Hour Exam III answers posted on the website
Earlier Hour Exam answers also posted.

Office hrs NOT WED, but Thurs (normal) and Fri from 4pm to 5:30 or so.

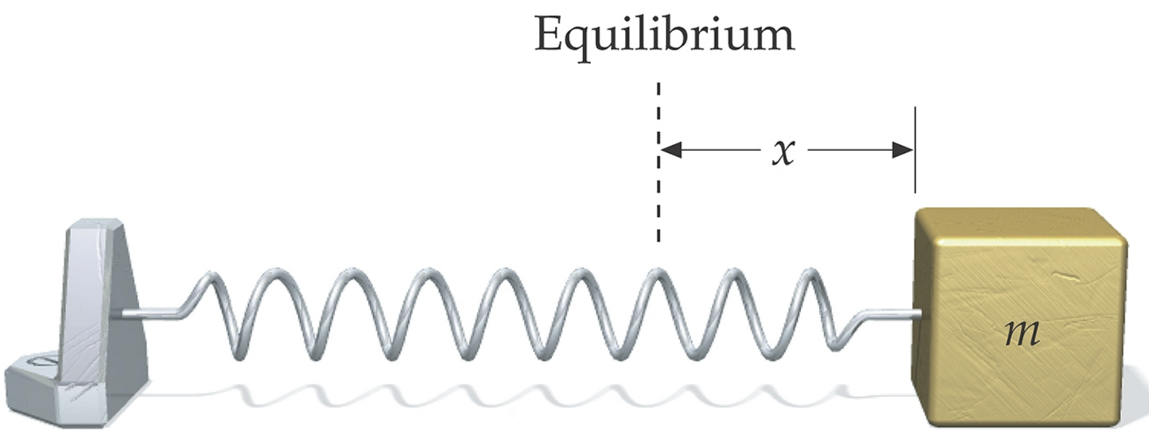
Final homework assignment due Friday Dec 7

Final exam Monday Dec 10 at 0830 here.

Oscillators Ch 14.1—14.2

Simple Harmonic Motion

archetypical example – spring and mass:



$$F = -kx = ma = m\frac{d^2x}{dt^2}$$ **(Newton, Hooke)**

which gives the differential equation

$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x(t)$$ **for the function *x*(*t*).**

Before solving this equation, think about the motion in general terms….

1. **It is oscillatory (i.e. periodic).**
2. **It is fastest at the equilibrium position (think sign of the force)**
3. **We have a choice of amplitude, but for given *m* and *k* that is all.**

Demo - then clicker.

A mass, M, on a spring, with Hook’s law constant K, oscillates with periodic motion. Which is true?

A. **The frequency depends on M not K**
B. **The frequency depends on K not M**
C. **The frequency depends on K/M**
D. **The energy is not conserved, even close.**
E. **The frequency depends on K and M in a more complicated way than just K/M**

what about $\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x(t)$ **?**

(called the equation of motion)

We need to figure out a function x(t) whose 2nd derivative is the same function times a negative constant. (k and m are both always positive).

Draw some such functions.

discuss "initial conditions" (x(0) and dx/dt at t=0)

Solutions: $x(t) = A\sin(\omega t)$ **or** $x(t) = B\cos(\omega t)$ **or combo. (Two adjustable parameters, *A, B* will handle the initial conditions)**

Since $\frac{d}{dt}\cos(\omega t) = -\omega\sin(\omega t)$ **and**

$\frac{d}{dt}\sin(\omega t) = +\omega\cos(\omega t)$ **see that either one satisfies the d.e. and** $\omega^2 = k/m \rightarrow \omega = \pm\sqrt{k/m}$

Take + ω since *A* and *B* can handle case of the – option, so we use $\omega = +\sqrt{k/m}$

Most general solution is then

$x(t) = A\sin(\omega t) + B\cos(\omega t)$ **with A and B chosen to give the right x(0) and v(0). Examples.**

Other ways to write $x(t) = A\sin(\omega t) + B\cos(\omega t)$

use trig identities:

$$\sin(\omega t + \phi) = \sin(\omega t)\cos(\phi) + \sin(\phi)\cos(\omega t)$$

and

$$\cos(\omega t + \phi) = \cos(\omega t)\cos(\phi) - \sin(\phi)\sin(\omega t)$$

Which show you could use

$C\sin(\omega t + \phi_s)$ **or** $D\cos(\omega t + \phi_c)$

now the two adjustable parameters are *C* and ϕ or *D* and ϕ.

Generally this second way is handier, using your favorite of sine or cosine.

Amplitude: |*C*| or |*D*| or $\sqrt{A^2 + B^2}$

Phase: ϕ

Angular frequency: ω

What about period? *x*(*t*) = *x*(*t*+*T*)

The period *T* is shortest time this happens in.

$\sin(x) = \sin(x + 2n\pi)$, ***n* = integer, so**

$C\sin(\omega t + \phi_s) = C\sin(\omega[t + T] + \phi_s)$ **if** $\omega T = 2n\pi$

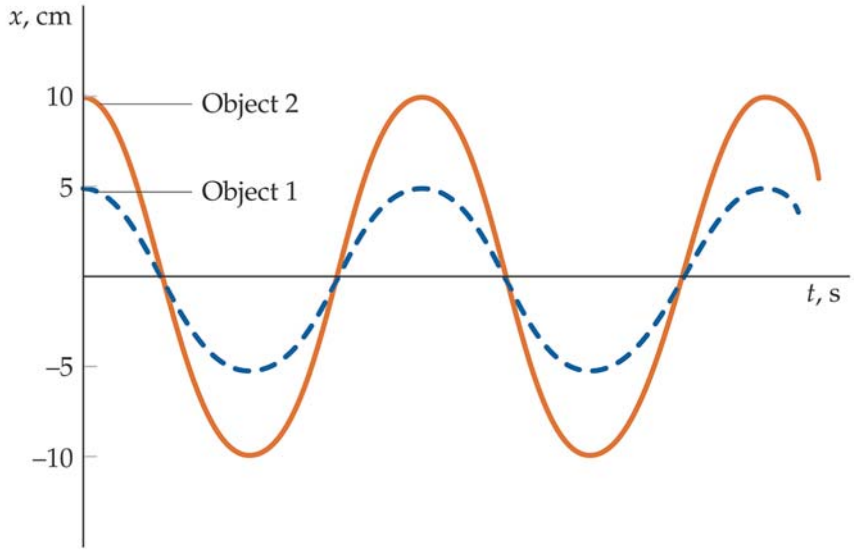
thus $T = 2\pi / \omega$ **which is what we had for circular motion too. Note ωt is an angle.**

Finally, frequency (how many cycles/second) is *f=1/T* (Note error in text eqn 14-11.)

$\omega = 2\pi \frac{1}{T} = 2\pi f$ **is correct. Clicker again**

A mass, M, on a spring, with Hook’s law constant K, oscillates with periodic motion. Which is true?

- **A. The frequency is independent of the amplitude**
- **B. The frequency increases with increasing amplitude**
- **C. The frequency decreases with increasing amplitude**
- **D. The frequency is proportional to the energy.**
- **E. The frequency is inversely proportional to the energy**



Motion is "sinusoidal" – which includes sines and cosines both.
Harmonic – frequencies that are multiples of the smallest. Simple harmonic – only one frequency, sinusoidal motion.

What about $v(t)$ and $a(t)$? for
$x(t) = A\cos(\omega t + \phi)$ **we take dx/dt = v**
$v(t) = -\omega A\sin(\omega t + \phi)$ **and dv/dt = a**
$a(t) = -\omega^2 A\cos(\omega t + \phi)$
Note amplitude increases by factor ω each step.
(takes care of the units.)
Phase changes by π/2 each step.
since cos(x+π/2) = -- sin(x)
Velocity is "out of phase" with amplitude and acceleration.
Acceleration is opposite to displacement.

Energy in S.H.M.

1. Energy is conserved, at least well enough that we can take it constant over one cycle.
2. At extreme displacement: $U = \frac{1}{2} k A^2$ and $K = 0$
3. At zero displacement: $U=0$ and $K = \frac{1}{2} m v_{max}^2$
   where $v_{max} = A\omega$,
   so at $x=0$, $K = \frac{1}{2} m A^2 \omega^2 = \frac{1}{2} mA^2 (k/m) = \frac{1}{2} kA^2$ too.
4. In general, $U = \frac{1}{2} kx^2$ $K = \frac{1}{2} mv^2$
   and $E = K + U = \frac{1}{2} kA^2$
   $x^2 = A^2 \cos^2(\omega t)$ and $v^2 = A^2 \omega^2 \sin^2(\omega t)$ so
   $$E = \frac{1}{2} kx^2 + \frac{1}{2} mv^2$$
   $$= \frac{A^2}{2} [k \sin^2(\omega t) + m\omega^2 \cos^2(\omega t)] = \frac{A^2 k}{2}$$
5. E proportional to A^2 also $U_{av} = K_{av} = E/2$

We have x, v, a, as functions of t. What about v and a as functions of x, skipping t?

Relate x^2 and v^2 from E equation above (v can be either way for any x)

$a = F/m = - kx/m$ was where we started.