The Immersed Interface Method – A Numerical Approach for Partial Differential Equations with Interfaces

by

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Abstract

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This thesis describes the Immersed Interface Method (IIM) for interface problems, in which the partial differential equations have discontinuities and singularities in the coefficients and the solutions. A typical example of such problems is heat conduction in different materials (discontinuous heat conductivity), or fluid interface problems where the surface tension gives a singular force that is supported only on the interface. The complexity of the interfaces makes it more difficult to develop efficient numerical methods.

Our immersed interface method is motivated by and related to Peskin's immersed boundary method (IBM) for solving incompressible Navier-Stokes equations with complicated boundaries. Our method, however, can apply to more general interface problems and usually attains second order accuracy.

We use uniform Cartesian grids so that we can take advantages of many conventional difference schemes for the regular grid points which are away from the interface. Hence attention is focused on developing difference schemes for the irregular grid points near the interface, which can cut through the grid in an arbitrary manner. Assuming a knowledge of jump conditions on the solution across the interface, which usually can be obtained either from the differential equation itself or by physical reasoning, we carefully choose the stencil and the coefficients of the difference scheme. By using local coordinate transformations and a modified undetermined coefficients method, we force the local truncation error to be $O(h^2)$ at regular grid points and $O(h)$ at irregular ones. For many interface problems, this leads to a second order accurate solution globally even if the solution is discontinuous. Cubic splines are used to represent and update the complicated interfaces.

We have implemented the immersed interface method for a number of interface problems including: general elliptic equations in one, two and three dimensions, alternating direction implicit methods for heat equations with singular sources or dipoles, the Stokes equations with a moving interface, and Stefan-like 1D moving interface problems in which the interface is determined by a nonlinear differential equation. Theoretical analysis and numerical examples are presented to show the efficiency of the immersed interface method. We believe that this methodology can be successfully applied to many other interface problems as well.

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