AMath 574
Homework 2
Due by 11:00pm on January 29, 2015

For submission instructions, see:

`http://faculty.washington.edu/rjl/classes/am574w2015/homework2.html`

Problem #4.1 in the book

Problem #4.2 in the book

Problem #6.1 in the book

Problem #6.2 in the book

Problem #6.3 in the book

Problem #6.4 in the book

Problem #6.5 in the book

Problem #6.6 in the book

Additional Problem:

Show that the flux-limiter method (6.40),(6.41) can be written as a *wave limiter* method as:

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}(\bar{u}^+ \mathcal{W}_{i-1/2} + \bar{u}^- \mathcal{W}_{i+1/2}) - \frac{\Delta t}{\Delta x}(\widetilde{F}_{i+1/2} - \widetilde{F}_{i-1/2}),$$

where $\mathcal{W}_{i-1/2} = Q_i^n - Q_{i-1}^n$ and the "correction flux" is

$$\widetilde{F}_{i-1/2} = \frac{1}{2}|\bar{u}| \left(1 - \frac{\Delta t}{\Delta x}|\bar{u}|\right) \widetilde{\mathcal{W}}_{i-1/2},$$

with the limited waves $\widetilde{\mathcal{W}}$ defined by

$$\widetilde{\mathcal{W}}_{i-1/2} = \phi(\theta_{i-1/2})\mathcal{W}_{i-1/2}.$$

The ratio $\theta_{i-1/2}$ is defined in (6.35) and the function ϕ might be one of limiters from (6.39).

Programming problem

Modify the IPython notebook `$AM574/labs/lab2/AdvectionTests.ipynb` to create a new notebook `AdvectionLimiters.ipynb` that illustrates your solutions to the following:

a. Modify the upwind code to use two ghost cells on each side rather than one. Check that this gives identical results to the original code for cases when the time `tfinal` is large enough that the periodic boundary conditions play a role. Include at least one of these tests in the notebook. Note that the method implemented in the next part will require two ghost cells.

b. Implement the wave limiter methods for advection, as described in the previous problem. Note that it's impossible to use half-integer indices, so you might want to declare arrays such as `Ftilde` in which `Ftilde[i]` holds the correction flux $\widetilde{F}_{i-1/2}$. (This is the convention used in Clawpack — the index i often refers to information at the left edge of the cell $x_{i-1/2}$.)

   Copy the `upwind` function definition to a new cell in the notebook and modify it to create a new function `wave_limiter` that has one additional argument `phi` in the calling sequence, so that a limiter function $\phi(\theta)$ can be passed in. The function ϕ might be one of those listed in (6.39a,b) in the book.

   For example:

```
def phi_minmod(theta):
    return(max(0., min(theta,1)))
```

   would define the minmax limiter, and then

```
q = wave_limiter(ubar,q0,x,tfinal,nsteps,phi_minmod)
```

   should solve the problem using the minmod wave-limiter method.

c. Test your function by reproducing Figures 6.2 and 6.3 from the book in your notebook.

Submit your notebook `AdvectionLimiters.ipynb`.