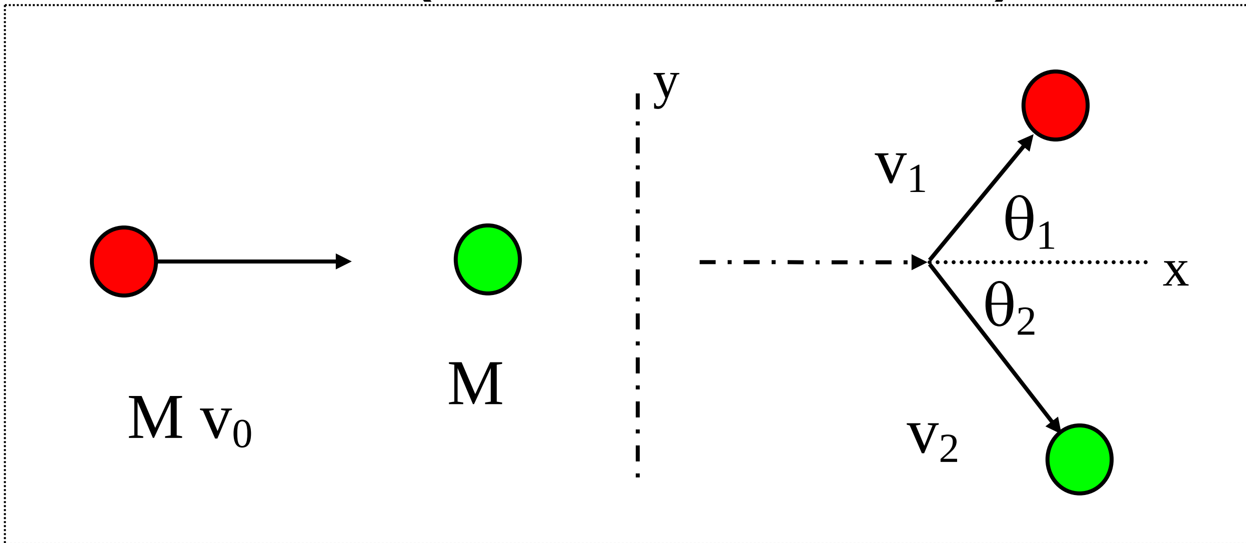
Next hour exam is Friday Nov 9 (next Fri)

Homework is due Wed Nov 7, and is posted on Tycho.

Comet

Billiard Balls **(ignore rotation for now)**
elastic collision (means K is conserved):

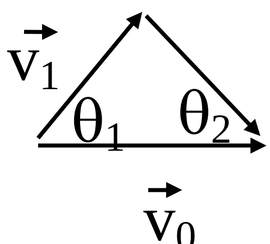


θ_1 can be a range of angles. What about θ_2?
What about v_1 and v_2?

Conservation of momentum: 2 equations
Conservation of energy: 1 equation.
3 equations → three unknowns, v_1, v_2 and θ_2

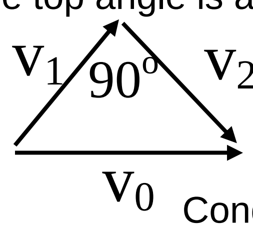
Challenging algebra.
(special trick when both have same M.)

$$M\vec{v}_0 = M\vec{v}_1 + M\vec{v}_2$$

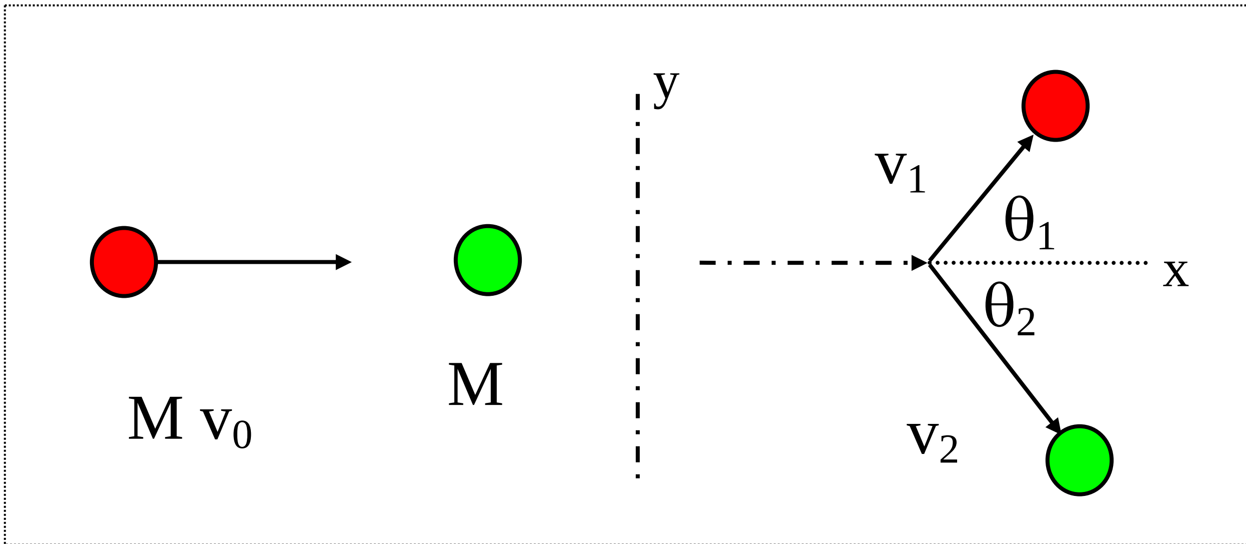


Conserve $\vec{P}$ (divide out M)
don't know top angle (yet)

$v_0^2 = v_1^2 + v_2^2$ for K conservation.
(divide out M/2) – apply Pythagoras
see top angle is a right angle



Congruent triangles. So all 3 angles known, and
$\theta_1+\theta_2 = 90^o$



Elastic, so $\theta_1+\theta_2= 90^o$ and then
$\mathrm{Sin}(\theta_2) = \mathrm{Cos}(\theta_1)$ and conservation of P_y gives
$0 = Mv_1\sin(\theta_1) - Mv_2\cos(\theta_1)$ so $v_2/v_1 = \tan(\theta_1)$

cons of P_x gives
$Mv_0 = Mv_1\cos(\theta_1) + Mv_2\sin(\theta_1)$ subst for v_2
$v_0 = v_1\cos(\theta_1) + v_1 \tan(\theta_1) \sin(\theta_1) = v_1 / \cos(\theta_1)$

Given θ_1 (must be < 90^o) get other 3 variables.
This depends on both masses being equal and collision must be elastic.

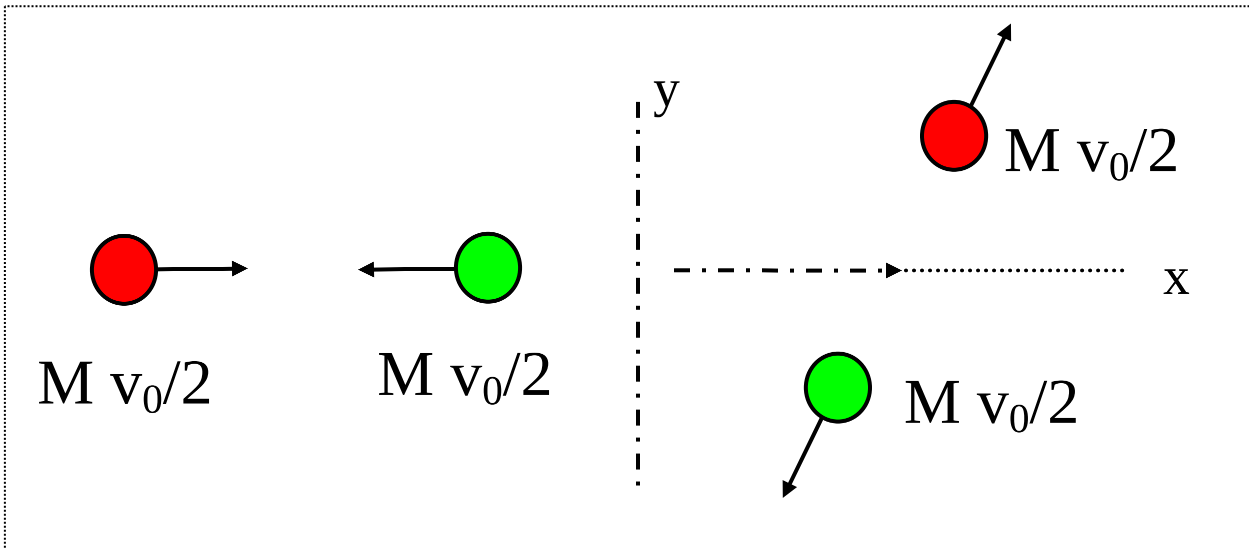
Section 8.4 collisions in CM

Moving with CM,

see $P_{syst} = Mv_{cm} = 0$ and P1 = -- P2

Example for equal masses, where

$$v_{cm} = \frac{v_0}{2} \quad \textbf{and} \quad u_{red} = -u_{green} = \frac{v_0}{2}\textbf{:}$$



Opposite momenta before and after collision.

$P_{syst} = 0$ before and remains so after collision.

Same speeds, to conserve K.

See textbook for examples with different masses

Clicker

Demo

A particle of mass $2m$ is moving to the right in projectile motion. At the top of its trajectory, an explosion breaks the particle into two equal parts. After the explosion, one part falls straight down with no horizontal motion. What is the direction of the motion of the other part just after the explosion?

A. up and to the left
B. stops moving
C. up and to the right
D. straight up
E. down and to the right

Section 8.5 – Rockets.

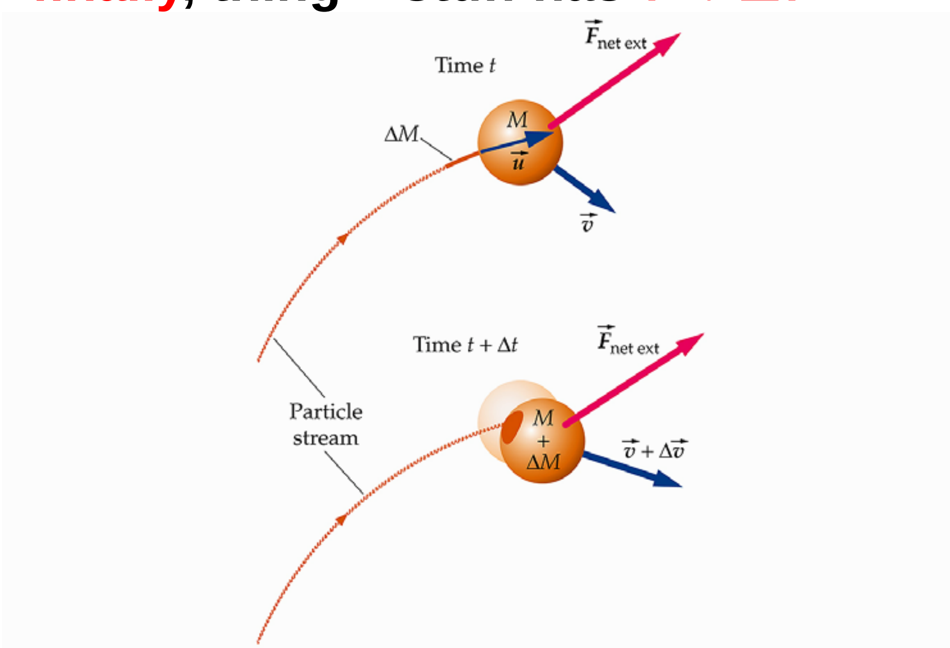
Since rockets squirt exhaust out and decrease in mass, to keep signs straight we start with a model of a thing getting stuff shot into it and sticking. A rocket reverses this process.

Have to consider change in mass and impulse:

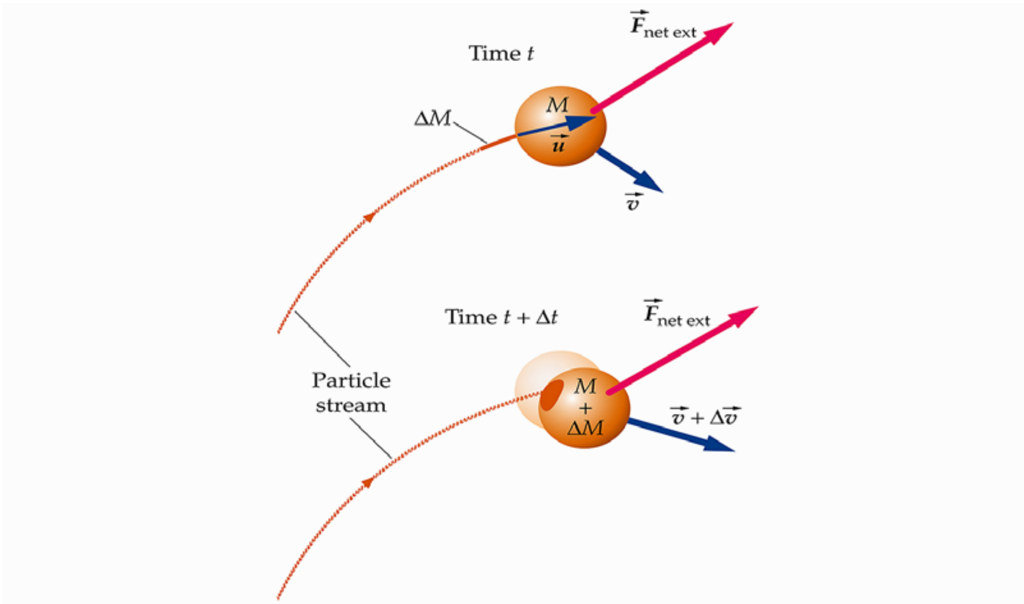
Interval Δt - thing's mass increases by ΔM

System:

thing (mass M) plus stuff stuck, (mass ΔM)
initially, thing has $\vec{V}$, stuff to be stuck has $\vec{U}$
finally, thing + stuff has $\vec{V} + \Delta\vec{V}$



Note that $\vec{F}_{net,ext}$ is EXTERNAL, not the result of accumulating ΔM. Could be gravity, for example.



Impulse – completely a result of ΔM hitting and sticking to M. Involves $\Delta\vec{u}$ and $\Delta\vec{v}$

$$\vec{F}_{\text{net,ext}}\Delta t = \Delta\vec{P} = (M + \Delta M)(\vec{v} + \Delta\vec{v}) - (M\vec{v} + \Delta M\vec{u})$$

Note ΔP is not caused directly just by ΔM and ΔP is the change in system momentum, not just M's. do some algebra (see text) and take limit Δt →0

$$\vec{F}_{\text{net,ext}} + \frac{dM}{dt}\vec{v}_{\text{rel}} = M\frac{d\vec{v}}{dt} \text{ and } \vec{v}_{\text{rel}} = \vec{u} - \vec{v}$$

positive v_{rel} has stuff hitting M.

For a Rocket, both $\frac{dM}{dt}$ and $\vec{v}_{\text{rel}}$ are negative.

Thrust is $\left|\vec{F}_{\text{th}}\right| = \left|\frac{dM}{dt}\right|\left|u_{\text{exh}}\right|$ where absolute value avoids confusion about signs.

Alternate approach.

Be in CM of rocket and bit of fuel to be burned.
Rocket expels ΔM at exhaust speed u_{exh}.with respect to rocket.
Leave external forces out until later.

In limit of small ΔM, speed of exhaust in CM is u_{exh} and to conserve momentum we get
$\Delta P_{rocket} = \Delta M\, u_{exh} = F_{th}\, \Delta t$ using impulse
in the direction opposite to that of the exhaust

Then we get $\left|\vec{F}_{th}\right| = \left|\frac{dM}{dt}\right|\left|u_{exh}\right|$ **in the limit $\Delta t \rightarrow 0$.**

Net force on rocket is $\vec{F}_{net,ext} + \vec{F}_{th}$ **where** $\vec{F}_{th}$ **points opposite the direction the exhaust is shot.**

To get motion of rocket going straight up

$$F_{net} = -Mg + F_{th} = -Mg + \left|\frac{dM}{dt}\right|\left|u_{exh}\right| = M\frac{dv_y}{dt}$$

acceleration is $\frac{dv_y}{dt} = -g + \frac{1}{M}\left|\frac{dM}{dt}\right|\left|u_{exh}\right|$ **and**

$$M(t) = M_0 + \frac{dM}{dt}t = M_0 - Rt$$ **(*R* is burn rate)**

put *M*(*t*) into the equation for acceleration

$$\frac{dv_y}{dt} = \frac{R\left|u_{exh}\right|}{M_0 - Rt} - g$$

If the rocket starts at rest at t=0, we can integrate this to get $v_y = u_{\text{exh}} \ln\left(\frac{M_0}{M_0 - Rt}\right) - gt$

which is also $v_y = u_{\text{exh}} \ln\left(\frac{1}{1 - Rt/M_0}\right) - gt$

some features:

The bigger u_{exh} the better

The ratio R/M_0 is what counts

Since the final mass (payload) is $M_0 - Rt_{\text{final}} = M_p$ and the fuel mass is Rt_{final} the velocity at burn out is

$$v_{y,\max} = u_{\text{exh}} \ln\left(\frac{M_{\text{fuel}} + M_p}{M_p}\right) - gt_{\text{burn-out}}$$

so it is good to maximize M_{fuel} / M_p

because ln(x) increases slowly with x, it takes a lot of fuel to work a rocket.

Demo