AMath 574
Homework 4
Due by 11:00pm on February 26, 2015

For submission instructions, see:

`http://faculty.washington.edu/rjl/classes/am574w2015/homework4.html`

Problem 1.

Consider the scalar conservation law $q_t + f(q)_x = 0$ with $f(q) = \sqrt{q}$, which is convex as long as we only consider states $q > 0$.

1(a). Consider this problem with data

$$q(x,0) = \begin{cases} 4 & \text{if } 0 < x < 1, \\ 1 & \text{otherwise} \ . \end{cases} \tag{1}$$

Determine the time t_s when the shock and rarefaction wave first begin to interact and the solution $q(x, t_s)$.

1(b). Use Clawpack to verify your solution. Set up a problem similar to what you did on the Programming problem of Homework #3. Choose the domain, mesh size, and limiter to illustrate the solution in a convincing manner. Save the code required in a directory `hw4/sqrtflux` and add a file `README.txt` with any instructions or comments on the solution.

1(c). Repeat parts (a) and (b) for the initial data

$$q(x,0) = \begin{cases} 4 & \text{if } 0 < x < 1, \\ 0.01 & \text{otherwise} \ . \end{cases} \tag{2}$$

For the programming part, also do the following:

- Set `clawdata.verbosity = 1` in `setrun.py` so that it prints out information every time step about the size of the time step. Comment on what you observe relative to a similar experiment with the initial data (1).
- Try using the Lax-Wendroff method (no limiter) for this problem and comment on what happens, relative to a similar experiment with the initial data (1).

If the results to the second part are mysterious, here are some hints:

- You might check the file `_output/fort.q0001` to see what the values the solution takes.
- In `setrun.py` if you set `clawdata.output_style = 1`, you can have it output results every time step for a certain number of steps.
- Fortran does not always trap arithmetic exceptions unless you tell it to, for example by setting
  ```
  FFLAGS =  -ffpe-trap=invalid,overflow,zero
  ```

in the Makefile and then doing `make new` to recompile everything. (These flags are for `gfortran`.)

See `http://www.clawpack.org/fortran_compilers.html` for some other useful debugging flags.

You do not need to check in code for Problem 1(c), but include some discussion in your written solutions about what you observe, including figures if you'd like.

Make sure you have committed any files to your repository that are needed to run your code and produce the plots. You do **not** need to commit the output files or plots, or the files produced by compiling the fortran code (`*.o, xclaw`).

Problem 2.

Consider the linear system $q_t + Aq_x = 0$ with

$$A = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}.$$

2(a). Determine the eigenvalues and eigenvectors of A and sketch the integral curves of the eigenvectors in the phase plane. (And recall that for a linear system these are also the Hugoniot loci.)

2(b). Consider the Cauchy problem for this system (no boundaries) with initial data

$$q(x,0) = \begin{cases} [4,4]^T & \text{if } x < 1, \\ [1,1]^T & \text{if } 1 \le x \le 3, \\ [2,1]^T & \text{if } x > 3. \end{cases}$$

2(c). Set up a Clawpack Riemann solver for this problem and use the initial condition above as a test of your code. (You might want to use the code in `$CLAW/classic/examples/acoustics_1d_example1` as a model for a linear system of two equations.) Solve it over a large enough domain to see all the states you expect to see in the exact solution, and use extrapolation boundary conditions.

Include some plots from your solution in your writeup.

Commit the files needed to produce these plots, in a directory `hw4/linsys`.

Please make sure comments in the code (and labels on plots) are relevant to the problem being solved and clean up things not needed for this code (e.g. parts of the acoustics code dealing with the physical parameters that are not relevant to this problem).

Problem 13.11 in the book, parts (a)-(c).

Also think about how the Riemann solution looks in the phase plane (q–u plane in this case) and think about part (d) in this context. You don't need to turn in anything, but try to understand the phase plane for this problem.

Problem 15.2 in the book.