AMath 584 / Math 584
Homework #2
Due 11:00pm PDT
Tuesday, October 25, 2016

Name: Your Name Here

Netid: Your NetID Here

Problem 1. Find (by hand) both the reduced and full SVD of the matrix

$$A = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ 0 & 4 \end{bmatrix}.$$

Remember that things should be ordered so that $\sigma_1 \geq \sigma_2$.

Use the SVD to determine the following:

- The best rank-1 approximation to A.
- The 2-norm of A.
- A basis for the linear subspace $\{x \in \mathbb{R}^2 : \|Ax\|_2 = \|A\|_2 \|x\|_2\}$ and the dimension of this space.
- A basis for the orthogonal complement of the range of A.
- A basis for the null space of A^T.

Problem 2. Determine by hand the SVD of the matrix

$$A = \begin{bmatrix} 17 & 1 \\ 6 & 18 \end{bmatrix}.$$

Hint: one right singular vector is

$$v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}.$$

Problem 3. Exercise 6.1 in Trefethen and Bau. In addition, illustrate with a sketch showing the effect of $A = I - 2P$ on a typical vector v for the case

$$P = \frac{1}{5} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}.$$

Also explain why a matrix A of this form is sometimes called a "reflector". **Hint:** See also Figure 10.2 in the book and the discussion for the case of a "Householder reflector".

Problem 4. Exercise 6.3 in Trefethen and Bau. Use the SVD to do this.

Problem 5. Exercise 6.4 in Trefethen and Bau.

Problem 6. Exercise 7.1 in Trefethen and Bau.

Problem 7. Exercise 7.2 in Trefethen and Bau.

Problem 8. Exercise 8.2 in Trefethen and Bau.

You can write a Python function instead of Matlab if you prefer.

Test your routine on the matrices from Exercise 6.4 of Trefethen and Bau and submit your output.

Test it on other matrices of different sizes to convince yourself it is working properly. You do not need to turn in more output, but submit the code in a file `mgs.m` or `mgs.py` in a way that we can test it on other matrices of our choosing.

Please write readable code!