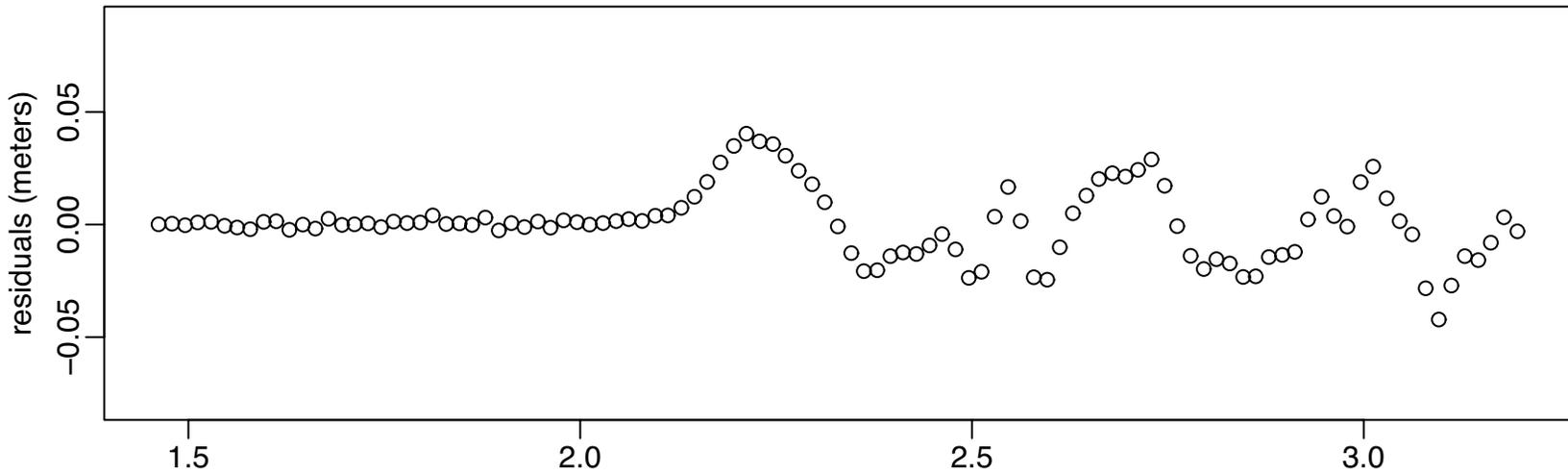
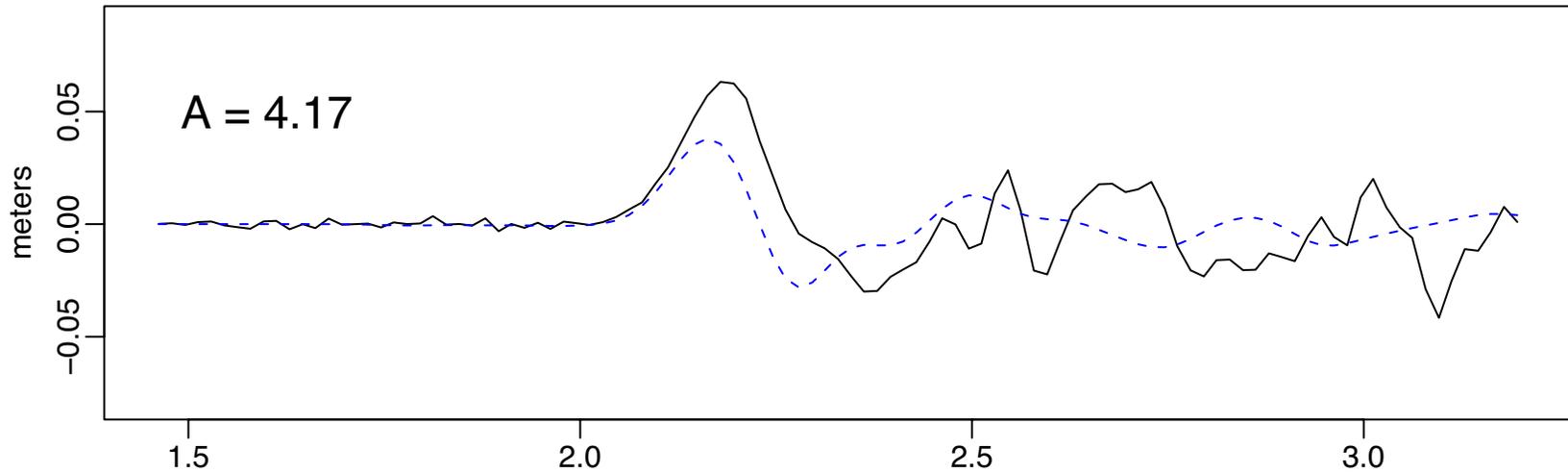


- model as AR process Y_t with amplitude modulation?
- used buoy geometry to adjust for different buoys?

Distorted Tsunami Signal: I

- motivation: residuals from fitted models exhibit low frequency patterns due to errors in unit-slip models etc
- consider example from 26 July seminar (fitted model is blue dashed line, data is black solid line)

Least Squares Estimate \hat{A}_2 of Slip for a12 & 21414



Distorted Tsunami Signal: II

- let Z_t be a stochastic process that models the distortions
- assume Z_t is an AR(1) process with time varying parameters:

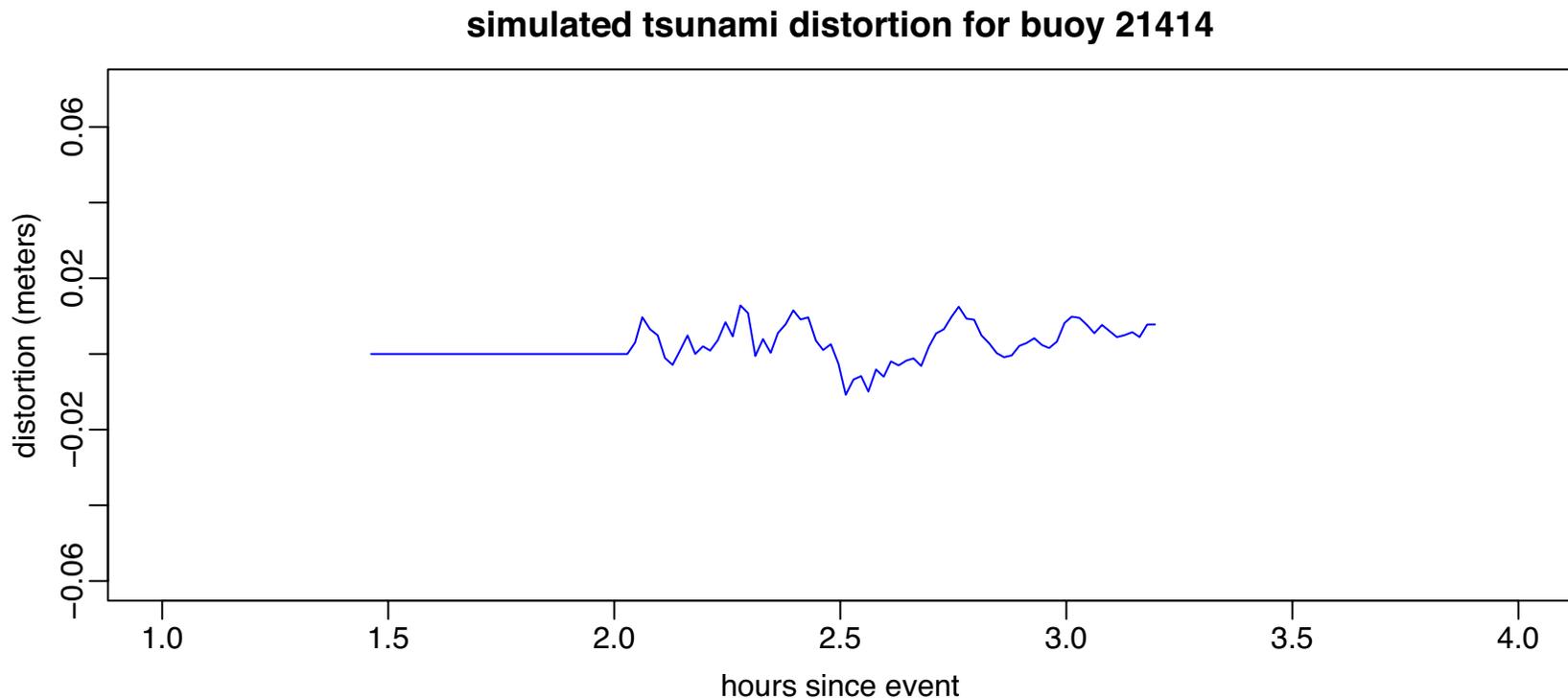
$$Z_t = \phi_{D,t} Z_{t-1} + \epsilon_t^{(D)}, \quad t = 0, 1, \dots, N - 1,$$

where $\epsilon_t^{(D)}$ is a Gaussian white noise process with zero mean and variance $\sigma_{D,t}^2$

- will assume that
 - $\phi_{D,t}$ and $\sigma_{D,t}^2$ depend upon a small number of parameters
 - parameters are same for all buoys (reasonable?)
 - distortions at different buoys are uncorrelated (?)

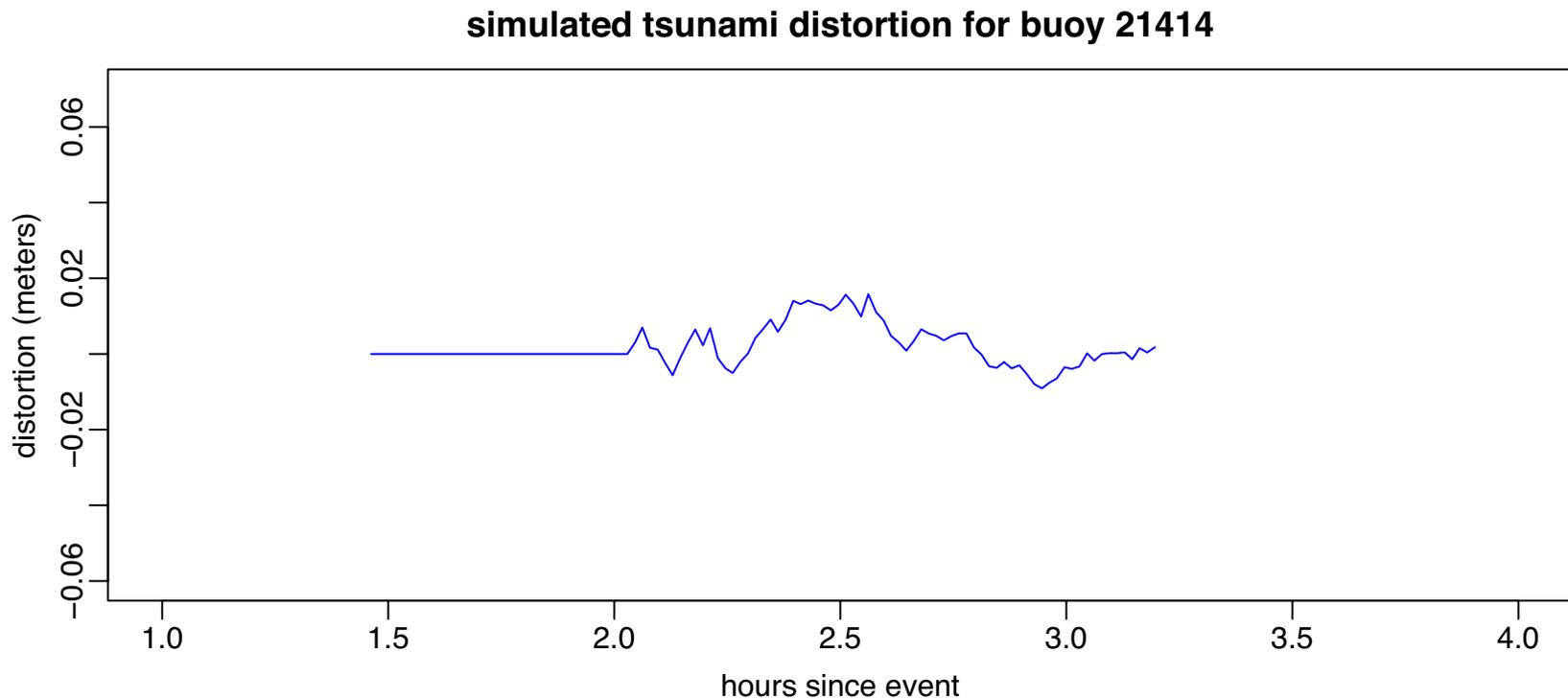
Distorted Tsunami Signal: III

- simulated distortion based upon analysis of residuals from fit to buoy 21414



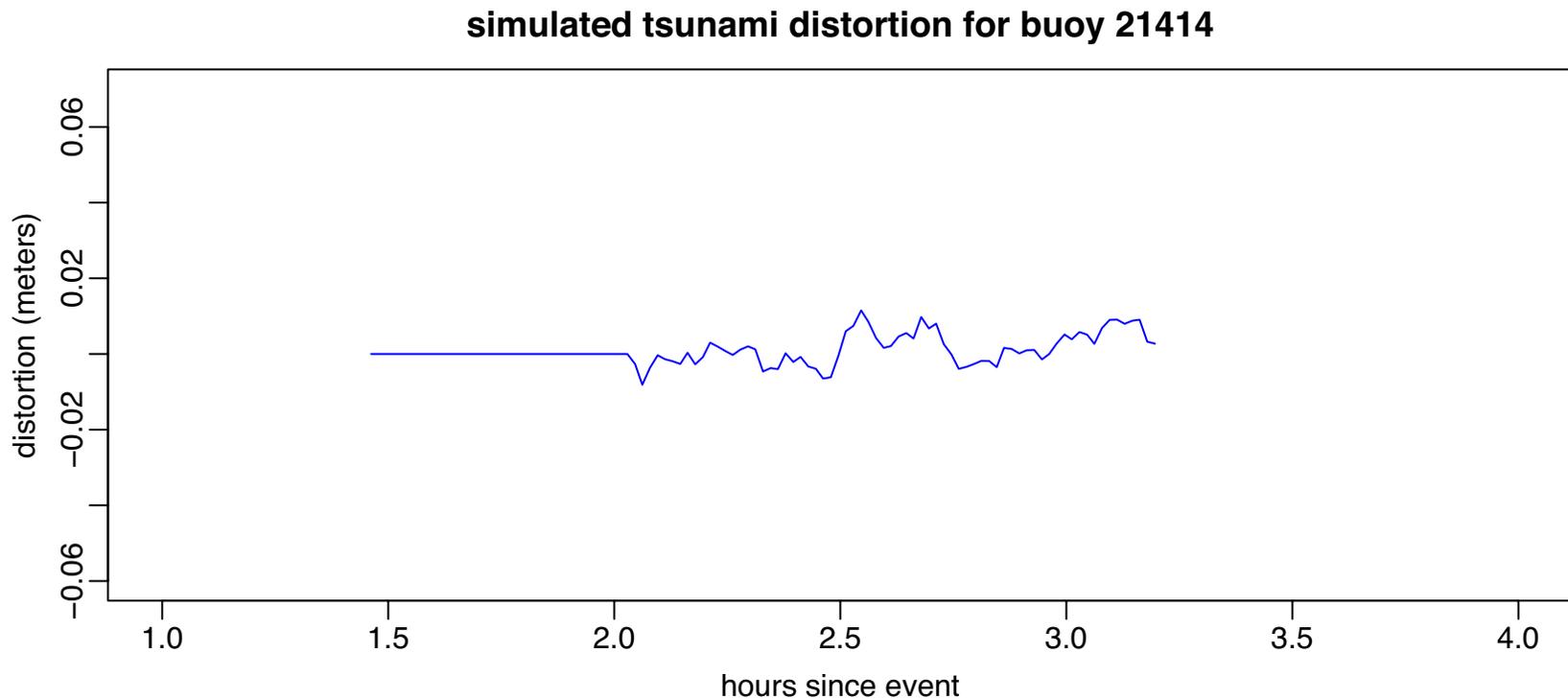
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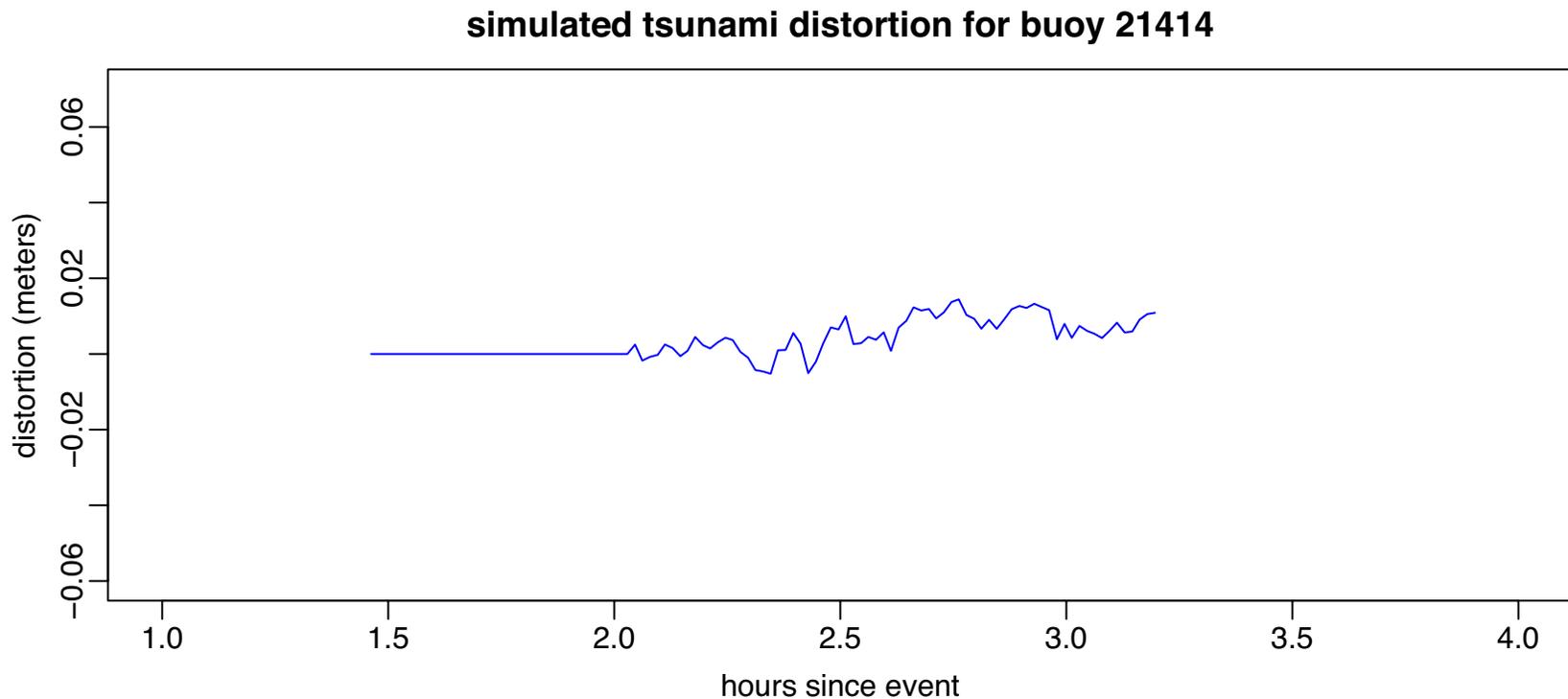
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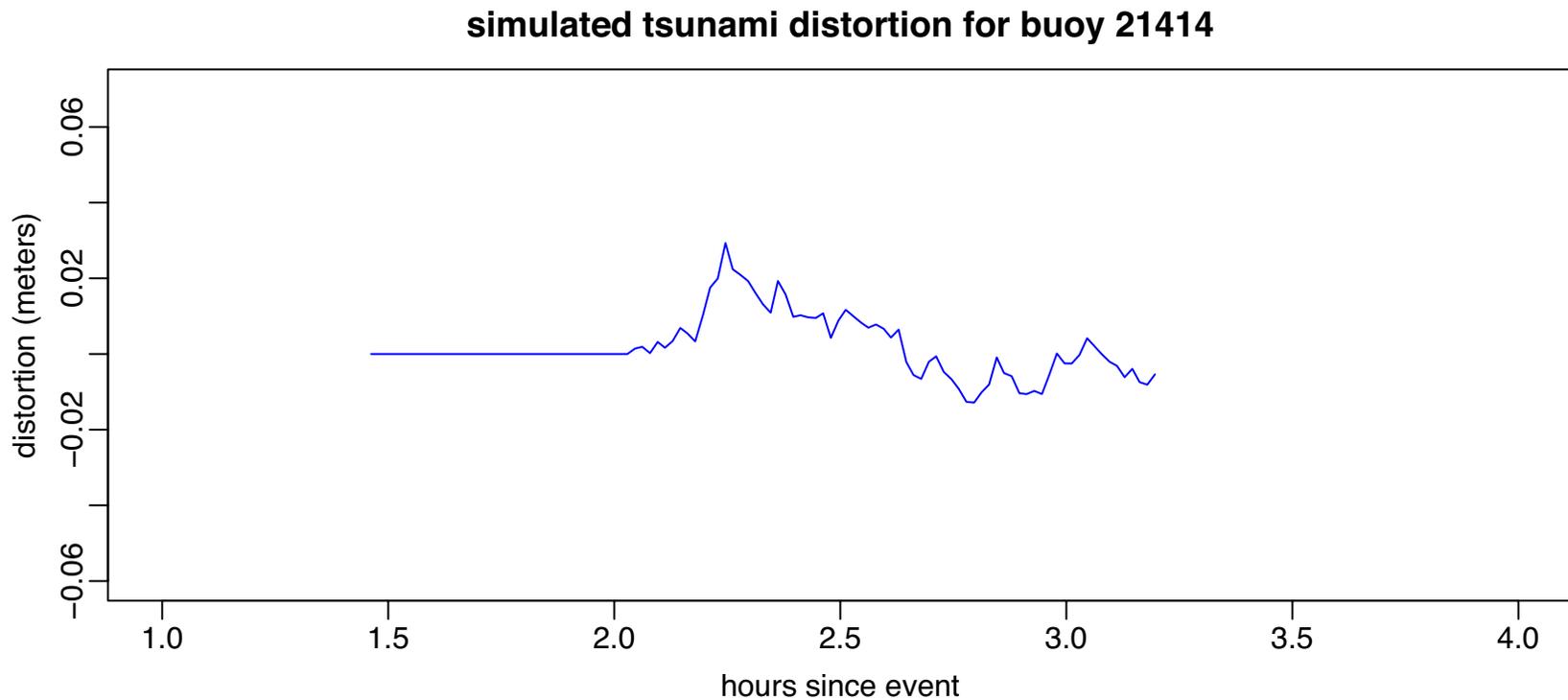
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Distorted Tsunami Signal: III

- simulated distortion based upon analysis of residuals from fit to buoy 21414



Putting It All Together

- simulated DART buoy data given by

$$\sum_{k=1}^K A_k g_k(t - a_k) + X_t + Y_t + Z_t$$

where

- $g_k(t)$ is the k th unit source model for a given DART buoy;
- A_k is the slip for the k th unit source
- a_k is either zero or a uniformly distributed random variable over an interval of ± 3 minutes (same for all buoys?)
- X_t is the background noise
- Y_t is the seismic noise
- Z_t is the distorted tsunami signal