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Streamflow Properties from Time Series of Surface Velocity and Stage

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Abstract

Time series of surface velocity and stage have been collected on the Cowlitz River at Castle Rock, WA between December 2002 and March 2003. Surface velocity was measured using an array of newly developed continuous-wave microwave sensors. Stage was obtained from the standard USGS measurements at the site, which yield the river elevation using a pressure transducer accurate to 3 mm. The depth of the river was measured several times during our experiments using sounding weights. Surface velocities measured by the microwave sensors agreed well with near-surface velocities measured by an acoustic Doppler current profiler and a current meter. The data plotted in the form of surface velocity versus stage for each of the eight microwave sensors clearly showed that the point of zero flow was not the bottom at the measurement site, indicating that a downstream control exists. Fathometer measurements confirmed this finding. A model of the surface velocity expected at a site having a downstream control was developed by combining a model of the friction velocity with a logarithmic velocity profile. The model shows that the standard form for the friction velocity which varies as the square root of stage times surface slope does not apply to sites where a downstream control exists. This model fit our measured surface velocity versus stage plots very well with reasonable values of the parameters. Discharges computed using mean velocities derived from the surface velocities along with the measured depths matched the USGS rating curve for the site very well. Values of depth-weighted mean velocities derived from our data did not agree with those expected from Manning's equation due to the downstream control. These results suggest that if real-time surface velocities were available at a gaging station, shifts of the bed either at the station or at the downstream control could be detected.

1 Surface Velocity and Stage

Doppler shifts of microwave signals backscattered from a river surface can be used to measure surface velocity (Okamoto, 1999; Costa et al., 2000; Melcher et al., 2002). Velocity sensors that detect these Doppler shifts do not contact the water and are therefore much less likely to be damaged than more conventional in-situ measurements of velocity. Furthermore, the relationship between surface velocity and mean velocity changes little as stage changes, unlike the relationship between the velocity at a fixed point and the mean velocity. We have recently installed eight continuous-wave microwave sensors operating at 24 GHz on the Cowlitz River bridge at Castle Rock, Washington (see Figure 1). The sensors were designed and constructed by the Applied Physics Laboratory of the University of Washington. Surface velocities measured by these sensors are sent back to our office within hours of being collected. Details of the operation of these sensors will be presented elsewhere. Briefly, signals transmitted by each sensor are scattered back to the antenna of the sensor by the surface roughness of the river. The scattering process is Bragg scattering from centimeter-length surface waves (Plant, 1990), so two Bragg lines may be produced in the Doppler spectrum of the backscattered signal. The midpoint between these Bragg lines is shifted from zero frequency by the surface current of the river. The relationship between the surface velocity V_s and the Doppler shift of the midpoint f_d is

$$f_d = 2V_s \sin \theta \cos \phi / \lambda \quad (1)$$

where θ is the incidence angle of the antenna, ϕ is its azimuth angle with respect to the flow direction, and λ is the wavelength of the microwave signal, here 1.25 cm. From this equation surface velocity can be obtained from a measurement of the midpoint frequency shift.

The midpoint of the two Bragg lines can generally be determined when the river is turbulent or when rain is falling. For slowly moving rivers in the absence of rain, however, surface roughness is generated by the wind. When the wind is blowing either directly toward or away from the antennas, only a single Bragg line may be detected. In this case, the midpoint may be either above or below the frequency of this single line. Thus the surface current can have either of two values; it is ambiguous. This ambiguity can often be removed if wind direction is known but this information was not always available in the experiments reported here. Thus we attempted to remove the ambiguity by comparison with unambiguous velocity measurements from the microwave sensors at nearby times.

We compared our measured surface velocities with those obtained from an acoustic Doppler current profiler (ADCP) and a current meter. Results are shown in Figure 2. The current meter measurements were made using the standard U.S. Geological Survey (USGS) technique (Rantz, 1982a,b). A Price AA current meter was suspended from the bridge on a cable that had a lead weight on the end to keep it taut. The measurements shown in Figure 2 were made six inches below the surface. The ADCP is a new technology which consists of miniature transducers riding on a floating styrofoam board (a "Boogie" board). One transducer points downward at a shallow angle and is capable of measuring velocities very close to the water surface. Figure 2 shows that the various sensors yield currents that are generally within 10 % of each other.



Figure 1: Microwave surface velocity sensors installed on the Cowlitz River bridge at Castle Rock, WA. The white antennas in the figure are four of the eight sensors that were installed on the bridge at various distances across the river.

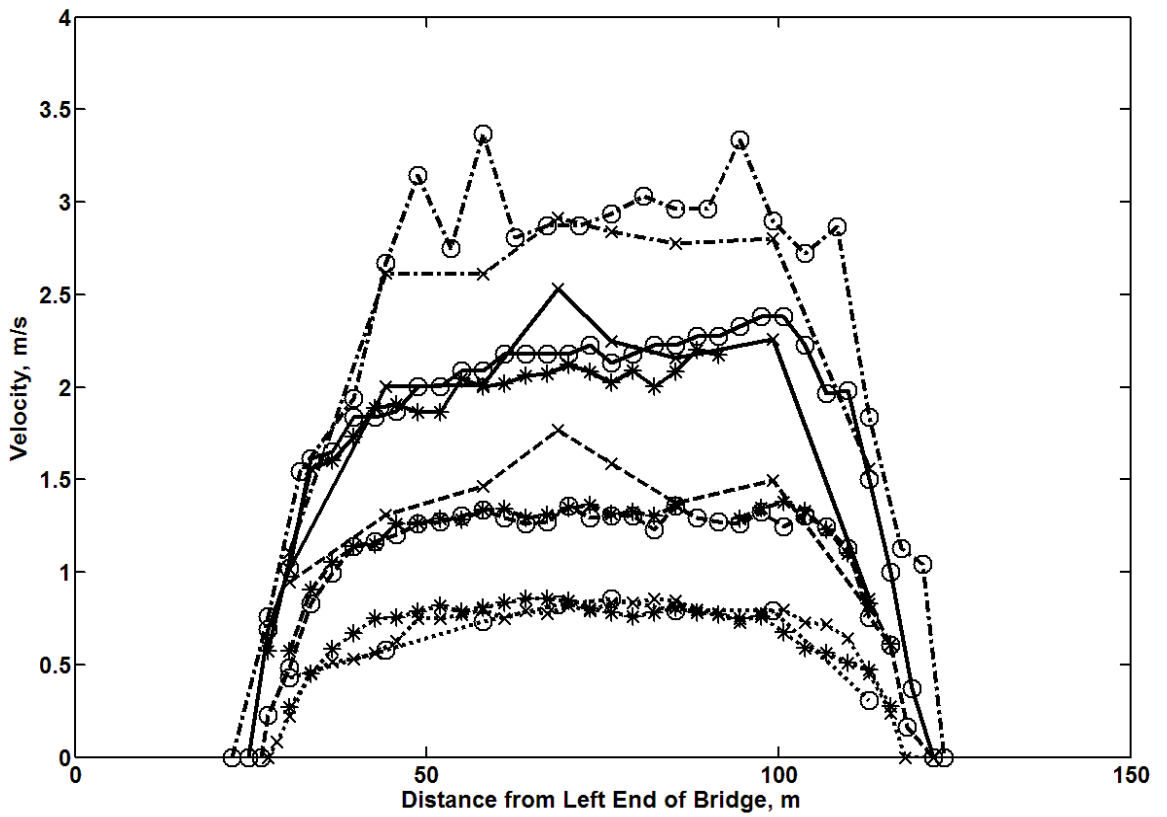


Figure 2: Surface and near-surface velocities across the Cowlitz River on four different days. X = microwave sensors, O = current meters, * = ADCP on "Boogie" board. Dotted lines connect data taken on Nov. 17, 2002, dashed lines connect data taken on Jan. 14, 2003, solid lines connect data taken on Feb. 4, 2003, and dash-dotted lines connect data taken on Feb. 1, 2003.

The Cowlitz River bridge at Castle Rock is also the site of a USGS streamgaging station (USGS station number 14243000). The data acquisition system for our microwave sensors occupies the same instrument shelter as the USGS equipment. The microwave sensors have now been operated for several months and have produced a semi-continuous record of surface velocity, V_s , over that period. USGS measurements have produced a continuous record of river stage, S , over the same period. Cross-section depths are measured periodically at the site by the USGS. Figure 3 shows time series of S along with time series of V_s from our microwave units located at different distances across the river.

It is straightforward to plot surface velocity versus stage for each of the eight microwave sensors. Figure 4 shows such plots for the unambiguous surface velocity measurements. Clearly a linear extrapolation of these data at low velocities indicates that the surface velocity will fall to zero at a stage between 8.5 and 8.8 m in most cases. Figure 5 shows the stage at the river bed measured by the USGS using sounding weights during the period of surface velocity data collection. This figure shows the river bed elevation to be considerably lower than the stage at which surface velocities appear to fall to zero. The reason for this was determined from fathometer measurements: a shallow area exists downstream of the measurement site and controls the flow of the river at the site. Figure 6 is a photograph taken from a point near this control looking upstream toward the bridge. It shows that the river widens at this point.

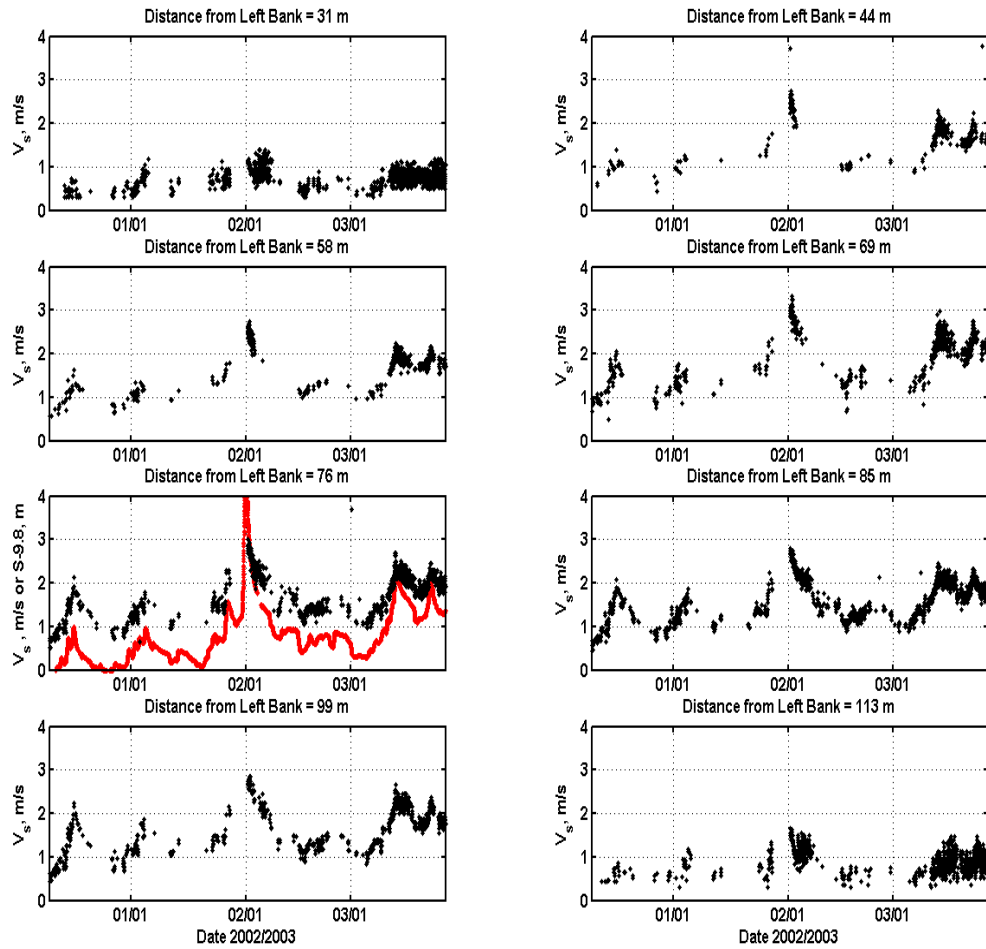


Figure 3: Time series of stage (minus 9.8 m) from standard USGS measurements (red) and surface velocity from the microwave sensors (black). Data are shown for the four month period from December 1, 2002 through March 31, 2003. The gap in the velocity record near February 1 was due to a computer malfunction; other gaps represent times when the velocity sensors could not produce a good value.

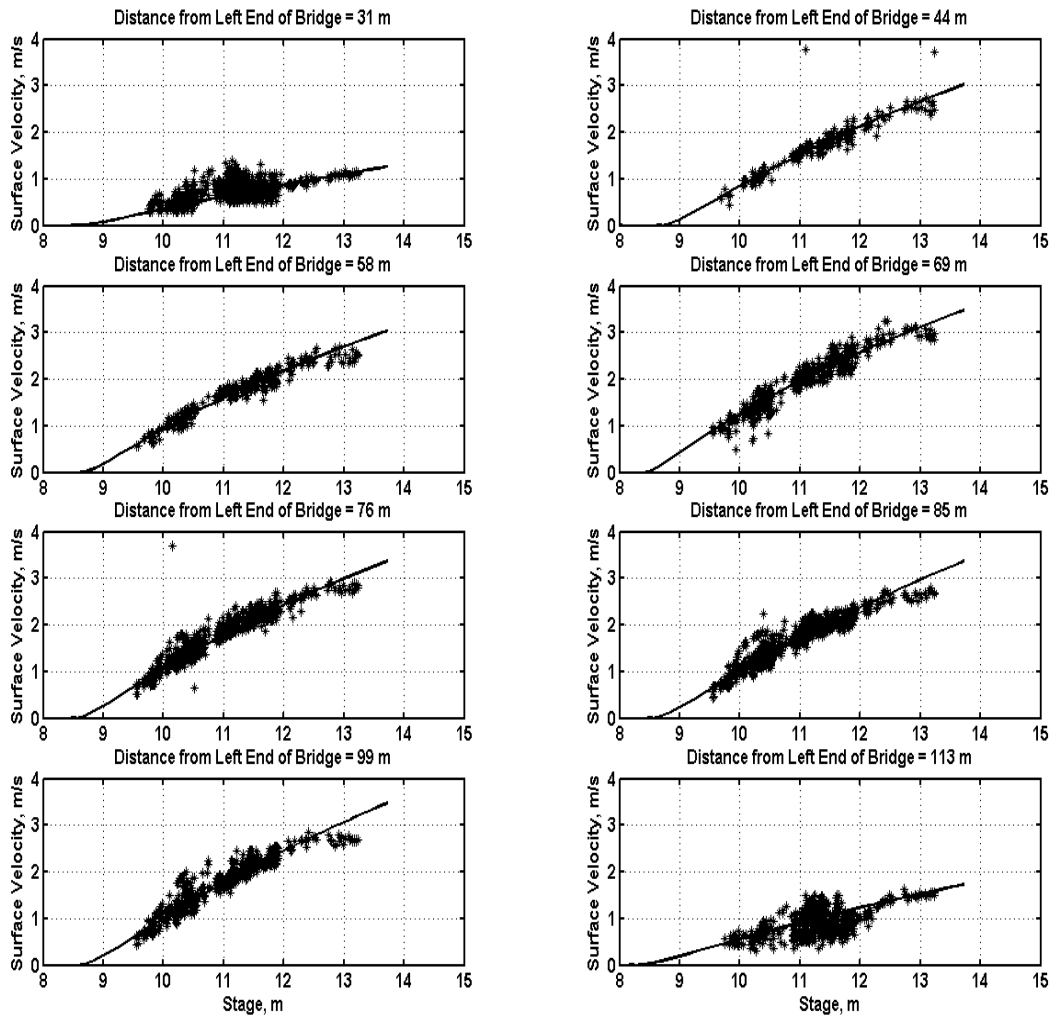


Figure 4: Surface velocity versus stage at the locations of the microwave sensors. Black asterisks are data while curves are fits to the theory of Section 2. Only unambiguous velocity measurements were used in these plots.

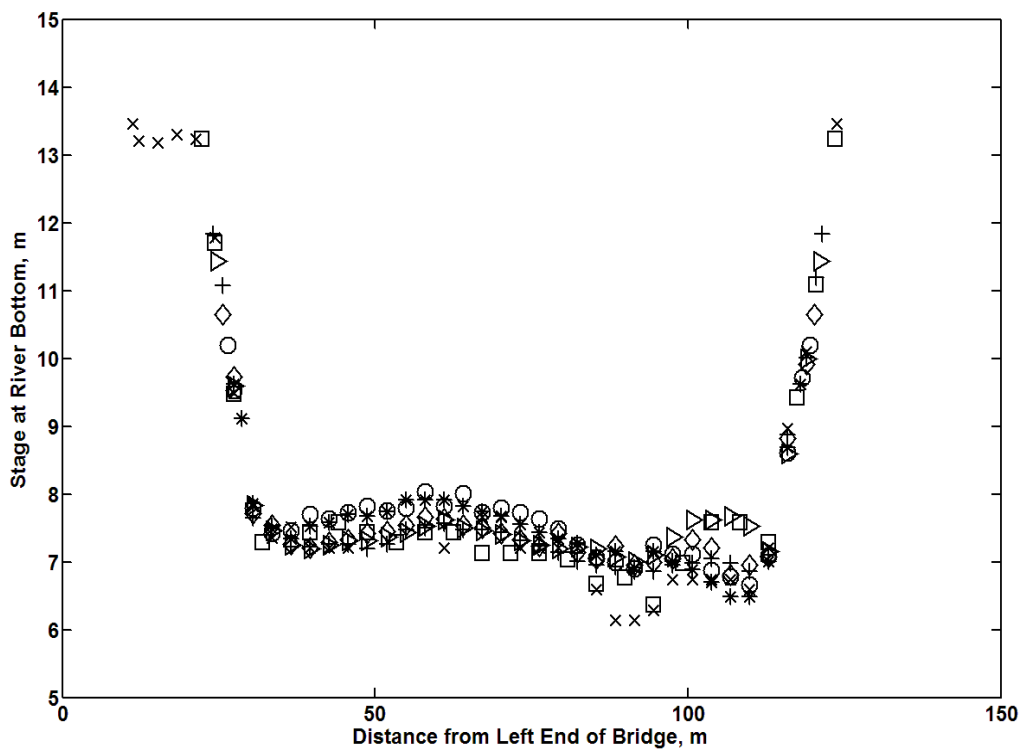


Figure 5: Depths measured by the USGS using sounding weights on the Cowlitz River at Castle Rock, Washington. Symbols indicate the following measurement dates: asterisks = November 21, 2002; circles = January 14, 2003; x = January 31, 2003; squares = February 1, 2003; triangles = February 4, 2003; diamonds = February 11, 2003; + = March 13, 2003.



Figure 6: Photograph of the Cowlitz River looking upstream from control area toward the bridge at Castle Rock, WA.

2 Surface Velocities Upstream from a Control

Consider a river with a large width-to-depth ratio and ignore possible small wind drift effects on surface velocity. Figure 7 shows a diagram of a cross section of the river and indicates the coordinate system. The vertical coordinate z is negative downward and $z = 0$ is the mean water surface. V_s is the distribution of surface velocity across the river while D is the (positive) depth. Assuming turbulent flow in the river, the vertical velocity profile at any point x across the river will be logarithmic:

$$V(z) = 2.5V_f \ln((z_o + z + D)/z_o) \quad (2)$$

where V_f is the friction velocity and z_o is the roughness length of the bed, taken to be positive. This equation agrees with the more common form of $\ln(z/z_o)$ (see Chow, 1959, p.201) where $z = 0$ is the bottom of the roughness elements since we take $z = 0$ to be the water surface and D is the water depth to the top of the rough elements. The USGS stage measurement S is the height above a constant point. Therefore it is related to depth by

$$D = S - S_o \quad (3)$$

where S_o is the stage at the bottom of the river. At the surface, $z = 0$ so the surface velocity is given by

$$V_s = 2.5V_f \ln(1 + D/z_o) \quad (4)$$

V_s is zero when $S = S_o$ if V_f does not fall to zero first. Over time, both stage and surface velocity vary so that plots of V_s versus S such as Figure 4 can be made. Clearly, such plots will change if S_o changes.

These plots are not necessarily logarithmic because the friction velocity is a function of depth. We may easily see why this must be so by considering the balance of forces in the flow. Water flowing downhill is driven by a component of the gravitational force per unit volume given by $\rho g S_l$, where ρ is the density of water, g is gravitational acceleration, and S_l is the slope of the river bottom. At points x away from the sides of the river, this may be integrated over the vertical water column to yield the gravitational force per unit area on the bottom, which is balanced by the drag of the bottom, ρV_f^2 . Thus this simple model yields

$$V_f = \sqrt{g S_l D} \quad (5)$$

which is clearly dependent on depth.

This model will apply if no downstream control exerts an influence on the flow at the measurement site. If such a control exists, however, then this simple expression for V_f will be changed because a force due to the control will add to the bottom drag to balance the downhill force of gravity. The easiest way to account for this is by invoking conservation of mass: the same amount of water must pass the measurement site and the control. Let us

assume a simple, rectangular river cross section at both the control and the measurement site. Then,

$$WD \langle V \rangle = W_c D_c \langle V_c \rangle \quad (6)$$

where W is width, D is depth, $\langle V \rangle$ is the vertical mean of the horizontal velocity, and subscripts "c" indicate values at the control site. For the logarithmic profile of (2), integration of z from $-D$ to 0 and division by D yields a mean velocity given by

$$\langle V \rangle = 2.5V_f [(1 + z_o/D) \ln(1 + D/z_o) - 1] \quad (7)$$

Thus, we have for the friction velocity at the measurement site,

$$V_f = \sqrt{gS_l D_c} \left(\frac{W_c D_c}{WD} \right) \left[\frac{(1 + z_{oc}/D_c) \ln(1 + D_c/z_{oc}) - 1}{(1 + z_o/D) \ln(1 + D/z_o) - 1} \right] \quad (8)$$

If S_{oc} is the stage of the river bottom at the control, then

$$D_c = S - S_l X - S_{oc} \quad (9)$$

where X is the distance between the measurement section and the control (recall that S is measured at the measurement section). Thus,

$$D - D_c = S_l X + S_{oc} - S_o \quad (10)$$

Since the cross section of the Cowlitz River is not truly rectangular (though it is close; see Figure 5), we applied the above ideas by considering the river to be divided into rectangular channels centered on the locations of our velocity sensors. These channels become wider at the control so that the ratio of their width to that at the measurement site is the same as it is for the river as a whole. While water can flow through the sides of these channels so that mass is not conserved in any one channel, we consider this leakage to be small and apply the above idea to each channel. Obviously mass must be conserved in the sum of the channels. We fit our measured velocity versus stage data to the form (4) in each channel using a friction velocity given by (8). We took S_o to be the mean of the USGS values at the locations of our velocity measurements. From our fathometer measurements, we found that the depth to zero flow was approximately one meter when the stage was 9.8 m, implying that $S_l X + S_{oc} \approx 8.8$ m. From survey measurements, we found that $X = 370$ m, $S_l = 0.0002$, and $W_c = 137$ m when $W = 91.5$ m at a stage of 9.8 m. We varied the values of S_{oc} , z_o , z_{oc} at each measurement location to obtain the best fit to the surface velocity measurements shown in Figure 4. The results are given by the curves shown in Figure 4; parameters necessary to produce these curves are given in Table 1. The very large values of z_{oc} near the sides of the river are probably a result of neglecting the drag of the sides in our formulations. They occur because of the much smaller variations of velocity with stage that occur near the sides. The highest velocities in plots away from the riverbank clearly fall below the curves. These values were measured after the computer malfunctioned as the stage was dropping and after the peak stage was reached (see Figure 3). It may be that the velocities are lower while the stage is falling because the water-surface slope is smaller than the slope at similar stages during rising water.

Location, m	30	44	58	69	76	85	99	113
Mean S_o , m	7.73	7.41	7.62	7.51	7.32	6.97	7.06	7.13
S_{oc} , m	8.41	8.62	8.56	8.34	8.47	8.47	8.53	8.16
$S_l X + S_{oc} - S_o$	0.75	1.25	1.01	0.90	1.22	1.57	1.54	1.10
z_o , mm	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1
z_{oc} , mm	82.4	0.30	0.46	0.21	0.15	0.09	0.06	21.3

Table 1: Parameters used to produce the curves shown in Figure 4.

3 Mean Velocities, Surface Velocities, and Discharges

It is interesting to note that substituting (8) into (7) yields

$$\langle V \rangle = 2.5\sqrt{gS_l D_c} \left(\frac{W_c D_c}{WD} \right) [(1 + z_{oc}/D_c) \ln(1 + D_c/z_{oc}) - 1] \quad (11)$$

which says that the mean velocity at the measurement site does not depend on the roughness parameters at this site but at the downstream control. On the other hand, we have

$$\frac{\langle V \rangle}{V_s} = 1 + z_o/D - 1/\ln(1 + D/z_o) \quad (12)$$

which shows that the ratio of mean to surface velocity is determined by the depth-to-surface-roughness ratio at the measurement site; the control does not enter into this ratio. This ratio is well known to be between 0.8 and 0.9 (Rantz, 1982a,b). Our current meter measurements showed that it is, in fact, approximately 0.85 at the Cowlitz River site.

Figure 8a shows plots of V_s and $\langle V \rangle$ versus stage. The lines in Figure 8a were obtained from (4) and (7) using (8) for the friction velocity. We took z_o to be 6.1 mm in order that the mean velocity be approximately 0.85 time the surface velocity over the range of stages of most interest. Figure 8b shows how the ratio of mean to surface velocity varies with roughness length and stage. The circles in Figure 8a are mean velocities produced by using the standard USGS method of estimating mean velocity from the average of velocity measurements at $z = -0.2D$ and $z = -0.8D$. Clearly this method gives correct mean velocities if the profile is logarithmic.

Although our velocity probes were spaced somewhat farther apart than would be optimum for calculating the discharge Q , we attempted to determine a stage/discharge relationship from our measurements. Discharge is given by

$$Q = \int D(x) \langle V(x) \rangle dx \quad (13)$$

Taking D from measured values and $\langle V \rangle$ from (11), essentially $0.85V_s$ (see Figure 8b), we obtained the stage/discharge relationship shown in Figure 9a. The standard USGS rating

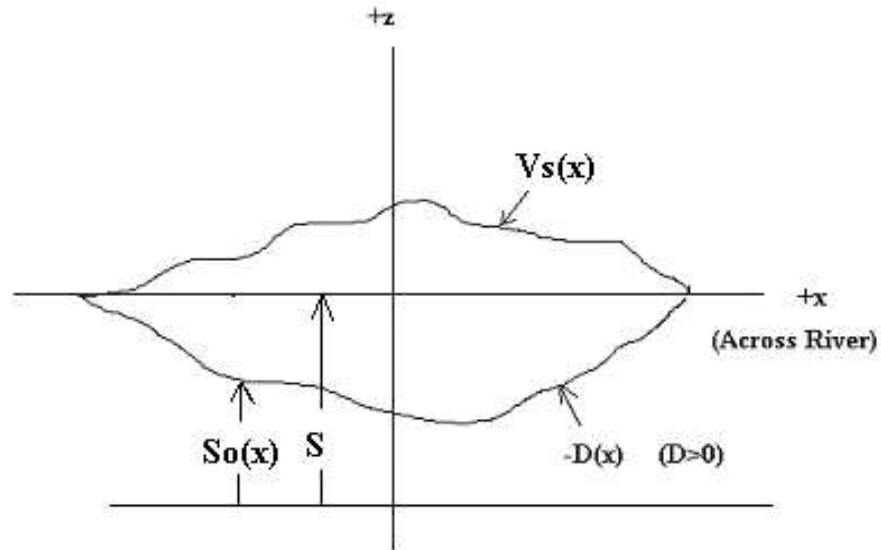


Figure 7: Diagram of river cross section defining coordinates. Flow is into page.

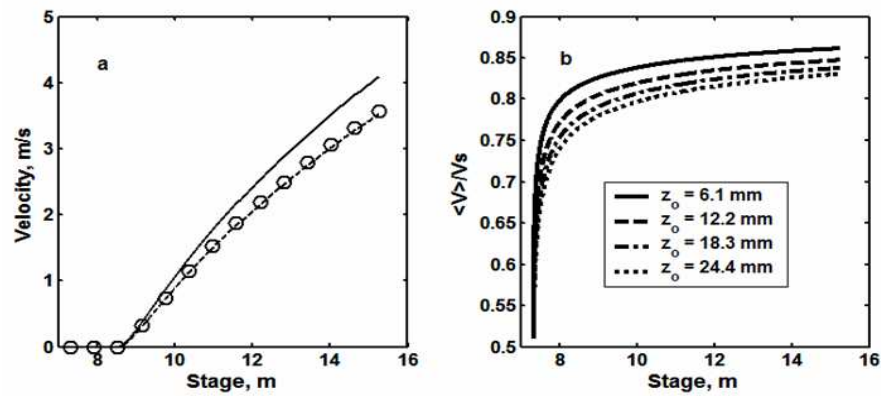


Figure 8: a) Surface velocities, V_s (solid), and mean velocities, \bar{V} (dashed), versus depth at a location 86.5 m from the left bank. The open circles show mean velocities, V_u , obtained from a logarithmic profile using the standard USGS method of averaging $V(z = -0.2D)$ and $V(z = -0.8D)$. b) Ratio of mean to surface velocity using measured depths and various roughness lengths.

curve for this site is also shown in the figure along with discharge values measured with a current meter and sounding weight between December 2002 and March 2003. The agreement between the two curves and the data is excellent. The fact that the extrapolation of the USGS curve to zero discharge occurs at a stage similar to those measured by the velocity probes reinforces the finding that the velocity drops to zero before the river is dry.

Finally, we computed the depth-weighted mean velocity, $V_m = Q/A$, where

$$A = \int D(x)dx \quad (14)$$

and compared it with that obtained from Manning's equation

$$V_m = (1.49/n)R^{2/3}S_l^{1/2} \quad (15)$$

where Manning's coefficient n is taken to be 0.025 and S_l is again 0.0002. We approximated the river shape by a rectangle to get the hydraulic radius R :

$$R = 320 < D > / (320 + 2 < D >) \quad (16)$$

and we took $< D >$ to be S minus the average of the USGS S_o values across the river. The results are shown in Figure 9b. Clearly Manning's equation does not describe the flow well at a site with a downstream control. We would expect better agreement at sites where no control exists downstream.

4 Conclusion

From time series of stage and surface velocity on the Cowlitz River at Castle Rock, WA, we plotted surface velocity versus stage at eight locations across the river. These plots clearly indicate that the river stops flowing at this site before it is dry. This suggests that a control exists downstream from the measurement site where the bed is higher than the bed at the site. Fathometer and survey measurements confirmed this suggestion. Thus the Cowlitz River at this site would pool if upstream flow ceased.

We developed a model for the dependence of the friction velocity on stage at a site where a downstream control exists. This dependence is different than the standard model of a square-root dependence of friction velocity on stage. With values for the depth of the bed at the control and measurement sites that agreed with in situ measurements, and with the right adjustment of roughness lengths, this model was shown to fit our measured surface velocities very well when inserted into a turbulent-flow logarithmic profile for the surface velocity. The value of the roughness length at the measurement site was set by requiring that the ratio of mean-to-surface-velocity be approximately 0.85 while the values of roughness length at the control were determined by fitting the surface velocity data. They are given in Table 1.

These results show the value of making non-contact surface velocity measurements that can be accessed in real time in conjunction with stage measurements. They indicate that if the bed of the river either at the measurement site or at the control shifts, the surface

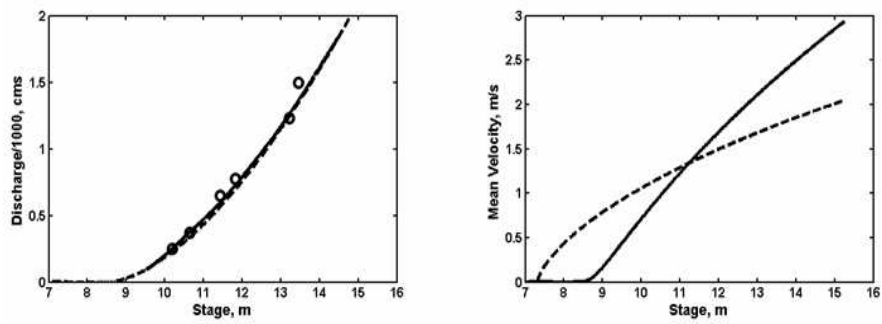


Figure 9: a) Discharge versus stage from the method outlined here (dashed), from the USGS rating curve for the site (solid), and from measurements with a current meter and sounding weight (circles). b) Depth-weighted mean velocity versus stage from surface velocity measurements (solid) and from Manning's equation (dashed).

velocity associated with a given stage will change. The equations given here indicate that if the water surface slope changes significantly over a flood peak, then the relationship of surface velocity to stage will change. When the stage is rapidly rising or falling, this effect may be confused with changes in bed elevation. However, once the rate of change of stage decreases sufficiently, changes due to scour or filling during the flood will be evident. Thus, non-contact surface velocity measurements could at least indicate when the bed shifts and at best provide a better means of gaging unstable streams.

5 Acknowledgements

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