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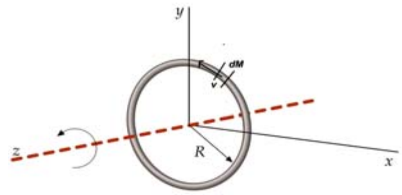
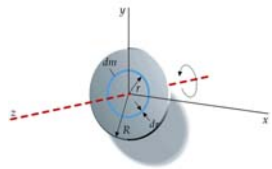
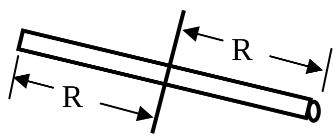
For reference, $I = fmR^2$ where f is given in the table:

Ring about axis through its center perpendicular to the plane it lies in.	1	
Disk, about axis perpendicular to it	1/2	
Sphere, about axis thru center	2/5	
Thin rod of length 2R about axis through center and perpendicular to the rod	1/3	

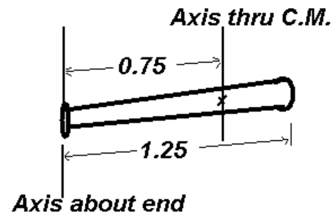
I. Multiple Choice
The following 9 questions are each worth 6 points, for a total of 54 points.

1. A 2 kg sphere of 0.10 m radius rolls without slipping down a ramp at 45 degrees to the horizontal, starting at rest. What is the ratio of its rotational kinetic energy to its total kinetic energy after it has rolled 1 m?

Rotational $K_r = \frac{1}{2} I\omega^2$ and translational $Kt = \frac{1}{2} mv^2$ while for non-slip rotation $\omega = v/R$. Since $I = fmR^2$ we can put these together and see $\frac{K_r}{K_r + K_t} = \frac{f}{1+f} = \frac{2/5}{7/5} = 2/7 = .29$ All the other figures are irrelevant.

A 0.40 B. 0.29 C. 0.08 D. 0.57 E. 1.00

2. A 2.0 kg bat has a moment of inertia about an axis perpendicular to it through its center of mass of 0.50 kg m^2. It is 1.25 m long and the C. M. is 0.75 m from the end where it is held. What is the moment of inertia about an axis perpendicular to it at the end where it is held?



Apply the parallel axis theorem, $I_A = I_{cm} + MX_{cm}^2 = 0.50 + 2.0(0.75)^2 = 1.63$ kg m^2

A. 0.50 kg m^2 B. 2.00 kg m^2 C. 1.13 kg m^2 D. 1.63 kg m^2 E. 1.50 kg m^2

Name (please print): ______________________________UW ID#_______________ score____
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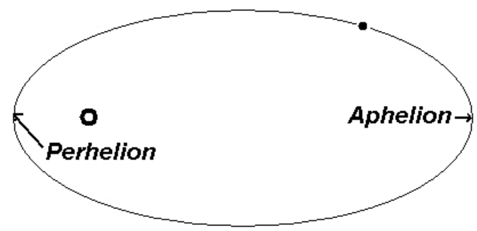
3. Satellites with mass m moving in any circular orbit around the Earth have the same value of which of the following? (Orbit period is T, radius R, orbital angular velocity of satellite is ω)

Keppler says (and it is because of Newton's law of gravity) that T^2 is proportional to R^3 so R^3/T^2 is a constant, and since $\omega = 2\pi/T$, $\omega^2 R^3$ is a constant. This applies to any orbit about the same massive body, with the body's mass determining the constant.

A. $T^2 R^3$ B. $\omega^2 R^3$ C. $\omega^2 R^3/m$ D. $\omega^2 R^3 m$ E. $T^2 R^3 m$

4. A comet has an orbit with aphelion (furthest distance to the Sun) of 5.0 astronomical units and perihelion (closest approach to the Sun) of 0.50 astronomical units. (1 astronomical unit is the average radius of the Earth's orbit about the Sun.) The ratio of its speed at aphelion to that at perihelion is



Because angular momentum is conserved by the comet in its orbit, when $\vec{r}$ is perpendicular to $\vec{v}$ the angular momentum is just r times v. Thus $r_a v_a = r_p v_p$ so $v_a / v_p = r_p / r_a = 0.5/5.0 = 0.10$.

A 0.10 B. 0.010 C. 1.0 D. 10. E. 3.16

5. For that same comet (previous question), the ratio of its angular momentum about the Sun at aphelion to that at perihelion is

Since angular momentum is conserved, it is the same everywhere and the ratio is 1.0

A 0.10 B. 0.010 C. 1.0 D. 10. E. 3.16

For 6 and 7, The Earth's period of rotation (the day) is gradually increasing (angular velocity is decreasing) because of torque applied by the moon via tides. (Assume the Earth-Moon system continues on the same circular orbit around the Sun, and can be treated as an isolated system.)

6. Which of the following is true for the Earth-Moon system?

The torque provided by the tides is an internal torque, so total angular momentum of the system is conserved, while the tidal forces are dissipative and reduce the total kinetic energy of the system. Since the day is lengthening, the internal torques are transferring angular momentum from the Earth to the Moon

A. Total angular momentum is conserved, and the rotational angular momentum the Earth is losing is transferred to the Moon (mostly to the orbit).
B. Total rotational energy is conserved for the system and the rotational energy the Earth is losing is transferred to the Moon (mostly to the orbit). – No, energy is not conserved.
C. Neither the angular momentum nor the rotational energy of the system is conserved, because of the dissipation of the tides. No, angular momentum is conserved.
D. The tides are on the Earth, and so the Moon is not affected by them. Newton's third law says the moon and Earth are connected.
E. Both total rotational kinetic energy and total angular momentum of the system are conserved. No, the tides dissipate energy.

7. Which of the following is true?

The angular momentum taken from the rotating Earth increases the angular momentum of the Moon's orbit (the rotational angular momentum of the moon is small, since it has the same ω as the orbit, and the momentum of inertia of the moon as a sphere is very small compared to the momentum of inertia of the moon in the orbit with a big radius). A circular orbit in a gravitational field has angular momentum increasing with radius. (Speed is proportional to R/T and Keppler gives T proportional to $R^{3/2}$. Thus speed decreases with R. So angular momentum, Rv, is proportional to $R^{1/2}$.)

A. The Moon's orbit radius is increasing and the total rotational energy of the system is decreasing.
B. The Moon's orbit radius is not changing. No, the radius must increase as L does.
C. The Moon's orbit radius is increasing and its kinetic energy is increasing Although it is hard to see how the kinetic energy could increase as a result of the tidal dissipation, we can see that with the orbit increasing the speed is decreasing so the kinetic energy is decreasing too.
D. The Moon's orbit radius is decreasing and its total energy is decreasing No, the radius must increase as L does.
E. The Moon's orbit radius is decreasing and its total energy in increasing No, the radius must increase as L does.

Name (please print): ______________________________UW ID#_______________ score____
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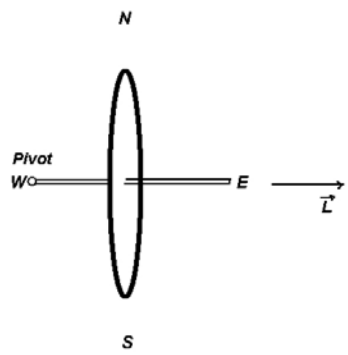
8. The spin angular momentum of a wheel along its axle points East. The wheel can pivot around the West end of its axle. To cause this vector to rotate toward the South, in which direction must a force be exerted on the East end of the axle?



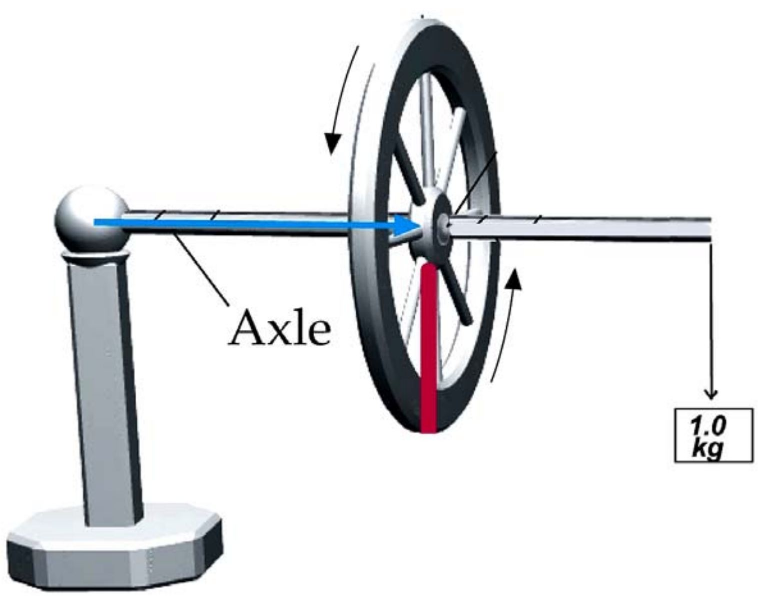
To make the angular momentum rotate to the South, we must add some angular momentum pointing to the South, and therefore we must provide a torque pointing toward the South. For the R vector from the pivot pointing East, an upward force will make such a torque.

A. East B. down C. North D. South E. up

9. A 0.50 kg gyroscope, consisting of a wheel mounted symmetrically between the ends of an axle, is spinning with the axle in the horizontal plane. It is supported at one end of the axle and is precessing at 0.10 rad/s When a 1.0 kg weight is attached to the free end of the axle, how fast will the gyroscope precess?

The initial precession is a result of the torque due to the gyroscope's weight acting at the c.m. one half the total axle length from the pivot. The added weight is twice as big and has twice the R, so the torque is 4 times the original. Thus the precession will increase by 4 times the original to a total of 5 times the original.

A. –0.10 rad/s B. 0.10 rad/s C. 0.30 rad/s D. 0.40 rad/s E. 0.50 rad/s



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II. (26 pts) Gyroscope

A motor driven gyroscope starts spinning from rest. The motor applies a constant torque of 0.5 Nm and the gyroscope has a moment of inertia of 0.2 kg m^2. The mass is mostly contained in a disk of radius 7 cm (0.07 m).

A. (5 pts) How long does it take for the gyroscope to spin up to 500 rad/s?
$\omega = \alpha t$ will relate time, change in speed (final speed from a start at rest) and angular acceleration. Use $\tau = I\alpha$ to find the angular acceleration. Thus
$t = \omega / \alpha = \omega I / \tau = 500(0.2)/0.5 = 200\ s$

B. (4 pts) How many revolutions per minute is 500 rad/s?
Remember that $\omega = 2\pi f$ so f = 500/2π=79.6 r.p.s. Or if you don't remember that, use the fact that 2π radians is a full circle. Then multiply by 60 s/min to get 4775 r.p.m. Note, since the question explicitly asked for "revolutions per minute" an answer with no specification of units is acceptable.

C. (6 pts) Once the gyroscope is spinning at 500 rad/s, the motor is turned off and somebody tries to stop the spinning by rubbing on the rim of the disk with his finger. He is able to apply a frictional force of 2 N. How many revolutions does the gyroscope make after the finger is applied and before it stops?
The torque is *FR* where *F* is the 2 N and *R* is the 0.07 m radius of the disk. As in A., we need α and could use $\omega_f^2 = \omega_0^2 + 2\alpha\theta$ with 0 for ω_f, 500 for ω_0, and get α from $\tau = I\alpha = FR$. Since we are slowing the thing, α is negative relative to θ and we get $\theta = \dfrac{\omega_0^2 I}{2FR} = \dfrac{500^2(0.2)}{2(2)(0.07)} = 179 \times 10^3$ radians. Since the number of revolutions is θ/(2π), it is 28 x 10^3 revolutions. As a check, note the average angular speed as it is brought to a stop is 500/2 = 250 rad/s so it takes 716 s to stop the wheel. This is 3.57 times the result in A., which corresponds to the torque ratio, 0.5/0.14, being 3.57 also.

D. (5 pts) What was the rotational energy of the gyroscope when it was going 500 rad/s, and where did this energy go when the operator stopped it?
$K = \frac{1}{2} I\omega^2 = \frac{1}{2}(.2)500^2 = 25\ \text{kJ}$ which went into heat, perhaps burning the operator's finger.

E. (6 pts) The wire to the motor breaks, so the operator will spin the gyroscope (starting from rest) by winding a string around the axle (of 3 mm radius) and pulling the string. It is possible to wind 1.0 m of string on the axle and pull it without slipping on the axle. How much force would the operator have to apply to the 1.0 m string to get the gyroscope to spin at 500 rad/s?
Although it is tempting to use angular information to do this, the easiest way is to note that you know the required kinetic energy, you have a distance, and the pull force times distance is energy. Hence you need work of 25 kJ = F s with s the 1 meter of string. Thus F=25 kN. (about 2.5 tons.) It is unlikely the operator (or the string) is strong enough to apply this, so he will fail in achieving the 500 rad/s desired.

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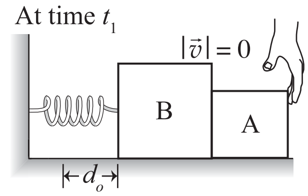
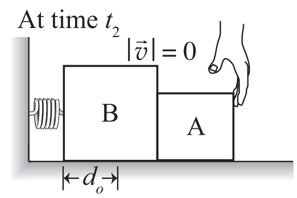
IV. [20 points total] Blocks A and B are at rest on a level, frictionless surface. Block B is attached to an ideal massless spring of constant k, which is initially at its equilibrium length. Block B has greater mass than block A ($m_B > m_A$).



During the interval from t_1 to t_2, a hand pushes block A to the left with a constant force of magnitude F_o, compressing the spring a distance d_o, as shown at right. At time t_2, both blocks again have a speed of zero.

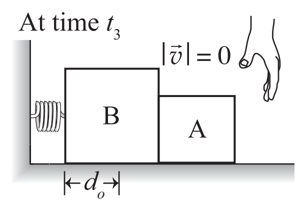


Consider system A consisting of block A and system BS consisting of block B and the spring.

A. [5 pts] During the interval from t_1 to t_2, is the magnitude of the net work done on system BS by external forces, *greater than, less than,* or *equal to* that done on system A by external forces? Explain.

$W_{net,ext} = \Delta E_{tot} = \Delta K + \Delta U$. *The change in kinetic energy of both systems is zero. Only system BS had a non-zero change in potential energy. Therefore, the absolute value of the net work done on system BS is greater than that done on system A.*

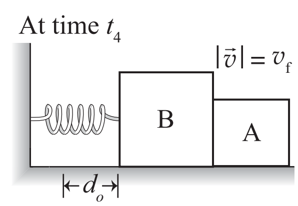
B. At a later time t_3, both blocks have a speed of zero, and the hand releases block A. At time t_4, the spring has returned to its equilibrium length and the blocks have speed v_f, as shown at right.



Consider the interval from t_3 to t_4:

i. [5 pts] Is the change in kinetic energy of system BS *greater than, less than,* or *equal to* that of system A? Explain.

$\Delta K = \frac{1}{2} m(v_f^2 - v_i^2)$. *Both systems have the same initial and final speeds, but system BS has greater mass than system A. Therefore, the change in kinetic energy of system BS is greater than that of system A.*



ii. [5 pts] Is the absolute value of the net work done on system BS by external forces *greater than, less than,* or *equal to* that done on system A by external forces? Explain.

The only external force that does work on system BS is the normal force on BS by A. The only external force that does work on system A is the normal force on A by BS. These two forces are equal in magnitude and opposite in direction by Newton's 3rd law. The displacement of the point of application of the force is the same for both systems. Therefore, the absolute value of the net work done on system BS by external forces is equal to that done on system A by external forces.

iii. [5 pts] Is the change in total energy of system BS *positive, negative,* or *zero?* Explain.

$\Delta E_{tot} = W_{net,ext}$. *From part ii, the only external force that does work on system BS is the normal force on BS by A. Since the force and the displacement of the point of application of the force are in opposite directions, the net work done by external forces on system BS is negative, and so is the change in total energy.*