

Exam answers posted on class website

Clicker Scores now posted on TYCHO. Look in grades for “clicker”

Tycho Typos. 60 not 80 pts for #7, and #8 heading about dates, number of problems, etc is incorrect.

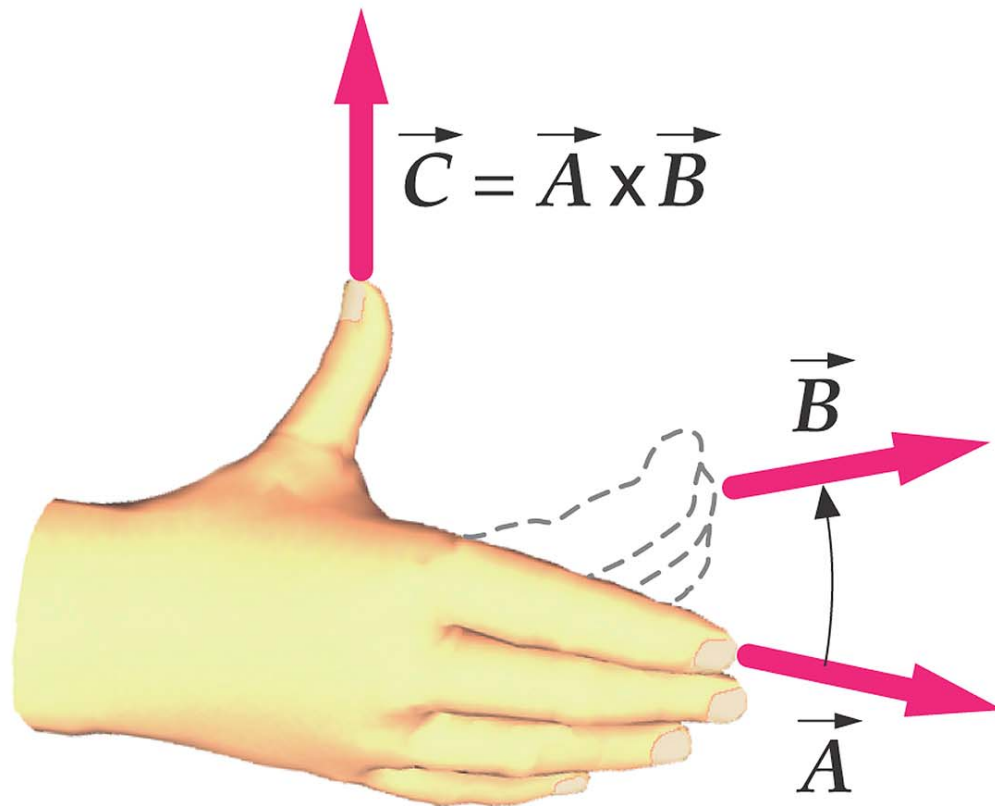
Homework #8 due next Wed (Nov 21) at midnight

Graded exams to be returned at end of class

Getting exam grades posted.....

Overview of HW #8 “hanging sign” and “disk and string”

Chapter 10. Polar vectors



\vec{C} is perpendicular to each of \vec{A}, \vec{B}

Magnitude is $AB\sin(\phi)$. C is Max when \vec{A} and \vec{B} are perpendicular (opposite from dot product).

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \text{ (which may seem peculiar)}$$

$\vec{\omega} = \vec{r} \times \vec{v} / r^2$ so **COUNTER** Clockwise rotation (as you look at it) points toward you.

I.e. the hands of a clock have $\vec{\omega}$ pointing into the clock, away from the viewer.

This is an arbitrary convention.

$\hat{i} \times \hat{j} = \hat{k}$ for “right handed coordinates”

$\hat{i} \times \hat{j} = \hat{k}$, $\hat{k} \times \hat{i} = \hat{j}$, $\hat{j} \times \hat{k} = \hat{i}$ (cyclic perm.)
 $\hat{j} \times \hat{i} = -\hat{k}$, etc. and $\hat{i} \times \hat{i} = 0$, etc.

For vectors written as $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{A} \times \vec{B} =$

$(A_x B_y - A_y B_x) \hat{k} + (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j}$

Note pattern in this.

If you like determinants:

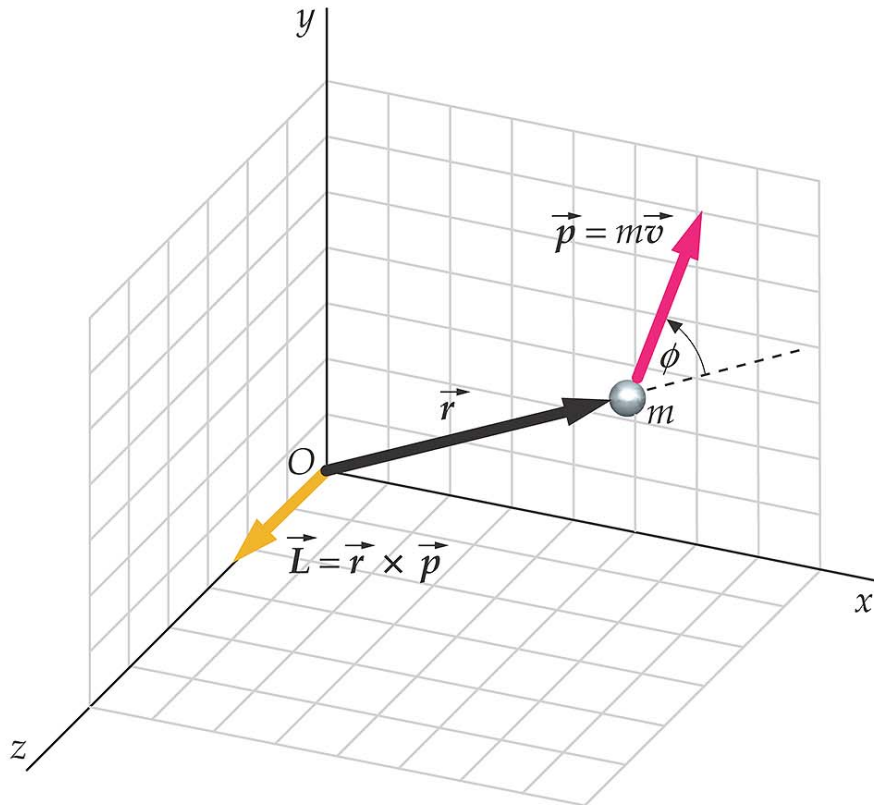
$$\vec{A} \times \vec{B} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{pmatrix}$$

Angular Momentum

Define, $\vec{L} = \vec{r} \times \vec{p}$ for a point particle and note

that Torque: $\vec{\tau} = \vec{r} \times \vec{F}$ and $\vec{\omega} = \vec{r} \times \vec{v} / r^2$

the \vec{r} goes first, and the right hand rule works



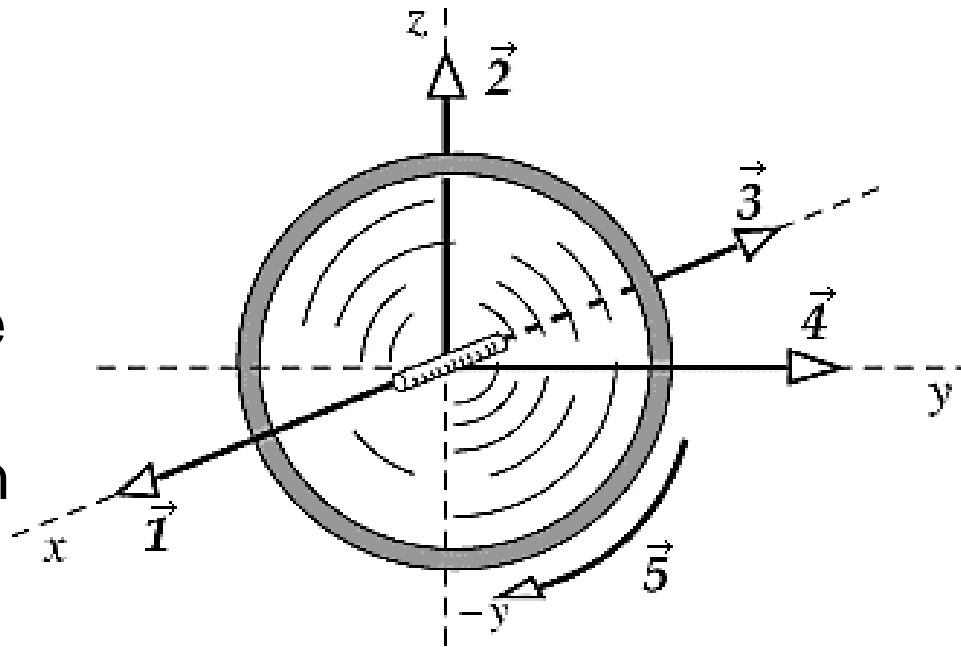
This particle may or may not be going in a circle about O . Often it won't be!!!

Consider a ring, rotating about its center at O .

$$\text{Then } \vec{L} = \vec{r} \times m\vec{v} = m\left(\frac{r^2}{r^2}\right)\vec{r} \times \vec{v} = mr^2\vec{\omega} = I\vec{\omega}$$

Clicker

A wheel is rotating clockwise as shown. Its rotation axis coincides with the x -axis. A torque that causes the wheel to slow down is best represented by the vector



A. $\vec{1}$

B. $\vec{2}$

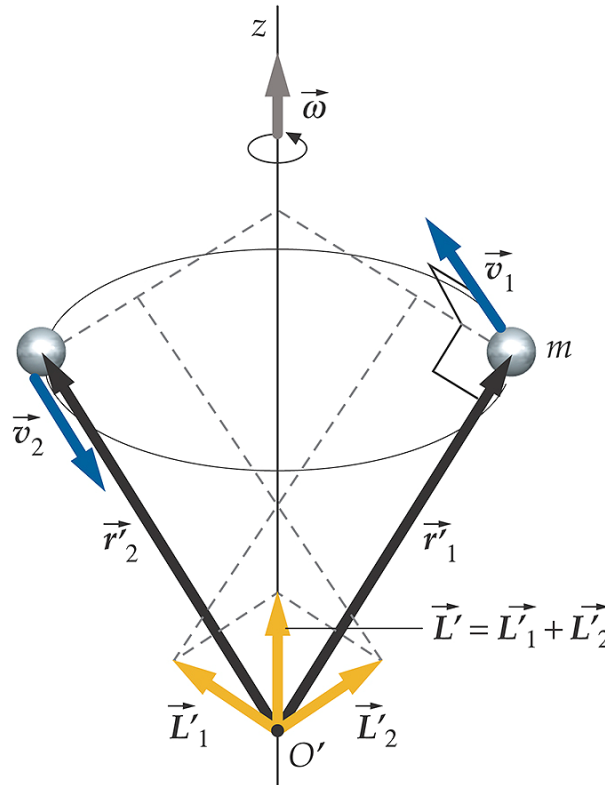
C. $\vec{3}$

D. $\vec{4}$

E. $\vec{5}$

If the ring is displaced from the x-y plane, but still rotates about the z-axis : $\vec{L} = I\vec{\omega}$ about the z-axis.

See cancellation of components.



In general, an **object rotating about a symmetry axis** has

$$\vec{L} = I\vec{\omega}$$

where I is moment of inertia **about that axis**.
(Life is complicated for non-symmetry axes.)

Exactly as was the case for linear momentum,
 where $\vec{F} = m\vec{a} \rightarrow F = \frac{d\vec{p}}{dt}$ we see

$$\vec{\tau} \equiv \vec{r} \times \vec{F} = I\vec{\alpha} \rightarrow \vec{\tau} = \frac{d\vec{L}}{dt} \text{ and}$$

$$\vec{\tau}_{\text{net,external}} = \frac{d\vec{L}_{\text{system}}}{dt} \quad \text{Likewise, angular impulse:}$$

$$\Delta\vec{L}_{\text{sys}} = \int_{t_i}^{t_f} \vec{\tau}_{\text{net,ext}} dt$$

Splitting motion into that **of C.M.** and **in C.M.**

$$\vec{L}_{\text{sys}} = \vec{r}_{\text{cm}} \times M\vec{v}_{\text{cm}} + \vec{L}_{\text{spin}} \quad \text{where}$$

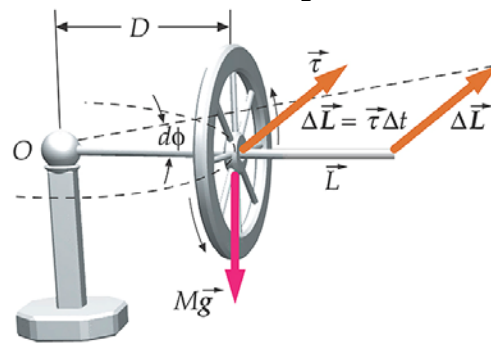
$\vec{r}_{\text{cm}} \times M\vec{v}_{\text{cm}}$ is called **“orbital”** angular momentum
 and is L of system associated with its C.M.
 moving with respect to some point.

\vec{L}_{spin} is the angular momentum **of** the system
about its C.M. – examples of Earth, Moon, Sun.

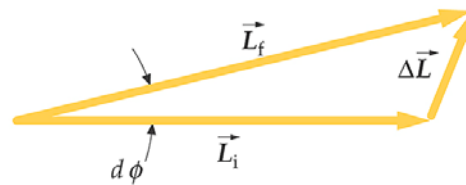
Gyroscope: (precession)

\vec{L}_{spin} is in horizontal plane. Gravity provides **torque**, also in horizontal plane, perpendicular to \vec{L}_{spin} .

Therefore $\Delta \vec{L}_{\text{spin}}$ changes **direction** of \vec{L}_{spin} , **not** its **magnitude** and it stays in horizontal plane.



(a)



(b)

$\Delta L = \tau \Delta t = MgD \Delta t$ -- angle axis points is ϕ :

$$\Delta \phi = \frac{\Delta L}{L_{\text{spin}}} = \frac{MgD \Delta t}{I_g \omega_{\text{spin}}}$$

so precession angular speed is

$$\omega_p = \frac{d\phi}{dt} = \frac{MgD}{I_g \omega_{\text{spin}}}$$