Wave Propagation Algorithms On Curved Manifolds With Applications To Relativistic Hydrodynamics

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This is to certify that I have examined this copy of a doctoral dissertation by

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and have found that it is complete and satisfactory in all respects,
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Abstract

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Many interesting physical systems are successfully modeled with time-dependent hyperbolic conservation laws on smooth manifolds. Examples of such systems include classical hydrodynamic flows in non-trivial geometry that are important in aerodynamic modeling, or the structure of shallow water flow on the surface of the earth important in climate modeling and oceanography. Another example arises in the study of relativistic flow of matter in the presence of compact objects like neutron stars and black holes. Astronomers wish to study this in order to to understand the high-energy radiation emitted from violent relativistic events such as the formation of bi-polar jets in active galactic nuclei, the collapse of very massive stars, and γ-ray bursts.

In this dissertation, we present a new high-resolution finite volume algorithm, an extension of the wave propagation method of LeVeque, to solve hyperbolic systems of conservation laws on general manifolds, $\mathscr{M}$. The representation of such equations on $\mathscr{M}$ introduces geometric terms due to the fact that the underlying basis will, in general, be spatially varying. For scalar components of the conservation laws, these geometric terms appear only in the flux function so that an autonomous flux function gains an explicit dependence on spatial variables through metric functions on $\mathscr{M}$. We develop a general approach to handle such spatially varying flux functions in the context of wave propagation.

In addition to spatially varying flux functions, vector components of the conservation laws will have geometrically induced source terms. The algorithm presented in this thesis avoids numerical complications that can arise from such source terms by treating the divergence operator in a local way. Cell averages are updated by solving modified Riemann problems. The modifications to Riemann data take care of source terms locally, thereby avoiding the need for an operator split approach for the sources.

The new algorithm has been implemented in the software package CLAWMAN, a freely available set of Fortran 90 routines. The implementation is general and can be used for different systems of conservation laws on a variety of manifolds. We demonstrate that the method works well on both flat and curved manifolds by presenting examples of the

algorithm and its implementation for both classical and relativistic conservation laws on various manifolds.

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