Fuzzy Supervised Optimal Regulator for Spacecraft Formation Flying

Kourosh Rahnamai

Western New England College
Electrical Engineering Department
1215 Wilbraham Road
Springfield, MA 01119
krahnama@wnec.edu

Payman Arabshahi, Andrew Gray

California Institute of Technology
Jet Propulsion Laboratory
4800 Oak Grove Drive, 238-343
Pasadena, CA 91109
{payman,gray}@jpl.nasa.gov

ABSTRACT

The Cassini-Huygens mission to Saturn is the end of an era for NASA; sending one large spacecraft equipped to carry out a multitude of scientific experiments. Future NASA missions will deploy many smaller spacecrafts in highly controlled spatial configurations in what is referred to as "formation flying." Among the many challenges to this approach are: maintaining precise relative-positions, attitude relative to desired target, and communication for information sharing among all spacecraft in formation. In this paper we will investigate the advantages of using an intelligent fuzzy supervisory unit to modify the optimal regulator developed to maintain the relative position between spacecraft. The fuzzy agent modifies the optimal regulator base on information received from the navigation, communication, and control systems, and relative trajectory of the formation. This fuzzy agent seamlessly schedules and nonlinearly interpolates the optimal control gains.

1. Introduction

To improve reliability and science return value, reduce cost, and extend mission life cycle, many near-earth and deep-space future missions will deploy two or more less-expensive cooperative spacecraft flying in formation as an alternative to a single, large, expensive spacecraft. These spacecraft will maintain a tight flying formation to take advantage of physically distributed sensors as shown in Figure 1. Unique measurements can be achieved by combining information from distributed sensors such as: stereoscopic view, simultaneous data collection from different angles, higher image resolution, to just name a few. NASA's EO-1 and Landsat-7 satellites, launched on November 21, 2000, are the first satellites used to demonstrate the capabilities of this new novel concept.

1.1 Relative Dynamics

The equation of motion of a satellite orbiting a plant in circular motion can be analyzed using the Clohessey-Wiltshire equations [1,7]. The motion of the spacecraft is described by

$$\ddot{\vec{r}} + \omega^2 \vec{r} = 0 \tag{1}$$

Where, $\vec{r}$ is the vector pointing from mass M to mass m, and ω is the angular velocity of the spacecraft.

1.2 Relative Equations of Motion for Two Spacecraft

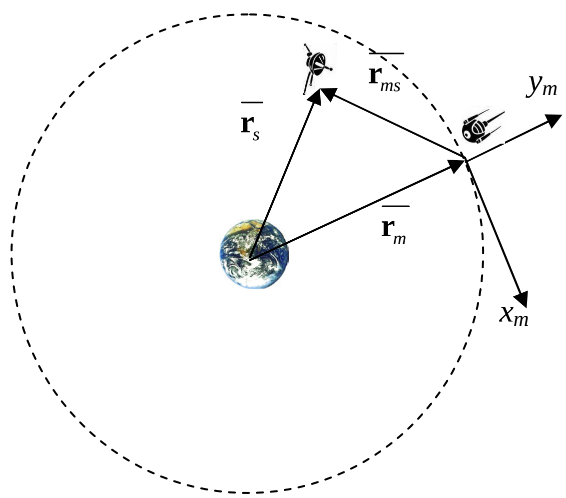

Figure 1: Master Slave Relative Motion

We can use Eq. (1) to develop the equations of motions for a pair of master-slave spacecraft shown in Figure 1. Subscripts m and s are used for master and slave spacecraft respectively. With the master spacecraft in circular orbit, the inertial frame of reference attached to the Earth, and a moving coordinate frame attached to the master spacecraft (x-axis opposite the tangential velocity, y-axis pointing in the same direction as instantaneous position of master satellite, and z-axis perpendicular to the x-y plane), the dynamic equation governing motion of the slave satellite with respect to the master is given by

$$\ddot{\vec r}_{ms} + \frac{\mu}{\left|r_m + r_{ms}\right|^3}\left(\vec r_m + \vec r_{ms}\right) - \frac{\mu}{r_m^{\ 3}}\vec r_m = \vec F \qquad (2)$$

$$\vec F = \vec F_m - \vec F_s$$

Where $\vec F_m$ and $\vec F_s$ are the external forces acting on the master and slave spacecraft respectively. Then $\vec F$ represents relative differential drag, solar pressure, gravitational perturbations and inputs applied to the slave spacecraft.

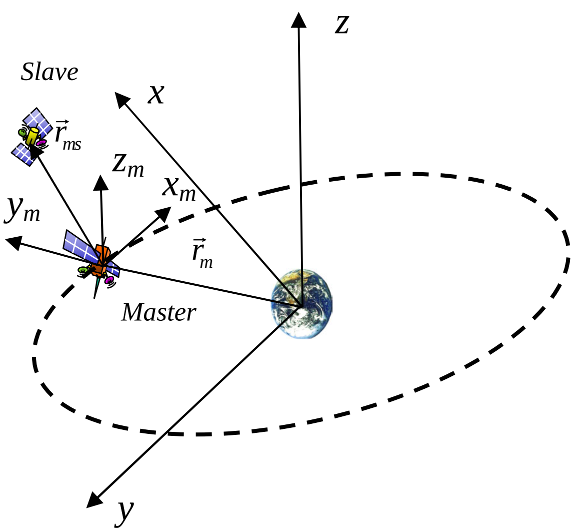

Figure 2: Master Slave Relative Coordinate Axis

The relative position of the slave spacecraft $\vec r_{ms}$ can be expressed in terms of a moving rectangular coordinate (x_m y_m z_m) system attached to the master spacecraft as shown in Figure 2:

$$\vec r_{ms} = x_m \hat i + y_m \hat j + z_m \hat k$$

$$\vec r_m = r\hat j$$

and the relative angular velocity vector

$$\vec\omega = \omega\hat k$$

Equation (2) will then yield

$$\vec F - \ddot{\vec r}_{ms} =$$

$$\omega^2\left\{\left[1+\frac{2y_m}{r}+\frac{\left(x_m^2+y_m^2+z_m^2\right)}{r^2}\right]^{-\frac{3}{2}}\left[\left(x_m\hat i+\left(y_m+r\right)\hat j+z_m\hat k\right)\right]-r\hat j\right\} \qquad (3)$$

Circular or near-circular motion of one spacecraft with respect to another can be analyzed using the linearized set of orbit equations by Clohessey-Wiltshire [11,12]. These equations, sometimes called Hill's equations, are

$$\vec r_{ms} = \begin{bmatrix} x_m & y_m & z_m \end{bmatrix}^T$$

$$\vec F = \vec F_m - \vec F_s = \begin{bmatrix} f_{x_m} & f_{y_m} & f_{z_m} \end{bmatrix}^T$$

$$\ddot{\vec r}_{ms} = \begin{bmatrix} \ddot x_m - 2\omega \dot y_m - \omega^2 x_m \\ \ddot y_m + 2\omega \dot x_m - \omega^2 y_m \\ \ddot z_m \end{bmatrix} =$$

$$\left(\ddot x_m - 2\omega \dot y_m - \omega^2 x_m\right)\hat i + \left(\ddot y_m + 2\omega \dot x_m - \omega^2 y_m\right)\hat j + \ddot z_m \hat k$$

Using these equations in Eq. (3) and collecting terms will result in relative nonlinear dynamics

$$\ddot x_m - 2\omega \dot y_m - \omega^2 x_m + \omega^2 g(x_m, y_m, z_m, r) = f_{x_m} \qquad (4)$$

$$\ddot y_m + 2\omega \dot x_m - \omega^2 y_m + \omega^2\left[g(x_m, y_m, z_m, r)(y_m + r) - r\right] = f_{y_m} \qquad (5)$$

$$\ddot z_m + \omega^2 g(x_m, y_m, z_m, r) z = f_{z_m} \qquad (6)$$

where,

$$g\left(x_m, y_m, z_m, r\right) = \left[1 + \frac{2y_m}{r} + \frac{\left(x_m^2 + y_m^2 + z_m^2\right)}{r^2}\right]^{-\frac{3}{2}}$$

Equations (4)-(6) are the nonlinear equations of motion for the relative position of the slave spacecraft with respect to the master. Since in most spacecraft flying formation scenarios $r >> (x_m, y_m, z_m)$, a set of linear equations of motion can be obtained by expanding $g(.)$ and neglecting higher-order terms

$$\ddot x_m - 2\omega \dot y_m = f_{x_m} \qquad (7)$$

$$\ddot{y}_m + 2\omega \dot{x}_m - 3\omega^2 y_m = f_{y_m} \quad (8)$$

$$\ddot{z}_m + \omega^2 z = f_{z_m} \quad (9)$$

It has been shown [1-7] that the open-loop relative spacecraft equations of motion (7)-(9) are unstable, and any disturbances will cause the spacecraft to drift apart and will result in an eventual loss of formation.

2. State Space Representation

It is clearly apparent that the in-plane dynamics (x_m, y_m) are decoupled from the out-of-plane dynamics (z_m). Defining

$$\mathbf{x}_i = \begin{bmatrix} x_m & y_m & \dot{x}_m & \dot{y}_m \end{bmatrix}^T, \mathbf{u}_i(t) = \begin{bmatrix} f_{x_m} & f_{y_m} \end{bmatrix}^T$$

$$\mathbf{u}_{ix}(t) = f_{x_m},\ \mathbf{u}_{iy}(t) = f_{y_m}, \mathbf{z}_i = \begin{bmatrix} z_m & \dot{z}_m \end{bmatrix}^T, \mathbf{u}_o(t) = f_{z_m}$$

and using these equations in Eqs. (7)-(9), the in-plane and out-of-plane state space representations can be written as:

$$\dot{\mathbf{x}}_i(t) = \mathbf{A}_i \mathbf{x}_i(t) + \mathbf{B}_i \mathbf{u}_i(t) \quad (10)$$

$$\dot{\mathbf{x}}_o(t) = \mathbf{A}_o \mathbf{x}_o(t) + \mathbf{B}_o \mathbf{u}_o(t) \quad (11)$$

where,

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2\omega \\ 0 & 3\omega^2 & -2\omega & 0 \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{B}_{ix} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T$$

$$\mathbf{B}_{iy} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$\mathbf{A}_o = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix}, \quad \mathbf{B}_o = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

3. Optimal Control Design

As can be seen from Eqs. (7)-(9), the relative in-plane and out-of-plane equations of motion are decoupled. For the remainder of this work we will concentrate on the control of in-plane dynamics. The in-plane state matrix, $\mathbf{A}_i$, is unstable with eigenvalues at 0,0, and $\pm j\omega$. It has been shown [5] that the reduced-order controllability pair ($\mathbf{A}_i$, $\mathbf{B}_{ix}$) can be used to design an optimal controller to stabilize the formation.

To reduce computational error and avoid working with an ill-conditioned state matrix, angular velocity is normalized to unity [1,5,6]. Note that in doing so, the orbital velocity ($v=\omega r$) is also unity. Non-dimensionalizing orbital radius will also broaden the result to all circular Keplerian orbits.

Discrete Linear quadratic regulator (LQR) theory is used to design the feedback gain matrix $\mathbf{K} = -\mathbf{X}_{id}\mathbf{U}_i$ such that the following cost function is minimized.

$$J_i = \sum_{k=0}^{\infty} \mathbf{x}_{id}^T(k)\mathbf{Q}_i\mathbf{x}_{id}(k) + \mathbf{u}_{ix}^T(k)\mathbf{R}_i\mathbf{u}_{ix}(k)$$

$$\mathbf{Q}_i = \alpha \mathbf{I}_{4x4}\ ,\ \mathbf{R}_i = \beta \mathbf{I}_{1x1} \quad (12)$$

$\mathbf{R}_i$ and $\mathbf{Q}_i$ are design-parameter matrices to emphasize the relative importance of fuel consumption verses trajectory optimization. By appropriate selection of these matrices, the eigenvalues of the closed-loop system, $\mathbf{A}$-$\mathbf{B}_{ix}\mathbf{G}$, can be moved to obtain stability, and a desired balance between state trajectories time constants and fuel consumption.

4. Fuzzy Supervisor

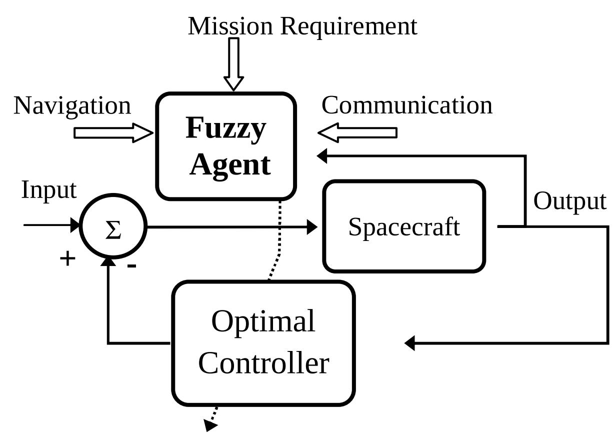

Figure 3: Fuzzy-Optimal Controller

We now present a fuzzy logic supervisory agent that would coordinate and change the controller behavior based on information received from navigation, communication, and mission requirement units as shown in Figure 3. The fuzzy agent modifies the optimal controller gains based on received information from other units. A matrix of optimal controller gains is calculated in the range from a very aggressive with less consideration for fuel expenditure to a very conservative, low-fuel consumption scenario. The fuzzy agent then modifies and interpolates the control system gains based on information that it receives from navigation, communication, and mission requirement modules. Flying formation requires cross-link communication between the spacecraft for navigation computation. If for any reason the navigation information is disrupted, the fuzzy agent will deploy a very conservative controller until such time that precise navigational information is received.

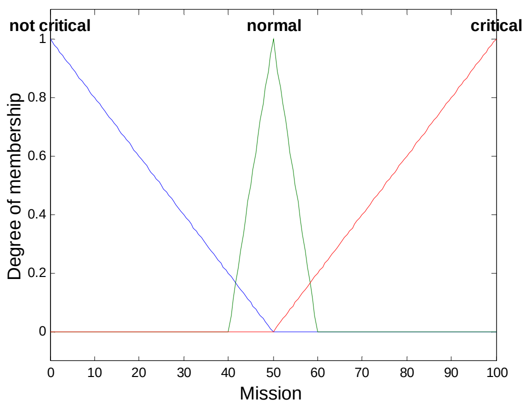

Figure 4: Mission membership functions

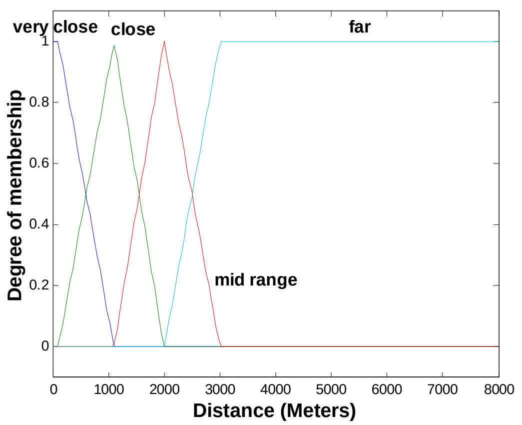

Figure 5: Relative distance memberships

As a sample of fuzzy agent task, using the Sugeno inference fuzzy system with mission requirement and the spacecraft relative distance as inputs and optimal feedback gains as the constant outputs of the fuzzy agent is considered and simulation programs are developed. The membership functions are shown in Figures 4 and 5.

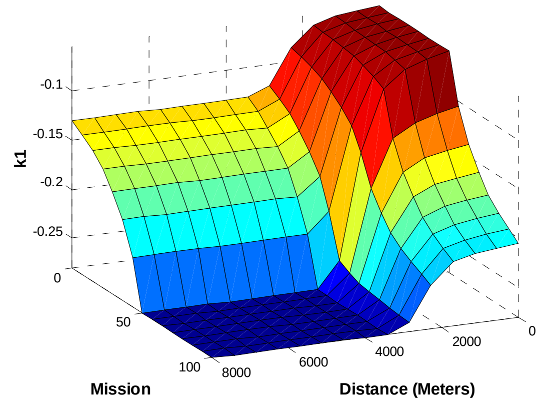

Figure 6: Optimal gain interpolation

Figure 6 shows the interpolation surface for the first optimal feedback gain. Note that the surface is nonlinear and the shape of this surface can change by modifying the fuzzy logic rule set. The simulation results of this system – with and without a fuzzy agent – are presented in the next section.

5. Simulation Results

Table 1: Optimal feedback gains

$\alpha = 1$	K_1	K_2	K_3	K_4
$\beta = 10$	-0.2804	-4.7655	2.2660	-1.4161
$\beta = 100$	-0.0935	-2.4996	1.3021	-0.4635
$\beta = 200$	-0.0669	-2.0256	1.0693	-0.3128
$\beta = 300$	-0.0550	-1.7847	0.9468	-0.2456

Four sets of optimal control gains were generated as shown in Table 1. Each row corresponds to one design and the design parameter is selected from a very aggressive fuel consumption, $\beta = 10$, to a very moderate, $\beta = 300$, value.

5.1 Simulation with Fuzzy Supervisory Agent

To test this strategy, a simulation of in-plane equation of motion was developed. The simulation generates the relative motion of salve spacecraft with respect to the master. A typical orbital radius, $r = 7153$ km, was used. Note that, the master spacecraft position need not be at the origin of state space and can be moved to any location with a simple coordinate transformation. The slave spacecraft response behavior was investigated over a range of initial conditions

$$39 \le x_{io}(1) \le 393 \quad \text{Meters}$$
$$5.5e-5 \le x_{io}(1) \le 5.5e-6 \quad \text{Nondimensionalized}$$

Response of the relative motion of slave spacecraft with respect to master with nondimensionalized initial conditions of $\mathbf{X}_{\text{io}}$ = [-5.5e-5 0 0 0]^T are shown in Figures 7-10. The slave spacecraft has only initial perturbation in the x_{m} direction. Because radial and tangent to orbital dynamics are coupled, we observe that the radial position of the slave spacecraft is perturbed from equilibrium point by a maximum of 90 meters before settling back to zero over a time period of 2.5 to 3.0 orbits.

Figures 7-10 also show the response of the formation with fuzzy intelligent agent acting as a supervisory unit on the controller. The radial perturbation of the slave spacecraft is used as an input to the supervisory unit. The fuzzy agent modifies the optimal gains based on relative radial position of the slave spacecraft and the mission criterion. Optimal gain values are interpolated based on input information to this unit. A typical response with mission priority set to 50 (normal operating conditions) is presented here. As can be seen from Figure 7, the response is a compromise between the four optimal designs. Fuzzy response starts very aggressive but gradually and nonlinearly, seen in Figure 8, changes to a very moderate response as the slave spacecraft approaches equilibrium point. Phase plane and individual radial and tangent to orbit trajectory plots are show in Figures 7-10.

Total fuel consumption is directly proportional to the integral of the applied input to the spacecraft. Figure 10 shows the required input when the fuzzy supervisor is used as well as the four optimal scenarios. Again fuzzy controlled system exhibits a compromise fuel consumption behavior as compared to the four optimal scenarios. Figure 11 shows the response of the system for mission criteria set to 100 (critical). As can be seen the system response is very close to the most aggressive optimal design.

6. Conclusion

As we gain more knowledge and experience in near-earth and deep-space exploration, our expectation for scientific returns of each mission are increasing. The advances in navigation, communication, controls methods and the requirements of future NASA missions are now at a point that would necessitate autonomous and intelligent self-reliant formation of spacecraft.

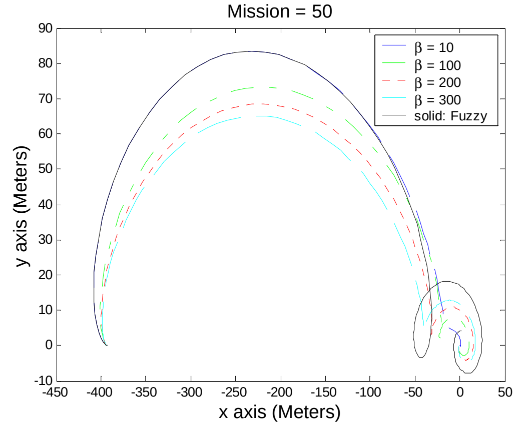

Figure 7: Optimal and fuzzy responses

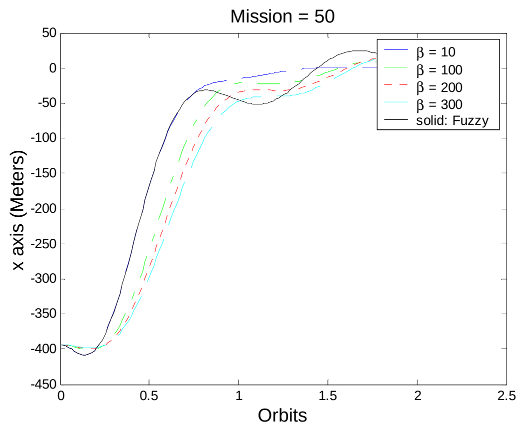

Figure 8: X-axis responses

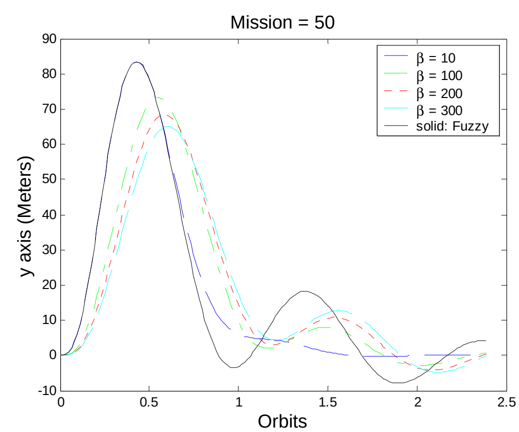

Figure 9: Y-axis responses

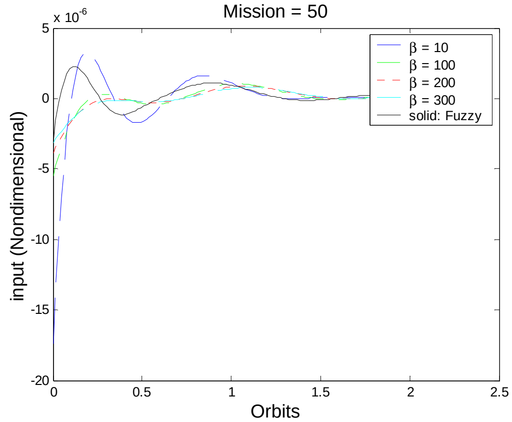

Figure 10: System input

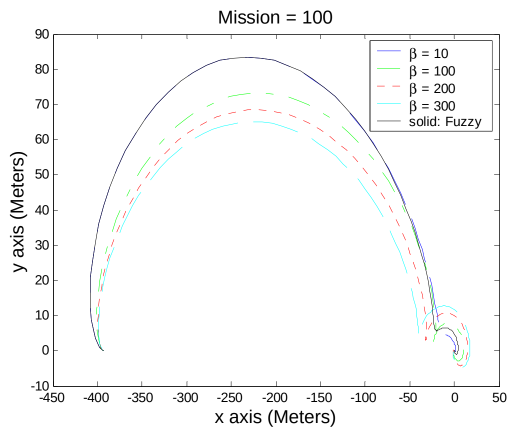

Figure 11: Slave responses (critical mission)

In this work we have shown a method of combining navigation and communication units to successfully modify the control law. An intelligent fuzzy logic-based agent is used to seamlessly and continuously interpolate between a set of optimal controller gains based on navigation and communication information. Fuzzy rules and embedded intelligence of the agent can easily be modified by adding or modified rules. Rules could even be conflicting, based on complexity and interactions between navigation, communication and control units.

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