Separating the Spatial, Seasonal, and Interannual Variability in Arctic Sea Ice Thickness

D. A. Rothrock, D. B. Percival, and M. Wensnahan
Applied Physics Laboratory, University of Washington, Seattle, Washington 98195 USA

Submitted to *Journal of Geophysical Research – Oceans*

30 March 2007

Abstract

Naval submarines have collected operational data of sea-ice draft (90% of thickness) in the Arctic Ocean since 1958. Data from 34 U.S. cruises are publicly archived. They span the years 1975 to 2000, are equally distributed in the spring and autumn, and cover roughly half the Arctic Ocean. The dataset is strong: 2203 values of draft averaged over nominal lengths of 50 km, values ranging from 0 to 6 m with a standard deviation of 0.99 m. Multiple regression is used to separate the interannual change, the annual cycle, and the spatial field. The solution gives an climatology for ice draft as a function of space and time. The residuals of the regression have a standard deviation of 0.46 m. The observational error has a standard deviation of 0.28 m. The overall mean of the solution is 2.97 m. Ice draft declined from a peak of 3.42 m in 1980 to a minimum of 2.29 m in 2000, a decrease of 1.13 m. The steepest rate of decrease is –0.08 m/yr in 1990. The rate slows to –0.007 m/yr at the end of the record. The annual cycle has a peak-to-peak amplitude of 1.06 m and a maximum on 1 May. The spatial contour map varies from a minimum of 2.2 m near Alaska to a maximum of over 4 m at the edge of the data release area 200 miles north of Ellesmere Island. This solution coalesces previous results focused on fewer aspects of the variability.

1. **Introduction**

For several decades, operational data from submarines have formed a basis of our observational knowledge of arctic sea-ice thickness. At first scientists used these data to characterize ice topography (pressure ridge statistics and the ice thickness distribution) and to characterize variability. By the 1980s enough data had accumulated to allow the spatial field of draft to be estimated, but it was clear that the contour maps had small scale structure and seasonal differences affected by undersampling in both space and time [*Bourke and Garrett*, 1987; *Bourke and McLaren*, 1992]. About 1989 investigators began to use submarine data to address the question of interannual change. Because the timing and tracks of submarine cruises were designed not to provide some optimal sampling of the spatial and temporal variability of sea ice but rather to meet military objectives, formulating analyses of the sparse and irregular data, either to map the field or to find a trend, has been problematic. There has been controversy about whether the dataset is sufficiently strong to extract any signal of long-term change from "natural variability"[*McLaren et al*., 1990; *Wadhams*, 1990]. Some studies have ignored the time of year altogether. Some have segregated the data into summer or winter seasons, losing the helpful link between summer and winter or suppressing the shape of the seasonal cycle. Some have focused on certain data-rich regions such as the North Pole or the strip from the pole to the Beaufort Sea roughly between 140° and 150°W. Some have compared data from two different clusters of years. Investigations focused on interannual change include *McLaren et al.* [1994], *Shy and Walsh* [1996], *Rothrock et al*. [1999], *Tucker et al*. [2001], *Winsor* [2001], *Wadhams and Davis* [2001]. Unanswered questions from these studies include, "Is the interannual signal truly discernible above the noise of 'natural variability'?" and, if so, "Is the interannual change one of continuing decline or is the signal more complicated?".

Over the decades, more and more data have become publicly available. Data on sea-ice draft from 37 submarine cruises within the Arctic Ocean are now available at the National Snow and Ice Data Center (NSIDC). The purpose of this paper is to analyze these data and determine what they tell us about sea ice variability. We purposely avoid any use here of other sea-ice information, in particular from sea-ice models. This analysis rests purely on the submarine data and has two strengths. First, the study makes use of data from 17 cruises recently placed at NSIDC, providing a fairly continual record in both spring and autumn from 1975 to 2000 [*Rothrock and Wensnahan*, 2007] for a total of 34 cruises. Second, it capitalizes on the opportunity provided by this expanded data set to analyze all the U.S. submarine data as a single dataset in order to separate from each other the dependencies on space, on season, and on year. In taking this approach we begin to fulfill the vision of *McLaren et al*. [1990] who saw that "A direct approach would involve statistical analysis by season, region and...comparable, basin-wide under-ice thickness distribution data obtained by U.S. and British nuclear submarines since 1958. Only then might genuine trends be distinguished from natural variability." We would add that only then will a spatial climatological field and annual cycle be identified.

We use multiple regression to determine how draft depends on the independent variables. The goal is to find a simple algebraic formula or regression model for draft as a function of space, season, and year, leaving residuals (discrepancies between the data and the regression model) that are small. One builds a regression model by starting with terms of low order and adding terms of higher order, until the addition of more terms in the model ceases to reduce the variance of the residuals significantly as determined by statistical tests. One says that the regression model "explains" a portion of the variance in the data, leaving the remaining variance in the residuals as "unexplained" variance that can be considered to be either error in the regression model or observational errors or both. Of course the form chosen for the model is somewhat subjective, guided by physical intuition, but, for instance, whether the spatial dependence should be linear or quadratic or cubic is determined by the data.

In §2, the dataset is described and the variables defined. Section 3 presents the best fit multiple regression model and the coefficients of the fit: the seasonal cycle, the spatial gradient and the interannual change. Discussion of these results in the light of previous results and concluding remarks are presented in §4.

2. The Data

The data used in this analysis are from 34 cruises of U.S. Navy submarines from 1975 to 2000. Each cruise lasted roughly a month; the distribution of cruises by year and month is shown in **Figure** 1 (one dot per cruise). Originally classified secret, the data have been declassified and released for public use mostly within a data release area (DRA), an irregular polygon (see **Figure** 2 and **Table** 1), that lies within the Arctic Ocean and outside the Exclusive Economic Zones of foreign countries. Data in the archive have been acquired by two different recording systems: digital and paper chart. We believe that the data extracted by scanning paper charts can be made equivalent (in the sense of being unbiased) to those acquired by digital recording [*Wensnahan and Rothrock*, 2005]. We do not use here archived data from British cruises, because there are not many of them in our study area, and they were derived manually from paper charts by a process that we know less about and may introduce a positive bias.

We use as our dependent variable the mean draft d in meters. The means are from nominal 50-km sections of a draft profile; for archived sections less than 50 km long, data from multiple sections within 75 km of each other are combined in a cluster such that the *sample* length is between 25 and 55 km. If no cluster can be formed to satisfy these criteria, data from a short section are discarded. These means include open water; they are not, as some investigators have considered, "ice-only" means that exclude from the average any ice thinner than some threshold, say, 30 cm.

The first independent variable, which models interannual variation, is the decimal year t; for example, the first instant of 1988 is $t = 1988.000$, which happens to be very nearly at the midpoint of the dataset's time span. The second variable is the decimal fraction of the year τ which marks the seasons; it ranges from 0 to 1 over the course of a calendar year and is the fractional part of t. To fit the annual cycle in the regression model we use the two terms $\sin(2\pi\tau)$ and $\cos(2\pi\tau)$ to represent the fundamental frequency; for easier interpretation, these are later converted to a single cosine function with a phase that gives the times of the annual maximum and minimum. The final two independent variables are spatial: x and y defined from latitude ϕ and longitude λ (in degrees) by

$$\begin{aligned} \rho &= 2R * \sin[(45° - 0.5\phi)\pi/180°] \\ x &= \rho * \cos[(\lambda - 35°)\pi/180°]/1000 \\ y &= \rho * \sin[(\lambda - 35°)\pi/180°]/1000 \end{aligned} \qquad (1)$$

where R = 6370 km is the nominal radius of the Earth. The (x, y) coordinate system has its origin at the North Pole, and the positive x-axis runs along 35°E. This transformation (Lambert azimuthal equivalent) maps the Earth's surface to a plane tangent at the North Pole; ρ is the straight-line distance from the Pole through the earth to a point (x,y) on the surface. The mapping conserves area. The units of x and y are nominally 1000 km, but the transformation shrinks latitudinal distance and expands longitudinal distance as one moves away from the pole.

At the pole, a degree of latitude is 111.17 km; at the extreme southern corner of the DRA (ϕ = 70°), a degree of latitude is 109.48 km.

3. The Result of the Multiple Regression

There are 2203 of these 50-km mean draft values , ranging from 0 to 6.09 m. The variance of these values is 0.98 m^2. Multiple regression allows us to determine how much of this variance in *d* can be explained by the four variables: *t, τ, x,* and *y*, and, conversely, how much cannot.

The first thing one might try is to see how the variables "individually" can explain the data. A regression model using a linear term in just the year *t* explains only 28% of the variance in the data. Using just the fundamental frequency of the season explains only 33% of the variance, and using just linear terms in *x* and *y* explains 26% of the variance. Clearly using all of these variables together in a multiple regression will do better, but how much better?

The simplest (linear) multiple regression equation treats the independent variables as separable

$$d(t,\tau,x,y) = C + I(t-1988) + A(\tau) + S(x,y) + \varepsilon(t,\tau,x,y) \qquad (2)$$

where C is a constant, $I(t\text{–}1988)$ describes the interannual change centered around 1988, $A(\tau)$ describes the annual cycle, and $S(x,y)$ is the spatial field. The inability of the those four terms to completely reproduce the data *d* is measured by the residuals or errors ε, which we assume to obey a multivariate Gaussian distribution with a common mean of zero and variance of σ^2_ε. The multiple regression method gives residuals that sum to zero and determines *C, I, A*, and *S* in (2) to minimize the sum of squares of the residuals. To find the multiple regression solution of Eq. (2) that fits the 2203 data points, we assume independence of data for different years and seasons. For data from the same season of the same year, we assume a correlation structure for the errors dictated by spatial long-range dependence [*Percival et al.*, submitted]. We used generalized least squares to fit the regression coefficients in (2), but we also tried ordinary least square regression with the same assumed error structure and found little change in the results.

In the following paragraphs we discuss the specific form of Eq. (2) and its solution. The form involves just the fundamental frequency in the annual cycle $A(\tau)$ and a cubic polynomial for *I(t–1988)*, while *S(x,y)* has some terms of 5th order, e.g., x^3y^2. The selected form involves coefficients that are statistically significant at a 95% confidence level, with higher order terms or omitted lower order terms statistically indistinguishable from zero. The solution has 14 coefficients, three for *I*, two for *A*, and eight for *S*. This solution explains 79% of the variance in the data, with the variance of the residuals, σ^2_ε, being 0.21 m^2.

The mean draft over the domain of our analysis is 2.97 m. The value of *C* is 3.63 m, but this is not the mean, because neither I nor S is zero-mean. The mean of *I* over the 26 years 1975–2000 is $\bar{I} = -0.12$ m, and the mean of *S* over the data release area is $\bar{S} = -0.54$ m. The annual cycle averages to $\bar{A} = 0$. So, the mean draft from the regression model, averaged over 26 years, over the data release area, and over a year, is $\bar{d} = C + \bar{I} + \bar{A} + \bar{S} = 2.97$ m.

The interannual change *I(t–1988)* is depicted in **Figure** 3. It represents the interannual change in mean draft averaged over a year ($\bar{A}$ = 0) and over the region of the data release area ($\bar{S} = -0.54$ m). The model draft rises for the first few years to a maximum of 3.42 m at year 1980.468, then falls by October 2000 to 2.29 m, a decrease of 1.13 m. Its steepest decline occurs at the end of 1990 and is –0.08 m yr^{-1}. By the end of the record the decline is much slower (–0.007 m yr^{-1}). There is no sign in the model curve or in the data of a recovery or rebound by 2000. The multiple regression solution for *I*(*t*–1988) is

$$\begin{aligned} &I(t-1988) = I_1(t-1988) + I_2(t-1988)^2 + I_3(t-1988)^3 \\ &I_1 = -0.0748 \\ &I_2 = -0.00219 \\ &I_3 = 0.000246 \end{aligned} \qquad (3)$$

The units of I_k are meters $(\text{year})^{-k}$.

The annual cycle *A(τ)* is shown in **Figure** 4. It represents the annual cycle averaged over the data release area and over the 26 years 1975–2000. The peak-to-peak amplitude is 1.06 m. The maximum occurs on May 1 ($\tau = 0.329$, day 121) and the minimum on October 31 ($\tau = 0.830$, day 304). The annual cycle is much larger than might be expected, given that this part of the ocean is mostly multiyear ice, and a mature ice slab has a much smaller thermodynamic annual cycle of thickness [~0.43 m, *Maykut and Untersteiner*, 1971]. Sea-ice models show an annual cycle that is asymmetric, falling more steeply in the middle of the year and growing more slowly in autumn, but one can see from the residuals plotted around *A(τ)*, that the data are not dense enough throughout the year to resolve any harmonics and are sparse in just the period when the melt would be fastest (June and July, $\tau \sim 0.4$ to 0.6). The multiple regression solution for $A(\tau)$ is

$$\begin{aligned} A(\tau) &= A_{s0}\sin(2\pi\tau) + A_{c0}\cos(2\pi\tau) = A_0\cos(2\pi[\tau - \tau_{\max}]) \\ A_{s0} &= 0.465 \\ A_{c0} &= -0.250 \\ A_0 &= 1.056 \\ \tau_{\max} &= 0.329 \end{aligned} \qquad (4)$$

The units of A_{s0}, A_{c0}, and A_0 are meters.

The spatial field of draft is shown in **Figure** 5. This represents the spatial dependence of the mean draft, averaged over an annual cycle and the 26 years of the data record 1975–2000. The draft varies from 2.2 m near Alaska to just over 4 m near Ellesmere Island. The multiple regression solution for $S(x,y)$ is (using the notation $S_{ij}x^i y^j$ for each term)

$$\begin{aligned} S(x,y) &= S_{10}x + S_{01}y + S_{20}x^2 + S_{30}x^3 + S_{40}x^4 + S_{22}x^2y^2 + S_{50}x^5 + S_{32}x^3y^2 \\ S_{10} &= -0.7425 \\ S_{01} &= -0.4548 \\ S_{20} &= -0.5616 \\ S_{30} &= 1.1719 \\ S_{40} &= 0.8308 \\ S_{22} &= 6.8515 \\ S_{50} &= 0.1389 \\ S_{32} &= 2.7062 \end{aligned} \qquad (5)$$

The units of S_{ij} are m/(10^3 km)$^{i+j}$. Other terms in powers of x and y up to order 5 and beyond are not significantly different from zero.

By the nature of our choice of the form of Eq. (2), the shape of the field never changes, although the values on the contours change. The field in Figure 5 also represents the 26-year mean field on January 30 ($\tau = 0.079$, day 30) and on July 31 ($\tau = 0.579$, day 212), the inflexion points of the sinusoidal annual cycle. If one wants a 26-year mean spatial field of draft for any time of year, one can construct it by adding 0.53 m to the map in Figure 5 for May 1, subtracting 0.53 m on October 31 or making an adjustment to any time of year by adding $A(\tau)$. Similarly, the mean annual field at any point between $t = 1975$ and $t = 2000$ can be computed by adding to the map in Figure 5 the quantity $I(t-1988)-\bar{I}$. If one wants the field averaged over a portion of the record from t_1 to t_2 (e.g., a period before the positive Arctic Oscillation anomaly in the early 1990s), one would add to the map $\int_{t_2}^{t_2} I(t-1988)dt-\bar{I}$.

We view the 0.98 m^2 of variance in the data as partitioned like this: 0.77 m^2 is explained by the regression model, Eq. (2), and 0.21 m^2 is not. How should we view the 0.21 m^2 of unexplained variance? The error in the measurement system has a standard deviation of 0.25 m [*Rothrock and Wensnahan*, 2007], or a variance of 0.063 m^2. The error in sampling due to long-range dependence in the sea-ice cover has a standard deviation of about 0.28 m [*Percival et al.,* 2007], or a variance of 0.078 m^2. It is not obvious how to regard these two sources of error. If we regard them as independent, we would add their variances (0.063 + 0.078) for an overall observational error of 0.141 m^2. So, the unexplained variance 0.21 m^2 would be partitioned into an observational error variance of 0.14 m^2 and a natural variance of 0.07 m^2 (SD = 0.26 m) unable to be captured by Eq. (2). We suppose that a better view is that the observational error is best represented by the larger source of error, the error due to long range dependence. Then the variance explained neither by Eq.(2), nor by observational errors, is 0.21 – 0.078 = 0.13 m^2 (standard deviation = 0.36 m); this value, then, represents the variability of the ice-cover not described by the regression model.

In any case, the unexplained variance is 0.21 m^2 (standard deviation = 0.46 m). This value seems to be a very strong upper bound on the observational error in the U.S. submarine ice draft data. It seems quite unlikely to us that the random observational error could be larger than this value. If it were, the data could not be represented by a smooth functions as with Eq.(2) with an unexplained variance as low as 0.21 m^2. There is also a bias in submarine data, which is estimated to be +0.29 m [*Rothrock and Wensnahan*, 2007]. The data must be reduced by 0.29 m when compared with any non-US-submarine observation or with ice model output.

4. Discussion and Summary

We have analyzed all available publicly archived data from U.S. submarines, separating from each other the interannual change, the annual cycle, and the climatological spatial field. We were surprised that the data supported regression models with polynomials of 5th order.

Working a few years ago with only eleven cruises and ten years of data, we found only the linear coefficients to be significant. With the present 26 years of data, we expected to find significant 2nd order terms. But in fact the data support 3rd order in time and 5th order spatial terms that show interesting and interpretable interannual and spatial structure. By separating temporal and spatial variation, the present formulation (2) does not quantify regional variations of interannual change and the annual cycle; that study should be attempted. To convert draft to thickness we multiply draft by 1.11.

The interannual response *I(t)* shows a high rate of decline in draft centered around 1991, preceded by a maximum in 1980 and a minimum in 2000 at the end of the record. The decline from the maximum to the minimum is 1.13 m. If we correct for the bias estimated by *Rothrock and Wensnahan* [2007] by subtracting 0.29 m from all values, this change represents a decline of 36% from the maximum. Whether this change is part of a cyclical or random variation or a stage in a continuing decline, it is a very large fractional change in ice thickness! Through 2000, we see no sign that ice thickness is rebounding in this large area of the Arctic Ocean. It is less than the 43% decline reported by *Rothrock et al.* [1999]. That analysis compared data from an earlier period (1958-1976) with data in the 1990s, and, in addition, the earlier data were manually digitized from paper charts and are likely of lower quality than the data used here and presently at NSIDC. The present analysis is based on a much more voluminous and higher quality data set, but over a shorter period. The timing of the steepest decline agrees with the findings of *Tucker et al.* [2001], who also noted that the decline was 1.5 m in the Canada Basin and insignificant at the North Pole. Our result is at odds with the conclusion of *Winsor* [2001] who reported no change during the 1990s. We take issue with two aspects of Winsor's result. First, to us it appears that the data in his Table 1 *do* show a considerable decline in the Beaufort Sea in contradiction to his stated conclusions. Second, in our opinion the result rests too strongly on the seasonal correction for data from mixed seasons (the spring data in 1991–4 and the fall data of 1993–7); the correction seems too large, as evidenced above in §3. Of older estimates of arctic ice thickness from Nansen's *Fram* expedition (1893–6), Koerner's British Trans-Arctic Expedition (1968–9), and the earliest submarine cruises (1958 ff.), none is thinner than the 3 m we find here, and several are closer to 4 m [*McLaren et al*., 1990].

The annual cycle $A(\tau)$ is quite large, 1.06 m peak-to-peak, over twice that of a thermodynamically mature slab of ice. We do not know of previous observational estimates of the large-scale annual cycle amplitude. The phase of the annual cycle is in line with other observations and the cycle in ice models.

Several previous investigators have produced contour maps of draft over sizeable portions of the Arctic Ocean. The spatial field in Figure 5 has structure that resembles some of these. The LeSchack field [Fig.1 in *Bourke and McLaren*, 1992] using data from the 1960s and 1970s shows a long-term mean field for the Pacific side of the Pole. Our field agrees with that estimate at the Pole, but differs by up to 1 m elsewhere. (For example, compared with our field the LeSchack field is +1 m at the location of maximum draft in the DRA off Ellesmere Is., +0.5 m at the southern tip of the DRA at Alaska, –0.6 m at the tip of the DRA pointed at the Laptev Sea.) The fields given by *Bourke and Garrett* [1987] (using 17 submarine cruises during 1960–1982 and other forms of data) are different from ours. Theirs is the "ice-only" mean draft—open water is excluded from their mean, although the threshold for exclusion is not given. The ice-only mean has the property that the annual cycle is almost inverted, although it is not clear to us why the inversion is so strong. In their Table 2, the minimum occurs in spring, the maximum in summer. The shape of their summer and autumn fields resemble the shape in our Figure 5. The contour maps of *Bourke and McLaren* [1992] (using data from 12 submarine cruises during 1958–1987) show detail that seems to arise from attempting to contour around disparate data from different cruises, where temporal change has occurred. We find no suggestion in our data of the 4-m ice they show in the southern Beaufort Sea and Chukchi Sea, but ice model results during periods of strong anticyclonic circulation show that thick ice is advected into those seas and into the East Siberian Sea. Note that both the papers by Bourke report results from outside the DRA; this was accomplished by working with classified data to obtain the contour maps which were then declassified. These data are not publicly archived.

Of the 0.98 m^2 of variance in the data, the multiple regression model explains all but 0.21 m^2 (21%) with a standard deviation = 0.46 m. We feel this gives an independent estimate of an upper bound for the observational error in the submarine data. But a reasonable error budget is that the observational error has a standard deviation of about 0.28 m, and that the signal in the data explained neither by the regression model nor the observational error has a standard deviation of 0.37 m, which might be thought of as "natural variability."

"How ubiquitous and widespread is the interannual change?" Without more data from outside the data release area, one cannot answer clearly the question of whether there is a "sloshing" mode such that ice at one time inside the DRA moves out into the area between the DRA and Canada, Ellesmere Is. and Greenland [*Holloway and Sou*, 2002; *Rothrock and Zhang*, 2005]. In this regard, our understanding of arctic sea ice thickness would greatly benefit by an updating of the analyses of LeSchack, Bourke, Garrett, and McLaren of *all* data from the Arctic Ocean, although from our enquiries it is doubtful that those data still exist. As for the present and future, it would be a tragedy for arctic science if the U.S. submarine fleet were unable to continue to collect and provide ice profiling data on future cruises.

Acknowledgement.

We gratefully acknowledge the Office of Polar Programs of the National Science Foundation for their generous support of our work in processing draft data (OPP-9910331) and in analyzing draft data (ARC-0453825) and also support under NASA Grant NNG04GH52G. We express our thanks to the staff of the Arctic Submarine Laboratory for their support of efforts to place in a public archive draft data that make studies such as this possible.

References

Bourke, R.H., and R.P. Garrett (1987), Sea ice thickness distribution in the Arctic Ocean, *Cold Regions Sci. and Technol., 13*, 259–280.

Bourke, R.H., and A.S. McLaren (1992), Contour mapping of arctic basin ice draft and roughness parameters, *J. Geophys. Res., 97*(C11), 17,715–17,728.

Maykut, G.A., and N. Untersteiner (1971), Some Results from a Time-Dependent Thermodynamic Model of Sea Ice, *J. Geophys. Res., 76*(6), 1550- 1575.

McLaren, A.S., R.G. Barry and R.H. Bourke (1990), Could Arctic ice be thinning?, *Nature, 345, 762.*

McLaren, A.S., J.E. Walsh, R.H. Bourke, R.L. Weaver, and W. Wittmann (1994), Variability in sea-ice thickness over the North Pole from 1977 to 1990, *Nature, 358*, 224–226.

McLaren, A.S., R.H. Bourke, J.E. Walsh, and R.L. Weaver (1994), Variability in sea-ice thickness over the North Pole from 1958 to 1992, in *Polar Oceans and Their Role in Shaping the Global Environment*, O.M. Johannessen, R.D. Muench, and J.E. Overland, eds., pp. 363-371, Amer. Geophys. Union.

Holloway, G., and T. Sou (2002), Has arctic sea ice rapidly thinned?, *J. Clim., 15*, 1691–1701.

Percival, D.B., D.A. Rothrock, A.S. Thorndike, and T. Gneiting (2006), The variance of mean sea-ice thickness: Effect of long-range dependence, *J. Geophys. Res*., submitted.

Rothrock, D. A., Y. Yu, and G. A. Maykut (1999), Thinning of the Arctic sea-ice cover, *Geophys. Res. Lett., 26*, 3469-3472.

Rothrock, D.A. and J. Zhang (2005), Arctic Ocean sea ice volume: What explains its recent depletion?, *J. Geophys. Res., 110,* C01002, doi:10.1029/2004JC002282.

Rothrock, D.A. and M. Wensnahan. (2007), The accuracy of sea-ice drafts measured from U. S. Navy submarines. Submitted to *J. Atmos. Oceanic Technol*.

Shy, T. L., and J.E. Walsh (1996), North Pole ice thickness and association with ice motion history 1977–1992, *Geophys. Res. Lett., 23*, 2975– 2978.

Tucker, W. B., J. W. Weatherly, D. T. Eppler, L. D. Farmer, and D. L. Bentley (2001), Evidence for rapid thinning of sea ice in the western Arctic Ocean at the end of the 1980s, *Geophys. Res. Lett., 28*, 2851-2854.

Wadhams, P. (1990), Evidence for thinning of the Arctic ice cover north of Greenland, *Nature*, 345, 795–797.

Wadhams, P. and N.R. Davis (2000), Further evidence of ice thinning in the Arctic Ocean, *Geophys. Res. Lett., 27*(24), 3973–3975.

Winsor, P. (2001), Artic sea ice thickness remained constant during the 1990s, *Geophys. Res. Lett., 28*, 1039– 1041.

Wensnahan, M., and D. A. Rothrock (2005), Sea-ice draft from submarine-based sonar: Establishing a consistent record from analog and digitally recorded data, *Geophys. Res. Lett., 32*, L11502, doi:10:1029/2005GL022507.

Figure Captions

Figure 1. Cruises from which sea ice draft data are available at NSIDC, by year and time of year [*after Rothrock and Wensnahan*, 2007, *submitted*].

Figure 2. Cruise tracks of U.S. Navy cruises for which data are available at NSIDC. The broad grey line outlines the data release area DRA: the "SCICEX Box", whose vertices are given in Table 1.

Figure 3. The interannual change in ice draft, $I(t-1988)+C+\overline{S}$, in meters, averaged over the data release area and over a year. The dots are the residuals (added to $I(t-1988)+C+\overline{S}$), black for summer/fall, grey for winter/spring.

Figure 4. The annual cycle of draft, $A(\tau)+C+\overline{I}+\overline{S}$, in meters, averaged over the data release area and over the 26 years 1975–2000. The dots are the residuals (added to $A(\tau)+C+\overline{I}+\overline{S}$), black for summer/fall, grey for winter/spring.

Figure 5. The spatial field of draft, $C+\overline{I}+S(x,y)$, in meters, averaged over the 26-years 1975–2000 and over an annual cycle.

Figure 6. The residuals of the data (upper) when *S(x,y)* is a linear polynomial, and (lower) for our solution when *S(x,y)* is a 5th order polynomial, black for summer/fall, grey for winter/spring. The solid curves are spline fits to the cluster of points.

Tables

Table 1. Coordinates of vertices in the data release area (DRA), known as the "SCICEX Box". The conversion between (lat, long) and (X, Y) is given in Eq. (1).

Latitude (°)	Longitude (°E:+, °W:–)	X (10^3 km)	Y (10^3 km)
87.00	–15.00	0.214 366	–0.255 471
86.58	–60.00	–0.033 104	–0.378 391
80.00	–130.00	–1.072 528	–0.287 386
80.00	–141.00	–1.107 658	–0.077 458
70.00	–141.00	–2.206 887	–0.154 326
72.00	–155.00	–1.962 697	0.346 071
75.50	175.00	–1.231 624	1.033 459
78.50	172.00	–0.933 494	0.870 501
80.50	163.00	–0.649 506	0.831 333
78.50	126.00	–0.022 275	1.276 201
84.33	110.00	0.163 001	0.608 326
84.42	80.00	0.438 730	0.438 730
85.17	57.00	0.498 047	0.201 224
83.83	33.00	0.684 883	–0.023 917
84.08	8.00	0.585 876	–0.298 519

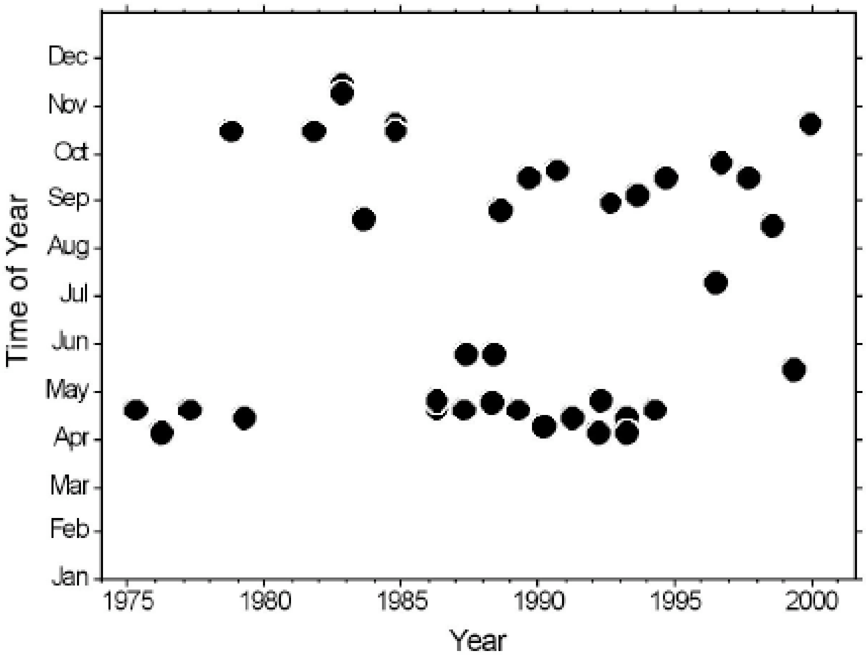

Figure 1. Cruises from which sea ice draft data are available at NSIDC, by year and time of year [*after Rothrock and Wensnahan*, 2007, *submitted*].

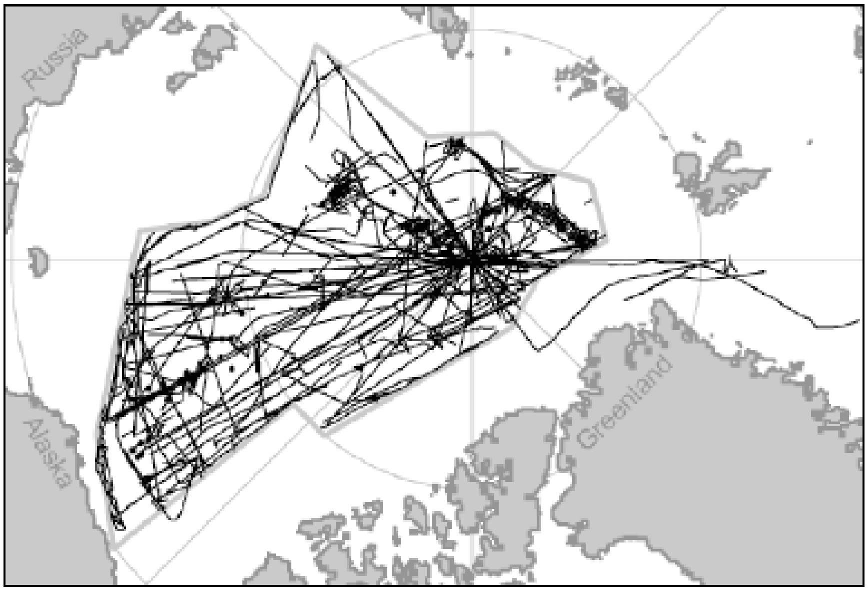

Figure 2. Cruise tracks of U.S. Navy cruises for which data are available at NSIDC. The broad grey line outlines the data release area DRA: the "SCICEX Box", whose vertices are given in Table 1.

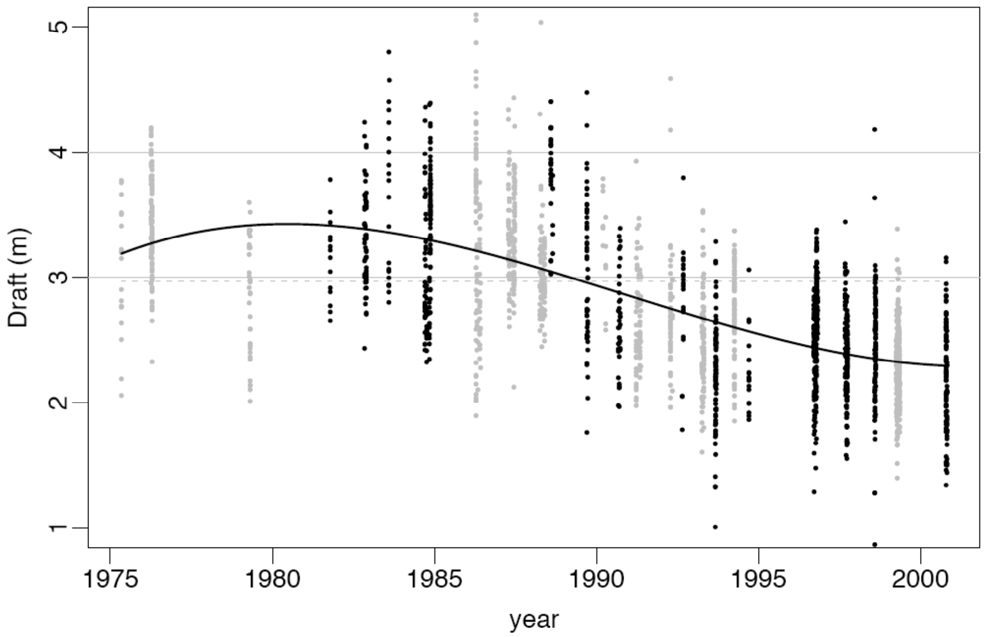

Figure 3. The interannual change in ice draft, $I(t-1988)+C+\bar{S}$, in meters, averaged over the data release area and over a year. The dots are the residuals (added to $I(t-1988)+C+\bar{S}$), black for summer/fall, grey for winter/spring.

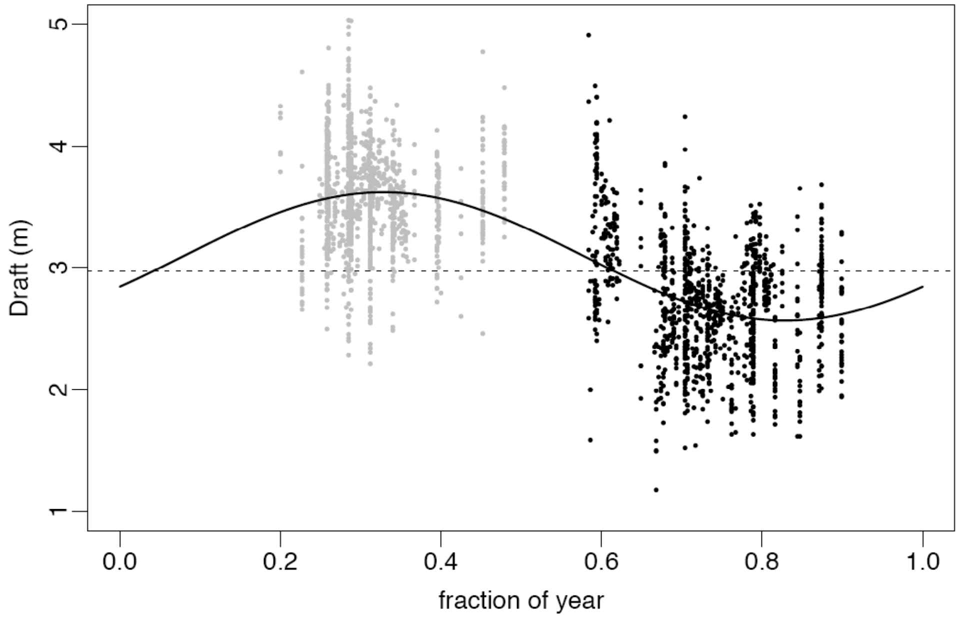

Figure 4. The annual cycle of draft, $A(\tau) + C + \bar{I} + \bar{S}$, in meters, averaged over the data release area and over the 26 years 1975–2000. The dots are the residuals (added to $A(\tau) + C + \bar{I} + \bar{S}$), black for summer/fall, grey for winter/spring.

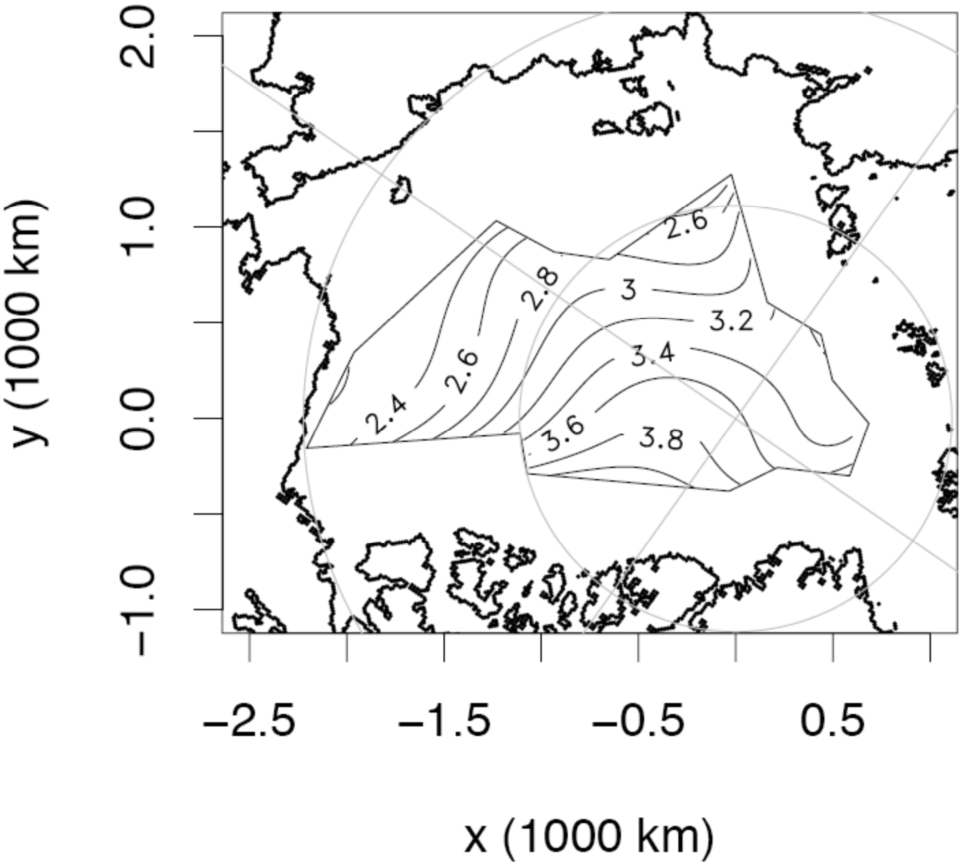

Figure 5. The spatial field of draft, $C + \bar{I} + S(x,y)$, in meters, averaged over the 26-years 1975–2000 and over an annual cycle.

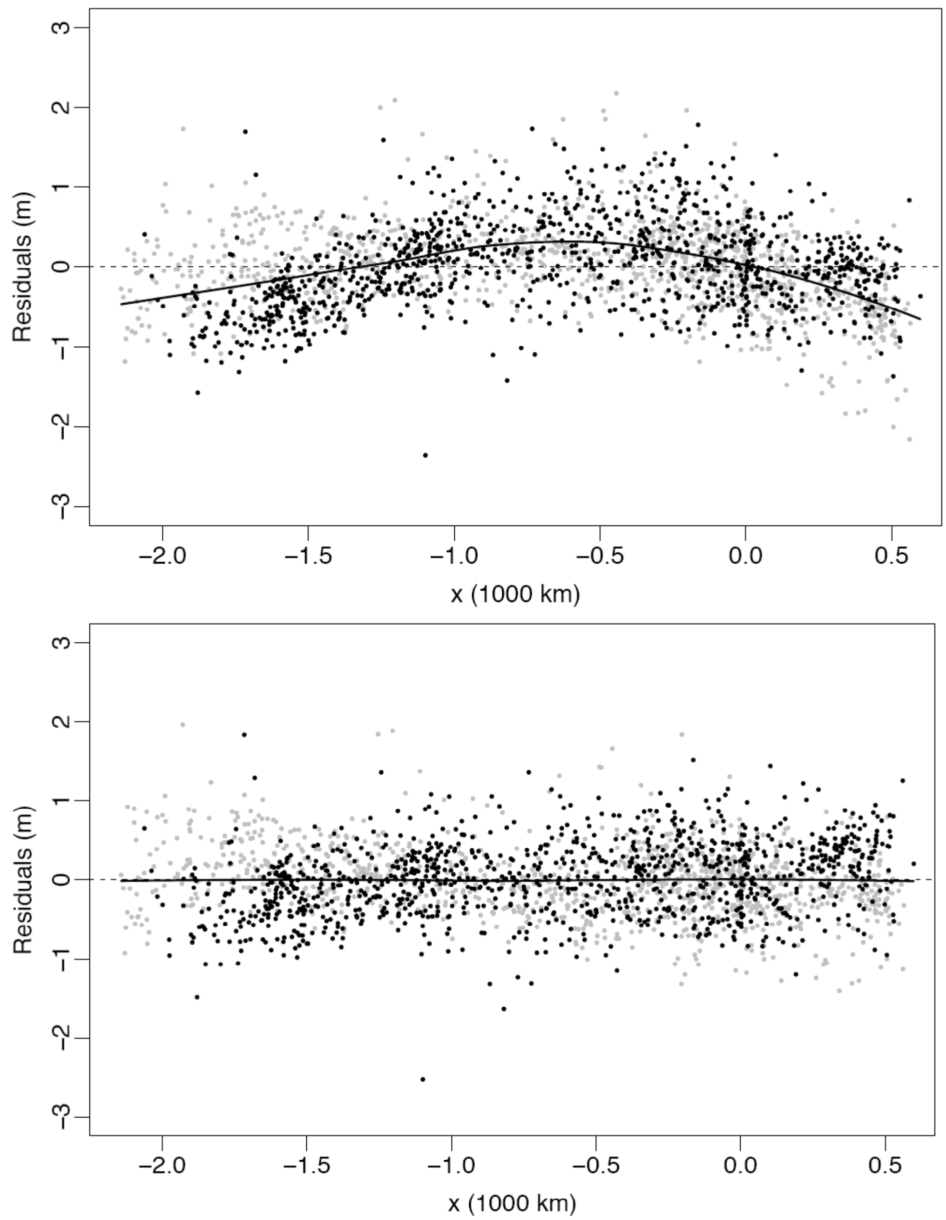

Figure 6. The residuals of the data (upper) when *S(x,y)* is a linear polynomial, and (lower) for our solution when *S(x,y)* is a 5th order polynomial, black for summer/fall, grey for winter/spring. The solid curves are spline fits to the cluster of points.