OPTIMAL SEQUENCING OF URGENT SURGICAL CASES

Scheduling Cases using Operating Room Information Systems

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ABSTRACT. Optimal sequencing of urgent cases (i.e., selecting which urgent case should be performed first and which second) may enhance patient safety, increase patient satisfaction with timeliness of surgery, and minimize surgeons’ complaints. Before determining the optimal sequence of urgent cases, an operating room (OR) suite must identify the primary scheduling objective to be satisfied when prioritizing pending urgent cases. These scheduling objectives may include: 1) perform the cases in the order that they were submitted; 2) perform the cases in the sequence that minimizes the average length of time each surgeon and patient waits; or 3) perform the cases based on medical priority, as prioritized by an OR director, or surgeons discussing the cases among themselves.

We provide mathematical structure which can be used to program a computerized surgical services information system to assist in optimizing the sequence of urgent cases. We use an example to illustrate that the optimal sequence varies depending on the scheduling objective chosen.

KEY WORDS. Operating rooms, operating room information systems.

INTRODUCTION

We consider the following scenario. An operating room (OR) suite staffs one OR on weekends for urgent cases. This OR suite “opens” a second OR only if the first OR is utilized and an additional case is “added-on” that needs to start before the case in the first OR can be completed. An anesthesiologist and two OR nurses are on-call to complete the add-on non-elective surgical cases in one OR. Three surgeons have each requested OR time to operate on patients for the following procedures: an appendectomy, an open repair of a femoral neck fracture, and an intracerebral aneurysm clipping.

Which case should be done first, which second, and which third? The surgeons have designated all three of the cases to be “urgent.” However, there are six different ways to sequence the three cases. The goal of this article is to provide mathematical structure to assist in choosing the optimal sequence to perform the urgent cases.

Optimal sequencing of urgent cases may be important to maintain patient safety, enhance patient satisfaction with the timeliness of their surgery, and minimize surgeons’ complaints. For purposes of this study, we consider a case to be “urgent” if the surgeons report that the case cannot wait to be completed during regularly scheduled OR hours. In this context, the term “urgent” is synonymous with terms such as “emergent,” “life or
limb at immediate risk,” or “add-on” that may be used at other hospitals.

Prior to determining the optimal sequence of urgent cases, the OR suite must choose the primary objective to be achieved in sequencing the cases. We evaluate three objectives that can be used to sequence urgent cases. (1) Perform the cases in the sequence that minimized the average length of time each surgeon and patient waits. (2) Perform the cases in the order that they were posted or submitted to the OR suite. (3) Perform the cases based on medical priority to minimize the chance of a poor patient outcome. Medical priority would be prioritized by the OR director, or the surgeons discussing the cases among themselves, either through formal or informal mechanisms. We assume that regardless of which of these three objectives is used to sequence the cases, an OR suite will mobilize staff and resources such that all medical deadlines as to when a case should start are met. Thus, the sequence chosen would come as close as possible to meeting one of the three objectives while assuring that all medical deadlines are met.

In this article, we provide mathematical structure on how to sequence optimally urgent cases based on the three objectives. The mathematical structure is necessary to program an OR information system to produce the optimal sequence of urgent cases based on each of these three objectives. We construct an example to illustrate this sequencing scheme.

Objective 1. Sequence cases to minimize the average time patients wait to have surgery

Sequencing cases to minimize the average time patients wait to have surgery will minimize the average and total length of hospital stay for patients undergoing urgent surgery. The sequence of cases that minimizes the average time patients have to wait is to perform the shortest case first, the second shortest case second, and so on in order of increasing case duration [1]. This solution is optimal as long as no medical deadlines are broken. Then, satisfying medical deadlines would take precedence. Below, we build an example to illustrate this sequencing scheme.

1.1 Data required for sequencing

We let \( n \) = the number of urgent cases to be sequenced. They are labeled with subscripts \( i = 1, 2, \ldots, n \), which refer to the order in which the cases were submitted by the surgeons. The method to sequence these \( n \) cases depends on the objective used by the OR suite in scheduling the cases. As such, OR suites need to define which scheduling objective is appropriate for urgent cases in their OR suite. Once the scheduling objective for urgent cases has been identified, the optimal sequence can be determined.

A surgeon can look at the sequence of cases and quickly understand why his/her patient is waiting relative to the other patients in the queue.

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**ANALYSIS**

**Example**

We consider an example of three urgent surgical cases that might be submitted to an OR suite. We let \( t \) = the current time, which for our Example is 8 A.M. on Sunday November 8, 1998 (Table 1). Using the data obtained from the surgeons and summarized in Table 1, the cases are sequenced as described below.

What is the optimal sequence to do these three cases? “Optimal” means the sequence of cases that comes the closest to meeting the OR suite’s objective. In each of the three sections below, labeled as objectives 1, 2, and 3, we provide a mathematical framework that can be programmed in a computer information system to sequence the urgent cases so as to best satisfy each objective. The three objectives are to perform the cases either (1) in the sequence that minimizes the average length of time each surgeon and patient waits, (2) in the order that they were posted or submitted to the OR suite, or (3) based on medical priority to minimize the chance of a poor patient outcome.

Our Example (Table 1) uses values for estimated case durations, times between occurrence and posting, and times between posting and deadline to start cases that will vary among OR suites. Our mathematical structure can be applied to all possible data values.

**Objective 1. Sequence cases to minimize the average time patients wait to have surgery**

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1.1 Data required for sequencing

We let \( n \) = the number of urgent cases to be sequenced. They are labeled with subscripts \( i = 1, 2, \ldots, n \), which refer to the order in which the cases were submitted by the surgeons. The method to sequence these \( n \) cases depends on the order in which the cases were submitted by the surgeons. The method to sequence these \( n \) cases determines which case should go first, second, and so forth. When a surgeon contacts the OR suite about adding an urgent case to the queue, answers to three questions are required. Using the answers to these questions, three data are calculated for each case. These three data are used along with the stated objective to provide a recommended sequence of the \( n \) cases.

Question #1. *What is the scheduled procedure?*

For our example, the submitted cases are: (i = 1) intracerebral aneurysm clipping by Dr Franklin; (i = 2) repair of a proximal femoral fracture by Dr Smith; and (i = 3) appendectomy in a 12 year old by Dr Jones.
Data #1. Estimated case duration.

The definition of case duration includes the case’s concomitant setup time and clean-up time. The OR information system would provide the surgeon’s historical average case duration for the same scheduled procedure. We let $c_i$ be the estimated case duration of the $i$th case. For example, if Dr Franklin’s average historical case duration including setup and clean-up times for cases submitted as intracerebral aneurysm clipping is 6.0 hours, then $c_1 = 6.0$ hours. For our examples, we consider the femoral fracture $c_2 = 2.5$ hours and appendectomy $c_3 = 2.0$ hours. Some OR suites may choose to adjust the average historical case duration up or down by $0\%-30\%$, based on the surgeon’s estimation of case complexity [2, 3].

Question #2. When was the occurrence of the disorder requiring surgery in your patient?

For example, for the intracerebral aneurysm clipping case ($i = 1$), the delay from the occurrence of symptoms from the subarachnoid hemorrhage to when the neurosurgeon posts the case may be 16 hours. For our examples, we consider the femoral fracture $c_2 = 2.5$ hours and appendectomy $c_3 = 2.0$ hours. Some OR suites may choose to adjust the average historical case duration up or down by $0\%-30\%$, based on the surgeon’s estimation of case complexity [2, 3].

Data #2. Time of occurrence of the disorder requiring surgery.

We use $o_i$ the time of occurrence of the disorder in the $i$th patient. The patient requiring aneurysm clipping sustained a subarachnoid hemorrhage at 6 A.M. November 7. This case ($i = 1$) was posted 16 hours later at 10 P.M. November 7. The femur fracture ($i = 2$) occurred at 6 P.M. November 7. The case was submitted to the OR suite 7 hours later at 1 A.M. November 8. The third case ($i = 3$, appendectomy) was posted at the current time $t = 8$ A.M. November 8. We consider symptoms for this case to have started 24 hours earlier at 8 A.M. November 7.

Question #3. What delay in starting your case is associated with an increased risk of morbidity?

This question could be asked as: ‘by when must we start your case?’ The answer to this question may be specific to the disease suffered by the patient that requires surgical treatment. The answer may require judgement by the surgeon as to the severity of the patient’s illness. If only one surgeon can perform the case, and the surgeon will not be available after a certain time, then availability of the surgeon may influence the deadline.

Data #3. Medical deadline by which time the case must have started or the patient is at risk for morbidity due to the delay in starting surgery.

We let $d_i$ be the medical deadline for the $i$th case. For example, we use $d_1 = 6$ A.M. November 9 to refer to the deadline for the start of the aneurysm clipping case. If Dr Smith specifies that surgery to repair the femur fracture should start within 17 hours of posting the case, then $d_2 = 6$ P.M. November 8 ((time of case posting) + (17 hours)). If Dr Jones specifies for her patient requiring an appendectomy that surgery should start within 13 hours, then $d_3 = 9$ P.M. November 8 ((time of case posting) + (13 hours)).

Using the data obtained from the surgeons, the cases are sequenced as described below.

### Table 1.

<table>
<thead>
<tr>
<th>Case label</th>
<th>Surgical procedure</th>
<th>Estimated case duration ($c_i$; hours)</th>
<th>Time of occurrence ($o_i$)</th>
<th>Time case is posted</th>
<th>Deadline to start the case ($d_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i = 1</td>
<td>Aneurysm clipping</td>
<td>6.0</td>
<td>6 A.M. November 7</td>
<td>10 P.M. November 7</td>
<td>6 A.M. November 9</td>
</tr>
<tr>
<td>i = 2</td>
<td>Femur fracture</td>
<td>2.5</td>
<td>6 P.M. November 7</td>
<td>1 A.M. November 8</td>
<td>6 P.M. November 8</td>
</tr>
<tr>
<td>i = 3</td>
<td>Appendectomy</td>
<td>2.0</td>
<td>8 A.M. November 7</td>
<td>8 A.M. November 8</td>
<td>9 P.M. November 8</td>
</tr>
</tbody>
</table>

1.2 Sequence the cases to most nearly satisfy the scheduling objective

In this section, we consider the scheduling objective for sequencing the urgent cases which (a) starts each case on average as soon as possible after the surgeon contacts the OR suite while (b) satisfying all deadlines. Let us suppose that the sequence with the shortest average waiting time would cause a medical deadline to not be satisfied. For example, a case may need to start within six hours because the patient would then have an increased risk of suffering morbidity by waiting longer. Then, the sequence used would be the sequence that is as close as possible to the sequence that minimizes average waiting time while assuring that all medical deadlines are satisfied. If all cases cannot be completed
by their deadlines, a second OR would need to be opened.

The mathematical structure for the objective in sequencing the urgent cases is as follows [1]. We let $s_i$ be the time at which the $i$th case is scheduled to start. The predicted start times $\{s_1, s_2, \ldots, s_n\}$ are calculated. Then, the objective is to identify the sequence that minimizes the average of all start times:

$$\text{minimize } n^{-1} \sum_{i=1}^{n} s_i,$$

subject to constraint $d_i \geq s_i$ for all $i = 1, 2, \ldots, n$.

The constraint $d_i \geq s_i$ specifies ‘satisfies all deadlines.’ The $n^{-1} \sum_{i=1}^{n} s_i$ term specifies “on average.” By minimizing the average $s_i$, we effectively say that we want to “start each case...as soon as possible.” We consider each of these three terms in the following two paragraphs.

The constraint $d_i \geq s_i$ specifies that all deadlines must be satisfied. We include this requirement because the deadlines are defined as those for which a patient may suffer increased morbidity due to a delay in starting surgery. If the cases can be completed sufficiently quickly such that the patients’ outcomes would not be affected by waiting until other urgent cases are completed, not opening a second OR is appropriate (see Discussion).

The optimal sequence of cases is the sequence with the smallest value for the average of all start times (Equation (1), the objective function) which satisfies the constraint that all medical deadlines be satisfied. The OR suite can determine this sequence by considering all possible sequences of cases. For each sequence, the objective function (i.e., mean time patients have to wait to have surgery) is calculated and the sequence is checked to ensure that the constraint is satisfied. This method is referred to as “exhaustive enumeration.” Although not sophisticated, it is practical for our problem. We chose an example with only $n = 3$ cases or $3 \times 2 \times 1 = 6$ different sequences so that we could show how optimal sequencing of urgent surgical cases would work; consideration of 6 different sequences can be done without a computer. A list of $n = 5$ cases is a large number of urgent cases waiting to be done in one OR. There are $5 \times 4 \times 3 \times 2 \times 1 = 120$ different sequences of $n = 5$ cases. This number of different sequences to be evaluated may be too many for a busy OR manager to consider by hand. However, for a computer, this is a trivial number of different sequences. If the number of urgent cases were to exceed 10, on the other hand, a more sophisticated algorithm than exhaustive enumeration may be needed to evaluate the sequences, since

### Table 2.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1] = 3</td>
<td>First case in the sequence (the appendectomy) is case $i = 3$.</td>
</tr>
<tr>
<td>[2] = 2</td>
<td>Second case in the sequence (the femur fracture) is case $i = 2$.</td>
</tr>
<tr>
<td>[3] = 1</td>
<td>Third case in the sequence (the aneurysm clipping) is case $i = 1$.</td>
</tr>
</tbody>
</table>

There are 10 × 9 × ⋯ × 1 = 3.6 million different possible sequences.

### 1.3 Scenario: All deadlines will be satisfied by the sequence that minimizes the average time that patients wait to have surgery

We let the notation $[k] = i$ specify the order sequence of the cases. The $k$th case in the sequence of $n$ cases is the $i$th case (see Table 2).

The solution that minimizes the average of all start times (the objective given in Equation (1)) while neglecting the constraints is to sequence the cases in order of increasing case duration [1]: $\{c_1, c_2, \ldots, c_n\}$, where the brackets indicate the position of a case in this “shortest cases first sequence.” When two cases have the same case duration, the case that was submitted first would proceed first.

For our example (1.1), the optimal sequence is appendectomy first, femur fracture second, and aneurysm clipping third. It is straightforward to verify that all deadlines are satisfied by this desired sequence.

### 1.4 Scenario: The use of only one open operating room is insufficient to satisfy all deadlines

There are situations where all of the cases cannot be performed in only one OR while satisfying all of the medical deadlines. In this scenario, the goal of minimizing the average patient waiting time is secondary. The anesthesiologists and nurses’ would need to “call in” more people to help complete the cases.

To evaluate whether all deadlines can be satisfied, we let $f_i = d_i + c_i$ = the finish time for the $i$th case provided the case starts right at its deadline $d_i$. The finish time for one case is the earliest time the next case in a sequence can be started. Cases are sequenced in increasing order of their finish times. The case with the smallest $f_i$ is sequenced first and started immediately. The case with
the second smallest $f_i$ is sequenced to start second, and so forth. Mathematically, $f_{[1]} < f_{[2]} < \cdots < f_{[n]}$. If this so called “earliest due date sequence” results in one or more cases which has not been started by its deadline, then no sequence exists which will allow all cases to be started by their deadlines [1]. We know immediately that one OR is insufficient to complete the cases by their deadlines. An additional OR needs to be opened. Expressing this result mathematically, $c_{[i]}$ refers to the case duration and turnover time of the case with the nearest finish time, $c_{[2]}$ refers to the case duration and turnover time of the case with the next nearest finish time, and so forth to $c_{[n]}$. If $t + c_{[i]} > d_{[i]}$, then the case with the next nearest finish time cannot be started on time. The same argument applies to checking whether $t + c_{[2]} > d_{[2]}$ and so forth to checking whether $t + c_{[i]} = d_{[i]}$, for any of the $k = 2, \ldots, n$, then one OR is insufficient to start the cases by their deadlines, and an additional OR needs to be opened.

To show that this scenario is not applicable to our example, we calculate the finish times of our three example cases: $f_1 = d_1 + c_1 = 6$ A.M. November 9 + 6.0 hours = 12 noon November 9, $f_2 = d_2 + c_2 = 6$ P.M. November 8 + 2.5 hours = 8:30 P.M. November 8, and $f_3 = d_3 + c_3 = 9$ P.M. November 8 + 2.0 hours = 11 P.M. November 8. We sort the finish times: $f_{[1]} = 8:30$ P.M. November 8 < $f_{[2]} = 11$ P.M. November 8 < $f_{[3]} = 12$ noon November 9. The patient with a fractured femur $i = 2$ goes first, appendectomy $i = 3$ second, and aneurysm clipping $i = 1$ third. The current time $t = 8$ A.M. November 8. The value ($t + c_{[1]}$) = (8 A.M. November 8 + 2.5 hours for the femur fracture) = 10:30 A.M. November 8, which is earlier than $d_{[2]} = 9$ P.M. November 8. Likewise, ($t + c_{[2]} + c_{[3]}$) = (8 A.M. September 5 + 2.5 hours for the femur fracture + 2.0 hours for the appendectomy) = 12:30 P.M. November 8, which is earlier than $d_{[3]} = 6$ A.M. November 9. Thus, for our example, all deadlines can be satisfied. One OR is sufficient to complete the sequence of cases.

1.5 Change in medical condition or submission of a new case

A patient’s deadline for surgery may change due to a change in the patient’s condition. Typically the deadline would become sooner, resulting in a smaller value for the difference between deadline and the occurrence time. The case may then achieve a higher relative priority in the queue. If a surgeon contacts the OR with a change in a deadline, the optimal sequence of cases must be recalculated.

A new case may be submitted while a case is in the OR. Then, the value for the number of cases $n$ is increased by 1. A new sequence is developed based on Objective #1. Once the OR becomes available, the first case in the revised sequence would proceed to the OR.

1.6 Impact of variability in actual case durations from estimated case durations

Surgeons may take a variable length of time to complete an urgent case; cases sometimes take much longer than expected. Because of differences between actual and estimated case durations, the recommended optimal sequence of urgent cases could include data on the differences between estimated start times and deadlines.

The analysis assumes that case durations are known precisely. To compensate, an OR suite can establish a policy which requires that $d_i - s_i > b$ hours for all cases, where we let the parameter $b =$ the buffer time to provide a safety cushion for variability in case durations. If a predicted start time for a case were within some time $b$ of a deadline (e.g., $b = 1$ or $b = 2$ hours), additional personnel may be necessary to open another OR.

The sequence of cases recommended by the mathematical analysis should then be modified as necessary based on medical judgement. An OR suite should also monitor retrospectively actual start times and deadlines to ensure deadlines are being satisfied.

OR suites that choose to minimize the average of all start times (Equation (1)) and use a buffer time $b$ may not need a large buffer time. Long cases characteristically have more variability in case duration than briefer cases [4–6]. Referring to our example cases, an appendectomy with an estimated duration $c_3 = 2.0$ hours is less likely to finish 1.5 hours late than is an aneurysm clipping with an estimated duration $c_1 = 6.0$ hours.

From Section (1.3), provided that all deadlines will be satisfied, the sequence of cases that minimizes the average time that patients wait to have surgery (Equation (1)) is the sequence of increasing case duration. This sequence will also characteristically assure that cases with reliable estimates of case duration proceed first.

1.7 More than one operating room available for urgent cases

If there is more than one (typically two) OR staffed and available for urgent cases, the sequence of cases satisfying the objective need not change. The first case in the sequence proceeds to the available OR. The next time that an OR becomes available, the cases would be sequenced and the new first case in the sequence would
proceed to the open OR. As considered in Section (1.2), so long as the total number of cases to be done \( n \leq 10 \), the use of exhaustive enumeration to sequence the cases is practical.

1.8 Monitoring surgeons’ deadlines so as to decrease “gaming” of the scheduling system

A surgeon could alter deadlines to get a higher priority for their case. Methods to monitor for gaming strategies may be important at some OR suites. To audit surgeons, two of the data obtained in Section (1.1) can be used: the time of occurrence \( o_i \) of the disorder requiring surgery and the reported deadline \( d_i \) (Table 1). The difference between these two data, \((d_i - o_i)\), refers to the delay from the time of occurrence of the disorder to the time at which the patient is at increased risk of morbidity. A group of peer physicians, or an OR director, can track \((d_i - o_i)\). These values can be compared to the medical literature, to ensure that surgeons are not altering times to get higher priorities for their urgent cases. Extreme examples of gaming are easy to identify when each difference is listed next to its corresponding surgical procedure.

1.9 Starting urgent cases during weekdays in the late afternoon after completion of elective cases

Some OR suites use all operating rooms for elective cases during regularly scheduled hours. Our analysis applies fully to this scenario, after the “current time \( t \)” is adjusted to equal the time in the late afternoon when the first of the urgent cases in the queue is expected to start. OR suites should examine the deadlines to ensure that all deadlines will be satisfied by postponing the start of the first case.

Objective 2. Sequence cases in the order that they were submitted to the OR suite

This objective is simple to implement. However, using first-come first-served sequencing may cause a medical deadline to not be satisfied. In this situation, the sequence used would be the sequence that is as close as possible to being first-come first-served. We describe below how the phrase “as close as possible” can be described in mathematical language so that a computerized information system can produce the optimal sequence.

The same three questions described in Section (1.1) are asked of each surgeon to obtain the same three data: estimated case duration \( c_i \), time of occurrence \( o_i \) of the disorder requiring surgery in the \( i \)th patient, and deadline \( d_i \) by which time the \( i \)th case must have started or the patient is at risk for additional morbidity related to the delay in starting surgery (Table 1). The predicted start times \( \{s_1, s_2, \ldots, s_n\} \) are the output of the method of sequencing the \( n \) cases.

The objective considered for sequencing the cases is to determine the sequence which both satisfies all deadlines and minimizes the average discrepancy from first-come first-served. This implies, described mathematically [1],

\[
\text{minimize } n^{-1} \sum_{i=1}^n |[i] - i|, \quad \text{subject to constraint } d_i \geq s_i \text{ for all } i = 1, 2, \ldots, n.
\]

Equation (2) differs from Equation (1) with respect to the term within the summation. The \([i] - i\) term specifies “discrepancy from first-come first-served” or that each case should be done as close as possible to the sequence submitted. The sequence of cases to use is the one with the smallest value for objective function (2) which satisfies the constraint. Just as in Section (1.2), this sequence would be determined by considering all possible sequences of cases and, for each, calculating the objective function and checking whether the constraint is satisfied. If all deadlines will be satisfied by the sequence that minimizes the Objective #2, then the sequence would be the order in which each case was submitted. Issues regarding satisfaction of deadlines, changes in a patient’s medical condition, variability in case durations from estimated case durations, more than one OR, monitoring surgeons’ deadlines, and starting urgent cases after completion of elective cases are identical to those considered in Sections (1.4–1.9).

For our Example, the sequence of cases is the sequence in which the cases were submitted: case \( i = 1 \) or aneurysm clipping goes first, case \( i = 2 \) or femur fracture goes second, and case \( i = 3 \) or appendectomy proceeds third. Importantly, this “optimal” sequence is different from the “optimal” sequence derived from Objective #1 and given in Section (1.3).

Objective 3. Sequence the cases based on medical priority, using evidence from the medical literature

Finally, we consider the objective of performing the cases based on medical priority, as decided by the surgeons or an OR director, based on evidence from the medical literature. The same three data obtained from the surgeons in Section (1.1) are needed for se-
quencing the n cases based on this third separate objective: estimated case duration \( c_i \), time of occurrence \( o_i \) of the disorder requiring surgery in the ith patient, and deadline \( d_i \) by which time the ith case must have started or the patient is at risk for morbidity (Table 1). However, the deadline \( d_i \) would be determined in a manner different than as in Section (1.1):

**Question #3. For the disease suffered by your patient, what delay from the time of occurrence of the disorder to the time of surgery is associated with an increased risk of morbidity?**

A review of the medical literature reveals that for many diseases requiring urgent surgery, delays from the time of occurrence of a disorder to the time of surgery that are associated with increased risk of morbidity have been studied (retrospectively). For example, the risk of cerebral infarction is greater in patients who undergo aneurysm surgery more than 48 hours after subarachnoid hemorrhage [7]. For a patient sustaining a proximal femoral fracture, the risk of mortality increases if the fracture is not repaired within 24 hours [8]. Among children requiring appendectomy, the risk of gangrenous and perforated appendicitis is larger among patients having surgery more than 37 hours after the onset of symptoms [9]. Our objective #3 quantifies the underlying continuums of risk that may exist, whereby every hour longer a patient waits for surgery the risk of morbidity increases. Nevertheless, the delays themselves are generally reported in the medical literature in terms of cut-points or thresholds (e.g., 48, 24, or 37 hours) as asked in this Question #3.

**Data #3. Calculate the deadline \( d_i \) by which time the ith case must have started or the patient is at risk for morbidity.**

For example, if Dr Franklin reports that for an aneurysm clipping \( (d_1 - o_1) = 48 \) hours, then \( d_1 = o_1 + 48 \) hours. If Dr Smith specifies for femoral fracture \( (d_2 - o_2) = 24 \) hours, then \( d_2 = o_2 + 24 \) hours. If Dr Jones reports for appendectomy that \( (d_3 - o_3) = 37 \) hours [9], then \( d_3 = o_3 + 37 \) hours.

The predicted start times \( \{s_1, s_2, \ldots, s_n\} \) are the output of the method of sequencing the n cases.

The objective in sequencing the cases is to satisfy all medical deadlines and start each case on average as early as possible relative to the case’s difference between deadline and occurrence time. Expressed mathematically,

\[
\begin{align*}
\text{minimize} & \quad n^{-1} \sum_{i=1}^{n} (s_i - d_i)/(d_i - o_i), \\
\text{subject to constraint} & \quad d_i \geq s_i \text{ for all } i = 1, 2, \ldots n.
\end{align*}
\]

Comparing Equation (3) to Equation (1), the numerator \((s_i - d_i)\) specifies “start each case as early as possible.” The denominator \((d_i - o_i)\) specifies “relative to the case’s difference between deadline and occurrence time.” The fraction \((s_i - d_i)/(d_i - o_i)\) specifies that each case should be started as early as possible relative to the case’s difference between deadline and occurrence time.

**Objective #3 incorporates the fact that small reductions in delay can have a potentially large impact on decreasing the risk of an adverse outcome when the \((d_i - o_i)\) time is relatively small. If all deadlines will be satisfied by the sequence that minimizes the Objective #3, the cases are sequenced in order of increasing weighted case duration [1]:

\[
\frac{c_1}{d_1 - o_1} = \frac{c_2}{d_2 - o_2} = \cdots = \frac{c_n}{d_n - o_n}.
\]

The shortest weighted case duration is sequenced first. The second shortest weighted case duration is sequenced second, and so on. When two cases have the same weighted case duration, the case that has waited the longest from the time at which the surgeon contacted the OR suite would proceed first. Issues regarding satisfaction of deadlines, changes in a patient’s medical condition, variability in case durations from estimated case durations, more than one OR, monitoring surgeons’ deadlines, and starting urgent cases after completion of elective cases are identical to those considered in Sections (1.4–1.9).

We now consider our example with three cases. The aneurysm clipping is scheduled to take \( c_1 = 6.0 \) hours, repair of a fractured femur \( c_2 = 2.5 \) hours, and appendectomy \( c_3 = 2.0 \) hours. Because the risk of cerebral infarction is greater in patients who undergo aneurysm surgery more than 48 hours after subarachnoid hemorrhage [7], \( (d_1 - o_1) = 48 \) hours. Likewise, \( (d_2 - o_2) = 24 \) hours [8] and \( (d_3 - o_3) = 37 \) hours [9]. To prioritize the cases based on medical priority, the shortest weighted case duration is sequenced to start first. For example, the femur fracture with a weighted case duration equal to \( 2.5 \times 24 = 60 \) hours would go first. The appendectomy with a weighted case duration of \( 2.0 \times 37 = 74 \) hours would proceed second, followed by the aneurysm clipping with a weighted case duration of \( 6.0 \times 48 = 288 \) hours.

A printout of the product of the case duration and the difference between the deadline and occurrence time would permit surgeons to quickly identify the basis for the sequence of cases.

**DISCUSSION**

The optimal sequence of urgent surgical cases depends on the objective that an OR suite chooses to use as a guideline when sequencing the cases. To illustrate this,
point, we presented a mathematical foundation for each scheduling objective and constructed an example whereby the recommended sequence varied depending on the chosen objective. The mathematics is necessary to write the computer program for urgent case scheduling in an OR information system. Using Objective #1, if cases are sequenced to minimize the average time patients wait to have surgery, the shortest case goes first. The optimal sequence is appendectomy \((c_3 = 2.0\) hours\), femur fracture \((c_2 = 2.5\) hours\), and then aneurysm clipping \((c_1 = 6.0\) hours\). Using objective #2, if cases are sequenced in the order that they were submitted to the OR suite, the optimal sequence is aneurysm clipping \((i = 1)\), femur fracture \((i = 2)\), and then appendectomy \((i = 3)\). Using Objective #3, if cases are sequenced based on medical priority to minimize the chance of a poor patient outcome, the optimal sequence is femur fracture \((c_2 (d_2 - o_2) = 2.5 \times 24 = 60 \text{ hours}^2)\), appendectomy \((c_3 (d_3 - o_3) = 2.0 \times 37 = 74 \text{ hours}^2)\), and then aneurysm clipping \((c_1(d_1 - o_1) = 6.0 \times 48 = 288 \text{ hours}^2)\).

Application of the three objectives

In some OR suites, the responsibility to sequence urgent cases is considered to be a “thankless” job. The stakeholders (e.g., surgeons, anesthesiologists, and nurses) can be difficult to please. A necessary condition for optimal sequencing of urgent cases is consensus as to the objective to be used when sequencing cases. An individual charged with the responsibility for sequencing urgent cases may find this task easier politically with implementation of one of the three “rules” (mathematical structure) described above. In this setting, the person responsible for deciding the order in which urgent cases should be done can limit their activities to assisting surgeons, whose patients are in the queue, in reviewing the rationale for the computer generated sequence.

More than one operating room

Section (1.7), which applies equally to all three objectives, considers the sequencing of cases when more than one OR is being used. However, the analysis assumes that all cases can be done in any of the operating rooms. OR suites with more than one OR available for urgent cases may need to assign cases to operating rooms based partly on the specialty experience of the staff; mathematical models that incorporate knowledge regarding staff expertise or comfort level when sequencing urgent cases would be difficult to implement, because it is difficult to program specialty experience into an OR information system. Our mathematical reasoning is meant to provide recommendations for sequencing of urgent cases, to be used in conjunction with medical judgement and the experience of OR personnel.

Economics of sequencing urgent cases

A rationale for our analysis is that if the urgent cases in a queue can be completed quickly enough such that patients’ outcomes will not be affected by waiting until other urgent cases are completed, then not opening an additional OR is appropriate. Not opening another OR can be economically preferable by reducing OR suite staffing costs. However, depending on how staff are paid, in some OR suites the cost of running a concurrent second OR is no greater than running one OR for a longer period of time. For example, some OR suites use part-time nurses (i.e., who do not receive overtime and who have no minimum number of hours of work a day) and have fee-for-service anesthesiologists. Under this compensation structure, no additional costs will be realized in opening an additional OR to complete the urgent cases sooner.

The economics of staffing one or two operating rooms for urgent cases requiring immediate care has recently been reviewed [10].

Fast algorithms to identify the sequence of cases that optimizes Objective #3

A sophisticated algorithm to obtain the optimum sequence to Equation (3) has been developed [1, 11]. A general purpose (commercial) OR information system would include this algorithm. For OR suites that routinely have more than 10 cases in a queue, implementation of this algorithm may be worthwhile. Nevertheless, as we consider in Section (1.2), most OR suites will have sufficiently few urgent cases on their add-on lists that their information system can perform the calculations described by exhaustive enumeration.

A fourth objective that could be used to sequence urgent cases

Another objective that could be used to sequence urgent cases is to modify the third objective by adding the time a patient has already waited for urgent surgery, providing for the objective of minimizing \(n^{-1} \sum_{i=1}^{n} (s_i - d_i)/(d_i - o_i - w_i)\) subject to the constraint \(d_i \geq s_i\) for all patients.
i = 1, 2, …n. Provided that all deadlines can be satisfied, the sequence that will minimize the objective given in Equation (4) is \[ c [1](d [1] - o [1] - w [1]) \leq c [2](d [2] - o [2] - w [2]) \leq \cdots \leq c [n](d [n] - o [n] - w [n]). \] Even if a case has a relatively large difference between the deadline and occurrence time, the case can still have a relatively high priority if it has already waited a long time.

Although this objective seems to make intuitive sense, we question the reasonableness of the medical implications of this objective. For example, we consider the sequencing of two urgent cases: a kidney transplant and a fasciotomy for extremity trauma (c6 = 2.5 hours). Survival of renal allografts is lower for cold ischemia times exceeding 4 hours [12]. The risk of infection is higher for patients undergoing fasciotomy within 12 hours of extremity trauma [13]. Suppose that the surgeons contacted the OR suites about the cases w6 = 28 hours and w5 = 0 hours ago. Let us also suppose that both cases have deadlines 4 hours in the future. Excluding waiting times, the fasciotomy would go first because 2.5 × 12 < 2.5 × 36. Including waiting times, the kidney transplant would proceed first because 2.5 × (36 - 28) < 2.5 × (12 - 0).

**Summary**

Before determining the optimal sequence of urgent cases, an OR suite must identify the primary scheduling objective to be satisfied when prioritizing a group of urgent cases. We have considered three different objectives for sequencing urgent cases. The recommended sequence should ensure that all medical deadlines are satisfied. To minimize the average patient waiting time, the primary statistic to determine the sequence in which to do the urgent cases is the case duration. A second objective may be to perform the cases on a first come first served basis. For this objective to be met, the primary statistic is the order in which the cases were submitted. Finally, to minimize the patient waiting time weighted by the time course appropriate for the patient’s medical condition, the product of (i) case duration in hours and (ii) the difference between deadline and occurrence time in hours is the key statistic to compare among urgent cases. The latter criteria incorporates medical issues into the method of sequencing cases. If either of the first two methods are used (minimize average patient waiting time or use first-come first-served), then medical judgement is left to the surgeon’s assessment of the deadline.

**NOTES**

2. To show that the algorithm works, we consider two cases labeled i = 1 and i = 2. Let w1 and w2 refer to times that the 1st and 2nd cases have waited since the surgeons submitted the cases to the OR suite. The corresponding case times are ci and ci. If case i = 1 goes first, the average patient waiting time = (w1 + c1 + w2)/2. If case i = 2 goes first, the average patient waiting time = (w2 + c2 + w1)/2. Comparing these two average waiting times, the values of w1 and w2 do not affect the results. The shortest case should go first.
3. To explain the logic of objective #2, we consider two cases labeled i = 1 and i = 2, submitted in that order. If the cases are performed in that sequence, then \[ n^{1} \sum_{i=1}^{2} |i| - i = 2^{−1} ((1| - 1) + |2| - 2) = 2^{−1} (|1| - 1) + |2| - 2 = 2^{−1} (0 + 0) = 0. \] If the cases are performed in the opposite order, then \[ n^{1} \sum_{i=1}^{2} |i| - i = 2^{−1} ((|1| - 1) + |2| - 2) = 2^{−1} (|2| - 1) = 2^{−1} (1 + 1) = 1. \]
4. Our observation holds for aneurysm clipping [7], proximal femoral fracture repair [8], and appendectomy [9]. Additional examples include the following. The risk of renal allograft failure is greater for cold ischemia times exceeding 36 hours [12]. The risk of infection is greater for patients undergoing fasciotomy longer than 12 hours of extremity trauma [13]. The risk of hypoxic encephalopathy from umbilical occlusion is greater than 0.4 hours after onset of acute fetal distress [14]. The risk of mortality is greater among patients with penetrating colon injuries who undergo operative repair longer than 12 hours after injury [15]. The risk of mortality is greater among patients with necrotizing fasciitis of the vulva who undergo surgical debridement longer than 48 hours after presentation [16].
5. The goal of starting each case on average as early as possible is straightforward. However, the plan of meeting this objective relative to the case’s difference between deadline and occurrence time may not be as clear. We consider two hypothetical surgical cases. The first is acute fetal distress from umbilical occlusion with (d4 - o4) = 0.4 hours. The second is femur fracture from trauma with (d2 - o2) = 24 hours. While it is desirable to start either case earlier rather than later, a gain of 0.1 hours in the situation of acute fetal distress represents a gain of 25% relative to its (d4 - o4) = 0.4 hours. In contrast, starting the femur fracture 0.1 hours early represents a gain of less than 1% relative to its (d2 - o2) = 24 hours. This example illustrates that the absence of precise knowledge of (d2 - o2) from the medical literature has little effect on the analysis. Whether or not the deadline for femur fracture (d2 - o2) = 24 hours or 36 hours, it is much longer than the deadline for hypoxic encephalopathy from occluded umbilical cord.
REFERENCES

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