Another look at Electromagnetism as a Local Gauge Symmetry: Calculating Cross Sections

We have been calculating rates for the broken symmetry (massive vector boson) charged and neutral current interactions. Here we will focus on the interactions given by the remaining unbroken U(1) symmetry, i.e., QED interactions. We have already studied the formal structure of U(1) symmetric gauge theories. In this lecture we want to translate that structure (the Lagrangian and Feynman rules) into cross sections for specific scattering processes. From the previous lecture we have the relevant Feynman rules for fermion and photon propagators (in Feynman gauge) and the fermion-photon vertex.

\[
\begin{align*}
\text{Fermion} & \quad \frac{i}{(\gamma^\mu q_\mu - m)} = \frac{i(\gamma^\mu q_\mu + m)}{q^2 - m^2} \\
\text{Photon} & \quad \frac{-ig^{\alpha\beta}}{q^2} \\
\alpha & \quad -iQ_f \gamma^\mu \\
\beta & \quad \gamma^\mu
\end{align*}
\]

where, as in Eq. (19.2), the symbol $Q_f$ stands for the electric charge of the fermion (labeled $Q_e$ in Eq. (19.2)). So for the electron – $Q_f$ is really $|e|$, the magnitude of the electric charge of an electron. As a specific first example consider the annihilation process $e^+e^- \rightarrow \mu^+\mu^-$. The lowest order (tree level) electromagnetic process is indicated in the figure below. To evaluate the cross section for the process we need to remember some of the kinematics that we learned last quarter, which we review here. Recall that in Lecture 6 we introduced the concept of a cross section and determined that the general expression for a 2 to n process is

\[
d\sigma_{ab\cdots n} = \frac{(2\pi)^4 |M_{ab\cdots n}|^2}{4\sqrt{(p^\mu_{a\mu})^2 - m_a^2 m_b^2}} d\Phi_n(p_1, \cdots, p_n, p_a + p_b), \quad (20.1)
\]

where the last factor is the phase space for the n particles in the final state,
\[ d\Phi = \delta^4\left(p_a + p_b - \sum_{i=1}^n p_i\right) \prod_{i=1}^n \frac{d^3p_i}{(2\pi)^3 2E_i}. \] (20.2)

In this lecture we will be able to calculate the appropriate expressions for the matrix element \( \mathcal{M} \) in the context of E&M.

For the special case of 2 to 2 processes (e.g., \( a \ b \rightarrow 1 \ 2 \)), recall that we have the following definitions of useful Lorentz invariants

\[
\begin{align*}
    s &= \left( p_a + p_b \right)^2 = \left( p_1 + p_2 \right)^2, \\
    t &= \left( p_a - p_1 \right)^2 = \left( p_b - p_2 \right)^2, \\
    u &= \left( p_a - p_2 \right)^2 = \left( p_b - p_1 \right)^2,
\end{align*}
\] (20.3)

which satisfy total energy momentum conservation,

\[ s + t + u = m_a^2 + m_b^2 + m_1^2 + m_2^2. \] (20.4)

The flux factor has the following forms in the two relevant frames

\[
\sqrt{\left(p_a^\mu p_b^\mu\right)^2 - m_a^2 m_b^2} = \left| \vec{p}_{\text{LAB}} \right| m_b \ (\text{LAB frame})
\]

\[ = \left| \vec{p}_{\text{CM}} \right| \sqrt{s} \ (\text{CM frame}). \] (20.5)

Recall that in the LAB frame one of the initial particles (the target) is sitting still, while in the CM frame the total 3-momentum of the 2-particle system vanishes. In the CM frame the 2 incoming particles collide in a head-on configuration with a common magnitude for their 3-momentum,

\[ p_{\text{CM}} = \sqrt{\left(s - (m_a - m_b)^2\right)\left(s - (m_a + m_b)^2\right)} \]

\[ = \frac{2\sqrt{s}}{2\sqrt{s}}. \] (20.6)

In the final state the corresponding common magnitude is
\[ p'_{\text{CM}} = \sqrt{s - (m_1 + m_2)^2} \left( s - (m_1 - m_2)^2 \right) / 2\sqrt{s}. \] (20.7)

Recall also that in the CM for 2 to 2 processes the only (unconstrained) dynamical degrees of freedom are the angles,

\[
\int d\Phi_{2,\text{CM}} = \int d\Omega_{\text{CM}} \frac{p_i^2 dp_i \delta(p'_{\text{CM}} - p_1)}{\sqrt{p_i^2 + m_i^2} \sqrt{p_1^2 + m_1^2} \left( \frac{p_1}{\sqrt{p_1^2 + m_1^2}} + \frac{p_1}{\sqrt{p_1^2 + m_1^2}} \right)}
\] (20.8)

Thus the interesting quantity is the differential angular cross section,

\[
\frac{d\sigma}{d\Omega_{\text{CM}}} \bigg|_{a+b\rightarrow a1\gamma} \equiv \frac{\bar{M}_{a+b\rightarrow a1\gamma}}{64\pi^2 s} p'_{\text{CM}}.
\] (20.9)

We can rewrite this in invariant notation as

\[
\frac{d\sigma}{dt} = \frac{\bar{M}_{a+b\rightarrow a1\gamma}^2}{64\pi s p_{\text{CM}}^2} = \frac{\left| \bar{M}_{a+b\rightarrow a1\gamma} \right|^2}{16\pi \left( s - (m_a + m_b)^2 \right) \left( s - (m_a - m_b)^2 \right)}
\] (20.10)

Now let us focus on the calculation of the matrix element \( \bar{M} \) in QED. The interesting structure arises from the spin of the leptons and the photons and the “bar” symbol on the matrix element is to remind us about this point. By definition an initial state that is unpolarized involves an incoherent average over the possible incoming spin states. If no spin dependent measurements are performed on the final states (e.g., we measure only the direction of the outgoing leptons as in the cross sections defined above), we must sum over the possible spin states of the outgoing leptons. The building blocks of the amplitude calculations are the Dirac spinors for fermions and
antifermions, i.e., the solutions of the Dirac equation without the overall plane wave factor, $e^{-ip\cdot x}$.

\[
\begin{align*}
(\not{p} - m)u(s, p) &= 0 : \not{u}(s, p)(\not{p} - m) = 0 \text{ (fermions)}, \\
(\not{p} + m)v(s, p) &= 0 : \not{v}(s, p)(\not{p} + m) = 0 \text{ (antifermions)}. 
\end{align*}
\] (20.11)

It follows that the sums over spin states yield

\[
\sum_s u(s, p)\not{u}(s, p) = \not{p} + m, \quad \sum_s v(s, p)\not{v}(s, p) = \not{p} - m.
\] (20.12)

Using the Feynman rules the scattering amplitude has the form

\[
\mathcal{M}_{e^+e^-\to\mu^+\mu^-} = (i|e|^2 \not{\nabla}(\not{\bar{v}}) (b) \gamma_\mu \mathcal{u}(e) (a) \not{\bar{u}}(\mu) (2) \gamma_\nu \mathcal{v}(\not{\bar{v}}) (1) \frac{-ig^{\mu\nu}}{s},
\] (20.13)

where there is a spinor $u$ for each incoming fermion, a $\not{u}$ for each outgoing fermion, a $v$ for each outgoing antifermion and a $\not{v}$ for each incoming antifermion (recall that incoming fermions are something like outgoing antifermions).

The appropriately spin averaged and summed amplitude squared is

\[
|\mathcal{M}_{e^+e^-\to\mu^+\mu^-}|^2 = \left( \frac{1}{2} \right)^2 \frac{e^4}{s^3} \sum_{s_1, s_2} \not{\nabla}(\not{\bar{v}}) (s_1, p_2) \gamma_\mu \mathcal{u}(e) (s_1, p_1) \not{u}(\mu) (s_2, p_2) \mathcal{v}(\not{\bar{v}}) (s_2, p_1) \\
\times \sum_{s_1, s_2} \not{u}(\mu) (s_2, p_2) \gamma_\nu \mathcal{v}(\not{\bar{v}}) (s_1, p_1) \not{v}(\not{\bar{v}}) (s_2, p_2) \mathcal{u}(e) (s_1, p_1) \\
\equiv \left( \frac{4\pi\alpha}{s} \right)^2 L^{(e)}_{\mu\nu} L^{(u)\mu\nu}.
\] (20.14)

We have introduced two polarization tensors to separately describe the coupling of the photon to the electron and to the muon. This “factorization” is characteristic of simple exchange processes. We can evaluate the spin sums using the definitions given earlier and the concept of the trace, as introduced in the previous lecture,
\[ L_{\mu \nu} = \frac{1}{2} \sum_{s_a, s_b} \bar{v}(s_a, p_a) \gamma_\mu u(s_a, p_a) u(s_b, p_b) \bar{u}(s_b, p_b) \gamma_\nu v(s_b, p_b) \]

\[ = \frac{1}{2} \text{Tr} \left[ (\not{p}_b - m) \gamma_\mu (\not{p}_a + m) \gamma_\nu \right] \]

\[ = 2 \left[ p_{a \mu} p_{b \nu} + p_{a \nu} p_{b \mu} - g_{\mu \nu} \left( p_a \cdot p_b + m^2 \right) \right], \tag{20.15} \]

where we used the trace identity

\[ \text{Tr} \left[ \gamma^a \gamma^b \gamma^\mu \gamma^\nu \right] = 4 \left[ g^{a \beta} g^{\mu \nu} - g^{a \mu} g^{\beta \nu} + g^{a \nu} g^{\beta \mu} \right]. \tag{20.16} \]

A brief summary of such useful identities is provided at the end of this lecture.

Note that

\[ q_{\mu} L_{\mu \mu} = q_{\nu} L_{\mu \nu} = 0, \tag{20.17} \]

where

\[ q^\mu = p_a^\mu + p_b^\mu, \quad q \cdot p_a = q \cdot p_b = m_e^2 + p_a \cdot p_b. \tag{20.18} \]

This result is to be expected from the fact that the electromagnetic current is conserved. A similar expression describes the muon coupling to the photon, but with \( m_\mu \) replacing \( m_e \). We can use these results to work out the form of the square of the averaged matrix element,

\[ \left| \overline{M}_{e^- e^+ \rightarrow \mu^+ \mu^-} \right|^2 = 4 \left( \frac{4 \pi \alpha}{s} \right)^2 \left[ 2 \left( p_a \cdot p_1, p_b \cdot p_2 + p_a \cdot p_2, p_b \cdot p_1 \right) - 2 \left( p_1 \cdot p_2 \right) \left( p_a \cdot p_b + m_e^2 \right) + 4 \left( p_a \cdot p_b + m_e^2 \right) \left( p_1 \cdot p_2 + m_\mu^2 \right) \right] \]

\[ = \frac{32 \pi^2 \alpha^2}{s^2} \left[ t^2 + u^2 - 4(t+u)(m_e^2 + m_\mu^2) + 6(m_e^2 + m_\mu^2)^2 \right]. \tag{20.19} \]

It is also instructive to rewrite this scattering amplitude in terms of the usual relativistic velocity fractions,

\[ \beta \equiv \frac{v}{c} = \frac{p}{E}. \tag{20.20} \]
In terms of this variable the CM kinematics look like

\[ E_a = E_b = E_1 = E_2 = \frac{\sqrt{s}}{2}, \]

\[ |\vec{p}_a| = |\vec{p}_b| = \beta_e \frac{\sqrt{s}}{4}, \beta_e = \sqrt{1 - \frac{4m_e^2}{s}}, \]

\[ |\vec{p}_1| = |\vec{p}_2| = \beta_\mu \frac{\sqrt{s}}{4}, \beta_\mu = \sqrt{1 - \frac{4m_\mu^2}{s}}. \]  

(20.21)

Since the production of the muon pair requires \( \sqrt{s}/2 \geq m_\mu = 207m_e \), \( \beta_e \) is equal to 1 to better than 1 part in \( 10^5 \) in the kinematic region of interest. In these variables as applied to the CM system we have

\[ |\vec{M}_{e^+e^- \rightarrow \mu^+\mu^-}|^2 = 16\pi^2 \alpha^2 \left[ 3 - \beta_e^2 - \beta_\mu^2 + \beta_e^2 \beta_\mu^2 \cos^2 \theta_{CM} \right] \]

\[ \approx 16\pi^2 \alpha^2 \left[ 2 - \beta_\mu^2 + \beta_\mu^2 \cos^2 \theta_{CM} \right], \]

(20.22)

where \( \theta_{CM} \) is the angle between the direction of the incoming electron and the outgoing muon. We also note the following relationships involving the variables of interest

\[ p_a \cdot p_b = \frac{s - 2m_e^2}{2} = \frac{s}{4} \left( 1 + \beta_e^2 \right) = \frac{s}{4} \]

\[ p_1 \cdot p_2 = \frac{s - 2m_\mu^2}{2} = \frac{s}{4} \left( 1 + \beta_\mu^2 \right), \]

(20.23)

\[ p_a \cdot p_1 = p_b \cdot p_2 = \frac{m_e^2 + m_\mu^2 - t}{2} = \frac{s}{4} \left( 1 - \beta_e \beta_\mu \cos \theta_{CM} \right) = \frac{s}{4} \left( 1 - \beta_e \beta_\mu \cos \theta_{CM} \right), \]

\[ p_a \cdot p_2 = p_b \cdot p_1 = \frac{m_e^2 + m_\mu^2 - u}{2} = \frac{s}{4} \left( 1 + \beta_e \beta_\mu \cos \theta_{CM} \right) = \frac{s}{4} \left( 1 + \beta_e \beta_\mu \cos \theta_{CM} \right), \]

where, as noted above, the approximation involved in the last expressions on each line is a very good one. Thus in the nonrelativistic limit, \( \beta_\mu \rightarrow 0 \), the last three of the 4-D dot products are \( s/4 \) with no angular dependence, while in the relativistic limit, \( \beta \rightarrow 1 \), the angular dependence becomes relevant. With these results we can now write down the form of the scattering cross section in the CM, (from now on we ignore the mass of the electron)
\[
\frac{d\sigma}{d\Omega_{CM}} \bigg|_{e^+e^-\rightarrow \mu^+\mu^-} = \left| \mathcal{M}_{e^+e^-\rightarrow \mu^+\mu^-} \right|^2 \frac{p_{CM}'}{64\pi^2 s} p_{CM}
\]

\[
= \alpha^2 \beta_\mu \left[ 2 - \beta_\mu^2 + \beta_\mu^2 \cos^2 \theta_{CM} \right]
\]  

(20.24)

\[
\frac{\beta_{\mu \rightarrow 1}}{2\alpha^2} \rightarrow \frac{\beta_\mu}{4s},
\]

\[
\frac{\beta_{\mu \rightarrow 1}}{4s} \rightarrow \alpha^2 \left[ 1 + \cos^2 \theta_{CM} \right].
\]

The nonrelativistic limit tells us that just above threshold for muon pair production the cross section is vanishing like the first power of the velocity (or the momentum) of the muons. In the other limit of \( s \gg m_\mu^2 \) we see the characteristic \((1 + \cos^2 \theta)\) behavior of a photon in the s-channel. The relativistic limit also has a compact form in terms of the invariants,

\[
\frac{d\sigma}{d\Omega_{CM}} \bigg|_{e^+e^-\rightarrow \mu^+\mu^-} \rightarrow \frac{\alpha^2}{2s} \left[ \frac{t^2 + u^2}{s^2} \right],
\]  

(20.25)

where, in this limit,

\[
\frac{t}{s} \approx -\frac{1 - \cos \theta_{CM}}{2}, \quad \frac{u}{s} \approx -\frac{1 + \cos \theta_{CM}}{2}.
\]  

(20.26)

We can also consider the invariant differential cross section

\[
\frac{d\sigma}{dt} \bigg|_{e^+e^-\rightarrow \mu^+\mu^-} = \left| \mathcal{M}_{e^+e^-\rightarrow \mu^+\mu^-} \right|^2 \frac{64\pi s p_{CM}^2}{64\pi s p_{CM}^2} \left[ t^2 + u^2 - 4(t + u)(m_\mu^2) + 6(m_\mu^2)^2 \right]
\]

(20.27)

\[
\frac{2\pi \alpha^2}{s^4} \left[ t^2 + u^2 \right].
\]

Finally the total cross section for this annihilation process can be obtained by integrating the differential cross section.
\[\sigma_{e^+e^+\to \mu^+\mu^-} = \int d\Omega_{CM} \frac{d\sigma}{d\Omega_{CM}}_{e^+e^+\to \mu^+\mu^-} = \frac{\pi\alpha^2}{s} \beta_\mu \left[ 2 - \beta_\mu^2 + \frac{\beta_\mu^2}{3} \right] = \frac{2\pi\alpha^2}{3s} \beta_\mu \left( 3 - \beta_\mu^2 \right) \] (20.28)

The interested student can obtain the same relativistic result by integrating the relativistic expression above for \(d\sigma/dt\), \(0 > t > -s\). Note the similarity with the weak interaction cross sections in the high energy limit where the vector boson masses do not matter.

Before closing this lecture we note that we can now use these results for the annihilation process to consider instead the scattering process \(e\mu \to e\mu\) with little added work. The trick here is to use the concept called “crossing” symmetry that we mentioned last quarter. This is represented in the figure to the right. The incoming positron has become an outgoing electron while the outgoing anti-muon has become an incoming muon. Furthermore the scattering amplitude is the same when written in terms of the dot products. What has changed is that we have switched momenta, \(p_b \leftrightarrow -p_1\), for the purposes of defining the invariants \(s, t\) and \(u\). For the scattering process the new identifications are: \(p_a\) is the incoming electron, \(-p_b\) is the outgoing electron, \(-p_1\) is the incoming muon and \(p_2\) is the outgoing muon. Hence the new invariants for the scattering process are identified as (along with the original annihilation process invariants)
\[ s_{\text{scatt}} = (p_a - p_t)^2 = m_{e}^2 + m_{\mu}^2 - 2p_a \cdot p_t \]
\[ = (p_b - p_2)^2 = m_{e}^2 + m_{\mu}^2 - 2p_b \cdot p_2 + t_{\text{annih}}; \]
\[ t_{\text{scatt}} = (p_a - \{-p_b\})^2 = 2m_{e}^2 + 2p_a \cdot p_b \]
\[ = (-p_1 - p_2)^2 = 2m_{\mu}^2 + 2p_1 \cdot p_2 = s_{\text{annih}}; \]
\[ u_{\text{scatt}} = (p_a - p_2)^2 = m_{e}^2 + m_{\mu}^2 - 2p_a \cdot p_2 \]
\[ = (-p_b - \{-p_1\})^2 = m_{e}^2 + m_{\mu}^2 - 2p_b \cdot p_1 = u_{\text{annih}}. \]

Thus crossing means that we should exchange \( t \) and \( s \) in the original annihilation amplitude squared,

\[
\left| \mathcal{M}_{e\mu \rightarrow e\mu} \right|^2 = 4 \left( \frac{4\pi\alpha}{t} \right)^2 \left[ 2(p_a \cdot p_b \cdot p_2 + p_a \cdot p_2 \cdot p_b \cdot p_1) \\
- 2(p_a \cdot p_b)(p_1 \cdot p_2 + m_{\mu}^2) - 2(p_1 \cdot p_2)(p_a \cdot p_b + m_{e}^2) \\
+ 4(2(p_a \cdot p_b + m_{e}^2)(p_1 \cdot p_2 + m_{\mu}^2) \right] \]
\[ = \frac{32 \pi^2 \alpha^2}{t^2} \left[ s^2 + u^2 - 4(s + u)(m_{e}^2 + m_{\mu}^2) + 6(m_{e}^2 + m_{\mu}^2)^2 \right] \]
\[ \approx \frac{32 \pi^2 \alpha^2}{t^2} \left[ s^2 + u^2 - 4(s + u)(m_{\mu}^2) + 6(m_{\mu}^2)^2 \right]. \]

The kinematics are a little different in the scattering process as compared to annihilation. In the latter there are equal mass fermions in the initial state and in the final state (although the masses are different in the two states), while in the former process there are (the same) unequal mass fermions in both the initial and final states. To work out the magnitudes of the momenta and energy in the scattering case we need to recall our evaluation of \( p_{\text{CM}} \) in the unequal mass case from last quarter (already quoted above). For the \( e\mu \) scattering process we have in the CM system

\[
p_{\text{CM}} = p'_{\text{CM}} = \frac{1}{2\sqrt{s}} \sqrt{s - (m_{e} + m_{\mu})^2} \sqrt{s - (m_{e} - m_{\mu})^2} \approx \frac{s - m_{\mu}^2}{2\sqrt{s}} \]
\[ = |\vec{p}_a| = |\vec{p}_b| = |\vec{p}_1| = |\vec{p}_2|. \]
The corresponding energies in the CM are

\[ E_{e,\text{CM}} = E_a = E_b = \frac{s + m_e^2 - m_\mu^2}{2\sqrt{s}} \simeq \frac{s - m_\mu^2}{2\sqrt{s}}, \]

\[ E_{\mu,\text{CM}} = E_1 = E_2 = \frac{s + m_\mu^2 - m_e^2}{2\sqrt{s}} \simeq \frac{s + m_\mu^2}{2\sqrt{s}}. \tag{20.32} \]

For now let us look at this cross section in the relativistic limit where all masses can be ignored. In this limit

\[ |\mathcal{M}_{\bar{e}^\prime \mu^* \rightarrow \bar{e}^\prime \mu^*}|^2 \rightarrow \frac{32\pi^2 \alpha^2}{t^2} \left[ s^2 + u^2 \right], \tag{20.33} \]

and

\[ \frac{d\sigma}{dt}_{\bar{e}^\prime \mu^* \rightarrow \bar{e}^\prime \mu^*} \rightarrow \frac{2\pi \alpha^2}{s^2 t^2} \left[ s^2 + u^2 \right], \tag{20.34} \]

which should be compared to Eq. (20.27). Defining a CM scattering angle as above in the relativistic limit, we find

\[ \frac{d\sigma}{d\Omega_{\text{CM}}}_{\bar{e}^\prime \mu^* \rightarrow \bar{e}^\prime \mu^*} \rightarrow \frac{\alpha^2}{2s} \left[ \frac{s^2 + u^2}{t^2} \right] \frac{\alpha^2}{2s} \left[ \frac{1 + \cos \theta_{\text{CM}}}{2} \right]^2 \left[ \frac{1 - \cos \theta_{\text{CM}}}{2} \right]^2 \tag{20.35} \]

which should be compared to Eq. (20.24). The singular behavior in the forward direction, \( \theta_{\text{CM}} \rightarrow 0, t \rightarrow 0 \), is what we expect from the exchange of a massless photon. The differential cross section is not formally integrable in this region. In real situations this divergence in the infrared, long wavelength regime is controlled by that fact that at long distances (e.g., on the scale of an atom) matter is neutral. Note that this divergent behavior is characteristic of (unbroken symmetry) gauge interactions, and we will see the analogue behavior when we study QCD.
However, it does not obtain for the weak interactions, where the vector bosons are massive.

We will consider the angular structure of these cross sections in more detail in the next lecture and the homework.

**USEFUL DETAILS:** Dirac matrix algebra

The numerators of the amplitudes involving fermions typically involve traces. The following relations arise from the definition of the Dirac matrices,

\[
\begin{align*}
\text{Tr}[1] &= 4, \\
\text{Tr}[\gamma^\alpha] &= \text{Tr}[\gamma^\alpha \gamma^\beta \gamma^\mu] = \text{Tr}[\text{odd number of } \gamma\text{'s}] = 0, \\
\text{Tr}[\gamma^\alpha \gamma^\beta] &= 4[g^{\alpha\beta}], \\
\text{Tr}[\gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu] &= 4[g^{\alpha\beta} g^{\mu\nu} - g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\mu}].
\end{align*}
\]

We can derive similar results for traces involving \( \gamma^5 \),

\[
\begin{align*}
\text{Tr}[\gamma^5] &= \text{Tr}[\gamma^\alpha \gamma^\beta \gamma^5] = 0, \\
\text{Tr}[\gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu \gamma^5] &= -4i\epsilon^{\alpha\beta\mu\nu}.
\end{align*}
\]

There are also simple results for contractions of the \( \gamma \) matrices. The results appropriate to 4 space-time dimensions are the following,

\[
\begin{align*}
\gamma^\mu \gamma_\mu &= 4, \\
\gamma^\mu \gamma^\alpha \gamma_\mu &= -2\gamma^\alpha, \\
\gamma^\mu \gamma^\alpha \gamma^\beta \gamma_\mu &= 4g^{\alpha\beta}, \\
\gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\nu \gamma_\mu &= -2\gamma^\nu \gamma^\beta \gamma^\alpha.
\end{align*}
\]