Lecture 7

In the Lecture we discuss matrices and here we focus on the many useful commands/functions for the manipulations of matrices defined in Mathematica. This commands become very helpful when we must deal with large matrices where the by-hand techniques become cumbersome. As we saw in the Lecture 6 notebook we can enter a matrix as an ordered (curly brackets) list. For example, from Eq. (7.31)

\[
\text{In}[1]:= A = \{\{a, 0, -b\}, \{0, 1, 0\}, \{b, c, a\}\}
\]

\[
\text{Out}[1]= \{\{a, 0, -b\}, \{0, 1, 0\}, \{b, c, a\}\}
\]

\[
\text{In}[2]:= \text{MatrixForm}[A]
\]

\[
\text{Out}[2]/\text{MatrixForm}=
\begin{bmatrix}
a & 0 & -b \\
0 & 1 & 0 \\
b & c & a
\end{bmatrix}
\]

Mathematica allows us to directly evaluate the determinant. We have

\[
\text{In}[3]:= \text{Det}[A]
\]

\[
\text{Out}[3]= a^2 + b^2
\]

Or see the explicit example in Eq. (7.23)

\[
\text{In}[4]:= \text{Det}[\{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}\}]
\]

\[
\text{Out}[4]= 0
\]

Back to the matrix A and its inverse

\[
\text{In}[5]:= AI = \text{Inverse}[A]
\]

\[
\text{Out}[5]= \{\{a^2 + b^2, -ac - b^2, b\}, \{0, 1, 0\}, \{-b, -ac - b^2, a\}\}
\]

\[
\text{In}[6]:= \text{MatrixForm}[AI]
\]

\[
\text{Out}[6]/\text{MatrixForm}=
\begin{bmatrix}
a & -ac - b^2 & b \\
0 & 1 & 0 \\
-b & -ac - b^2 & a
\end{bmatrix}
\]

Checking we have (note the notation for matrix multiplication, like vectors)

\[
\text{In}[7]:= A.AI
\]

\[
\text{Out}[7]= \{\{a^2 + b^2, b^2, 0, 0\}, \{0, 1, 0\}, \{c - \frac{a^2 c}{a^2 + b^2} - \frac{b^2 c}{a^2 + b^2}, c - \frac{a^2 + b^2}{a^2 + b^2}\}\}
\]

OK - must ask for simplification
In[8] = MatrixForm[Simplify[A.AI]]

Out[8]/MatrixForm =

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

And checking the other order

In[9] = MatrixForm[Simplify[AI.A]]

Out[9]/MatrixForm =

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

The various other typical transformations of matrices are also built-in

Transpose

In[10] = MatrixForm[Transpose[A]]

Out[10]/MatrixForm =

\[
\begin{pmatrix}
a & 0 & b \\
0 & 1 & c \\
-b & 0 & a
\end{pmatrix}
\]

Conjugate


Out[11]/MatrixForm =

\[
\begin{pmatrix}
\text{Conjugate}[a] & 0 & -\text{Conjugate}[b] \\
0 & 1 & 0 \\
\text{Conjugate}[b] & \text{Conjugate}[c] & \text{Conjugate}[a]
\end{pmatrix}
\]

Hermitian or conjugate transpose

In[12] = MatrixForm[ConjugateTranspose[A]]

Out[12]/MatrixForm =

\[
\begin{pmatrix}
\text{Conjugate}[a] & 0 & \text{Conjugate}[b] \\
0 & 1 & \text{Conjugate}[c] \\
-\text{Conjugate}[b] & 0 & \text{Conjugate}[a]
\end{pmatrix}
\]

Also the Trace

In[13] = Tr[A]

Out[13] = 1 + 2 a

Now check-out the explicit examples in Appendix B.