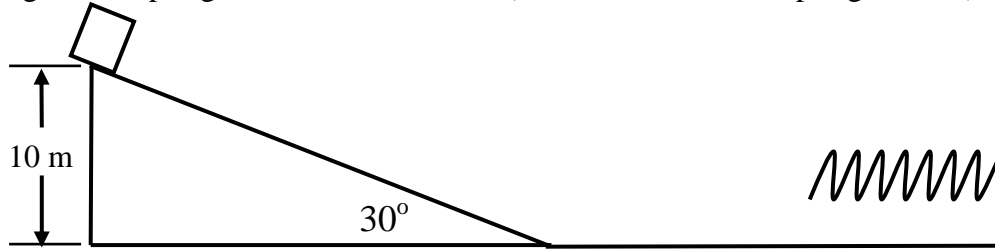


Question I. (20 pts) Energy

A 4.5-kg block slides down a 10-m high, 30° ramp with no friction. It starts at the top at rest. At the bottom it goes horizontally briefly, still with no friction, and then hits a spring whose spring constant is 100 N/m. (The other end of the spring is fixed).



1. (5 pts) How far does the spring compress?

Conservation of mechanical energy enables us to equate the initial potential energy (when the block is at rest with $K=0$ at the top of the ramp) with the final potential energy of the spring, when the block again has $K=0$. Thus $mgh = \frac{1}{2} kx^2$ so $x = \sqrt{2 m g h / k} = \sqrt{9 \cdot 9.8 \cdot 10 / 100} = 2.96 \text{ m} = 3.0 \text{ m}$ if we include the correct number of significant figures.

A 8.8 m B. 3.0 m C. 2.1 m D. 6.0 m E. 10.0 m

2. (5 pts) What is the kinetic energy of the block when the spring reaches the maximum compression?

When the spring is fully compressed the block has stopped and there is no K

A 98 J B. 9.8 J C 0.0 J D. 441 J E. 21 J

3. (5 pts) What is the kinetic energy of the block on the horizontal part before it hits the spring?

The block has descended the ramp and the original potential energy of mgh is turned into kinetic energy. The value of mgh is $4.5 \cdot 9.8 \cdot 10 = 441 \text{ J}$

A 98 J B. 9.8 J C 0.0 J D. 441 J E. 221 J

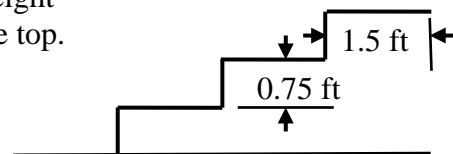
4. (5 pts) If there were some kinetic friction between the block and ramp, say $\mu_k = 0.1$, how would the spring compress, compared to the result in 1.?

Some friction would provide dissipation (provided friction were not so large that the block just stuck at the top and didn't slide) so the K at the bottom would be less than that that above. This would turn into U by compressing the spring, but less U than above, hence less compression.

A farther B. not as far C. the same

Question II (15 pts) Work and Power

A 110 pound student runs up 10 steps each of 0.75 ft height and 1.5 ft length. It takes her 4.0 s, and she stops at the top.



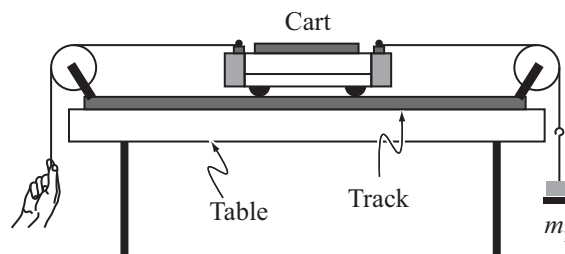
5. (5 pts) How much work does she do?
She does work against gravity, and by going up a height h this is mgh . (The work changes her gravitational potential energy.) There are 10 steps of 0.75 ft each, giving $h=7.5$ ft. Her weight is given in pounds, which is mg . So the work is $110 \cdot 7.5 = 825$ ft-lb. There is no friction because she isn't sliding on anything. Her legs do the work, lifting her C.M. up one step each time.
A 1650 ft-lb B. 825 ft-lb C 412 ft-lb D. 3300 ft-lb E. 206 ft-lb
6. (5 pts) Given that one horsepower (hp) is 550 ft-lb/s, how many horsepower does she produce while running up the stairs ?
We have the work from 5., and power is work/time. Power is then $825/4$ ft-lb/s. To get hp, we divide by the 550 given.
A 3.0 hp B. 0.38 hp C 0.19 hp D. 1.5 hp E. 0.09 hp
7. (5 pts) If there were 20 of the same size steps, and she took 8.0 s. Compared to the two answers above, which statement is true?
The work is doubled (twice the displacement) and the time was doubled too, so the power, work per unit time, is unchanged.
A. double work, same power B. same work, double power
C. double work and double power D. same work and same power

last

first

- I. [25 pts total] A cart of mass $m_C = 1.5 \text{ kg}$ is attached to two massless inextensible strings, each of which is placed over a pulley. One string is pulled with a constant force of 3.0 N by a hand as shown at right. The other string is attached to a piece of metal with mass $m_R = 0.5 \text{ kg}$. Ignore friction. Let $g = 10 \text{ m/s}^2$.

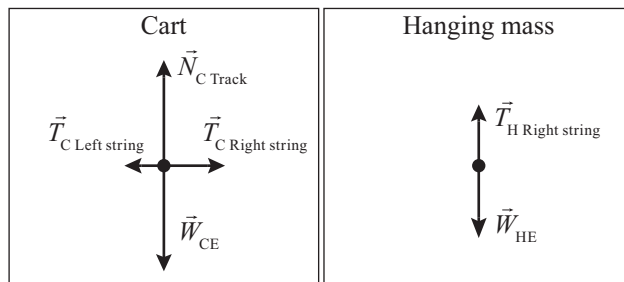
Consider the time starting just after the cart is released from rest and ending just before the cart hits the pulley.



- A. [6 pts] Draw separate free-body diagrams for the hanging mass and the cart.

For each force indicate the following information:

- the type of force
- the object on which each force is exerted
- and the object exerting each force.



- B. [5 pts] Rank the magnitudes of the following quantities in order from largest to smallest:

- a_L , the acceleration of the hand,
- a_R , the acceleration of the hanging mass,
- a_C , the acceleration of the cart,

Explain.

Since the strings are inextensible, [the lengths of the strings are constant, so any displacement of the cart must be accompanied by the same magnitude displacement of the hand and hanging mass. Because these displacements would happen over the same time interval, the speeds of the three objects would all be the same. Thus any change in speed of the cart over an interval must be the same as the change in speed of either of the other two objects in the same interval, meaning] all three accelerations have the same magnitude.

Note: The reasoning inside the [] is not required for full credit.

- C. [7 pts] Determine the magnitude and direction of the acceleration of the cart. Show your work, including any equations you used.

Take the positive direction to be to the right for the cart and down for the hanging mass. Consider the system containing the cart, the hanging mass and the string connected to both. The external forces acting on this system are: $T_L = 3 \text{ N}$ in the negative direction and $W_R = 5 \text{ N}$ in the positive direction, thus the net force is 2 N in the positive direction. By Newton's 2nd law, $F_{\text{net}} = (m_C + m_R)a$ or $2 \text{ N} = (2 \text{ kg})a$, therefore $a = 1 \text{ m/s}^2$ in the positive direction (to the right).

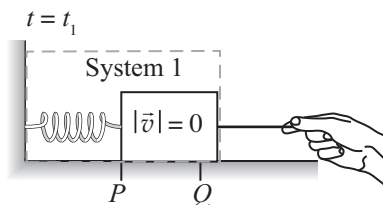
- D. [7 pts] Determine the magnitude of the tension force on the hanging mass by the right string. Show your work, including any equations you used.

*The forces on the hanging mass are T_R in the negative direction and W_R in the positive direction, so $F_{\text{net}} = W_R - T_R$ for the hanging mass. By Newton's 2nd law $W_R - T_R = m_R a$, or $T_R = W_R - m_R a$. Thus $T_R = 5 \text{ N} - 0.5 \text{ kg} * 1 \text{ m/s}^2 = 4.5 \text{ N}$.*

last

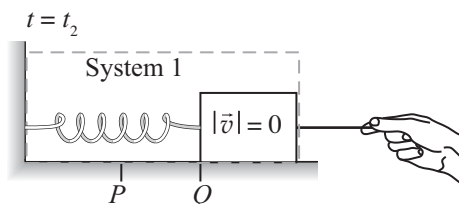
first

- IV. [20 pts] A block sits on a frictionless table and is connected to an ideal spring of negligible mass. A rope begins to exert a constant force on the block to the right, as shown in the diagram. At time t_1 , the block is at point P , its speed is zero, and the spring is at its equilibrium length. At time t_2 the block is at point Q and its instantaneous speed is again zero.



- A. [5 pts] During the interval from t_1 to t_2 is the acceleration of the block constant? Explain.

All forces acting on the block are constant except for the force by the spring, thus the net force on the block is not constant during the interval. By Newton's 2nd law, the acceleration of the block must also not be constant.



- B. For the following questions consider System 1, consisting of the block and the spring.

- i. [5 pts] For an instant between t_1 and t_2 , list all forces acting on System 1. For each force, indicate the object exerting that force and the type of force.

T_{IW} - Spring force by the wall (or tension force since the spring is stretched)

N_{IT} - Normal force by the table

W_{IE} - Gravitational force by the earth

T_{IR} - Tension force by the rope

- ii. [5 pts] For each force you listed in part A, state whether the work done on System 1 by that force is *positive*, *negative*, or *zero* as the block moves from P to Q . Explain.

The displacement of the point of application of N_{IT} , W_{IE} , and T_{IR} points directly to the right. Neither the normal nor the gravitational force has a component in this direction, so they do zero work. The tension force by the rope also points to the right, so it does positive work. The displacement of the point of application of the spring force by the wall (the left edge of the spring) is zero, so this force does zero work.

- iii. [5 pts] Suppose the rope is detached from the block at $t = t_2$ and the block begins to move back toward point P . At $t = t_3$ the block reaches point P . Note: the spring is at its equilibrium length when the block is at point P .

Is the magnitude of the net work done on System 1 from t_1 to t_2 **greater than**, **less than**, or **equal** to the magnitude of the net work done on System 1 from t_2 to t_3 ? Explain.

The forces acting on System 1 from t_2 to t_3 are the normal force by the table, the gravitational force by the earth and the spring force by the wall. For the same reasons as in part ii. above, these forces do zero work. Thus zero net work was done on the system during the interval from t_2 to t_3 . This is less than the positive net work done on System 1 from t_1 to t_2 (as discussed above).