Some Final Comments on QCD: Nonperturbative Structure and the Vacuum (See, e.g., Chapters 16 and 17 of Rolnick.)

In the previous (5) lectures we focused on the (simple) perturbative structure of QCD where the particle content of the Lagrangian (the quarks and gluons) is most evident. However, as we have already discussed, the low energy regime of normal nuclear interactions (the spatially asymptotic states of the high energy analysis) is dominated by the infrared confining properties of the theory. In this regime we need to apply a different set of tools to understand what is happening. In particular, we must pay close attention to the structure of the vacuum, especially with respect to symmetry issues. In this closing lecture we will (only) touch on a short list of concepts that are useful in the low energy regime and (hopefully) will make contact with subjects that we have already mentioned.

The first concept is that of chiral symmetry, which we have mentioned several times before. The central idea is that the real world is usefully approximated by a model world where the masses of the $u$, $d$ and $s$ quarks are set to zero (compared to $\Lambda_{\text{QCD}}$). Of course, this is exactly the numerical approximation we used in most of our perturbative calculations. Here we wish to ponder the deeper implications of such a limit. Thus, for now, we want to turn off the spontaneous symmetry breaking supplied by the vev of the Higgs field, at least for the light quarks. We consider a Lagrangian density of the form (recall Lecture 30)

$$L_{\text{chiral}}^{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^{QCD,k} F^{QCD,\mu\nu,k} + \sum_{f=u, d, s} \bar{\psi}_{a,a,f} \left( i D_{\mu,ab}^{QCD,\nu} \psi_{b,b,f} \right), \quad (36.1)$$

where

$$F_{\mu\nu}^{QCD,k} = \left( \partial_{\mu} G_{\nu}^{k} - \partial_{\nu} G_{\mu}^{k} \right) - g f^{klm} G_{\mu}^{l} G_{\nu}^{m}. \quad (36.2)$$

With no mass term we can rewrite this Lagrangian with separate terms for the left-handed and right-handed quark states (suppressing non-essential indices)

$$L_{\text{chiral}}^{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^{QCD,k} F^{QCD,\mu\nu,k}$$

$$+ \sum_{f=u, d, s} \left\{ \bar{\psi}_{L,f} \left( i D_{\mu}^{QCD} \gamma^{\mu} \right) \psi_{L,f} + \bar{\psi}_{R,f} \left( i D_{\mu}^{QCD} \gamma^{\mu} \right) \psi_{R,f} \right\}, \quad (36.3)$$
In this chiral limit we see that this Lagrangian is invariant under two groups of global unitary transformations
\[ \psi_L \rightarrow U_L \psi_L, \psi_R \rightarrow U_R \psi_R, \quad (36.4) \]
which separately mix the flavors of the left-handed quarks and the flavors of the right-handed quarks. This Lagrangian is said to have a \( U(3)_L \times U(3)_R \) chiral symmetry. In turn, we expect the states of the true QCD Lagrangian (with small quark masses) to exhibit the corresponding approximate chiral symmetry, since the masses of the 3 lightest quarks are small compared to the characteristic QCD momentum scale provided by, for example, \( \Lambda_{QCD} \sim 200 \text{ MeV} \). Here we want to pursue the implications of this expectation.

Our previous discussions of global symmetries (in the autumn) suggest that this chiral theory should exhibit \( 9 + 9 = 18 \) conserved Noether currents corresponding to these symmetries. (Each \( U(3) \) is an \( SU(3) \) with 8 generators plus 1 for the remaining \( U(1) \).) To discuss these expected currents it is convenient to switch from the \( L-R \) basis to the \( V \)-\( A \) (vector-axial vector) basis that we have used in our earlier discussions of currents, where \( V = R + L \) and \( A = R - L \). Using the standard “5” notation for the latter, the expected currents look like
\[
\left( \begin{array}{c}
\psi \\
\gamma_5 \psi
\end{array} \right),
\quad \left( \begin{array}{c}
\psi \\
\gamma_5 \psi
\end{array} \right)
\]
\[
SU(3)_V \times U(1)_V : J^{a}_\mu = \bar{\psi} \gamma_\mu \lambda^a 2 \psi, \quad J^{a}_\mu = \bar{\psi} \gamma_\mu \lambda^a 2 \psi,
\quad (36.5)
\]
where the \( \lambda^a, a = 1-8 \), are the Gell-Mann matrices representing the 8 generators of \( SU(3) \). (Note that here they are applied to a global flavor symmetry and not the local color symmetry.) Based on the symmetry of the chiral Lagrangian we expect these currents to be conserved,
\[
\partial^\mu J^{a}_\mu = \partial^\mu J^{a}_\mu = \partial^\mu J^{a}_\mu = \partial^\mu J^{a}_\mu = 0.
\quad (36.6)
\]
In the “real” world we observe that the corresponding vector symmetries of (global) flavor \( SU(3) \) invariance and baryon number \( U(1) \) conservation are nearly exact and are understood to be exact in the limit of zero quarks masses (no Higgs vev). For example, recall from autumn quarter that the observed hadrons come in complete representations of flavor \( SU(3) \), i.e., the content of the quark model, with their mass degeneracy broken by the nonzero quark masses (the Gell-Mann-Okubo mass...
formula). (Note that the SU(2) subgroup here is the strong isospin symmetry group and not the local gauge symmetry group of the electro-weak interactions.) On the other hand, the full chiral symmetry (both $V$ and $A$) does not seem to be realized in the “real” world. For the full symmetry we would require that the observed fermions, e.g., the proton, either be massless or appear in parity doublets, e.g., the $\frac{1}{2}^+$ proton should have a $\frac{1}{2}^-$ partner. Neither scenario is even approximately true independent of the size of quark masses. (Recall that the bulk of the proton’s mass comes from the binding, i.e., the “gluon cloud”.) Thus we conclude that, even in the limit of zero quark masses when we turn off the Higgs mechanism, something else must be going on in “real” QCD in order to break the chiral symmetry.

What is believed to be happening (although not all of the details have been thoroughly confirmed experimentally or via lattice calculations) is that QCD exhibits dynamical symmetry breaking (recall Lecture 17). Similarly to the Higgs mechanism, it is the structure of the vacuum (not the Lagrangian) that provides the breaking. We believe that the lowest energy state corresponds not to an empty vacuum but rather to a scalar condensate (think Cooper pairs) of quark-anti-quark pairs (traced over flavor),

$$\langle 0 | \overline{\psi} \psi | 0 \rangle = \langle 0 | \overline{\psi}_R \psi_L + \overline{\psi}_L \psi_R | 0 \rangle \neq 0.$$  \hspace{1cm} (36.7)

(This is the mechanism that is thought to be at work in the context of the so-called Technicolor interaction that replaces the Higgs mechanism in electro-weak symmetry breaking in the Technicolor theory.) Thus the vacuum is no longer invariant under separate rotations of the left-handed and right-handed quarks. Rather it is invariant under only the combined transformations where both the left- and right-handed states rotate together. We expect the original chiral symmetry to be broken down to just the observed vector symmetry (in the absence of quark masses)

$$SU(3)_L \times SU(3)_R \xrightarrow{\langle \overline{\psi} \psi \rangle \neq 0} SU(3)_V.$$  \hspace{1cm} (36.8)

From our discussion about symmetry breaking, we know that the breaking of such a global symmetry implies the appearance of corresponding massless Goldstone bosons. These are identified with the observed octet of pseudoscalar $0^-$ mesons (the pions and kaons). Of course, in the real world (with a nonzero Higgs vev) the quarks have nonzero masses and the chiral symmetry is only approximate. In this case, the dynamical breaking of the approximate chiral symmetry yields so-called “pseudo-Goldstone” bosons that are very light, rather than massless. Further, as we have
already observed, the masses of the pseudo-Goldstone bosons should vanish in the limit of zero mass for the quarks. Since the SU(2) chiral symmetry is closer to being respected than the full SU(3) symmetry (the u and d quarks are more nearly massless than the s quark), we expect the pion to be quite light and it is! The structure of nuclear physics is driven by the small pion mass and thus the approximate chiral symmetry. This connection between the non-conservation of the axial current and the pion mass is encoded in the pion matrix element of the divergence of the axial current,

\[ \langle 0 | \partial^\mu j^a_\mu | \pi^b \rangle = \delta^{ab} f_\pi m_\pi^2, \quad (36.9) \]

where we recognize the right-hand-side as involving the well-known pion decay constant \((f_\pi)\) and the pion mass. The symmetric limit, where the current is conserved, is just the Goldstone limit where the pion mass vanishes (along with the quark masses). The study of the above defined currents (Current Algebra) and especially the nearly conserved axial current, PCAC (partial conserved axial current), affords many (still useful) avenues to analyze the low energy strong interactions. Unfortunately, we do not have time to pursue this subject further here.

So now we understand that within the Standard Model we expect symmetry breaking both of the spontaneous Higgs variety (a vev for a scalar field) and of the dynamical variety (a vev for a composite quark-anti-quark pair). The former breaks the local gauge symmetry of the electro-weak interactions and the latter breaks the global chiral/flavor symmetry of QCD. But the story is not yet complete. We have not yet discussed the U(1) axial current. The scenario above implies that chiral symmetry breaking (including the U(1)A) should result in 9 (not 8) pseudo-Goldstone bosons, which become massless in the massless quark limit. The obvious candidate for the ninth pseudo-Goldstone boson, the \(\eta^\prime\), is slightly heavier than the proton (958 MeV) and is a very poor match to our desired properties. Clearly something else must be going on. In fact, several effects are contributing to this channel and we will only briefly touch on them here.

One relevant issue is that of the axial anomaly, which is the classically unexpected contribution to the divergence of the axial current that we discussed in Lecture 21 in the context of the electro-weak theory. In that Lecture the anomaly was used to motivate the observed structure of the fermion generations in the Standard Model in order that the coefficient of the anomaly vanishes. This cancellation between the various contributions is essential in order to preserve the renormalizability expected from the underlying gauge symmetry. Even with this disaster averted the triangle diagram of the electro-weak anomaly still plays a role in the context of PCAC. The
decay of the neutral pion into 2 photons, which we discussed in autumn quarter, arises from exactly the (anomaly) triangle diagram (1 axial current and 2 vector ones, which we interpret as having the particle content of the pion coupling to 2 photons).

The appearance of the axial anomaly suggests that maybe the answer to the so-called “U(1) problem” (the missing 9th pseudo-Goldstone boson) is simply that the axial current was never conserved in the first place (i.e., even when the quarks are massless the anomaly can still yield a nonzero divergence for the axial current). While that observation is relevant, it is still not the full story! Consider the QCD version of the anomaly equation (presented here without derivation) in the massless quark limit

\[ \partial^\mu J_{5\mu} = -n_f \frac{g^2}{8\pi^2} \text{Tr} \left[ F_{\mu\nu}^{QCD} \tilde{F}_{\mu\nu}^{QCD} \right]; F_{\mu\nu}^{QCD} = F_{\mu\nu}^{QCD,\lambda} \frac{\lambda^k}{2}. \] (36.10)

Recall that the dual tensor is defined by

\[ \tilde{F}_{\mu\nu}^{QCD} \equiv \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}^{QCD}. \] (36.11)

Next define the gluon field including the matrix representation of the generator,

\[ G_\mu = G_\mu^k \frac{\lambda^k}{2}, \] (36.12)

and define the following current

\[ K_\mu \equiv n_f \frac{g^2}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left[ G_\nu \partial_\alpha G_\beta + \frac{2}{3} ig G_\nu G_\alpha G_\beta \right]. \] (36.13)

A little arithmetic confirms that the divergence of this current is of the form above, modulo the sign,

\[ \partial^\mu K_\mu = n_f \frac{g^2}{8\pi^2} \text{Tr} \left[ F_{\mu\nu}^{QCD} \tilde{F}_{\mu\nu}^{QCD} \right]. \] (36.14)

Hence we might expect to be able to define a new, but conserved (in the limit of massless quarks) current

\[ \hat{J}_5^\mu \equiv J_{5\mu}^\mu + K_\mu, \] (36.15)
and again proceed to look for a 9\textsuperscript{th} Goldstone boson. To actually proceed we need to be able to define the corresponding conserved charge (\(\hat{Q}_5 \sim \int d^3 x \hat{J}_5^0\)), which we require to be localized (\textit{i.e.}, localizable). This in turn requires that \(K^0\) vanish at spatial infinity. This brings us to the interesting but confusing root of the problem. What really does (can?) happen at infinity in QCD. Clearly for finite energy states, \textit{e.g.}, the vacuum, the energy density must go to zero as \(|x| \to \infty\) (think about Euclidean space and consider going to infinity in all directions). However, this does not require that the gluon field, \(G^\mu\), vanishes at infinity. Rather, we need only require that the asymptotic vector gauge field can be transformed to the zero field. If \(\Omega^{QCD}(x)\) is a QCD gauge transformation, than

\[
G_\mu \rightarrow -i g \left(\Omega^{QCD}(x)^{-1} \left( \partial_\mu \Omega^{QCD}(x) \right)\right),
\]

(36.16)
a so-called “pure gauge” configuration, is an acceptable asymptotic form for the gluon field. But, in general, this configuration will not yield a vanishing value for \(K^\mu\). The educated student will now say, “but how can a pure gauge have a physical effect?” Can’t we just “gauge it away”? The answer, surprisingly, is no, not always! And now things get interesting.

To understand this remarkable result we must think about the topology of the field configurations and, in particular, recall our discussion of the relationship between SO(3) and SU(2) back in Lecture 5 of Physics 557. That discussion involved (at least) two important ideas: the concept of a mapping between the group space and the parameter space, and the idea that the group space (and the mapping) may not be simply connected. In the current context we are mapping the space at infinity, which we take to be a 3-D sphere, \(S_3\), in 4-D Euclidean space, onto the group space of SU(3), \textit{i.e.}, at each point on the \(S_3\) we have a pure gauge field configuration. It is possible that as we integrate over \(S_3\), the corresponding path in the group space makes several “windings” around the group space. The mapping of one topological space onto another is characterized by the homotopy class of the mapping. Each class is then labeled by a “winding number”. Those gauge transformations that are continuously deformable to the origin (the identity operator) in group space are said to belong to the homotopy class with winding number 0. (The SO(3) analog are those rotations through angles strictly less than \(\pi\). Recall that, once we get out to \(\pi\), we can come back to the origin in group space along the opposite direction and define a path with winding number 1.) Thus we cannot use a nonsingular gauge transformation to
change a field configuration with nonzero winding number to one with zero winding number. The winding number (only poorly defined here) of the pure gauge configuration is, in fact, a conserved topological charge. It is also closely related to the charge associated with the current $K^\mu$,

$$n = -\frac{1}{2n_f} \oint dS_\mu K^\mu = -\frac{1}{2n_f} \int d^4x \partial_\mu K^\mu. \quad (36.17)$$

Thus, if the winding number is nonzero, we cannot gauge away the divergence of $K^\mu$.

Since this discussion is pretty abstract, let us consider a concrete example. First take the gauge group SU(2). The handy feature in this case is that, if we use both the identity matrix and the Pauli matrices (the representation of the generators), we can map the group space directly onto configuration space. Recall that we are working in 4-D Euclidean space (and time has become $x_4$). For larger gauge groups, e.g., SU(3), we can still apply the following example by focusing on an SU(2) subgroup of the larger group. In the SU(3) case simply replace the Pauli matrices by, for example, the first 3 Gell-Mann matrices, $\sigma_1, \sigma_2, \sigma_3 \rightarrow \lambda_1, \lambda_2, \lambda_3$. Returning to the SU(2) case, consider the following (singular) gauge transformation

$$\Omega_{n=1}(x) \rightarrow x_4 - i\vec{\sigma} \cdot \vec{x} / |\vec{x}|, |\vec{x}| = \sqrt{x_4^2 + x_1^2 + x_2^2 + x_3^2}. \quad (36.18)$$

In some sense the transformation “points” in the same direction in group space as the location vector points in configuration space (modulo the sign). (For this reason this configuration is sometimes called a “hedgehog” configuration.) Now imagine trying to transform this configuration smoothly to the identity transformation. In order to move away from this configuration with radial orientation everywhere we must introduce a singular transformation somewhere. In other words, “you can’t comb a tennis ball without using a part”. This is just the point. You cannot transform away this configuration using nonsingular gauge transformations because it winds fully once around the group space. To unwind this configuration requires that we “break” it somewhere, i.e., introduce a singular transformation.

Now let us work out some of the details. A little algebra yields the following form for the asymptotic vector field
\[ G^{n=1}_\mu \rightarrow -\frac{i}{g} \Omega^{-1}_n(x) \left( \partial_\mu \Omega^{n=1}_n(x) \right) \Rightarrow \]
\[ G^{n=1}_4 = \frac{\vec{\sigma} \cdot \vec{x}}{g x^2}, \]
\[ G^{n=1}_1 = \frac{1}{g x^2} \left[ x_3 \sigma_2 - x_2 \sigma_3 - x_4 \sigma_1 \right], \]
\[ G^{n=1}_2 = \frac{1}{g x^2} \left[ x_1 \sigma_3 - x_3 \sigma_1 - x_4 \sigma_4 \right], \]
\[ G^{n=1}_3 = \frac{1}{g x^2} \left[ x_2 \sigma_1 - x_1 \sigma_2 - x_4 \sigma_3 \right]. \tag{36.19} \]

As the interested reader can confirm, the corresponding vector \( K^\mu \) is asymptotically radially oriented and of constant magnitude over the asymptotic 3-sphere. Thus we only need to evaluate it at one point on the sphere and we choose the simplifying case \( x_i = R, \bar{x} = 0 \). Some more algebra yields

\[ G^{n=1}_4 \bigg|_{x_i = 0, x_4 = R \rightarrow \infty} = 0, \]
\[ \bar{G}^{n=1} \bigg|_{x_i = 0, x_4 = R \rightarrow \infty} = -\frac{\vec{\sigma}}{g x}, \tag{36.20} \]
\[ \partial_k G^{n=1}_l \bigg|_{x_i = 0, x_4 = R \rightarrow \infty} = \epsilon_{kln} \frac{\sigma_m}{g x^2}, \]

and

\[ \frac{1}{2n_f} K^4 \bigg|_{x_i = 0, x_4 = R \rightarrow \infty} = \frac{g^2}{8\pi^2} e^{4\omega a^2} \text{Tr} \left[ G^{n=1}_v \partial_\alpha G^{n=1}_\beta + \frac{2}{3} i g G^{n=1}_v G^{n=1}_\alpha G^{n=1}_\beta \right] \bigg|_{x_i = 0, x_4 = R \rightarrow \infty} \]
\[ = \frac{g^2}{8\pi^2} \cdot 6 \cdot \text{Tr} \left[ G^{n=1}_1 \partial_2 G^{n=1}_3 + \frac{2}{3} i g G^{n=1}_1 G^{n=1}_2 G^{n=1}_3 \right] \bigg|_{x_i = 0, x_4 = R \rightarrow \infty} \]
\[ = \frac{3g^2}{4\pi^2} \cdot \frac{1}{g^2 R^3} \cdot \text{Tr} \left[ (-\sigma_1) \sigma_2 + \frac{2}{3} i (-\sigma_1)(-\sigma_2)(-\sigma_3) \right] \]
\[ = \frac{3}{4\pi^2 R^3} \left[ -2 + \frac{4}{3} \right] = -\frac{1}{2\pi^2 R^2}. \tag{36.21} \]
Finally we can calculate the winding number in this configuration,

\[ n = -\frac{1}{2n_f} \oint dS \mu K^\mu = \frac{1}{2\pi^2 R^3} \oint dS = 1. \]  

(36.22)

In the last step we used the fact that the “surface” of the 3-D sphere in 4-D Euclidean space is \(2\pi^2 R^3\) (recall our discussion in Lecture 30, Appendix B of dimensional regularization).

Returning to the main discussion, we still might ask, if we start in the perturbative regime where \(G^\mu = 0\) at infinity, do we have to worry about configurations where \(G^\mu \neq 0\) at infinity with nonzero topological charge? Vacua with nonzero topological charges may exist but maybe we never see them. It turns out that we cannot ignore these vacua because quantum tunneling always mixes them in. There are, in fact, field configurations with finite action that interpolate between the various vacua. These field configurations are localized in both time and space (in our Euclidean geometry) and are referred to as instantons (the 4-D analogues of solitons in 3-D). A specific example of an instanton field of “size” \(\rho\) corresponding to the example above is given by

\[ G^{\text{Instanton}}_{\mu} = -\frac{i}{g} \frac{|x|^2}{|x|^2 + \rho^2} \Omega_{n=1}^{-1}(x) \left( \partial_\mu \Omega_{n=1}(x) \right). \]  

(36.23)

A calculation similar to that above shows that this field configuration is of finite action and interpolates between two vacua differing in winding number by 1.

Thus we start with an infinite set of apparently degenerate vacua \(|n\rangle\), labeled by their topological quantum number, and now we (must) include tunneling between these states. Thus the true vacuum state will be a symmetrical linear combination of these states that is invariant under the tunneling operator \(T\) defined by

\[ T|n\rangle = |n+1\rangle. \]  

(36.24)

We can write such an invariant vacuum state in terms of a single parameter

\[ |\theta\rangle = \sum_n e^{-in\theta} |n\rangle \]  

(36.25)

such that
\[
T |\theta\rangle = e^{i\theta} |\theta\rangle. \tag{36.26}
\]

Since this overall phase factor is generally not a physical observable, such a state is effectively invariant under the operation of \(T\). This form is why this state is often called the \(\theta\)-vacuum”. This complex structure in the vacuum is equivalently re-expressed in terms of a simple vacuum, say the state with \(n = 0\), plus an extra term, \(n\theta\), in the action or the following term in the QCD Lagrangian density

\[
\mathcal{L}_\theta = \theta \frac{g^2}{16\pi^2} \text{Tr} \left[ F^{QCD}_{\mu\nu} \tilde{F}^{QCD,\mu\nu} \right]. \tag{36.27}
\]

This result raises the question, why didn’t we include such a term in the first place? It has all the correct gauge and Lorentz properties to be included. As outlined above, we know that this term can be expressed as a total derivative and we would have previously argued that it cannot influence the physics. However, due to the quantum tunneling between the various vacua (i.e., due to the instantons), it is no longer possible to make such an argument. Rather we are required to include such a term and allow a new free parameter \(\theta\). The bad news is that such a term respects neither parity (recall the \(\varepsilon\) factor in the definition of \(\tilde{F}\)) nor \(CP\). Thus, although we earlier thought that the strong interactions respected both \(P\) and \(CP\) from a theoretical standpoint, this is apparently not the case. Of course, it remains true experimentally. The bound from the experimental limit on the electric dipole moment of the neutron implies that

\[
\theta \leq 10^{-9}. \tag{36.28}
\]

So now we face a new problem. Why is this QCD parameter so small? The most attractive solution was offered by Peccei and Quinn. They noted that, if we make the Lagrangian invariant under an extra \(U(1)_{R-L}\), we can use this \(U(1)_{PQ}\) to rotate \(\theta\) to zero. The trick for ensuring that the Yukawa terms (\(\sim h \bar{q}_L \nu \nu\)) are invariant is to specify that the Higgs fields also transform (i.e., obtain an appropriate phase) under the \(U(1)_{PQ}\) transformation. Most importantly, to make it all work, we must include a second Higgs doublet. After both of the doublets acquire vev’s and 3 degrees of freedom are “eaten” by the now massive \(W^\pm\) and \(Z\) (the Higgs mechanism) the remaining 5 degrees of freedom correspond to 2 neutral and 2 charged Higgs particles and one “true”, nearly massless, pseudo-Goldstone boson. This last particle is called the axion and is expected to have a mass of order
\[ m_a \approx \frac{m_\pi f_\pi}{v} \approx 100 \text{ KeV.} \] (36.29)

Unfortunately searches for this particle have not been successful yet and the simplest version of Peccei-Quinn is effectively ruled out. More complicated versions with further extra scalar fields and more vev’s at larger energy scales (as in the context of GUTS) yield scenarios that cannot be ruled with current experiments and there is a first class experiment, ADMX, closing in on its final phase at CENPA. It is clear that this is a story that is far from over.

In summary, the constraints arising from symmetries and how we break them remain some of the most useful tools for analyzing the behavior of the Standard Model (and beyond) at both small and large energies.

Finally we close with a brief list of the issues that must be dealt with by physics beyond the Standard Model (i.e., these are thesis topics):

- Explain how gravity fits into all this;
- Explain the nature of dark matter and dark energy;
- Explain the genesis of the (~ 20) parameters of Standard Model, (Yukawa couplings, \( \theta_W \), \( \theta_{QCD} \), etc.);
- Explain the hierarchy of energy scales;
- Explain why we have 3 generations;
- Etc.;
- Etc.