Conservation Laws and Finite Volume Methods
AMath 574
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Notes:

Outline

Today:
- 2D hyperbolic systems (on board)
- 2D Advection (on board)
- Clawpack
- Acoustics
- 2D examples

Friday:
- Software: Clawpack prerequisites and VM

Reading: Start Chapter 3 for Monday

Notes:

CLAWPACK — Conservation Laws Package

- Open source, 1d, 2d, 3d   **www.clawpack.org**
- Originally f77 with Matlab graphics.
- Moving to f95 with Python.
- Adaptive mesh refinement.
- OpenMP and MPI.

User supplies:
- Riemann solver, splitting data into waves and speeds
  (Need not be in conservation form)

- Boundary condition routine to extend data to ghost cells
  Standard `bc1.f` routine includes many standard BC's

- Initial conditions — `qinit.f`

- Source terms — `src1.f`

Notes:

Some applications where CLAWPACK has been used

- Aerodynamics, supersonic flows
- Seismic waves, tsunamis, flow on the sphere
- Volcanic flows, dusty gas jets, pyroclastic surges
- Ultrasound, lithotripsy, shock wave therapy
- Plasticity, nonlinear elasticity
- Chemotaxis and pattern formation
- Semiconductor modeling
- Multi-fluids, multi-phase flows, bubbly flow
- Combustion, detonation waves
- Astrophysics: binary stars, planetary nebulae, jets,
- Magnetohydrodynamics, plasmas, relativistic flow
- Numerical relativity — gravitational waves, cosmology

Notes:

Options for using Clawpack

1. Install from tar file or Subversion: **Instructions**.

   Requires some **prerequisites:** Fortran, Python modules.

   rerequisites exist on some AMath Linux computers.
2. Use the **VirtualClaw** virtual machine.
3. For some applications, use **EagleClaw**
   (Easy Access Graphical Laboratory for Exploring Conservation Laws)

Read the **documentation**!

Also perhaps useful:

AMath 583 Class notes on Python, Fortran, version control, etc.

Notes:

Advection examples

- **$CLAW/apps/advection/1d/example1/README.html**
- **Advection in EagleClaw**

Notes:

Example: Linear acoustics in a 1d tube

$$q = \begin{bmatrix} p \\ u \end{bmatrix} \qquad \begin{array}{l} p(x,t) = \text{pressure perturbation} \\ u(x,t) = \text{velocity} \end{array}$$

Equations:

$$\begin{array}{ll} p_t + \kappa u_x = 0 & \kappa = \text{bulk modulus} \\ \rho u_t + p_x = 0 & \rho = \text{density} \end{array}$$

or

$$\begin{bmatrix} p \\ u \end{bmatrix}_t + \begin{bmatrix} 0 & \kappa \\ 1/\rho & 0 \end{bmatrix} \begin{bmatrix} p \\ u \end{bmatrix}_x = 0.$$

Eigenvalues: $\lambda = \pm c$, where $c = \sqrt{\kappa/\rho} = $ sound speed

Second order form: Can combine equations to obtain

$$p_{tt} = c^2 p_{xx}$$

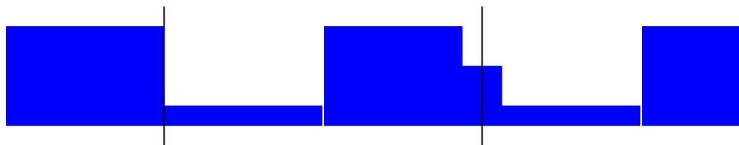## Riemann Problem

Special initial data:

$$q(x,0) = \begin{cases} q_l & \text{if } x < 0 \\ q_r & \text{if } x > 0 \end{cases}$$

Example: Acoustics with bursting diaphram
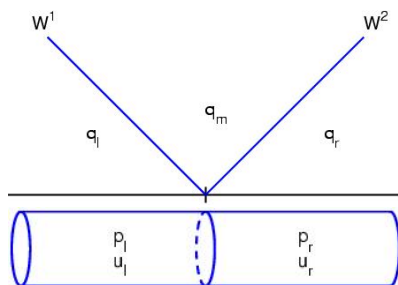

Pressure:

Acoustic waves propagate with speeds $\pm c$.

Riemann Problem for acoustics

Waves propagating in x–t space:

Left-going wave $\mathcal{W}^1 = q_m - q_l$ and
right-going wave $\mathcal{W}^2 = q_r - q_m$ are eigenvectors of A.

Acoustics examples

- **$CLAW/apps/acoustics/1d/example2/README.html**
- **Acoustics in EagleClaw**

Notes:

Two-dimensional advection examples

See the Clawpack gallery:
www.clawpack.org/doc/apps.html

Notes:

Equations of linear elasticity (in 2d)

$$
\begin{aligned}
\sigma_t^{11} & - (\lambda + 2\mu)u_x - \lambda v_y & = 0 \\
\sigma_t^{22} & - \lambda u_x - (\lambda + 2\mu)v_y & = 0 \\
\sigma_t^{12} & - \mu(v_x + u_y) & = 0 \\
\rho u_t & - \sigma_x^{11} - \sigma_y^{12} & = 0 \\
\rho v_t & - \sigma_x^{12} - \sigma_y^{22} & = 0
\end{aligned}
$$

where $\lambda(x,y)$ and $\mu(x,y)$ are Lamé parameters.
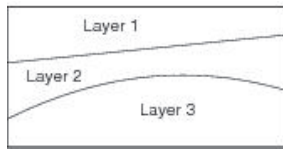
This has the form $q_t + Aq_x + Bq_y = 0$.

The matrix $(A\cos\theta + B\sin\theta)$ has eigenvalues $-c_p, \; -c_s, \; 0, \; c_s, \; c_p$

P-wave (dilatational) speed: $c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}$

S-wave (shear) speed: $c_s = \sqrt{\frac{\mu}{\rho}}$

Notes:

Seismic waves in layered earth

Layers 1 and 3: $\rho = 2, \ \lambda = 1, \ \mu = 1, \ c_p \approx 1.2, \ c_s \approx 0.7$

Layer 2: $\rho = 5, \ \lambda = 10, \ \mu = 5, \ c_p = 2.0, \ c_s = 1$

Impulse at top surface at $t = 0$.

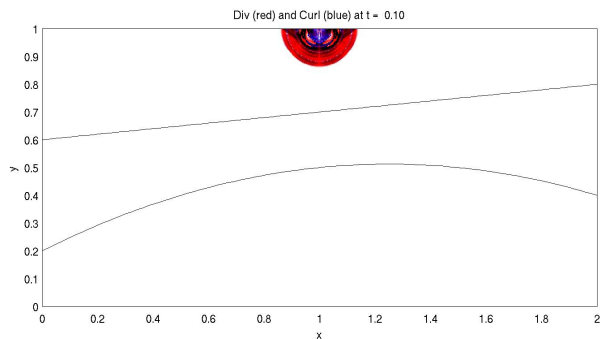Solved on uniform Cartesian grid (600×300).

Cell average of material parameters used in each finite volume cell.
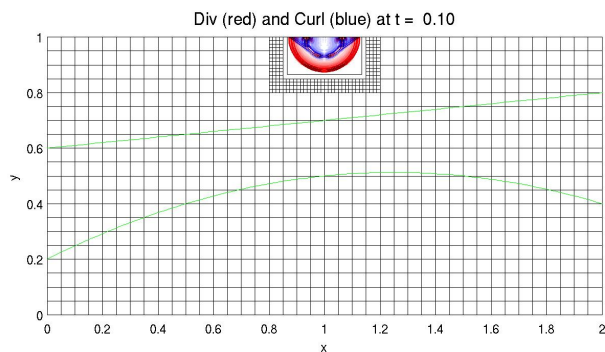
Extrapolation at computational boundaries.

Notes:

Seismic wave in layered medium

Red = div(u) [P-waves],   Blue = curl(u) [S-waves]

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