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Homework #9 due next Wed (Nov 28).
Hour exam Friday, Nov 30

First homework problem "Earth & Moon" had R_{moon} incorrect. Value in m was labeled km. This has been corrected. (1740 km is correct)

Kepler (1571-1630) and laws of motion of planets.
Kepler used data by Tycho Brahe. (d. 1601)
These are empirical laws (based on observation).

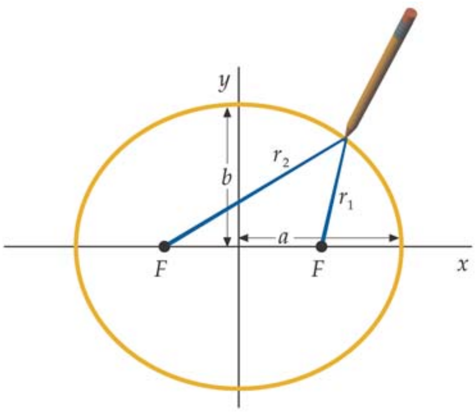
1. **Planets move in elliptical orbits with the Sun at a focus. (1609)**
2. **Radius to Sun sweeps out equal areas in equal times (1609)**
3. **The orbit period squared is proportional to the semi-major axis cubed. (1619)**
4. **The 4th law had to do with the relative sizes of the orbits of the six known planets. Kepler tried various schemes and could not successfully find a law.**

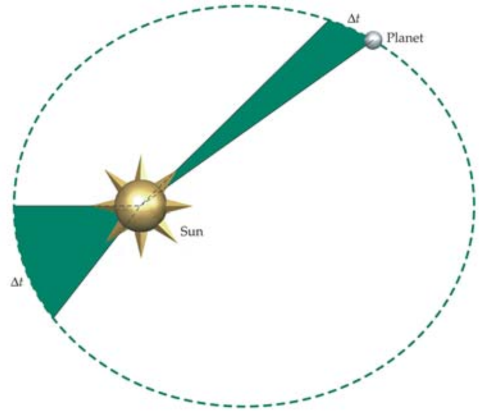
Historical framework:

Copernicus (1543) had replaced Ptolemy's earth centered reference frame with one centered on the Sun. This was controversial before Kepler's time.

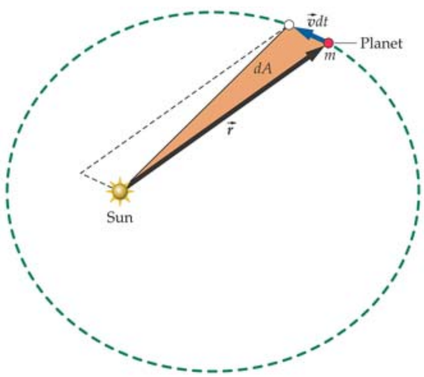
Copernicus liked circular orbits. Kepler liked Copernican scheme, and tried to demonstrate its validity.

Mars' orbit is particularly elliptical, and forced Kepler to modify Copernican viewpoint.





Ellipse **equal area in equal time**



Central force, so planet's *L* is conserved. Area is

$$\frac{1}{2}\left|\vec{r}\times\vec{v}\right|\Delta t = \frac{1}{2}(L/m)\Delta t$$

thus K's 2nd follows from conservation of *L* with central force.

Why an ellipse? (Kepler's 1st)

Why is T^2/a^3 a constant? (Kepler's 3rd)

Newton: law of gravitation: $F = \frac{GM_1M_2}{r_{12}^2}$

attractive between the two objects.
Based on comparing g at surface of Earth with a_c determined from Moon's orbit R and ω.
Approximate orbit as circle, use average *R*

$M_M a_c = M_M \omega^2 R = GM_E M_M / R^2$

$= M_M (GM_E/R_E^2) R_E^2/R^2. = M_M g R_E^2/R^2$

so can relate ω, R, and R_E to check this idea.

$$\omega^2 = \frac{g(R_E / R)^2}{R}$$

$$\rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{(R_E / R)} \sqrt{\frac{R}{g}}$$

For moon's orbit is $R \approx 60R_E$

$$T \approx 2\pi 60 \sqrt{\frac{60(6.4x10^6)}{9.8}} \sqrt{\frac{m}{m/s^2}} \frac{1}{8.6x10^4} \frac{d}{s} = 27d$$

Note $\omega^2 \propto 1/R^3 \rightarrow R^3 / T^2 = Const$ **is Keppler's 3rd (for circular orbit, anyway).**

Why should g at surface be that for case where all M is at center and we are floating at R_E?

Integrate over mass of earth (see sec 11-5)

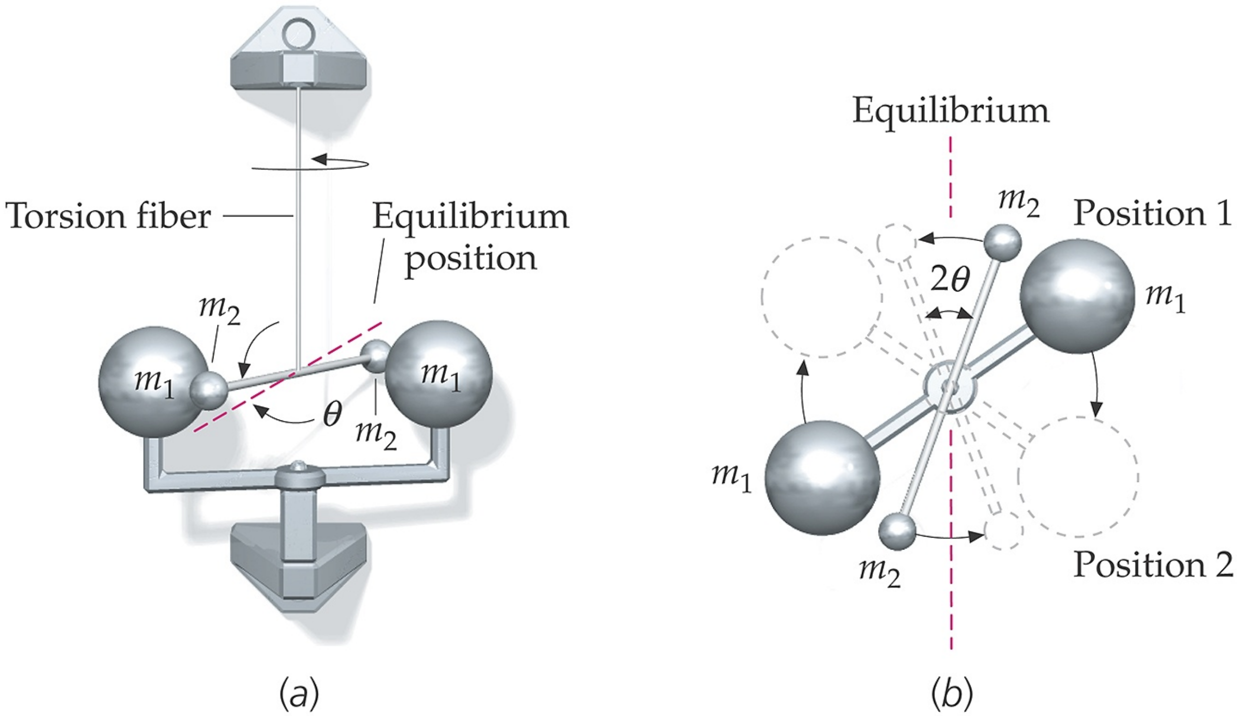
We are left to prove Keppler's 1st , which indeed follows from Newton's laws of motion and gravity, but is a bit complicated.

Clicker

Suppose a planet exists that has half the mass of earth and half its radius. On the surface of that planet, the acceleration due to gravity is

A. twice that on Earth.
B. the same as that on Earth.
C. half that on Earth.
D. one-fourth that on Earth.
E. none of these.

Measure G (best value to date obtained by Prof Jens Gundlach of this dept.)
Cavendish:



(a) (b)

Can determine "spring constant" from frequency of oscillations of torsion pendulum. Then twist is proportional to torque from source masses.

A difficult experiment. There are problems with the inelastic behavior of the torsion fiber. Also the source masses must be good, uniform spheres. Resulting G is 6.67390 x 10^{-11} Nm^2/kg^2 accuracy of .0014% (Gundlach and Merkowitz)

G is approximately 2/3 x 10^{-10} Nm^2/kg^2

Gravitational potential energy: **Special case of a big object attracting a small one.**
So C.M. of system is approx center of large object.
Small one located at r from the center of the big one.

$$U(r) = -\int_{r_0}^{r} \vec{F}\bullet d\vec{r} = -\int_{r_0}^{r} \left(\frac{-GMm}{r^2}\hat{r}\right)\bullet d\vec{r}$$

$$= GMm\int_{r_0}^{r}\frac{dr}{r^2} = GMm(-1/r + 1/r_0)$$

Pick constant r_0 so $U \rightarrow 0$ as $r \rightarrow \infty$
That is $r_0 = \infty$ Then $U(r) = -GMm/r$
Show $U(R_E + h) = U(R_E) + mgh$

Binding energy: **Object at rest at, say, R_E has negative energy. If you add an equal magnitude of positive energy, the object could be free, with net energy 0.**

Escape velocity: **v_{esc} (in the $\hat{r}$ direction)**
so ½ $mv_{esc}^2 = GM_Em/R_E$ for object on surface of Earth. (or appropriate *M* and *R* for other thing.)
$GM_E/R_E^2 = g$ so $v_{esc}^2 = 2g\,R_E$ →

$$v_{esc} = \sqrt{2(9.8)(6.4\text{x}10^6)} \approx 11 \text{ km/s}$$