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On the Usage of Musical Keys: A Descriptive Statistical Perspective

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Abstract

A great deal has been written about the affinity between composers and musical keys. For instance, it is well-known that Mozart composed the majority of his works in the key of C major, and that some of Beethoven's most popular works are in the key of C minor. But little is written about composers' least used keys, or more generally about their usage of all keys. Here, a methodology is proposed which allows one to 1) concisely describe a composer's usage of keys, and 2) compare different composers in terms of their key usage. The main tool underlying the former is the key histogram, and the latter is displayed in terms of the scatter plot of keys and further quantified by the correlation coefficient. The comparison itself is performed in two ways: 2a) pairwise, between all composers, and 2b) by comparing each composer's key histogram with a "gold standard" key histogram (e.g., key histogram across all composers). The former allows for a focused comparison of key usage between two composers, while the latter is useful for ranking of the composers in terms of their key usage. The method is demonstrated on a list of ten composers. For example, it was found that Mozart composed primarily in the keys of C, D, E, F, G, and E-flat; and with very few to no works in any other key. By contrast, Rachmaninoff's works appear in every key, with near-equal frequency (as assessed by the chi-squared statistic). The pairwise

comparisons show that some expected similarities between composers (e.g., Haydn and Mozart), as well as some expected dissimilarities (e.g., Haydn and Rachmaninoff), can be placed on a firm, quantitative setting. The comparison of the individual composers' key usage with the key usage across all ten composers, suggests that Rachmaninoff and Schubert occupy the extremes, respectively least similar and most similar to the key usage across all ten composers.

Introduction

The concept of the musical key (key, for short) has played an important role in shaping the history of Western music[1]. The Greeks utilized a system which was abandoned after the fall of the Roman Empire[2]. Throughout the Middle Ages, musicians experimented with various modes, but by the end of the Baroque era the notion of a key had become commonplace[3]. The resulting system consisted of a series of seven "natural" notes, each augmented by lower and higher notes referred to as "flat" and "sharp," respectively. A key specifies whether each note is to be played in the natural, flat, or sharp. Most of western classical music employs about 12 keys (referred to as diatonic) which are usually specified by one of the letters, A, B, C, D, E, F, G, followed by "sharp", "flat," or nothing. Additionally, these keys appear in a "minor" or "major" variety (with the "major" often not denoted), leading to a total of 24 keys. Works written in minor keys are generally considered sad or melancholic, while major keys normally invoke happiness and excitement. Buelow[4] thoroughly explains the use and the affects of such musical devices. With the seven letters A through G, along with the specifications "natural/sharp/flat", and "major/minor", one can construct more than 24 keys, but many of the keys are in fact tonally equivalent. For example, C-flat major is tonally equivalent to B major. Therefore, 24 is the number of distinct keys.

For a composer the choice of a key is a complex matter including mood (happy versus sad), and the choice of the instrument because different instruments have an affinity for different keys. As a result, the list of all keys in which a composer has composed is dictated by a wide range of factors. In spite of the complexity in choosing a key, most composers have some "favorite" keys and some keys in which they rarely compose. For example, many of Bach's most significant works are in the key of D minor (e.g., Art of Fugue BWV 1080, or Chaconne from the solo violin sonata BWV 1004), and Beethoven is known to have used the key of C minor for his most dramatic works (e.g., Symphony No. 5, Op. 67).[5]

The most frequently used key, and the least frequently used key, are only extremes of a more general concept, called a key histogram. For a specific composer, to build a key histogram one simply counts the number of works in each key, and then plots that number versus the keys. The key with the smallest count can be considered the least favorite, while the most favored key would be the one with the largest count. The entire histogram, however, conveys more information; it displays not only the least or most favored keys, but the composer's sentiment regarding all keys. As shown below, some composers tend to write in many keys and with comparable frequency, while other composers tend to write in only a specific set of keys, and not at all in other keys. Also, the histogram of keys for a specific composer is unique to that composer, because it is extremely unlikely that two composers would write exactly the same number of pieces in the same set of keys. As such, the histogram of keys for a given composer is characteristic of the composer in the same way a fingerprint is a unique characteristic of a person. For this reason, a histogram is a useful tool for quantifying a composer's usage of keys. (The proposal to study composers in terms of their key histogram is not new; for example, it has been used for comparing Schubert's major and minor works.[6])

Associating a composer with a unique key histogram offers the possibility of quantitatively comparing two or more composers in terms of their key usage. Indeed, the comparison can be done in multiple ways. On the one hand, one can compare two composers directly in terms of their respective histograms. Although there exist many methods for comparing two histograms, one intuitive method is based on the correlation between histograms, and is described in the next section. On the other hand, such pair-wise comparison of composers involves many comparisons; for ten composers, there exist 45 (i.e., 10 choose 2) comparisons: 1^{st} with 2^{nd}, 1^{st} with 3^{rd}, ..., 1^{st} with 10^{th} , 2^{nd} with 3^{rd}, ..., 9^{th} with 10^{th}. Although these comparisons may be useful if two specific composers are to be compared, an alternative method of comparing composers is to compare each composer's histogram with a single "gold standard" histogram. The choice of the gold standard is ambiguous, but two possibilities are described in the next section.

In this paper, it is proposed that a composer's key usage can be characterized in terms of a histogram of the keys, and that these histograms not only summarize a given composers sentiment regarding all keys, but also offer the possibility of comparing composers in terms of their key usage. The next section presents details of the proposed method, and demonstrates it on ten composers. The conclusion section presents

the results specific to the ten composers examined here, and is followed by a discussion and ways in which the method can be generalized.

Materials and Methods

Data

The connection between composers and keys is partly dictated by the instruments for which they compose. For instance, for a composer who writes primarily for the Piano, the key of C Major is the simplest key because it corresponds to all of the white keys on the keyboard. A "difficult" key is C-sharp major because every note is sharped. For most wind instruments the key of E Major is a difficult key, and so an orchestral work in that key is somewhat unnatural[7]. As a result, the relationship between composers and their key usage is confounded by the choice of instruments. In order to avoid this complexity, and still be able to compare composers in terms of their key usage, the focus here is placed only on composers who have written works for a wide range of instruments. Based on this criterion, ten relatively well-known composers are selected; see Table 1.

Composer	Years	Number of works
J. S. Bach	1685-1750	889
J. Haydn	1732-1809	393
W. Mozart	1756-1791	572
L. V. Beethoven	1770-1827	186
F. Schubert	1797-1828	1041
J. Brahms	1833-1897	378
A. Dvorak	1841-1904	242
F. Mendelssohn	1809-1847	555
P. I. Tchaikovsky	1840-1893	381
S. Rachmaninoff	1873-1943	217
Total	-	4,854

Table 1. List of ten composers employed for demonstrating the proposed method. Also shown are the period during which each composer lived, and the total number of works for each composer (as defined in the text).

For each composer, the International Music Score Library Project (IMSLP)[8] is consulted for obtaining a list of works, and the associated keys. Obtaining a count of the number of works in a given key is somewhat ambiguous. For example, many composers have revisited a work years after it was conceived, in which case the key associated with

the work would be counted twice. Such ambiguities plague even the total number of works written by a composer. For example, Beethoven has 137 works with unique Opus numbers, but there are many works without an opus number (denoted with the symbol “WoO” for “Without Opus”). The latter are generally considered early pieces, and often neglected in concert or recording repertoire. For that reason, here WoO are excluded from analysis. However, the Hungarian Dances of Brahms, though well-known and well-recorded are in fact works without an opus number, because they were composed over the span of a decade. For the current paper, these works are included in the analysis. Some works with an opus number are not listed as having a key (e.g., Brahms' Alto Rhapsody), and are therefore excluded from the analysis. Finally, it should be noted that the single key associated with a work is not necessarily the only key appearing in that work; most works explore a wide range of keys in spite of a unique key associated with the work as a whole. The final count of works for each composer is listed in Table 1, and the analysis is performed on the single key reported by IMSLP.

Method

For each composer, the histogram of keys is computed. It provides a visual display of the composer's key preference. In order to simplify the task of comparing histograms, instead of plotting the count of each key, it is customary to plot the proportion of the works in each key.[9]

The comparison of key histograms between composers is more complex because it can be done in one of two ways: a pair-wise comparison, or a comparison of each composer’s histogram with a “gold standard” histogram. One possible gold standard is a histogram consisting of an equal number of works for each key. Such a histogram implies that the corresponding (fictitious) composer has no affinity for any specific key, and so, has composed the same number of works in each and every key. This gold standard is the basis of the chi-squared test, wherein for a given composer the count associated with each key is compared with what one would expect if the composer had no affinity with any particular key. Said differently, this gold standard is relevant if one aims to test whether a given composer has no affinity for any key. An alternative gold standard is the histogram of keys across all composers. Such a histogram can be considered as an “average key histogram” across all composers. Equivalently, it can be viewed as the key histogram of an “average composer.” With this choice of the gold standard, one can quantify how far each composer is from the “average composer” in terms of key usage.

The visual similarity of two histograms can be displayed in a more

objective fashion. Each histogram is essentially a list of 24 numbers (or proportions), one for each of the keys. And so the comparison of one histogram with another is tantamount to the comparison of a list of 24 numbers with another list of 24 numbers. One appropriate tool for that purpose is the scatterplot[9], wherein one represents each key with a point whose *x* and *y* coordinates are the numbers in the two lists. Generally, any linear pattern of points in such a scatter plot is indicative of the similarity of the underlying histograms. A perfect agreement between the two lists (histograms) would manifest itself as 24 points along a straight line on such a scatter plot. At the other extreme, complete dissimilarity between two histograms would lead to a scatter plot of 24 randomly distributed points.

The linear pattern in a scatter plot can be summarized/quantified in terms of a single number, called the correlation coefficient[9], denoted by the symbol *r*. A perfectly linear relationship, with all of the points in the scatter plot falling on a straight line, leads to $r=\pm 1$ (plus or minus depending on the slope of the linear pattern), while a random pattern of points scattered across the scatter plot corresponds to $r=0$. The correlation coefficients for all 45 pairwise comparisons are computed. Also computed are the correlation coefficients between each composer's histogram and the aforementioned “average histogram”.

Results

For the data at hand, the overall histogram of the 4,854 works is shown in Figure 1. The symbols on the *x*-axis denote the 24 keys as follows: “C” and “Cm” denote C major and C minor, respectively. The symbols “#” and “b” denote sharp and flat, respectively. For example, “Ebm” denotes the key of E-flat minor. The dashed horizontal line is the histogram of a fictitious composer who composes with equal proportion ($1/24 \approx 0.042$) in each and every key.

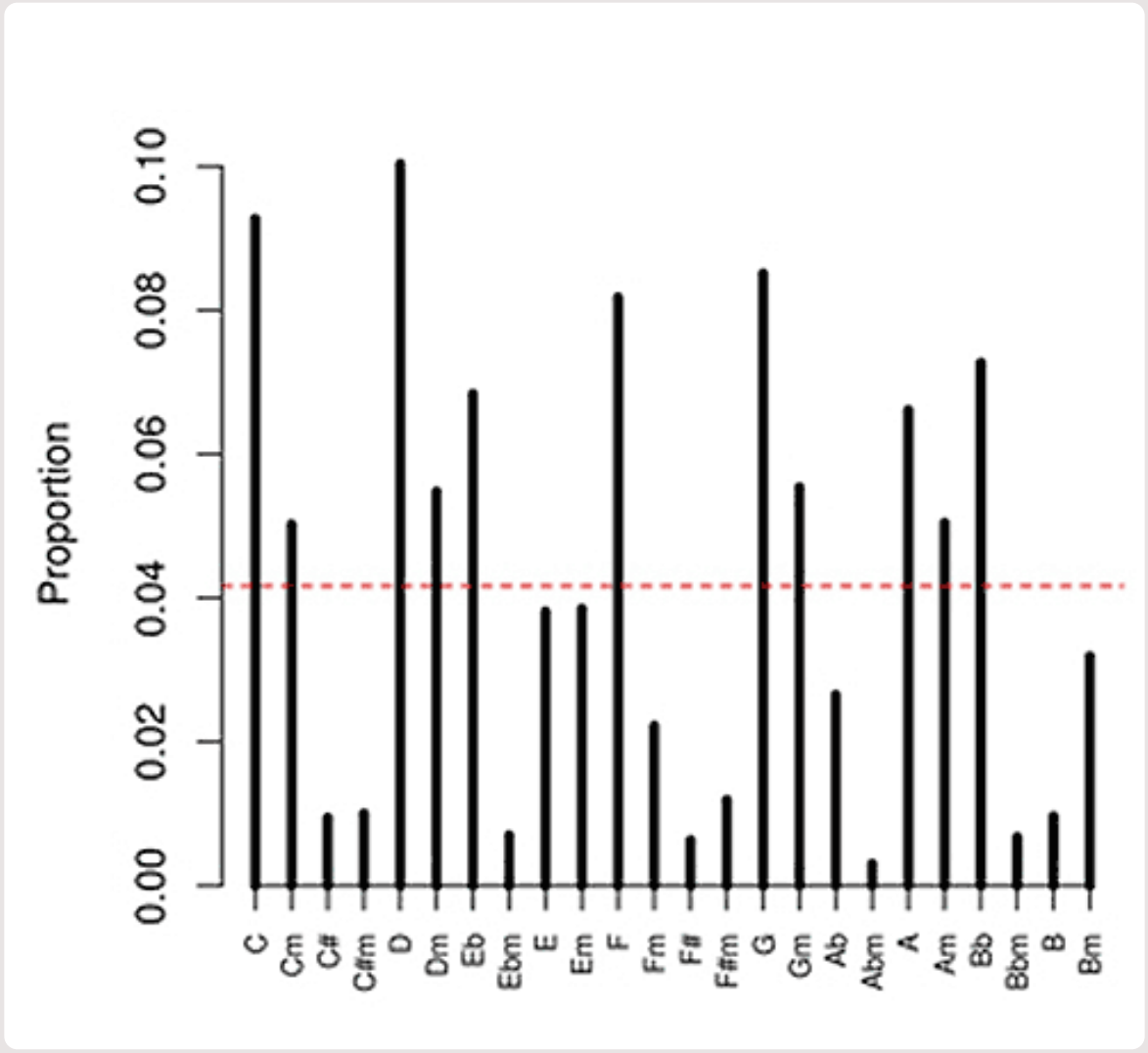


Figure 1. The histogram of the keys for each of the ten composers in this study. (The order of the composers is explained below.) Recall that each of these histograms can be viewed as a "fingerprint" characterizing the composer's usage of keys. This analogy is clearly visible in Figure 2 where no two histograms are identical.

Each panel in Figure 2 also shows the number of works for each compose (n), and the value of the chi-squared statistic (chi-sqd). The latter is a measure of how much the histogram deviates from the dashed horizontal line. A small value (e.g., 85.5 for Brahms) implies that the composer wrote with nearly equal proportion in each key. A large value (e.g., 1165.8 for Mozart) suggests that the works are far less-evenly distributed across all keys. The corresponding p-values (testing the statistical significance of the difference between the histogram and the dashed line) are all less than 0.001. Therefore, none of the 10 composers examined here have written truly evenly across all keys. In other words, each composer has an affinity for some set of keys.

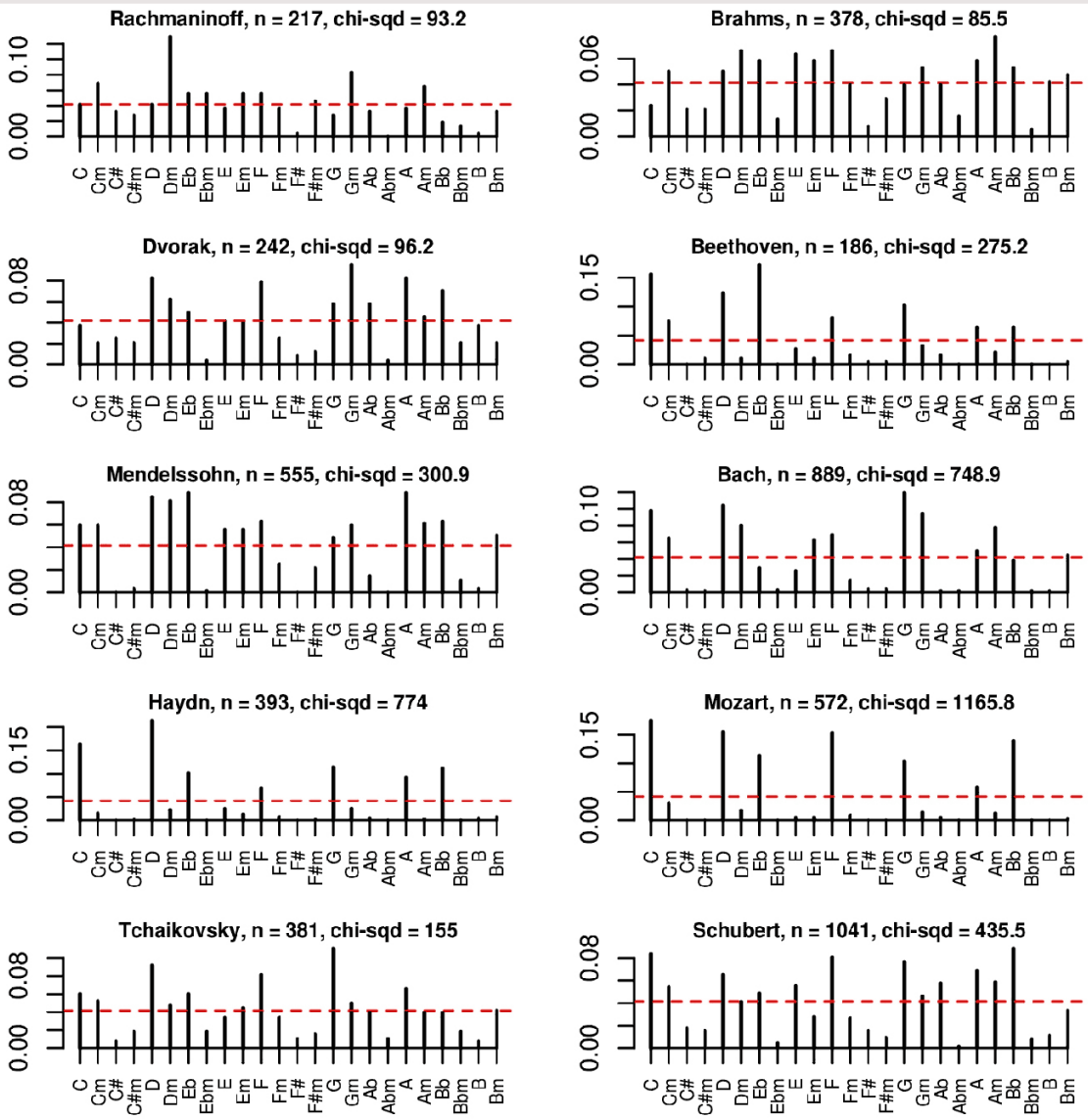
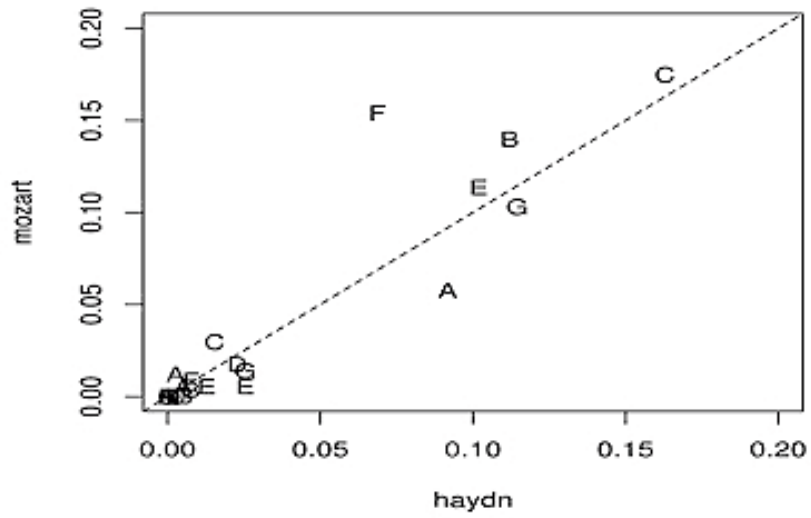


Figure 2. Histogram of keys for each of the 10 composers.

Figure 3 shows two scatter plots for pairwise comparisons; the diagonal (dashed) line has been added to aid in assessing any linear relationship. The top panel is a scatter plot of the histogram of keys for Mozart versus that of Haydn. This scatter plot displays one other piece of information, which will be discussed in the next paragraph. That information is related to the fact that points in the scatter plot have been replaced by the letter of the corresponding key. Ignoring that information, the close proximity of the points (letters) to the diagonal line indicates that there is a linear pattern. By contrast, the relatively large amount of scatter of points in the bottom panel suggests no linear relationship between the histogram of keys for Rachmaninoff and Haydn.



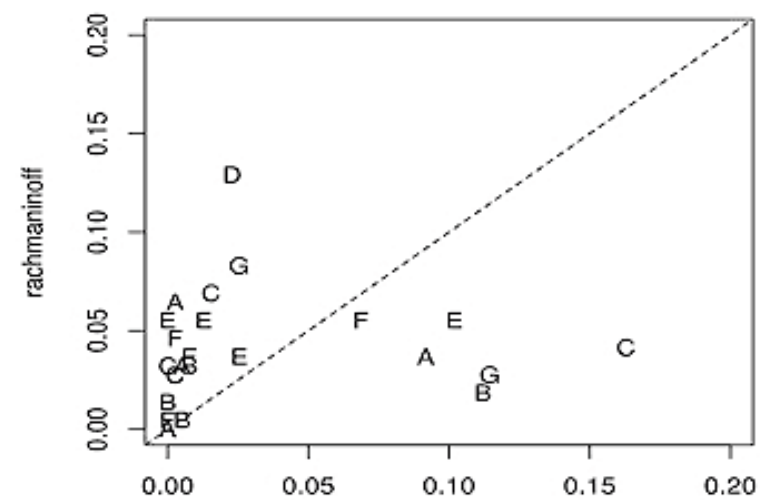


Figure 3. Two scatterplots of key histograms. The values on the axes are the y-axis (i.e., proportion of works written in each key) of the histograms shown in Figure 1. The dashed diagonal line has been added to aid in assessing any linear relationship between the axes.

It was noted that points on these scatter plots have been displayed with letters denoting the corresponding key (the sharp/flat and major/minor specifications have been suppressed for visual clarity). For example, in the top panel, consider the "C" in the upper/right corner of the scatter plot The coordinates of this point are (0.16, 0.17). In other words, 16% of the works of Haydn are in the key of C, and 17% of the works of Mozart are in that key. The closeness of these two percentages is reflected in the closeness of that point to the diagonal line. The display of the keys in a scatter plot aids in pinpointing the differences between composers' key usage.

As mentioned above, in order to minimize the number of pairwise comparisons one can compare each composer's histogram to the overall histogram (Figure 1). Recall that this comparison essentially measures how much each composer deviates from the "average" (or "typical")

composer. The resulting scatter plots, and the corresponding correlation coefficients (*r*) are shown in Figure 4. Note the increasing pattern of the *r*-values, which is in fact the reason for the order of the composers in this figure.

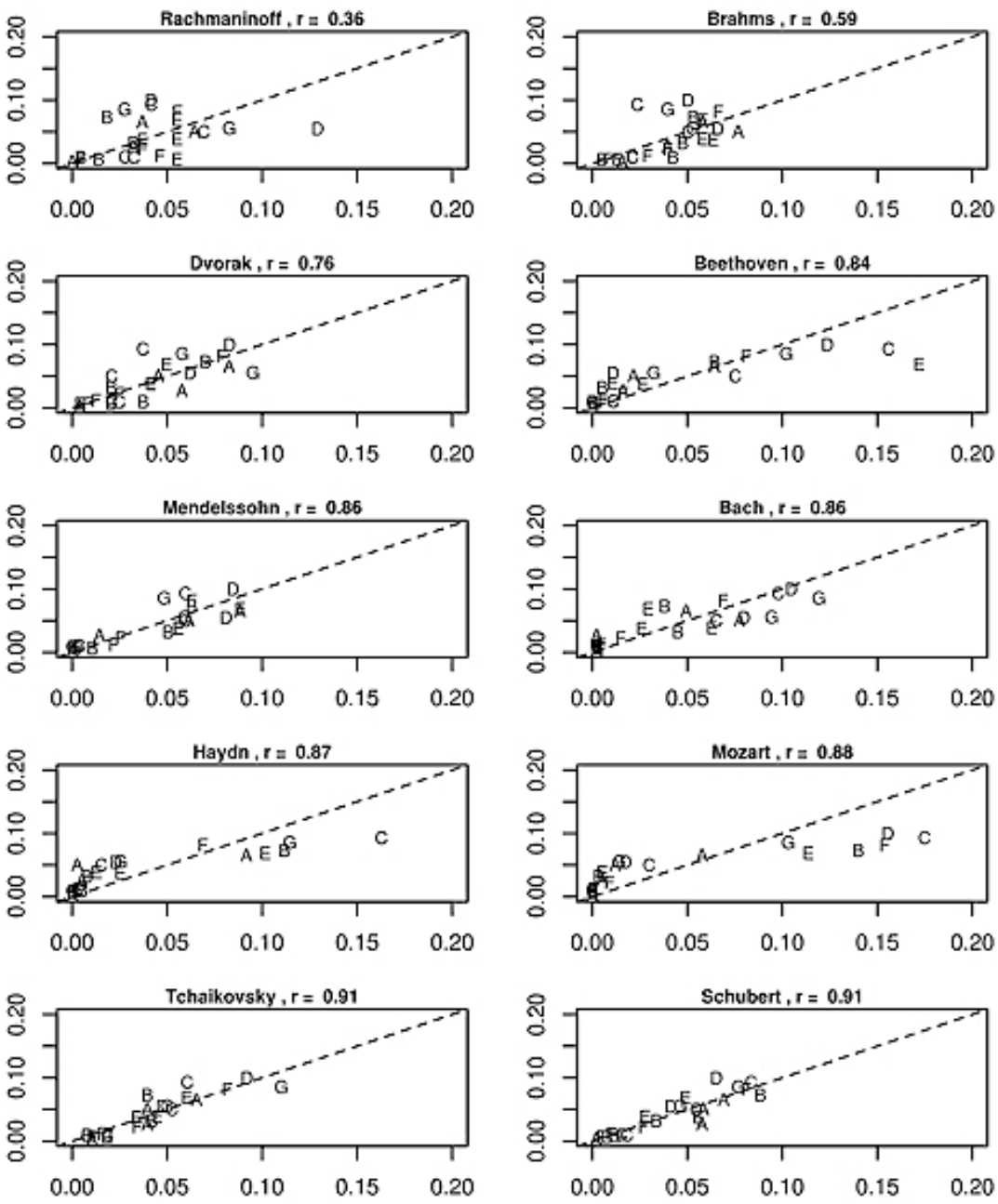


Figure 4. Comparison of each composers key usage with the "average" key usage. The numbers appearing above each panel are the correlation coefficient

Discussion

A method is put forth for quantitatively assessing and comparing composers in terms of their key usage. The method is based on several common statistical tools: the key usage itself is quantified with a histogram, and comparisons between composers are performed in terms

of scatter plots and correlation coefficients. The notion of an "average composer" is introduced as a means of comparing all of the composers with a single "gold standard." The method is illustrated by application to ten composers. Some of the specific conclusions in this particular application are as follows: Rachmaninoff and Brahms composed in a wide range of keys, while Haydn, Mozart, and Beethoven had strong preferences in favor of the keys C, D, and G, and against the keys C-sharp, A-flat minor, B-flat minor, and E-flat minor.

According Figure 1 shows that about 10% of all 4,854 works written by the composers studied here are in the key of D (the most frequent key), closely followed by C (9.2%); the least frequent key is A-flat minor (0.3%). Some of the features of this histogram can be explained by the definition of each key. For example, the fact that A-flat minor is the least frequent key across the 4,854 works of the 10 composers can be attributed to the 7 flats associated with that key - difficult to play on any instrument.

The key histogram for each of the ten composers (Figure 2) has a wide range of implications. Consider two extreme examples - Brahms and Mozart. It is evident that Brahms composed most evenly in almost all keys, while Mozart clearly has some "favorite" keys, namely C and D, followed closely by F and B-flat. This dissimilarity between Brahms' and Mozart's histogram can be contrasted by the similarity of Brahms' and Rachmaninoff's histogram.

As mentioned previously, displaying the key corresponding to the 24 points in the scatter plots is useful. For example, the linear relationship in the top panel of Figure 3 suggests that Haydn and Mozart have a similar key preference; but one can also conclude that the most prominent difference between the two composers is in their usage of the key of F. That conclusion follows from the fact that the point labeled "F" is the farthest from the diagonal line. Its coordinates are (0.07, 0.15), implying that whereas Haydn wrote only 7% of his works in the key of F, Mozart wrote nearly double that percentage (i.e., 15%) in that key. Conversely, the two composers' usage of the keys of C, E, D, and A are similar, as seen from the proximity of these points to the diagonal line.

Figure 3 shows only two of the 45 possible scatter plots; they are not all shown here, but all 45 correlation coefficients have been computed. They range from 0.25 to 0.95, and are all statistically significant (p-value $<$ 0.001). These two extremes correspond to the two comparisons shown in Figure 3. In other words, the two most dissimilar composers (among the ten considered here) are Rachmaninoff and Haydn, and the two most similar composers are Haydn and Mozart.

As shown above, an alternative to pairwise comparisons is to compare each composer's histogram to the histogram of all works. The corresponding scatter plots are shown in Figure 4; the corresponding *r*-values range from 0.36 to 0.91 (all with p-values < 0.001). Given the large scatter of points in the top/left panel, it follows that Rachmaninoff's key usage is the least similar to the overall key usage ($r = 0.36$). Said differently, Rachmaninoff can be considered the most atypical of the ten composers examined here. By contrast, both Schubert and Tchaikovsky have a key usage which is very similar to the overall key usage ($r = 0.91$). As such, they can be considered the most typical of the ten composers.

The work presented here can be extended in a number of ways. Here, the key associated with each work is that officially assigned to that work according to IMSLP. For example, the key associated with Beethoven's 5^{th} symphony is C minor, regardless of the key changes within the first movement, or across the various movements. It will be interesting to apply the above analysis to each and every key that makes an appearance anywhere in a work. Other possible extensions would take into account the musical form (e.g., symphony versus solo-instrument), the choice of the instrument (e.g., piano versus clarinet), and the musical era to which the composer belongs.

Finally, one can supplement the above analysis by providing "theoretical" arguments that may explain the conclusions found here. For example, the ranking of the composers in Figure 4 begs some explanation. The manner in which Rachmaninoff emerges as an atypical composer may be attributed to the fact that he belongs to a different era than the rest of the composers. However, that explanation is not entirely satisfactory for two reasons: 1) Brahms, a transition composer between the Classical era and the Romantic era, is the second-ranking atypical composer, while 2) Bach, a Baroque composer, emerges as a relatively typical composer ($r = 0.86$). In other words, the composer's era appears to be unrelated to the ranking in Figure 4. In way of a different explanation, one may suspect the ranking is a consequence of the total number of works written by each composer. If that were the case, then the number of works displayed atop each panel in Figure 2 would display some pattern (increasing or decreasing); but they do not. As such, there appears to be no simple theoretical explanation for the ranking of the composers in Figure 4. Another possible explanation may follow from analyzing the historically established relationships between composers; for example, it is well-known that Mendelssohn was influenced by Bach[10]; this may explain why both composers appear with comparable correlation coefficients (r=0.86) in Figure 4. It will be important to establish other

such historical connections in order to explain the conclusions found here.

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