Physics 558 – Lecture 24

Introduction to SuperSymmetry:  (See, e.g., Chapter 12 in Griffiths or Chapter 17 in Rolnick.)

Before discussing in detail the properties of QCD we will take a couple of detours to introduce some interesting corners of particle physics. As we have noted several times the proposed goal of the LHC, besides finding the Higgs boson, is to identify what physics lies Beyond the Standard Model (BSM). One popular, but as yet experimentally unconfirmed, possibility is that BSM physics is described by Supersymmetry. So here we take a brief detour and extend our previous symmetry discussions to include the concepts of supersymmetry.

Last quarter we considered situations (i.e., Lagrangians) where the physics was the same for (i.e., the Lagrangian was symmetric under the interchange of) states of differing $I_3$ (isospin symmetry), $J_3$ (rotational symmetry), strangeness (flavor SU(3) symmetry) or color (color symmetry). In each case the physical states appear in degenerate multiplets of the appropriate symmetry. So the question arises, under what conditions do we see degenerate multiplets of states with different spin, e.g., of spin $\frac{1}{2}$ and 1 or spin $\frac{1}{2}$ and 0. Systems that display invariance under the interchange of states with differing spin, in particular, of states whose spins differ by $\frac{1}{2}$, are said to be supersymmetric. The underlying supersymmetric algebra will involve both bosonic and fermionic operators, i.e. operators that satisfy relations based on both commutators and anti-commutators.

The study of such systems is interesting for at least three reasons. The first is simply that this symmetry between bosons and fermions is intrinsically interesting. The second is that string theory studies suggest that supersymmetry, hereafter referred to as SUSY, plays a role in particle physics, at least at very short distances. The third, and most direct, reason is associated with wanting relatively low mass scalar particles, e.g., Higgs bosons, in the theory. This latter point deserves some level of explanation, which will allow us to begin discussing some other concepts important to the way we look at modern particle physics. As we discussed last quarter, both vector particles and spin $\frac{1}{2}$ particles naturally display symmetries that can ensure that they remain massless, even as we include quantum corrections. In the former case it is a gauge symmetry that plays this role. As long as the gauge symmetry remains unbroken, there can be no interactions in the theory (i.e., terms in the Lagrangian, including a bare mass term) that give the vector gauge boson a mass. The symmetry also ensures that this situation remains true even at higher order in perturbation theory. In particular, radiative corrections in the form of loop diagrams (to be
explained more thoroughly below) will not cause the vector gauge boson to acquire a mass. Likewise a Lagrangian with chiral symmetry (i.e., a Lagrangian where the right-handed fermions are treated independently from the left-handed ones) will exhibit massless (Dirac) fermions to all orders in perturbation theory. Recall that the typical terms in the Lagrangian for a gauge theory, except for fermion mass terms, can be written separately for the different chiral components. For example, the gauge covariant derivative is diagonal in the chiral basis,

\[ i \bar{\Psi} \partial_i \Psi \rightarrow i \bar{\Psi}_R \partial_R \Psi_R + i \bar{\Psi}_L \partial_L \Psi_L, \]

(24.1)

while a mass term is off-diagonal in the same basis,

\[ m \bar{\Psi} \Psi \rightarrow m \bar{\Psi}_L \Psi_R + m \bar{\Psi}_R \Psi_L. \]

(24.2)

While Eq. (24.1) allows independent transformations of the chiral states,

\[ \Psi_R \rightarrow U_R \Psi_R, \quad \Psi_L \rightarrow U_L \Psi_L, \quad U_R \neq U_L, \]

(24.3)

Eq. (24.2) does not. Hence the requirement of chiral symmetry means that fermion masses are not allowed in the Lagrangian and, more importantly, will not arise from (perturbative) quantum corrections at higher orders. If the lowest order interactions (those in the Lagrangian) respect the chiral symmetry, the higher order interactions will also. As we have discussed briefly, the symmetry in the Lagrangian can be broken only by “spontaneous” effects (the structure of the vacuum), or by “dynamical” effects (nonperturbative bound-state structure), or by the anomalies we discussed in Lecture 21.

On the other hand, scalar particles may have zero “bare” mass (i.e., no mass term in the Lagrangian) but they will generally pick up a mass from their self-interaction at higher orders. Consider a theory with quartic interactions for the scalar field (as in a \( \phi^4 \) theory and the theories for the Higgs “potential” that we have been discussing). There are radiative corrections to the scalar field inverse propagator (i.e., the mass\(^2\)) of the form (here \( g^2 \) is the strength of the quartic coupling)

\[ g^2 \left( \frac{2\pi}{4} \right)^4 \int \frac{d^4k}{k^2} \propto g^2 \Lambda^2, \]

(24.4)
where $k$ is the momentum running around the loop and the $1/k^2$ is the propagator for the scalar particle in the loop. (For now, do not worry about the details of how this result is obtained. Dimensional analysis is enough.) This integral is clearly quadratically divergent in the UV (at the upper limit). Here we have simply put in a cutoff $\Lambda$. We interpret this result as saying that, if the underlying theory is meant to be valid up to a scale $M_{\text{GUT}}$ or $M_{\text{Planck}}$, where a higher symmetry and/or gravity become relevant, the “natural” scale for the renormalized mass of the scalar field (modulo possible factors of $g^2$) is $M_{\text{GUT}}$ or $M_{\text{Planck}}$ (unless we are willing to allow some high-powered fine tuning). Unfortunately, to allow the Higgs mechanism in the Standard Model to work its magic, we want the Higgs mass scale to be around 1 TeV (e.g., 125 GeV) in order to explain the observed electro-weak symmetry breaking scale (and the results from the LHC). Without a fix for this issue it is difficult to see how the Higgs mechanism can work in the context of the Standard Model alone.

SUSY offers at least the possibility of a fix. The basic idea is based on the fact in a SUSY world every scalar field degree of freedom will have a SUSY related fermion degree of freedom and in the SUSY limit the two will have the same mass. Thus, if the fermion’s mass is protected by a chiral symmetry, SUSY will protect the scalar’s mass also. In detail, every divergent diagram like that above with a scalar loop will be matched by a diagram with a fermion loop,

$$
\left(\frac{2\pi}{\lambda}\right)^{d-2} \frac{1}{\lambda} \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k}\right)^2 \sim -g^2 \Lambda^2.
$$

As indicated in Eq. (24.5) although the diagram looks different from the scalar one in Eq. (24.4), the fermion loop is kinematically very similar to the scalar one since each of the two fermion propagators goes like $1/k$. Its coefficient, however, will differ by an overall factor of $-1$, due essentially to the Fermi statistics of the corresponding operators (we will consider this in more detail later). Thus the divergences in the two diagrams will cancel. Since we do not experimentally observe degenerate superpartners at current energies, the world we live in does not respect SUSY at energy scales below 1 TeV. However, the supersymmetric cancellation noted above could still “protect” the mass of the Higgs boson down to 1 TeV, if the scale of SUSY breaking is not much above a TeV. As a result, if this SUSY scenario is correct, we should see super-quarks (squarks) and super-leptons (sleptons), etc., at the LHC! So there is now tension between the observation of a Higgs-like boson at 125 GeV and the lack of any evidence for super partners at the LHC.

With this somewhat mysterious “motivation”, let us try to understand the structure of a simple example of SUSY. Consider first a supersymmetric harmonic oscillator, i.e.,
let us start with SUSY quantum mechanics. We define the system to have the usual bosonic excitations with creation operator \( b^\dagger \) so that (in natural units)

\[
[b, b^\dagger] = 1, [b, b] = [b^\dagger, b^\dagger] = 0.
\] (24.6)

In the usual way it follows that

\[
(b^\dagger)^n |0\rangle = |n\rangle, b^\dagger b |n\rangle = n |n\rangle, bb^\dagger |n\rangle = (n + 1) |n\rangle.
\] (24.7)

Now define also a corresponding “fermionic” excitation with creation operator \( f^\dagger \) where (note that \([,]\) goes to \({,}\))

\[
\{ f, f^\dagger \} = 1, \{ f, f \} = \{ f^\dagger, f^\dagger \} = 0.
\] (24.8)

Thus we cannot create a state with two identical fermions,

\[
f^\dagger f^\dagger |0\rangle = 0,
\] (24.9)

as is required by Fermi statistics. The two different operators must commute, i.e., the two kinds of excitations are distinct,

\[
[b, f] = [b^\dagger, f^\dagger] = [b^\dagger, f] = [b, f^\dagger] = 0.
\] (24.10)

Now we define a Hamiltonian for this simple system in the form

\[
H = \frac{1}{2} \omega_B \{ b^\dagger, b \} + \frac{1}{2} \omega_F \{ f^\dagger, f \},
\] (24.11)

with eigenvalues

\[
E = \omega_B \left( n_B + \frac{1}{2} \right) + \omega_F \left( n_F - \frac{1}{2} \right),
\] (24.12)

where \( n_B \) and \( n_F \) count the number of bosonic and fermionic excitations.

The SUSY limit corresponds to the choice
\omega_B = \omega_F = \omega : \mathcal{E}_{\text{SUSY}} = \omega (n_B + n_F). \quad (24.13)

Note that, due to Fermi statistics, \( n_F \) is either 0 or 1 (but not larger). So the spectrum of these systems has a vacuum state \( n_B = n_F = 0 \), plus a tower of “superpartnered” states, \( i.e., \) degenerate pairs of bosons and fermions, for all \( n > 0 \):

<table>
<thead>
<tr>
<th>Boson</th>
<th>Fermion</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_B = n, n_F = 0 )</td>
<td>( n_B = n-1, n_F = 1 )</td>
<td>( n \omega )</td>
</tr>
</tbody>
</table>

To describe this system we can define a new symmetry operator that has the effect of changing the number of fermion excitations by 1 and changing the number of boson excitations by 1 in the opposite direction. The operator is typically defined to be self-conjugate so that it can take us both from a fermion state to a boson state and \( \text{vice versa} \). Note that the conventional symbol for this operator is \( Q \) but it is NOT the electric charge,

\[ Q \equiv \sqrt{\omega} \left( b^\dagger f + bf^\dagger \right) = Q^\dagger. \quad (24.14) \]

This is a symmetry operation because

\[ [Q, H] = [Q^\dagger, H] = 0. \quad (24.15) \]

In the SUSY limit the operator \( Q \) does not change the energy of a state (or any other internal quantum number, explaining the choice to define it to be self-conjugate). Rather it changes only the number of bosonic and fermionic excitations. To see Eq. (24.15) in detail, let us write it out,

\[ [Q, H] = \frac{1}{2} \left( \sqrt{\omega} \right)^3 \left( [b^\dagger f, \{b^\dagger, b\}] + [b^\dagger f, \{f^\dagger, f\}] + [bf^\dagger, \{b^\dagger, b\}] + [bf^\dagger, \{f^\dagger, f\}] \right). \quad (24.16) \]

The first term (a bosonic \( \omega_B \) term) yields

\[ [b^\dagger f, \{b^\dagger, b\}] = [b^\dagger f, b^\dagger b] + [b^\dagger f, bb^\dagger] = b^\dagger [b^\dagger, b] f + [b^\dagger, b] b^\dagger f = -2b^\dagger f, \quad (24.17) \]
while the second term (a fermionic $\omega_f$ term) yields

$$
\left[ b^\dagger f, [f^\dagger, f] \right] = \left[ b^\dagger f, f^\dagger f \right] - \left[ b^\dagger f, f f^\dagger \right] \\
= b^\dagger \left( f f^\dagger f - f^\dagger f f - f f^\dagger f + f^\dagger f f \right) \\
= 2b^\dagger f.
$$

(24.18)

Similarly the third and fourth terms yield

$$
\left[ b f^\dagger, \left\{ b^\dagger, b \right\} \right] = \left[ b f^\dagger, b^\dagger b \right] + \left[ b f^\dagger, b b^\dagger \right] \\
= \left[ b, b^\dagger \right] b f^\dagger + b \left[ b, b^\dagger \right] f^\dagger = 2b f^\dagger,
$$

(24.19)

and

$$
\left[ b f^\dagger, [f^\dagger, f] \right] = \left[ b f^\dagger, f^\dagger f \right] - \left[ b f^\dagger, f f^\dagger \right] \\
= b \left( f^\dagger f^\dagger f - f^\dagger f f^\dagger f - f^\dagger f f^\dagger + f^\dagger f f^\dagger f \right) \\
= -2b f^\dagger.
$$

(24.20)

To obtain the fermionic results we have used the fact that two contiguous fermionic annihilation operators annihilate any state, while $f f^\dagger f$ acts like $f$, 

$$
\begin{align*}
&f \ket{1} = \ket{0}, \quad f^\dagger \ket{0} = \ket{1}, \quad f \ket{0} = 0, \quad f^\dagger \ket{1} = 0, \\
&f^\dagger f \ket{1} = \ket{1}, \quad f^\dagger f \ket{0} = 0, \quad f f^\dagger \ket{1} = 0, \quad f f^\dagger \ket{0} = \ket{1}, \\
&f f^\dagger f f^\dagger \ket{1} = \ket{0}, \quad f f^\dagger f \ket{0} = 0, \\
&f^\dagger f f \ket{1} = f^\dagger f f \ket{0} = f f^\dagger f \ket{1} = f f^\dagger f \ket{0} = 0.
\end{align*}
$$

(24.21)

Likewise two contiguous fermionic creation operators annihilate any state, while $f^\dagger f$ acts like $f^\dagger$.

Thus we see that $Q$ commutes with $H$ precisely because of a cancellation between the bosonic contribution and the fermionic contribution (in the SUSY limit with identical frequencies)
\[
[Q,H] = \frac{1}{2} \left( \sqrt{\omega} \right)^3 \left( -2b^\dagger f + 2b^\dagger f + 2bf^\dagger - 2bf^\dagger \right) = 0. \tag{24.22}
\]

As suggested earlier, it is this cancellation between bosonic and fermionic degrees of freedom that is the hallmark of SUSY. Now comes the really interesting relation (and the reason for the choice of normalization for \( Q \)). We consider the following anticommutator

\[
\{Q,Q^\dagger\} = 2\omega \left( b^\dagger f bf^\dagger + bf^\dagger b^\dagger f + b^\dagger fb^\dagger f + bf^\dagger b f^\dagger \right) \\
= 2\omega \left( b^\dagger bf^\dagger + bb^\dagger f^\dagger \right) \\
= \omega \left( bb^\dagger - 1 \right) f^\dagger f + b^\dagger b \left( 1 - f^\dagger f \right) \\
+ bb^\dagger \left( 1 - f^\dagger f \right) \left( 1 + b^\dagger b \right) f^\dagger f \tag{24.23}
\]

\[
= \omega \left( b^\dagger b + bb^\dagger + f^\dagger f - f^\dagger f \right) = 2H.
\]

Again we have used the fact that \( f f \) and \( f^\dagger f^\dagger \) annihilate all states. This result suggests that we should enlarge the concept of an algebra, which up to now has involved only commutators of generators, to include also anticommutators. It also underlines the close connection between the “internal symmetry” generated by \( Q \) and space-time symmetries as indicated by \( H \) (recall that it was in this context of mixing internal and space-time symmetries that we first mentioned supersymmetry). In fact, the mathematicians have long studied such mixed algebras. The result is called a graded Lie algebra. An example is the above set of operators, \( Q, Q^\dagger, H \), which closes under a set of both \([,] \) and \( \{,\} \),

\[
[Q,H] = 0, [Q^\dagger,H] = 0, \{Q,Q^\dagger\} = 2H. \tag{24.24}
\]

If we label the usual bosonic operators as “even” operators and the new fermionic operators as “odd” operators, we having the following general structure for a graded Lie algebra

\[
[\text{even, even}] = \text{even}, \\
[\text{even, odd}] = \text{odd} \text{ (including 0)}, \\
\{\text{odd, odd}\} = \text{even}.
\]
When we move from the algebra to the full group, we expect to exponentiate the operator $Q$ to obtain $e^{iQ\alpha}$. Since the resulting operator should satisfy commutation relations \textit{(i.e., be “even” although $Q$ itself is “odd”)}, the number $\alpha$ must \textit{not} be an ordinary number. Instead we must introduce the concept of \textit{anticommuting} numbers. Again the mathematicians have a name for these, Grassmann numbers. For example, if $\alpha$ and $\beta$ are Grassmann numbers, then $\alpha\beta = -\beta\alpha$.

If we consider “supersymmetrizing” a locally gauge symmetric field theory (initially with chiral fermions and gauge bosons), there will generally be a scalar degree of freedom for every fermionic degree of freedom, \textit{e.g.}, for each chiral state, and a chiral degree of freedom for each gauge boson degree of freedom. With enough SUSY operators one should see spin 0, $\frac{1}{2}$, 1, 3/2 and 2 particles, all in degenerate multiplets.

In realistic applications of SUSY we want to enlarge to algebra to include not just $H$ but rather the full Poincaré algebra with generators (recall the discussion in Lecture 5)

$$P^\mu \left( \text{translations in 4-D, } P^0 = H \right),$$

$$\vec{J} = \left( \text{rotations - } J_{23}, J_{31}, J_{12} \text{ in Lecture 5, or sometimes } M_{23}, M_{31}, M_{12} \right),$$

$$\vec{K} = \left( \text{boosts - } J_{10}, J_{20}, J_{30} \text{ in Lecture 5, or sometimes } M_{01}, M_{02}, M_{03} \right).$$

If we have a typical \textit{internal} (even) symmetry, whose generators we represent by $T_a$ and whose internal space is disjoint from space-time, we have

$$\left[ P^\mu, T_a \right] = \left[ J^{\mu\nu}, T_a \right] = 0. \quad (24.25)$$

These commutation relations, which state that a Lie group containing both the Poincaré group and an internal symmetry group must be a direct product and not mix the dependence on space-time and internal degrees of freedom in a non-trivial way, was demonstrated as a “no-go” theorem in 1967 by Coleman and Mandula under quite general conditions. SUSY evades this theorem by including both commutation and anticommutation relations in the algebra, \textit{i.e.}, by using a graded Lie algebra.

To pursue this idea further consider a fermionic operator, $Q_{\alpha}$ ($\alpha=1-4$), which has the structure of a Majorana operator, \textit{i.e.}, is self-conjugate,

$$Q = Q^C = C\bar{Q}^T. \quad (24.26)$$
As we discussed earlier, such operators are, in some sense, half of a Dirac operator, \textit{i.e.}, represent only two degrees of freedom instead of 4. To see this connection we write a general Dirac field as

\[
\Psi = \frac{1}{\sqrt{2}} (\Psi_1 + i\Psi_2). \tag{24.27}
\]

The 2 components,

\[
\Psi_1 = \frac{1}{\sqrt{2}} (\Psi + \Psi^c), \Psi_2 = -\frac{i}{\sqrt{2}} (\Psi - \Psi^c) \tag{24.28}
\]

are independent Majorana spinors that each satisfy

\[
\Psi_i = \Psi_i^c, \tag{24.29}
\]

just as we require of the \( \mathcal{Q} \). Recall that the Majorana spinors can have \textit{no} nonzero quantum numbers (other than that they are fermions).

The fact that these are spinor operators is encoded in the commutator that specifies their properties under a Lorentz transformation,

\[
\left[ Q_\alpha, J^{\mu\nu} \right] = \frac{1}{2} (\sigma^{\mu\nu})_{\alpha\beta} Q_\beta, \tag{24.30}
\]

where we recall that

\[
\sigma^{\mu\nu} = \frac{i}{2} \left[ \gamma^\mu, \gamma^\nu \right]. \tag{24.31}
\]

This last object, which is an antisymmetric Lorentz tensor and a matrix in spinor space, provides a spinor representation of the Lorentz group.

The (necessarily true) Jacobi Identity of commutators

\[
\left[ \left[ Q_\alpha, P^\mu \right], P^\nu \right] + \left[ \left[ P^\nu, Q_\alpha \right], P^\mu \right] + \left[ \left[ P^\mu, P^\nu \right], Q_\alpha \right] = 0, \tag{24.32}
\]

requires that the operator \( Q_\alpha \) is \textit{translationally} invariant and represents a symmetry of the system,

\[
\left[ Q_\alpha, P^\mu \right] = 0. \tag{24.33}
\]
The graded Lie algebra is then closed by including the anticommutator noted earlier

\[ \{ Q_\alpha, \bar{Q}_\mu \} = 2 (\gamma^\mu)_{\alpha\beta} P_\mu, \]  

(24.34)

which exhibits the explicit mixing of space-time with internal symmetries that is intrinsic to SUSY. We will delay a full derivation of this remarkable result until we have a more fully developed grasp of field theory (but we will discuss the Wess-Zumino model briefly in the Appendix and the HW). In fact, this anticommutator, which is an even generator, a tensor with respect to the spinor indices \( \alpha\beta \), a Lorentz scalar and must be linear in one of the other generators, can only have the form shown or be proportional to \( \sigma^{\mu\nu} J_{\mu\nu} \). But the latter expression does not lead to a closed algebra. In the HW we will study an explicit example of this structure in the context of massless particles.

If we express the (formerly) 4-component Majorana spinor operator in terms of 2-component left-handed Weyl (or chiral) spinors, \( W \), with our convention for the Dirac matrices, we find

\[ Q = Q^c \equiv \begin{pmatrix} 0 \\ W \end{pmatrix} + C\gamma^{0T} \begin{pmatrix} 0 \\ W^* \end{pmatrix} = \begin{pmatrix} i\sigma_2 W^* \\ W \end{pmatrix}, \]  

(24.35)

where we used the fact that (in this notation) the conjugation operation looks like

\[ C\gamma^{0T} = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}. \]  

(24.36)

While we use the notation \( W^* \) here, you should probably think \( W^\dagger \) since we are really discussing operators. [This 2-component notation is used in Peskin and Schroeder, Chapter 22.4]

**ASIDE** - Recall the following properties of Dirac matrices: In the Appendix of Lecture 9 we introduced a specific representation of the Dirac matrices that are useful when using the language of chiral or Weyl fermions as here. We used the convention of the text by Rolnick,
To avoid confusion (or to maximize it) it is useful to note that in several other texts, for example, the Field Theory book by Peskin and Schroeder, a different convention is used. In this second convention the $\bar{\gamma}$ have changed sign so that $\gamma^5$ must also change sign and the matrices have the form

$$
\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix},
$$

(24.37)

$$
\gamma^3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.
$$

We only see differences between these choices when we express results in detail. In particular, when we want to write Dirac or Majorana 4 component spinors in terms of 2 component Weyl spinors, the identifications are different. In our convention of Eq. (24.37) we have

$$
\Psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix},
$$

(24.39)

while in the other convention of Eq.(24.38) (with the opposite sign for $\gamma^5$) we have
\[ \Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \]  

(24.40)

When we apply this formalism in the HW to a massless particle traveling in the \( \hat{z} \) direction, \( P^\mu = (E,0,0,E) \), we learn that the second component of the Weyl spinor and its conjugate, \( W_2^+, W_2 \), act just like the operator \( Q \) in the harmonic oscillator case. They commute with the Hamiltonian,

\[ \left[ W_2^{\dagger}, P^\mu \right] = 0, \]  

(24.41)

and their anticommutator is

\[ \{ W_2, W_2^{\dagger} \} = 4E. \]  

(24.42)

When applied to a left-handed massless particle, the helicity, and thus the spin, is changed by \( \frac{1}{2} \). In this case, the other components, \( W_1^+, W_1 \), carry no new information.

As applied to a Standard model with massless quarks, leptons, gauge bosons, Higgs bosons and gravitons, we expect SUSY to be realized by superpartners for the “observed” particles: spin 0 squarks and sleptons, spin \( \frac{1}{2} \) gauginos (photino, gluino, wino, bino, zino), spin \( \frac{1}{2} \) higgsino, and a spin 3/2 gravitino. The charged gauginos and higgsinos are called charginos while the neutral states are called neutralinos. Since some of the (yet to be) observed states will be linear combinations (i.e., mixing is likely), the labels wiggsino and ziggsino also sometimes used. Note that the super multiplets, for example, (the left-handed multiplet)

\[ \text{Chiral Multiplet} \quad \text{Vector (gauge) Multiplet} \]

\[ \begin{align*}
\text{fermion} & | \frac{1}{2}, -\frac{1}{2} \rangle \\
\text{sfermion} & | 0, 0 \rangle \\
\text{gauge boson} & | 1, -1 \rangle \\
\text{gaugino} & | \frac{1}{2}, -\frac{1}{2} \rangle
\end{align*} \]

must have all other quantum numbers identical within the super-multiplet. So in the simplest SUSY scenarios the massless matter fermions are not connected via SUSY to massless vector particles. The matter fermions are typically in the fundamental representation of the other internal symmetries (U(1), SU(2) and SU(3)) while the only allowed massless vector particles (i.e., the only ones that stay massless when radiative corrections are included and the only ones that yield renormalizable theories) are the gauge particles that are in the adjoint representations of the
symmetries. As in our discussion of the neutrino, CPT implies the presence of antiparticles with the opposite helicity.

**ASIDE** Massive super-multiplets: Without derivation we note that for the case of a massive particle in its rest frame, $P^\mu = (M, 0, 0, 0)$, both components of $W$ have a nonzero anticommutator,

$$\{W_1, W_1^*\} = \{W_2, W_2^*\} = 2M,$$  \hspace{1cm} (24.43)

and act as SUSY generators. There are now three distinct “SUSY raising” operators, $W_1^*, W_2^*, W_1^*W_2^* = -W_2^*W_1^*$, that can connect 3 states with a helicity structure suggested by the following figure.

Hence this super-multiplet has 1 scalar, 1 fermion and 1 vector, and all have the same mass $M$.

Clearly SUSY is broken in the universe we live in – we do not see degenerate super multiplets. However, if SUSY is a symmetry at short distances and serves to “protect” the Higgs boson from having a mass larger than 1 TeV, our expectation is that the masses of the broken-symmetry partners will be of order 1 TeV and therefore accessible at the LHC, although apparently not at the Tevatron. So, as noted above, there is tension raised but the current lack of super-partner signals at the LHC.

To see SUSY scenarios discussed in more detail, consult the PDG review article at [http://pdg.lbl.gov/2012/reviews/rpp2012-rev-susy-1-theory.pdf](http://pdg.lbl.gov/2012/reviews/rpp2012-rev-susy-1-theory.pdf). Here we will provide just a brief summary of some of the phenomenologically relevant details (relevant, for example, to the discussions at last week’s Terascale – Higgs
A particularly important issue is the vast number of parameters in a broken-SUSY model. While in the SUSY limit the couplings and masses of the superpartners are identical to the SM particles, in the broken case the masses of the (as yet, unseen) superpartners must be specified along with a description of the new, SUSY-breaking dynamics. Note, however, that the couplings of the superpartners to the SM particles are specified by the underlying SUSY (to be identical to the SM couplings). Further, the Higgs sector must include at least a second (complex) Higgs doublet (along with the corresponding fermionic Higgsinos) in order to avoid issues with anomalies. Note that this doubling of Higgs fields and vacuum expectation values allows the usual style Yukawa couplings to provide separate masses to “up” and “down” type quarks and charged leptons in fashion consistent with SUSY (see the corresponding labeling in the Table below). It is also explicitly introduces a new parameter $\beta$ defined by

$$\tan \beta \equiv \frac{v_u}{v_d}, \quad v_u^2 + v_d^2 = \frac{4M_W^2}{g^2} \approx (246 \text{ GeV})^2,$$

where $v_u$ and $v_d$ are the vacuum expectation values of the neutral components of the corresponding Higgs fields $H_u$ and $H_d$ (see the Table below). MSSM scenarios are then often organized in terms of being large $\tan \beta$ ($\gg 1$) or small $\tan \beta$ ($\sim 1$). The complex Higgs system is also typically organized into CP-even and CP-odd components, where the mass of the CP-odd Higgs is typically represented by the parameter $A^0$.

So, in principle, SUSY scenarios involve an enormous number of parameters. Much of the complexity of searches for SUSY signals arises from the challenge of organizing techniques to simplify this enormous parameter space. The simplest place to start is the so-called Minimal

**Table 1:** The fields of the MSSM and their SU(3)×SU(2)×U(1) quantum numbers are listed. Only one generation of quarks and leptons is exhibited. For each lepton, quark, and Higgs supermultiplet, there is a corresponding anti-particle multiplet of charge-conjugated fermions and their associated scalar partners.

<table>
<thead>
<tr>
<th>Super-Multiplets</th>
<th>Boson Fields</th>
<th>Fermionic Partners</th>
<th>SU(3)</th>
<th>SU(2)</th>
<th>U(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>gluon/gluino</td>
<td>$g$</td>
<td>$\tilde{g}$, $\bar{W}^0$</td>
<td>8</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$W^\pm$, $W^0$</td>
<td>$\tilde{g}$, $\bar{W}^0$</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$B$</td>
<td>$\tilde{B}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>slepton/lepton</td>
<td>$(\tilde{\nu}_L, \tilde{e}^-)_L$</td>
<td>$(\nu, e^-)_L$</td>
<td>1</td>
<td>2</td>
<td>$-$2</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\nu}_R$</td>
<td>$\nu_R$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>squark/quark</td>
<td>$(\tilde{u}_L, \tilde{d}_L)$</td>
<td>$(u, d)_L$</td>
<td>3</td>
<td>2</td>
<td>$1/3$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{u}_R$</td>
<td>$u_R$</td>
<td>3</td>
<td>1</td>
<td>$4/3$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{d}_R$</td>
<td>$d_R$</td>
<td>3</td>
<td>1</td>
<td>$-2/3$</td>
</tr>
<tr>
<td>Higgs/higgsino</td>
<td>$(H_d^0, H_d^-)$</td>
<td>$(\tilde{H}_d^0, \tilde{H}_d^-)$</td>
<td>1</td>
<td>2</td>
<td>$-1$</td>
</tr>
<tr>
<td></td>
<td>$(H_u^+, H_u^0)$</td>
<td>$(\tilde{H}_u^+, \tilde{H}_u^0)$</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
SuperSymmetric Standard Model or MSSM. Here the SM is “minimally” extended by including all of the superpartners of the SM particles, the 2 complex doublets of Higgs scalars with their fermionic Higgsino partners, the graviton and gravitino, and all the distinct anti-particles. This field content is summarized in the above table from the PDG review. As indicated in the table, the standard notation for the superpartner is to use the symbol for the SM particle with a tilde (~) on top, e.g., the gluino is represented by \( \tilde{g} \). The underlying symmetries and corresponding renormalizability (of the SM) are maintained along with the (hopefully) familiar conservation of \( B-L \) (baryon number minus lepton number). The SUSY-breaking physics is required to be “soft”, i.e., full SUSY is regained at energies well above the scale of the breaking (which is presumed to be near 1 TeV for the reasons discussed earlier). As a result of the \( B-L \) invariance, the MSSM exhibits a new multiplicative so-called \( R \)-parity invariance defined by

\[
R = (-1)^{3(B-L)+2S},
\]

(24.45)

for a particle of spin \( S \). This means that the familiar SM particles have even \( R \) parity (+), while the superpartners exhibit odd \( R \) parity (-). Thus, in the absence of explicit \( R \) parity violating (RPV) dynamics, the superpartners, the sparticles, are always produced by the SM interactions in sparticle-antisparticle pairs (recall the same was true of the heavy flavor quarks). Similarly the decays of heavy sparticles will correspond to cascades with an odd number (typically 1) of sparticles at each stage. The lowest mass sparticle, which is typically electrically and color neutral to avoid the energy in the corresponding fields, is stable and is generally a neutralino or the gravitino. It is referred to as the LSP (Lightest Super Particle), and, like the more familiar neutrino, the LSP is not detected at the LHC. Since the LSP appears at the end of the above mentioned decay cascades, events with large “missing \( E_\text{T} \)”, or large MET, are candidates to be events involving the production (and decay) of sparticles. (Of course, what is really missing is \( \vec{p}_T \), since that is the conserved quantity that is most readily measured.) Recall that it was this expectation that led to the (mis)interpretation of the “mono-jet” events at the earlier SppbarS CERN collider as a SUSY discovery (see the book “Nobel Dreams”). In fact, what was missing then arose from SM neutrinos. Moreover, the expected LSP mass scale, weak couplings and stability of the LSP make it an attractive candidate Dark Matter particle. The standard cosmological evolution would leave a remnant LSP density in good (within uncertainties) agreement with the observed dark matter densities.
After a discussion of the Wess-Zumino Model in the Appendix, we will return to our discussion of the Standard Model.