Physics 557 – Lecture 19

The Standard Model of the Electroweak Interactions Revisited – The Feynman Rules

To effectively summarize what we have learned so far about the electroweak interactions of the leptons let us discuss the Feynman rules for the interaction vertices in this theory that arise from the Lagrangian discussed in Lecture 18. Recall that we can obtain the interaction vertices by considering the terms in the quantity $i\mathcal{L}$ that are cubic, and higher, in the fields.

First consider the vertices describing the interaction of the matter fermions with the gauge bosons. From expanding the Lagrangian for the lepton with the correct definition of the $Z$ and $A$ fields we found in the last lecture that the neutral current sector looks like

$$i\mathcal{L}_{\text{neutral current}} = i e \bar{\nu} \gamma^\mu e A_\mu - i \sqrt{\frac{G_F M_Z^2}{2\sqrt{2}}} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu Z_\mu$$

$$-i \sqrt{\frac{G_F M_Z^2}{2\sqrt{2}}} \left[ R_e \bar{\nu} \gamma^\mu (1 + \gamma_5) e + L_e \bar{\nu} \gamma^\mu (1 - \gamma_5) e \right] Z_\mu. \tag{19.1}$$

Thus the electron couples to the photon via the vertex in the figure, which we have already seen, and which has the form

$$-iQ_e \bar{\nu} \gamma^\mu e A_\mu = i e \bar{\nu} \gamma^\mu e A_\mu. \tag{19.2}$$

We can similarly read off the corresponding coupling to the $Z$ (the neutral weak interaction) and find the following vertex,
The corresponding coupling of the $Z$ to neutrinos is the following,

$$
-i \sqrt{\frac{G_F M_Z^2}{2\sqrt{2}}} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu Z_\mu
$$

$$
= -i \frac{g}{4 \cos \theta_W} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu Z_\mu. \tag{19.4}
$$

The charged current vertex, which we have discussed already last quarter, arises from the charged current part of the Lagrangian that we introduced in Lecture 18, Eq. (18.33)

$$
i \mathcal{L}_{\text{charged current}} = -i \frac{g}{2\sqrt{2}} \left[ \bar{\nu} \gamma^\mu (1 - \gamma_5) e W_{\mu}^+ + \bar{e} \gamma^\mu (1 - \gamma_5) \nu W^-_{\mu} \right]. \tag{19.5}
$$

This yields the following vertices

$$
-i \frac{g}{2\sqrt{2}} \bar{\nu} \gamma^\mu (1 - \gamma_5) e W_{\mu}^+
$$

$$
= -i \sqrt{\frac{G_F M_W^2}{\sqrt{2}}} \bar{\nu} \gamma^\mu (1 - \gamma_5) e W_{\mu}^+. \tag{19.6}
$$

and the conjugate process
In comparing the $Z$ and $W$ vertices, note the missing factor of $\frac{1}{\sqrt{2}}$ and the substitution $M_Z^2 \to M_W^2$ (the factor of $\cos \theta_W$).

Next consider the various couplings between the gauge bosons themselves. These arise from the cubic and quartic terms in the original SU(2)$_L$ gauge Lagrangian, $-\frac{i}{4} F^{\mu \nu} F_{\mu \nu}^k$, where $F^{\mu \nu} = \partial_\mu W^\nu - \partial_\nu W^\mu - g_{\cal{E}}^{\mu \nu \alpha \beta} W^\alpha_{\mu} W^\beta_{\nu}$. Thus the generic cubic term looks like (there is a factor of 4 in the numerator from the 2 in the cross term and the 2 from the two terms in the curl)

$$i \mathcal{L}_{\text{gauge, cubic}} = -\frac{i}{4} (-g \cdot 2 \cdot 2 \cdot \partial_\mu W_\nu^k g_{\mu \alpha \nu \beta} g_{\epsilon}^{\kappa \lambda m} W^\lambda_\alpha W^m_\beta)$$

$$= i g g_{\mu \alpha} g^{\nu \beta} \epsilon^{\kappa \lambda m} \partial_\mu W_\nu^k W^\lambda_\alpha W^m_\beta.$$

We can define all the momenta as incoming so that $\partial_\mu W^k_\nu \to -iq^k_\mu W^k_\nu$, with $q^k_\mu$ the (incoming) momentum of vector boson with SU(2) index $k$ ($k, l, m$ must all be different due to the antisymmetric structure constant $\epsilon^{klm}$). There are 3! or 6 terms or coupling structures that we find when expanding out this antisymmetric product. We can write (all momenta defined as incoming)

$$i \mathcal{L}_{\text{gauge, cubic}} = g \left[ g^{\nu \beta} (q^\alpha_1 - q^\alpha_3) + g^{\nu \alpha} (q^\beta_2 - q^\beta_1) + g^{\alpha \beta} (q^\nu_3 - q^\nu_2) \right] W^1_\nu W^2_\alpha W^3_\beta.$$

To get to “physically” relevant vertices we must project $W^1$ and $W^2$ onto $W^+$ and $W$ and $W^3$ onto $Z$ and $A$. Recall that, since the final photon is a mixture, including a component from the original non-Abelian SU(2) theory, it also participates in the “non-Abelian-like” couplings. The former projection introduces a factor of $i$, while the latter introduces factors of $\cos \theta_W$ and $\sin \theta_W$, respectively.
The photon couples to a W boson pair in the following way. The arrows indicate the assumed flow of momentum for the definition given. We find the following form (the sign is the hard part, depending on definitions of incoming versus outgoing states and many different expressions are provided in the literature),

\[ i g \sin \theta_W \left[ g^{\alpha \beta} (k_+ - k_-)^\mu + g^{\alpha \mu} (p - k_+)^\beta \right. \\
\left. + g^{\beta \mu} (k_- - p)^\alpha \right] W_\alpha^+ W^-_\beta A_\mu. \tag{19.10} \]

Recall that the coupling for the A vertex can also be written as \( g \sin \theta_W = e \), i.e., the charge of the W is e. The corresponding vertex with the photon replaced by a Z differs simply by changing \( \sin \theta_W \) to \( \cos \theta_W \), (the other component of \( W^3 \))

\[ i g \cos \theta_W \left[ g^{\alpha \beta} (k_+ - k_-)^\mu + g^{\alpha \mu} (p - k_+)^\beta \right. \\
\left. + g^{\beta \mu} (k_- - p)^\alpha \right] W_\alpha^+ W^-_\beta Z_\mu. \tag{19.11} \]

Note that the coupling can also be written \( g \cos \theta_W = e \cot \theta_W \).

Next we come to the quartic couplings of the gauge bosons. These arise from the square of the purely non-Abelian term in the field strength tensor. The generic form is

\[ i \mathcal{L}_{\text{gauge, quartic}} = -i \frac{g^2}{4} \left[ \varepsilon^{ilm} \varepsilon^{krs} g^{\mu \alpha} g^{\nu \beta} \right] W^l_\mu W^m_\nu W^-_\alpha W^s_\beta. \tag{19.12} \]

With 4 formally identical fields (members of the same multiplet), there are 4! ways to construct this product. Of these, only 6 are actually distinct (canceling the factor of 4 in the denominator) and we can write
Due to the antisymmetric group factors, it cannot be the case that 3 of the coupled bosons are of the same type. Thus there are 2 general possibilities of the form $W^\alpha W^\beta W^\gamma W^\delta$ and $W^\alpha W^\beta W^\gamma W^\delta$. As we did with the cubic vertex, we must project onto the physical states, yielding 4 physically distinct cases of the form $W^+ W^\pm W^\pm W^\pm$. First consider the $W^+ W^\pm W^\pm W^\pm$ vertex, where again we define all bosons as incoming to yield

$$i \mathcal{L}_{gauge, quartic} = -ig^2 \left[ \varepsilon^{klm} \varepsilon^{krs} \left( g^{\mu \alpha} g^{\nu \beta} - g^{\mu \beta} g^{\nu \alpha} \right) + \varepsilon^{klr} \varepsilon^{kms} \left( g^{\mu \nu} g^{\alpha \beta} - g^{\mu \beta} g^{\nu \alpha} \right) + \varepsilon^{klm} \varepsilon^{krs} \left( g^{\mu \nu} g^{\alpha \beta} - g^{\mu \beta} g^{\nu \alpha} \right) \right] W_{\mu}^l W_{\nu}^m W_{\alpha}^r W_{\beta}^s. \tag{19.13}$$

For the $W^+ W^\pm W^\pm W^\pm$ vertex we have

$$-ig^2 \sin^2 \theta_W \left[ 2g^{\mu \nu} g^{\alpha \beta} - g^{\mu \alpha} g^{\nu \beta} - g^{\mu \beta} g^{\nu \alpha} \right] \times W_{\alpha}^\pm W_{\beta}^\pm A_{\mu} A_{\nu}, \tag{19.14}$$

where $g^2 \sin^2 \theta_W = e^2$.

The $W^+ W^\pm W^\pm W^\pm$ vertex looks like ($g^2 \sin \theta_W \cos \theta_W = e^2 \cot \theta_W$)

$$-ig^2 \sin \theta_W \cos \theta_W \left[ 2g^{\mu \nu} g^{\alpha \beta} - g^{\mu \alpha} g^{\nu \beta} - g^{\mu \beta} g^{\nu \alpha} \right] \times W_{\alpha}^\pm W_{\beta}^\pm A_{\mu} Z_{\nu}. \tag{19.15}$$

For the $W^+ W^\pm Z^\pm Z^\pm$ and $W^+ W^\pm W^\pm W^\pm$ vertices we have
where $g^2 \cos^2 \theta_W = e^2 \cot^2 \theta_W$, and

$$-ig^2 \left[ 2 g^\mu \gamma^\nu g^{\alpha \beta} - g^{\mu \alpha} g^{\nu \beta} - g^{\mu \beta} g^{\alpha \nu} \right] W^\alpha_\mu W^\beta_\nu Z_\mu Z_\nu,$$

(19.16)

where $g^2 = e^2 / \sin^2 \theta_W$.

Finally we want to consider the couplings of the scalar Higgs boson. Due to the purely Higgs part of the Lagrangian, the Higgs boson has both a cubic and a quartic coupling from the expansion of the term $-\lambda \left[ (v + h) / \sqrt{2} \right]^4$. In this case the $n!$ factor, due to all the ways the $n$ identical bosons can be identified with the external particles, is explicit. Thus the cubic term has a factor of 4 in the denominator from the $\sqrt{2}$ normalization factor, a factor 4 in the numerator from the expansion of the quartic polynomial and the 3! factor to yield,

$$-6i \lambda vhhh = -3i m_h^2 \sqrt{2} G_F \ hhh.$$  

(19.18)

The corresponding quartic terms has no extra factor of 4 from the expansion but the symmetry factor is now 4! to yield,
As we noted in the last lecture, since the coupling of fermions to the scalar vacuum expectation value via the Yukawa term provides the mass of the fermion, the coupling of a fermion to the physical Higgs particle is proportional to the fermion’s mass. The vertex has the general form

\[ -i \frac{m_f}{v} \bar{f} f = -i m_f \sqrt{2G_F} \bar{f} f. \]  

These fermion-antifermion channels are presumably the dominant decay channels for a Standard Model Higgs with a mass of 126 GeV. The mass is too small for the top-antitop channel, but the bottom-antibottom channel is expected to provide about 56% of the decays. Unfortunately this channel has an enormous background from QCD (no Higgs) processes and has not yet played a role in the “discovery”. However, the tau-antitau channel, which should be about 6% of the decays is part of the current analyses.

The coupling of the original scalar doublet to the gauge bosons means that the physical Higgs boson also should have such couplings. We can read these terms off by returning to the terms in the Higgs Lagrangian of Lecture 18 that gave masses to the vector bosons and replacing \( v \) by \( v + h \),

\[ i \mathcal{L}_{\text{scalar, vector bosons}} = i \frac{v^2}{8} \left[ g^2 \left| W^+ \right|^2 + g^2 \left| W^- \right|^2 + \left( g'^2 + g^2 \right) \left( Z \right)^2 \right]. \]

\[ \rightarrow i \frac{\left( v + 2vh + h^2 \right)}{8} \left[ g^2 \left| W^+ \right|^2 + g^2 \left| W^- \right|^2 + \left( g'^2 + g^2 \right) \left( Z \right)^2 \right]. \]

Thus we have couplings of a single Higgs boson to pairs of both the \( W \)'s and the \( Z \). Noting that the two \( W \) terms both contribute to the same vertex and the \( Z \) vertex has a \( 2! \) identical particle factor, we find
\[ i \frac{g^2}{2} g^{\alpha \beta} W_\alpha^+ W_\beta^- h = ig M_W g^{\alpha \beta} W_\alpha^+ W_\beta^- h = 2i M_W^2 \sqrt{2} G_F g^{\alpha \beta} W_\alpha^+ W_\beta^- h \]  
(19.22)

These vertices are written for an incoming Higgs and outgoing vector bosons but they are essentially the same if the vector bosons are incoming (only the \( \dagger \) goes away).

For a Higgs mass of 126 GeV the WW channel should provide about 23\% of the Higgs decays, but is a challenge to detect with large backgrounds for the W\'s decaying to hadrons and the neutrino being undetected for the leptonic W decays. The ZZ channel should be about 3\% of the decays and can yield 4 leptons, which are straight forward to detect and this channel is included in current LHC analyses. Note that, for both WW and ZZ with a 126 GeV Higgs, one of the bosons is necessarily virtual (not on the mass-shell).

Note that, from the original Lagrangian, there is no direct coupling of the Higgs to 2 photons. However, this can occur through a virtual loop of heavy fermions (from Eq. (19.20)) or from a virtual W loop (from Eq. (19.22)). The resulting 2 photon decay occurs rarely at about the 0.1\% level, but has a distinctive experimental signature and is an important part of the current LHC analyses. (There is a corresponding 2 gluon decay at about the 10\% level, but again there are huge backgrounds.)

Finally the \( h^2 \) term yields quartic couplings between Higgs pairs and vector bosons pairs. These have the form (with the extra 2! for the identical Higgs bosons)
In the next lecture we will return to the question of calculating with these Feynman rules.

For completeness let us restate here also the forms of the relevant propagators:

fermion \( \sim \frac{i}{\mathcal{K} - m} = i \frac{\mathcal{K} + m}{k^2 - m^2} \),

massless vector \( \sim -i \frac{g^{\mu \nu}}{k^2} \),

massive vector \( \sim -i \frac{g^{\mu \nu} - \frac{k^\mu k^\nu}{M^2}}{k^2 - M^2} \).