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Question I. (20 pts) Projectile motion

A ball is thrown at 30^o above the horizontal. The initial speed is 25 m/s. Ignore air resistance.

1. (5 pts) How much time does it take to reach its highest point?

   A 2.6 s B. 1.3 s C. 2.2 s D. 0.8 s E. 1.6 s

   We have initial velocity's vertical component. $v_{y0} = v_0 \sin(\theta)$ During the trip to the highest point the acceleration of gravity changes this vertical velocity component to 0. We know the change in velocity under constant acceleration is $\Delta\vec{V} = \vec{a}t$. So put in –g for the acceleration and you've got it.

2. (5 pts) What is its speed at its highest point?

   A 0.0 m/s B. 25 m/s C. 22 m/s D. 13 m/s E. 29 m/s

   At the highest point the vertical component of velocity is 0 and the horizontal component is the same as the initial horizontal component, since there is no horizontal acceleration. So it is $v_x = v_{x0} = v_0 \cos(\theta)$ and the velocity vector just has this one component, so the speed is the magnitude of it.

3. (5 pts) How high does it go?

   A 18 m B. 0.64 m C. 1.2 m D. 16 m E. 8.0 m

   Again, using the initial vertical component of velocity and noting that the acceleration of gravity changes this to 0, you can use the equation relating velocity, distance and acceleration: $V_f^2 = V_i^2 + 2a\Delta y$ where v_f=0, v_i=v_{y0}, a = -g and you get Δy. Alterntely, since you know how long it takes to get to the top, you can use the equation relating distance, initial velocity, and acceleration $\Delta y = v_{y0}t + \frac{1}{2}at^2$ where t is the answer from part 1.

4. (5 pts) How far horizontally does it go? (It starts and ends at the same height.)

   A 55 m B. 32 m C. 28 m D. 65 m E. 16 m

   Here you can use the total time of flight (2 times the answer to part 1, since the ball goes up in that time, and then comes back down) times the initial x component of velocity, since there is no acceleration in the x direction. This component of velocity is the answer to part 2.

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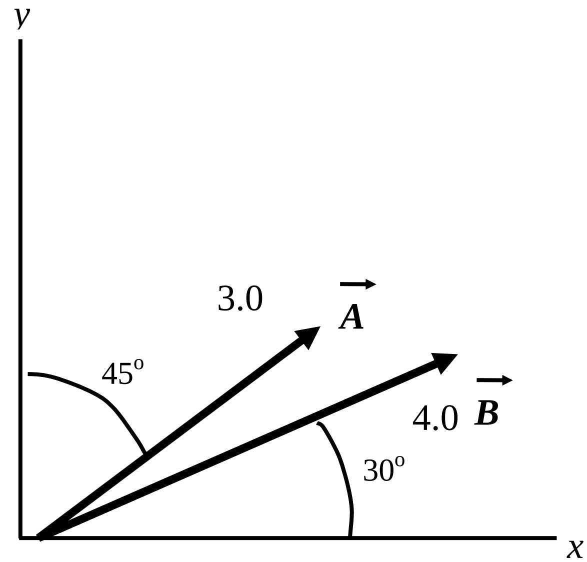
Question II (15 pts) Vector manipulation
Vectors ***A*** and ***B*** are shown on the diagram.



5. (5 pts) Which component is the largest?

   A. A_y B. A_x C. B_y D. B_x

   You can look at the picture, or if you don't trust it note that $B\cos(30^o)$ is greater than $B\sin(30^o)$, and also greater than $A\sin(45^o)$, since B is 4 and A is 3.

6. (5 pts) What is the magnitude of the y component of the vector ***A*** + ***B*** ?

   A. 2.0 B. 2.5 C. 5.0 D. 4.1 E. 5.5

   Here you have to find the y components of the two vectors and add them up. They are $A\sin(45^o)$ and $B\sin(30^o)$, which are 2.1 and 2.0, respectively.

7. (5 pts) What is the magnitude of the vector ***A*** – ***B*** ?

   A. 6.9 B. 1.3 C. – 1.3 D. – 6.9 E. 1.0

   Here you have to get both the components, and subtract them to get the components of the difference vector. Once you have those you get the magnitude from the square root of the sum of their squares.
   Note that the magnitude of a vector must not be negative, and if two vectors point sort of in the same direction, the magnitude of the difference should not be bigger or even near the sum of the magnitudes. So that leaves only B and E as plausible answers.

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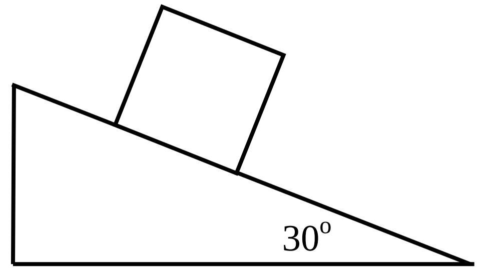
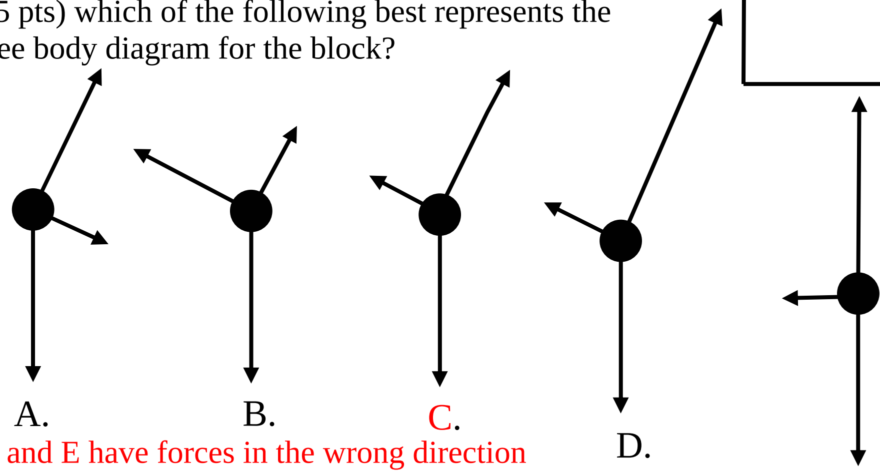
Question III (10 pts)
A block is sitting still (not accelerating) on an incline, as shown in the diagram:



8. (5 pts) which of the following best represents the free body diagram for the block?



A. B. C. D. E.

A and E have forces in the wrong direction
B has a friction force that is too big and D has a normal force that is too big. (The sum of the 3 forces should be 0, since the block is sitting still.) C is the only diagram where the force vectors sum to 0.

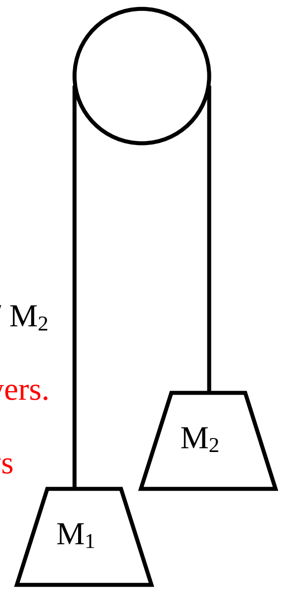
9. (5 pts) Which one of the following must be true?

A. The coefficient of kinetic friction is $\tan(30^o)$.
B. The coefficient of static friction is $\tan(30^o)$.
C. The coefficient of static friction is greater than or equal to $\tan(30^o)$
D. The coefficient of kinetic friction is greater than or equal to $\tan(30^o)$
E. There is no friction, because the block is at rest.

Taking free-body diagram C above, see that the weight has a component along the incline of $mg \sin(30^o)$ (pointing down the incline) and one of $mg \cos(30^o)$ pointing perpendicular to the incline's surface and downward. The first is canceled by the friction force, and the second by the normal force exerted by the surface on the block, F_n. The friction force is less than or equal to $\mu_s F_n$ so $\mu_s F_n \geq F_f = mg \sin(\theta)$ and $F_n = mg \cos(\theta)$. Put this F_n into the inequality, and divide both sides by $mg \cos(\theta)$. Recall $\tan(\theta) = \sin(\theta) / \cos(\theta)$.

Question IV (10 pts) Masses on a string

Two masses are attached to a string run over a frictionless, massless pulley as in the diagram. M_1 is moving upward at 1.0 m/s at t=0.



10. (5 pts) If $M_1 = M_2$, the tension in the string is

A $2M_1g$ B. M_1g C. $(M_1 + M_2)g$ D. $(M_1 + M_2)g/\sqrt{2}$

Remember that if a weight pulls on one end of a string and the string pulls back, the tension is equal to the weight. So the tension is either M_1g or M_2g, which are equal.

11. (5 pts) If $M_1 > M_2$, M1 accelerates downward with acceleration

A 2g B. 0.5g C. $(M_1 - M_2)g/ (M_1 + M_2)$ D. $(M_1 - M_2)g / M_2$

We could solve this problem, but it is more fun to eliminate wrong answers. If M_1 is equal to M_2 it won't accelerate. So A and B can't be right. If M_1 is much bigger than M_2 then it will accelerate nearly at g. But D says its acceleration would be much bigger than g. C has the right limiting behavior for nearly equal M's and for the case M_1 is much bigger than M_2 so it is the only choice.

12. There was no question 12.

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Question V (25 pts) Motion on a curve

A car of mass 1000 kg is traveling on a curve of radius 250 m and is slowing down with constant deceleration. The car enters the curve at 25 m/s and after going 75 m it has slowed to 20 m/s.

13. (6 pts) Compute the tangential acceleration of the car.

use distance, v_i, v_f , relation to get a, which is 1.5 m/s^2 in the direction opposite v.)

$$v_f^2 = v_i^2 + 2as \rightarrow a = \frac{v_f^2 - v_i^2}{2s} = \frac{20^2 - 25^2}{2(75)} = -1.5\text{m/s}^2$$

preferably give direction, but – sign is ok. Since the car is slowing with constant deceleration, and the tangential acceleration is doing the slowing, the tangential acceleration is constant, as is required by that equation.

14. (6 pts) Where on the curve does the car have the largest magnitude of acceleration?

At the beginning, where the centripetal acceleration is max., because the speed is max and centripetal acceleration is v^2/r. The "magnitude of acceleration" is the size of the whole acceleration vector which may have several components. In this case it has both a tangential and a centripetal component.

15. (6 pts) What is the largest magnitude of acceleration?

a_c is v^2/r which is 2.5 at the beginning: $a_c = v^2 / r = 25^2 / 250 = 2.5$

a_t = - 1.5 so magnitude is $\sqrt{1.5^2 + 2.5^2} = 2.9\text{m/s}^2$

16. (7 pts) If the magnitude of acceleration was 3.5 m/s^2, what must the minimum coefficient of static friction between the tires and road be if the car is not to skid? (Note, a moving car has the surface of the tire stationary at the point of contact between the tire and the road if it is not skidding.) The curve is not banked.

Friction force is μmg, need ma =3.5m = μmg, so $\mu = 3.5/g = 0.36$ Remember μ is dimensionless, since it relates one force to another and the forces have the same dimensions.
I gave a different magnitude of acceleration than the result in 15, so a solution to 15 wasn't required to do 16.

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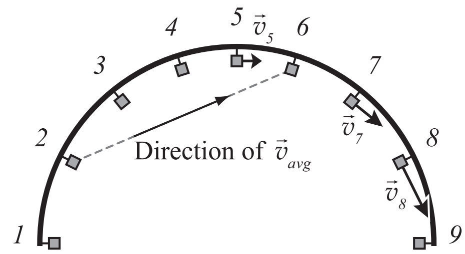
17. [20 points total] A hanging style rollercoaster car enters a semi-circular portion of track from the bottom left. A strobe photograph reveals the position of the car at equal time intervals, as shown at right.



A. [6 pts] On the diagram, draw vectors to indicate the instantaneous velocity of the car at points 5, *7,* and *8*. If the magnitude of any of the vectors is zero, state so explicitly. Explain why you drew the vectors the way you did.

The car slows down until it reaches the top, at which point it begins to speed up. This means the speed of the car is smallest at point 5 and largest at point 9. The relative speeds are indicated by the length of the arrows. Instantaneous velocity points in the direction of motion, which is tangent to the trajectory of the object.

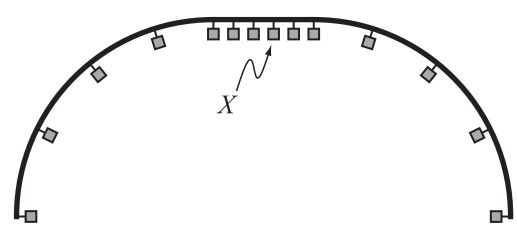
B. [4 pts] Is the magnitude of the average velocity between points *1* and *2* $\boxed{\textbf{\textit{greater than}}}$, *less than,* or *equal to* the magnitude of the average velocity between points 5 and 6*?* Explain.

Average velocity is defined as $\vec{v}_{avg} \equiv \Delta\vec{x}/\Delta t$. *The magnitude of the car's displacement from point 1 to point 2 is greater than that from point 5 to point 6, yet the time interval between points 1 and 2 is the same as that between points 5 and 6. Thus the magnitude of the average velocity between points 1 and 2 is* ***greater than*** *that between points 5 and 6.*

C. [4 pts] On the diagram above, draw an arrow to represent the direction of the average velocity of the car between points *2* and *6*. Explain why you drew the arrow the way you did.

From the definition above, the average velocity could only point in the same or the opposite direction as the displacement, depending on the sign of Δt. *(Physically time only increases, so) the denominator in the definition is always positive. Thus the average velocity always points in the same direction as the displacement, in this case from point 2 to point 6.*

D. [6 pts] Suppose a length of straight track were inserted as shown at right. A similar strobe photograph reveals the position of the car at equal time intervals. Again, the car enters from the bottom left of the track. . The speed of the car at point *X* is the same as it was at point *5* in the original situation.



Is the magnitude of the acceleration of the car at point *X* $\boxed{\textbf{\textit{greater than}}}$, *less than,* or *equal to* the magnitude of the acceleration of the car at point *5* in the original situation? Explain.

On the straight, horizontal portion of the track, the car's speed remains the same, and its direction of motion is not changing; therefore the magnitude of the acceleration of the car at point X is zero. Since the direction of the motion of the car is changing at point 5 in the original situation, the magnitude of the acceleration at point 5 must be greater than zero. Thus the magnitude of the acceleration of the car at point 5 is ***greater than*** *that at point X.*