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Question I. (20 pts) Projectile motion

A ball of mass 0.3 kg is thrown at an angle of 30^o above the horizontal. Ignore air resistance. It hits the ground 100 m from where it was released. Assume it starts and ends at the same height.

1. (5 pts) How fast was it going when it was released?
   To do this from basic principles, we note the vertical component of the initial velocity is v sin(30deg) and the horizontal one is v cos(30deg). The time the ball is in the air is 2 times the time it takes to reach its highest point, which we can get from the change in vertical velocity and the acceleration of gravity. v cos(30deg) = gt The ball is in the air twice this time, and goes 100 m, so 100m = 2 v sin(30deg) t. Solve the first eqn. for t, plug into the second and we get $100 = 2\ v^2\ \sin(30deg)\cos(30deg)/g$ which gives $v^2 = 100\ g / [2\ \sin(30)\ \cos(30)\ ]$ from which we can get v.
   A 95.1 m/s B. 24.3 m/s C. 33.6 m/s D. 31.3 m/s E. 47.6 m/s

2. (5 pts) An identical ball is thrown at the same angle with an initial speed of 40 m/s. How high does it go?
   Again, we take the vertical component, v sin(30deg) and see how far this goes with a negative acceleration of g. Since we have velocity, acceleration, and want distance, we use $v_f^2 = v_i^2 + 2a\ x$, giving $[v\ \sin(30deg)]^2 = 2g\ x$ or $x = [v\ \sin(30deg)]^2/(2g)$.
   A 57.7 m B. 20.4 m C. 31.3 m D. 33.6 m E. 24.3 m

3. (5 pts) How fast was this second ball (40 m/s initial speed) going at the highest point of its path?
   At the highest point the vertical component of velocity is 0 but the horizontal component is unchanged, and is v cos (30deg) = 40 (.866)
   A 0.0 m/s B. 25.0 m/s C. 20.0 m/s D. 34.6 m/s E. 40.0 m/s

4. (5 pts) If a ball of 0.6 kg were thrown with the same initial velocity as the one that went 100 m, from the same point, how far horizontally would it go? (Again, it starts and ends at the same height.)
   The trajectory is not dependent on the mass, so this ball goes the same distance. Note that "velocity" includes both magnitude and direction, so this ball also goes at 30 deg to the horizontal
   A 71 m B. 141 m C. 50 m D. 100 m E. 200 m

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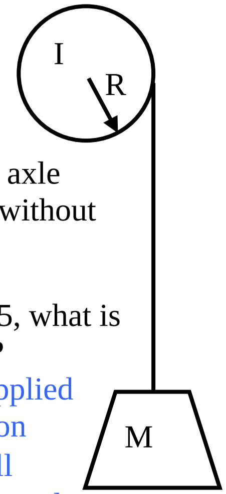
Question II (15 pts) Wheel and weight



A long string is wound on a wheel of radius R= 0.50 m and moment of inertia $I=2.0$ kg m^2. The string is attached to a weight of M=1 kg.
Initially the system is held at rest by a student whose finger is pressing on the rim of the wheel. Then the student removes the finger. The wheel has excellent axle bearings, so you can ignore rolling friction. The string unwinds from the wheel without slipping

5. (5 pts) If the coefficient of static friction between the finger and wheel is 0.5, what is the minimum normal force the student must apply to keep the system at rest?
   The finger must apply an equal magnitude and opposite sign torque to that applied by the string, namely MgR. The tangential force from the finger is the friction force, μF_N where F_N is the normal force we are after. The friction torque will automatically be opposite sign to the torque applied by the string, so we just need $\mu F_N R = MgR$ giving $F_N = Mg/\mu = 9.8/0.5$
   A. 4.9 N B. 9.8 N C. 19.6 N D. 0.5 N E. 39.2 N

6. (5 pts) One second after the finger is released, what is the angular acceleration of the wheel?
   Angular acceleration is produced by torque, and is the same any time after the finger is released. We have to calculate the tension in the string T, and there are 2 equations
   Torque on the wheel accelerating it: $TR = I\alpha$
   Force on the mass accelerating it. $Mg - T = Ma = MR\alpha$,
   since the angular and linear accelerations are related by $R\alpha = a$, because the string unwinds from the wheel. Solve the first for $T = I\alpha/R$ and substitute into the second:
   $Mg - I\alpha/R = MR\alpha$, which gives $\alpha = Mg\,R/(I + MR^2)$
   A. 2.18 rad/s^2 B. 2.45 rad/s^2 C. 19.6 rad/s^2 D. 3.27 rad/s^2 E. 6.54 rad/s^2

7. (5 pts) After the weight has fallen 1 m, what is its (the weight's) kinetic energy?
   We know the potential energy Mgh (h=1 meter) is converted to kinetic energy which is shared by the wheel and weight. Kinetic energies are
   $K_{rot} = ½ I\omega^2$
   $K_{trans} = ½ Mv^2$ with $\omega R = v$, because of the string. Thus the ratio of these is
   $K_{rot} / K_{trans} = I / MR^2$ so $K_{total} = K_{rot} + K_{trans} = (I / MR^2 + 1) K_{trans}$
   set this equal to Mgh and then $K_{trans} = Mgh/(I / MR^2 + 1)$
   Alternatively, we know the net work done on the weight is $W = h\,(Mg - T)$. From the previous problem, $TR = I\alpha$ with $\alpha = 2.18$ rad/s^2 giving $T = 8.72$ N.
   A. 9.81 J B. 29.4 J C. 2.18 J D. 4.91 J E. 1.09 J

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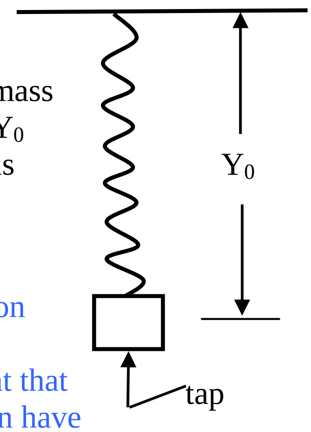
Question III (15 pts) Simple Harmonic Oscillator

A 2 kg mass is hanging from a spring which exerts 50 N for each meter of stretch, and obeys Hooke's law. The spring is attached to the ceiling and the distance from the ceiling to the mass, when the mass is stationary, is Y_0. The mass is initially at rest, and then it is given a small upward tap each time it reaches Y_0 and, after the first tap, only when it is moving upward. After ten such taps, it is moving at 3.0 m/s each time it passes Y_0 while going upward.



8. (5 pts) How fast is it going when it passes Y_0 going downward?
   The speed of a harmonic oscillator is the same magnitude at a given position regardless of the direction of motion. For example, the kinetic energy is a function of the position, and depends on velocity squared, so the velocity at that position has the same magnitude but depending on where in the cycle it can have either direction.
   A. 1.5 m/s B. 3.0 m/s C. 4.5 m/s
   D. 30.0 m/s E. 5.0 m/s

9. (5 pts) What is the amplitude of the oscillation?
   Using conservation of energy, we know the sum of K and U for the oscillator is a constant. The K the oscillator has at the equilibrium point, Y_0, is converted to U when it is stretched to the full amplitude. So $\frac{1}{2} kA^2 = \frac{1}{2} m v_0^2$ and from the statement of the problem, $k = 50$ N/m. Thus $A^2 = (m/k) v_0^2$
   A. 6.0 m B. 1.2 m C. 0.6 m D. 3.0 m E. 0.36 m

10. (5 pts) What was the total work done on the system of mass and spring by the 10 taps?
   Again, because energy is conserved, the total work is the kinetic energy at Y_0, which is $\frac{1}{2} mv^2$
   A. 90 J B. 9.0 J C. 18 J D. 100 J E. 50 J

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Question IV (10 pts) Orbits

The Hubble space telescope is in orbit around the Earth at an altitude of 590 km above the surface of the Earth, which has a radius of 6400 km. Assume the orbit is circular.

11. (5 pts) The value of g (acceleration due to Earth's gravity) at the Hubble's orbit is
Since $g(R) = GM_E / R^2$ gives g at any R and from this $g(R) / g = (R_E/R)^2$ where g is the usual value at the surface. The Hubble's radius is 6400+590 km, and squaring the ratio of R_E to this, times 9.8 m/s^2 gives the desired value
A. 9.8 m/s^2 B. 9.0 m/s^2 C. 8.2 m/s^2 D. 0.90 m/s^2 E. 0.98 m/s^2

12. (5 pts) The period of the orbit of the Hubble space telescope is 96 minutes. What would be the orbit radius (not altitude) for a satellite in a circular orbit about the Earth with period twice as long (193 minutes)?
If you remember Keppler's law, you know $T^2 \propto R^3$, or you can derive this using centripetal force = gravitational force; $\omega^2 Rm = GMm/R^2$, giving $\omega^2 R^3$ a constant for circular orbits around a given primary. Since $T \propto 1/\omega$ Keppler's law follows.
Thus if we double T we have to increase R by $2^{2/3}$. Thus the new radius is $2^{2/3}(6400+590)$ km.
A. 14.0×10^3 km B. 11.1×10^3 km C. 19.8×10^3 km
D 6.99×10^3 km E. 2.47×10^3 km

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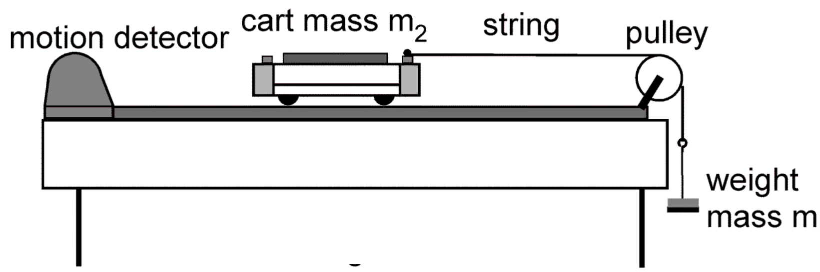
If you mark your answers on BOTH the bubble sheet and this page, you will have a record of your responses. Bubble sheets are not returned.

V. [25 points] 13 – 17: A hanging weight of total mass m_1 (including the carrier) causes the cart (mass m_2) shown in the figure to move with acceleration a. Neglect friction and the mass of the string.



13. (5pts) Assume the mass of the pulley is negligible. The acceleration a is given by which formula?

A. $a = \left(\frac{m_1}{m_2}\right)g$

B. $a = g$

C. $a = \left(\frac{m_1 + m_2}{m_1}\right)g$

D. $a = \left(\frac{m_2}{m_1 + m_2}\right)g$

E. $a = \left(\frac{m_1}{m_1 + m_2}\right)g$

If the tension is T, Newton's law for the two masses gives $m_1 g - T = m_1 a$ and $T = m_2 a$. The accelerations are the same because the masses are tied together with a string under tension. Eliminate T: $m_1 g - m_2 a = m_1 a$. Correct answer is E.

14. (5pts) Now consider the pulley mass to be **not** negligible. You carry out two experiments, experiment #1 with a pulley of mass M, and radius R, and experiment #2 with a pulley of mass $M/2$, and radius $2R$. Consider the pulleys to be like bicycle wheels, with all the mass at the rim. The pulley axle is positioned to make the string connected to the cart always horizontal. The masses m_1 and m_2 are unchanged. If the acceleration in experiment #1 is a_1, and in experiment #2 is a_2, which result should be obtained?

A. $a_1 > a_2$
B. $a_1 = a_2$
C. $a_1 < a_2$
D. Insufficient information to determine

If the pulley mass is not negligible, it is also being accelerated into angular rotation by the string (which does not slide on the pulley). The tension in the string to the left of the pulley is now less than to the right, and so m_2 accelerates more slowly than before. The masses are still tied together, so both accelerate the same. The force is gravity acting on m_1. The system being accelerated is m_1, m_2, and the pulley. For rotational motion $\tau = I\alpha$. With a tangential force F_T applied to the rim, the pulley rim has a tangential acceleration given by $F_T = \frac{I}{R^2}a$, and so $\frac{I}{R^2}$ serves as the mass in F=ma. The moment of inertia of the pulley is MR^2, so the radius squared drops out and the acceleration of the pulley just depends on its mass, which is less for the second wheel. Hence the correct answer is C.

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If you mark your answers on BOTH the bubble sheet and this page, you will have a record of your responses. Bubble sheets are not returned.

15. (5pts) You repeat your data for one of the experiments 3 times, obtaining:

a (Run#1) = 0.536 m/s^2
a (Run#2) = 0.551 m/s^2
a (Run#3) = 0.509 m/s^2

The mean acceleration a and the uncertainty in the mean are (choose the best answer):

A. 0.532 ± 0.005 m/s^2
B. 0.532 ± 0.012 m/s^2
C. 0.532 ± 0.014 m/s^2
D. 0.532 ± 0.017 m/s^2
E. 0.532 ± 0.027 m/s^2

First find the r.m.s. uncertainty: a) calculate the mean. b) calculate the difference of each measurement from the mean. c) square. d) average the 3 numbers. e) square root. That gives 0.017 m/s^2 which is the *"sample"* uncertainty (the uncertainty expected for a single measurement). The uncertainty in the *mean* is smaller by the square root of the number of measurements (best to use n-1 = 2, but 3 is ok). B is correct.

16. (5pts) You also determine the acceleration by measuring the time t it takes the cart to move a distance s along the track, starting from rest. If your measured values are s = 0.900 ± 0.001 m, and t = 1.854 ± 0.030 s, what is the uncertainty in the calculated acceleration (choose the best value)?

A. ± 0.1 %
B. ± 1 %
C. ± 1.6 %
D. ± 3 %
E. ± 4 %

The time, distance and acceleration are related by $s = \frac{1}{2}at^2$, so $a = \frac{2s}{t^2}$. For products and quotients, add the fractional uncertainties in quadrature. The fractional uncertainty in quantities raised to a power *n* is *n* times the fractional uncertainty in the quantity. The percentage uncertainty in *s* is 0.11% (which turns out to be negligible). The percentage uncertainty in *t* is 1.6%. The percentage uncertainty in t^2 is 3.2%. The correct answer is D.

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If you mark your answers on BOTH the bubble sheet and this page, you will have a record of your responses. Bubble sheets are not returned.

17. (5pts) Suppose the acceleration measured in an experiment like Question 15 is 0.608 ± 0.027 m/s^2 and the acceleration calculated from an experiment like Question 16 is 0.524 ± 0.027 m/s^2 . The two values

- **A.** are in agreement
- **B.** cannot be said either to agree or to disagree
- **C.** are not in agreement
- **D.** cannot be compared because one is experimental and one theoretical
- **E.** cannot be compared because one is random and the other systematic

To compare two numbers, see if the difference equals zero within the combined uncertainties. The difference here is $(0.084 \pm 0.038) \times 10^{-2}$ kg m^2 where the uncertainty is found by adding the individual uncertainties in quadrature (it's just √2 times one of the uncertainties, since they are the same). The numbers disagree by more than two standard deviations. The correct answer is C.

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~~IV.~~ [20 points total] In all of the following situations, identical rods are on a frictionless table. The rods are attached to strings and the strings are all exerting the same force. All the strings are oriented perpendicular to the rods.

Below, $|\vec{\tau}_{\text{net,rod}}|$ is the magnitude of the net torque on the rod about its center and $|\vec{F}_{\text{net,rod}}|$ is the magnitude of the net force on the rod.

~~1.~~ [5 pts] Choose the statement that best describes situation I:

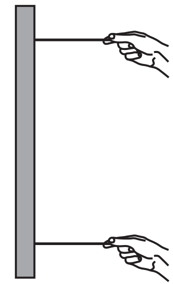
A. $|\vec{\tau}_{\text{net,rod}}| = 0, |\vec{F}_{\text{net,rod}}| = 0$

B. $|\vec{\tau}_{\text{net,rod}}| \neq 0, |\vec{F}_{\text{net,rod}}| = 0$

C. $\boxed{|\vec{\tau}_{\text{net,rod}}| = 0, |\vec{F}_{\text{net,rod}}| \neq 0}$

D. $|\vec{\tau}_{\text{net,rod}}| \neq 0, |\vec{F}_{\text{net,rod}}| \neq 0$

The top string exerts a torque into the page; the bottom string exerts a torque out of the page. Since the distances from the center of the rod to the points of application are the same, and the forces are the same magnitude, the two torques have the same magnitude. Thus the net torque is zero. The forces act in the same direction, so they can not sum to zero.



Situation I

~~2.~~ [5 pts] Choose the statement that best describes situation II:

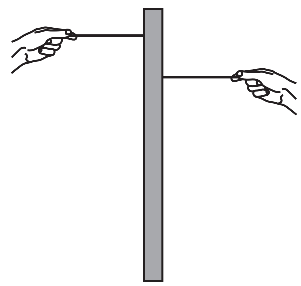
A. $|\vec{\tau}_{\text{net,rod}}| = 0, |\vec{F}_{\text{net,rod}}| = 0$

B. $\boxed{|\vec{\tau}_{\text{net,rod}}| \neq 0, |\vec{F}_{\text{net,rod}}| = 0}$

C. $|\vec{\tau}_{\text{net,rod}}| = 0, |\vec{F}_{\text{net,rod}}| \neq 0$

D. $|\vec{\tau}_{\text{net,rod}}| \neq 0, |\vec{F}_{\text{net,rod}}| \neq 0$

Since the distance from the center of the rod to the point of application of the force is greater for the top string, the two torques can not sum to zero even though the magnitudes of the forces are the same. The forces act in opposite directions and have the same magnitude and therefore sum to zero.



Situation II

~~3.~~ [5 pts] Choose the statement that best describes situation III:

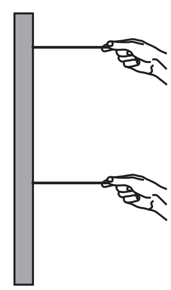
A. $|\vec{\tau}_{\text{net,rod}}| = 0, |\vec{F}_{\text{net,rod}}| = 0$

B. $|\vec{\tau}_{\text{net,rod}}| \neq 0, |\vec{F}_{\text{net,rod}}| = 0$

C. $|\vec{\tau}_{\text{net,rod}}| = 0, |\vec{F}_{\text{net,rod}}| \neq 0$

D. $\boxed{|\vec{\tau}_{\text{net,rod}}| \neq 0, |\vec{F}_{\text{net,rod}}| \neq 0}$

Since the distance from the center of the rod to the point of application of the force is greater for the top string, the two torques can not sum to zero even though the magnitudes of the forces are the same. The forces act in the same direction, so they can not sum to zero.



Situation III

~~4.~~ [5 pts] Choose the statement that best describes situation IV:

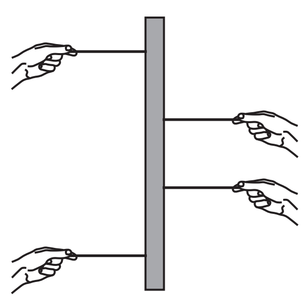
A. $\boxed{|\vec{\tau}_{\text{net,rod}}| = 0, |\vec{F}_{\text{net,rod}}| = 0}$

B. $|\vec{\tau}_{\text{net,rod}}| \neq 0, |\vec{F}_{\text{net,rod}}| = 0$

C. $|\vec{\tau}_{\text{net,rod}}| = 0, |\vec{F}_{\text{net,rod}}| \neq 0$

D. $|\vec{\tau}_{\text{net,rod}}| \neq 0, |\vec{F}_{\text{net,rod}}| \neq 0$

The two strings pulling to the left act at points the same distance, but opposite direction from, the center of the rod, and the magnitudes of the forces are the same so their torques cancel. Similarly, the torques by the two strings pulling to the right cancel. Since there are an equal number of forces to the left and to the right, the net force is zero.



Situation IV

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Question VII (25 pts)

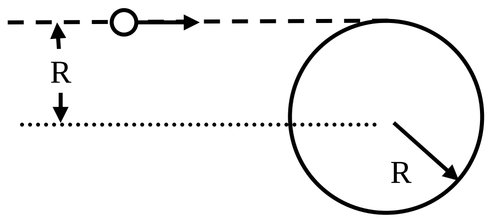
An object of mass 0.1 kg has kinetic energy of 24500 J, and is moving horizontally on a frictionless surface. It hits another object, of mass 1.0 kg, originally at rest on the same surface, and sticks to it.

A. (6 pts) What is the speed of the small mass before the collision?
$K = \frac{1}{2} m v^2$ so $v = (2K/m)^{1/2}$ = 700 m/s

B. (6 pts) What is the speed of the center of mass of the combined masses after the collision?
Momentum is conserved, so the initial momentum, mv_s, is equal to the final, $(m+M)v_c$ and here m is the little mass, M the big one, v_s is the speed of the small mass and v_c is the speed of the cm of the combined mass. So $v_c = mv_s/(m+M) = 0.1(700)/(1.0+0.1) = 700(0.1/1.1) = 63.6$ m/s

C. (6 pts) What is the translational kinetic energy after the collision?
The translational K is that of the cm, which is
$K = \frac{1}{2}(m+M)\, v_c^2 = \frac{1}{2}(1.1)(63.6)^2 = 2227$ J.

D. (7 pts) The large object is a uniform disk ($I = \frac{1}{2} mR^2$) of radius 0.2 meter, and the small one is moving tangentially to the disk when it hits the rim of the disk and sticks. What is the angular momentum of the combined disk and small (take it to be a point) object, about the center of the disk, after the collision?



Initial angular momentum is $\vec{R} \times \vec{p} = R_\perp p$ with $R_\perp$ =R.
This points into the page.
As there are no external torques, angular momentum is conserved, and the final angular momentum is also
$Rmv_s = 0.2(0.1)(700) = 14$ kg m^2 /s (pointing into the page)

Note that the final object will actually rotate nicely about its cm, not about the center of the disk. But we can calculate angular momentum about any point.

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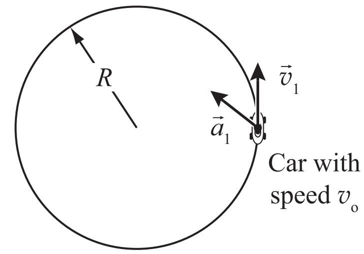
~~IV.~~ 1. [20 points] A race car drives in a counter-clockwise direction around a level circular track of radius R as shown at right and ***speeds up at a constant rate.*** At time t_1, the speed of the car is v_o.

$t = t_1$



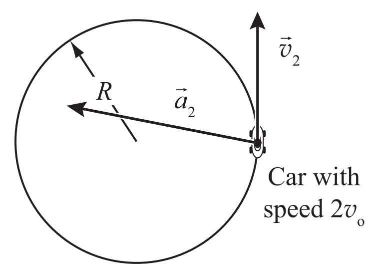
Top view

A. [5 pts] On the diagram, draw and label velocity and acceleration vectors for the car. (Draw both vectors with their tails on the object.) Explain.

The velocity vector will be tangent to the path as shown at right. The acceleration vector will have a tangential component since the car is speeding up. The acceleration vector will also have a perpendicular component since the car is changing direction. The resultant acceleration will make an acute angle with the velocity, as shown at right.

B. [12 pts] At time t_2, the car has traveled once around the circle and has a speed of $2v_o$ as shown below. (Recall that the car is speeding up at a constant rate.)

$t = t_2$



Top view

i. On the diagram at right, draw and label the velocity and acceleration vectors for the car.

ii. Is the direction of the acceleration of the car at time t_2 the same as that at t_1? If not, how it is different? Explain.

Since the car is speeding up at a constant rate, the parallel component of the acceleration remains the same. The perpendicular component is greater at $t = t_2$ since the speed is greater, therefore the angle between the acceleration and the velocity is greater at $t = t_2$.

iii. Is the magnitude of the acceleration of the car at times t_2 ***greater than***, *less than,* or *equal to* that at t_1? Explain.

Since the parallel components of the acceleration are the same at the two instants and the perpendicular component is greater at $t = t_2$, the magnitude of the acceleration is ***greater*** *at $t = t_2$.*

C. [3 pts] Suppose that the motion is repeated by a motorcycle with half the mass of the car. The motorcycle moves in exactly the same way as the car (*i.e.,* it has the same initial velocity and its speed increases at the same rate).

At time t_2, would the magnitude of the acceleration for the motorcycle be *greater than, less than,* or *equal to* the magnitude of the acceleration of the car? Explain.

Since the motions of the car and motorcycle are the same, their positions, velocities, and thus accelerations are also the same.