Exam answers posted on class website

TYCHO issue for some people for last part of billiard balls problem. (round off)

Email me your correct result if you didn't get it accepted by Tycho. Include your answers to all 4 parts and also your initial variables (mass, angles, v_0)

Homework deadline has been extended to Friday. (Thanks to Bristin) This should be posted on Tycho. Following week homework will be due Wed. Thurs is Thanksgiving holiday.

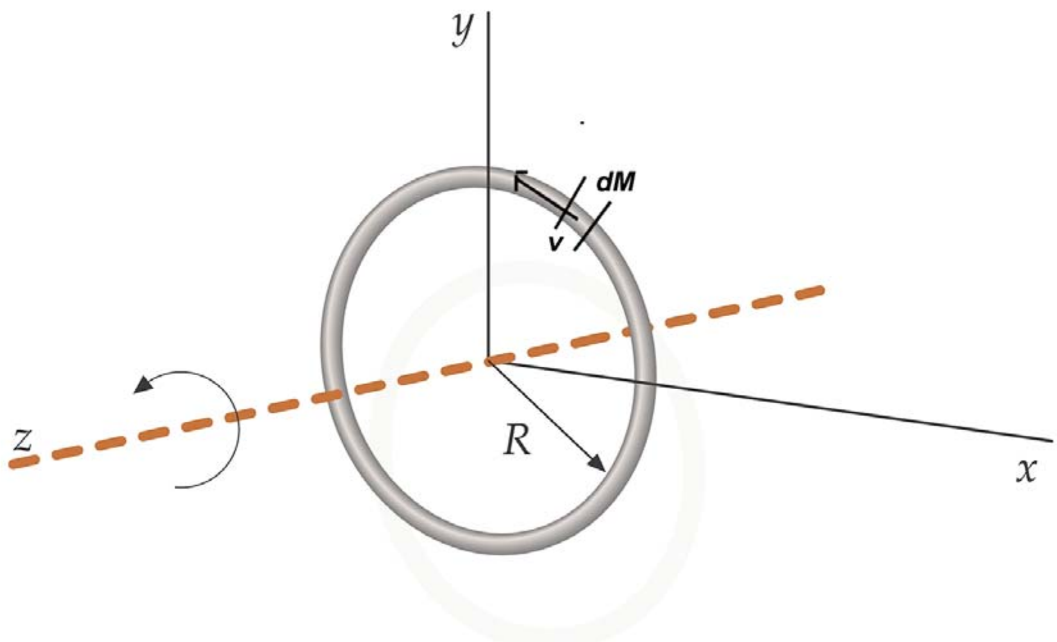
Chapter 9.2 – 9.3 Rotational inertia and energy

We will find many similarities between rotational and linear motion. Use ω instead of v.

What about m?

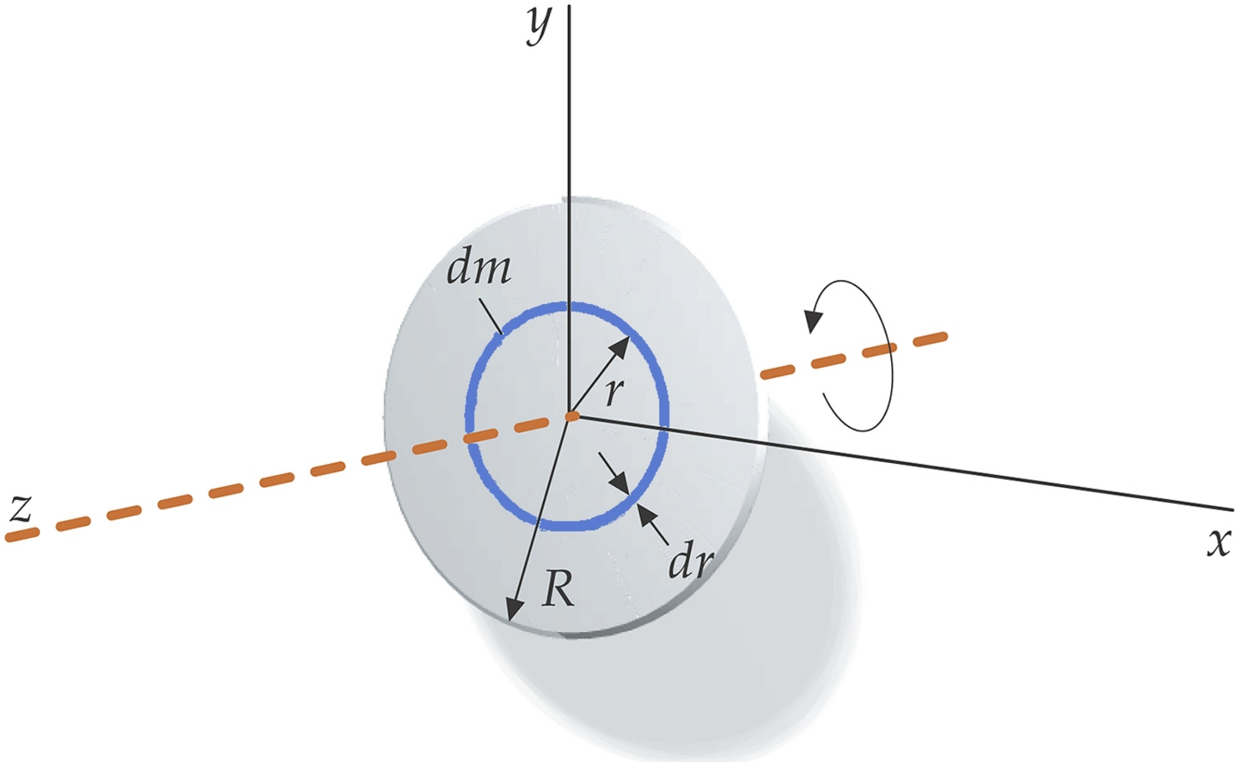
Kinetic Energy of a rotating solid object:
(First, we will consider symmetry axes)

A ring (e.g. bicycle wheel) of mass M– all the mass at R, rotates about an axis perp. to the plane of the ring with ang. velocity ω



Each bit of mass has speed $v = \omega R$ so
$K = \frac{1}{2} M(\omega R)^2 = \frac{1}{2} (MR^2) \omega^2$

A disk (or cylinder) of mass M, rotating about the axis of the cylinder:



Integrate differential rings from *r=0* to *r=R*

Each has speed $v(r) = \omega r$

(*r* is measured to axis, not to origin)

$$K = \frac{1}{2}\int_0^R v^2 dm = \frac{1}{2}\int_0^R (r\omega)^2 dm = \frac{1}{2}\int_0^R (r\omega)^2 (\rho 2\pi r t dr)$$

where *t* is thickness and ρ is density: $\rho = \dfrac{M}{\pi R^2 t}$

so $K = \dfrac{1}{2}\displaystyle\int_0^R (r\omega)^2 \left(\frac{M}{R^2} 2r dr\right) = \frac{1}{2}\frac{M}{R^2} 2\omega^2 \int_0^R (r)^3 dr$

$$K = \tfrac{1}{2}\left(\tfrac{1}{2} MR^2\right)\omega^2$$

Thin Disk about axis in plane, thru center (e.g. x axis)

$K = \frac{1}{2}\left(\frac{1}{4} MR^2\right)\omega^2$

Solid sphere, about axis thru center

$K = \frac{1}{2}\left[(2/5)\, MR^2\right]\omega^2$

In summary: An object, rotating about an axis has $K = \frac{1}{2}\left(f\, MR^2\right)\omega^2$

Where *f* depends on the geometry, i.e.

1. **The shape of the object (defines *R*) and**
2. **The location of the axis of rotation.**

There is a table for many objects on p 295.

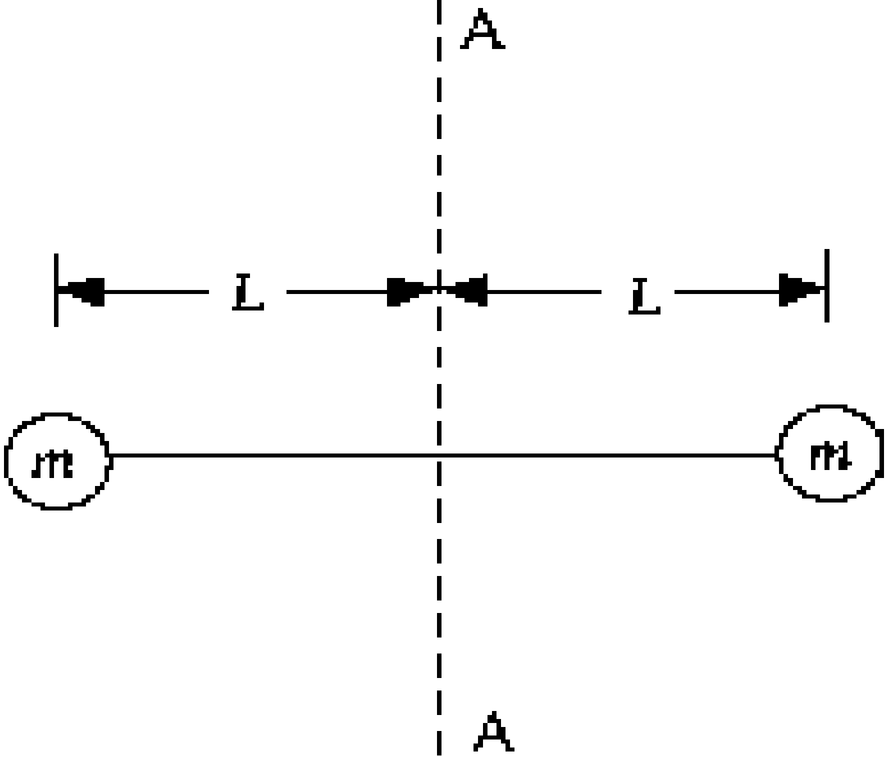
The quantity $f\, MR^2$ **(or** $f\, ML^2$ **for non-round objects), is the Moment of Inertia, *I*, and always** $K = \frac{1}{2} I \omega^2$ **note *I* has dimensions** $M \cdot L^2$

$$I = \int_{object} r^2 dm = \sum_i r_i^2 m_i$$

For *K*, use *I* instead of *m* and ω instead of *v*

Clicker

The moment of inertia of a set of dumbbells, considered as two mass points *m* separated by a distance 2*L* about the axis AA, is



A. mL^2

B. ½ mL^2

C. $2mL^2$

D. ¼ mL^2

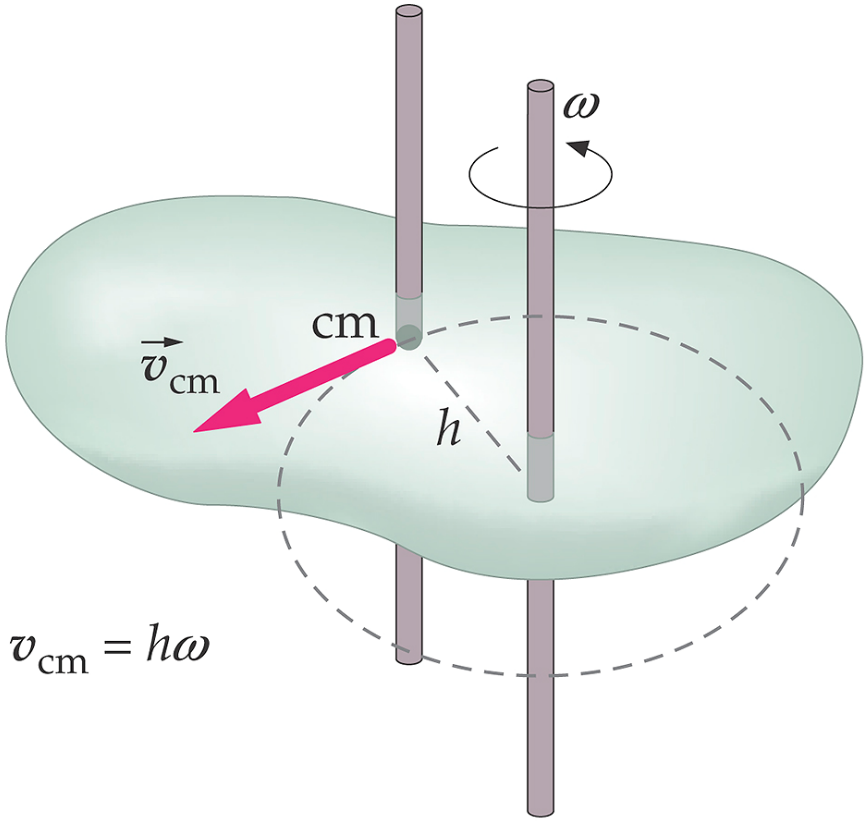
E. $4mL^2$

SYMMETRY AXES

1. **Go through C.M and**
2. **Are perpendicular or parallel to some main plane of the object.**

For rotation about axis thru C.M. the C.M stays at rest.

Consider rotation about axis parallel to a symmetry axis (thru C.M.):



Now C.M. is moving. In one revolution the C.M. goes around a circle of radius *h* and the thing also rotates once.

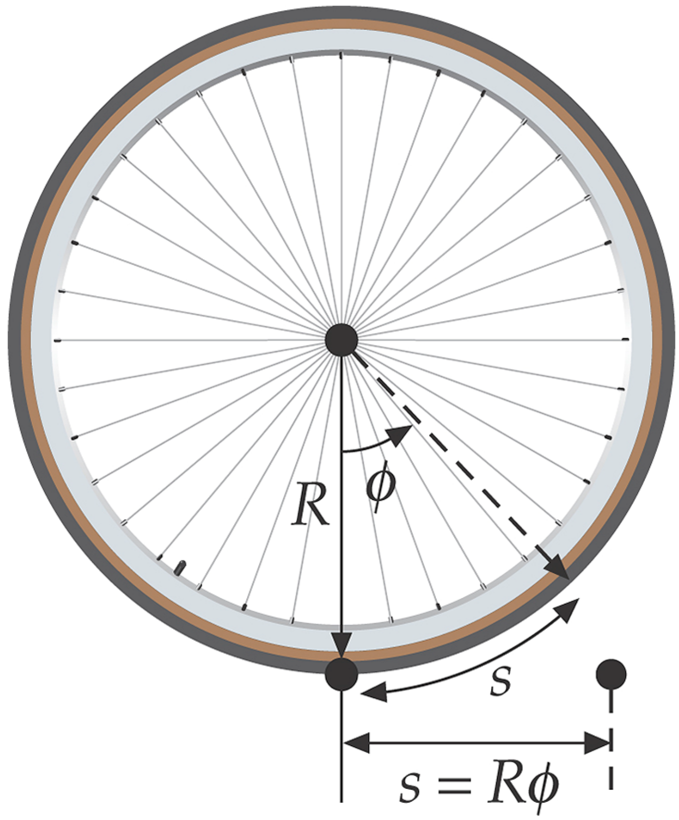
K **has two parts, rotation about axis thru C.M. and rotation of C.M. itself: (I_{cm} is for object rotation about axis thru C.M. and v_{cm} is speed of C.M.)**

$$K = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} M v_{cm}^2 = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} M(h\omega)^2$$

so $I = I_{cm} + Mh^2$ **Parallel Axis theorem.**

Rolling without slipping.

A (not-slipping) wheel (the center) goes linear distance *r* per radian of revolution so $v_{cm} = r\omega$



So it has two kinds of *K*,

1. **translational** = $\frac{1}{2} M v_{cm}^2 = \frac{1}{2} M(R\omega)^2$ **and**
2. **rotational** = $\frac{1}{2} I(\omega)^2$