The Swerve and the Flow:

Music's Relationship to Mathematics

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There are certainly perspectives on the relationship of music to mathematics from sociology and intellectual history, and perhaps even a natural history – the cats and dogs of music scholarship, so to speak, allegorizing the dichotomy between musicians who can and will do some mathematics, and those who can't or won't. Underlying these distinctions is a relationship – which I will not call essential – between the nature of music and the nature of mathematics.

We are all familiar with the influence of Pythagoreanism, a mighty bell once sounded whose dying reverberations pervade music history with the idea that Number is Music and Number is Cosmos, and that harmonic relations among numbers are privileged. This locates the nature of music in that of Number. However, we can look at such Pythagoreanism as one aspect of a larger thread or strand in pre-Socratic Greek philosophy, namely the tendency to identify an unchanging essence of things and to sort out the cosmos in terms of such essences: discourses of Being. Along this thread I would locate all atomisms from Democritus through Epicurus and Lucretius. Atoms *are* – tiny indivisible unchangeable Beings whose combinations account for the variety of the cosmos – atomism is founded on Being. Platonic Forms and Ideas are also along the thread of Being, as is almost all Christian theology.

Parmenides put the underlying ethos well:

> Being is without beginning and without end, whole, unique, imperturbable, and complete; it never "was" and never "will be," since it Is now altogether, one, coherent; for what genesis could you try to find? How and whence did it grow? For I will not allow you to speak or think from the standpoint of Non-being; for it can not be said or thought that it Is-not. What need would arouse it sooner or later, beginning from nothing, to start being? Therefore it must exist entirely or not at all.
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> Itself remaining in itself, it lies situated throughout itself and remains steadfastly in its place; for powerful Necessity holds it in the bonds of its furthest limit, which closes it around, wherefore it is not lawful that Being be unfulfilled to its limits. For if Being were in lack of something, it would lack everything. ...for nothing either is or will be separately, apart from being, since Fate bound it to be whole, inviolable.[i]

The beauty, and the convenience, of the strand of Being is that it provides flash-frozen slices of the universe (horizontal or vertical slices) which lie quiet so that their structure may be perceived and described – and of course, mathematics is the best tool for describing structure. Being provides a frame.

The problem with the strand of Being as an approach to music, or to life, is to accommodate within the matrix of Being not only apparent change in general, but also human will, free will, artistic will.

There is a second strand in philosophy that is radically different from the strand of Being: the strand of Becoming, existence rather than essence; change, flux, fire, plasma, flow. It begins with Heraclitus, who says among other things:[ii]

1. Nocturnal wanderers: magician-priests, Bacchants, wine-festival party people, mystical initiates.

2. The hidden harmony surpasses the apparent harmony.

3. Disbelief escapes and is not known.

4. Remember also the person who forgets how the road goes on.

5. Those who sleep are workers and co-workers in the universe of becoming.

6. Know that war is common to things that are, to justice and conflict, and everything needs to arise through strife.

7. They do not understand how something can go against itself, yet agree with itself; tuning by contrary motion, like a bow or lyre.

8. The thunderbolt steers everything.

9. One cannot go twice into the same river.

10. Nature likes to conceal itself.

11. The way up and down is one and the same.

Musicians, he us speaking to us! The vagabonds of the night, the magicians, the possessed, the bacchantes, the inspired! Clearly this is a different voice. We hear it from a few philosophers – certainly Nietzsche. The phenomenology and logic of Hegel are everywhere indebted to Heraclitus. Existentialists from Avicenna through Sartre sing in harmony with Heraclitus. Philosophers of flow and time such as Suzanne K Langer and Henri Bergson are in this strand. Deleuze and Guattari would like to be: in their brilliant

and influential book, *A Thousand Plateaus*, a nostalgia for fireflow, change, and metamorphosis coexists with a discourse of escape from frame.[iii] Music flows, and swirls madly.

But how does mathematics relate to flow as pure change? The first issue is the nature of change: to describe change in terms of states, at least two describable states are needed, before and after – perhaps almost infinitesimally after. But do two such static descriptions capture the feeling of change itself? A second issue is related to unpredictability and free will: there is a mathematics of turbulence, but is that suited to describe flow resulting from artistic governance during creation, steering the work amid the rapids, continuing acts of free artistic will (which are not stochastic)? We will explore these questions as we go on.

To sharpen our point, we must try to weave these two threads together, Being entwined with Becoming, state with change.

Lucretius, following Epicurus, modeled the universe as a frame of atoms of Being falling (naturally, according to their weight) forever in parallel lines, with this important tweak: occasionally, for no reason, an atom will *swerve* in its fall (the *clinamen* or *ekklisis*). The frame of structural (or divine) order and the Swerve of free and artistic will.

Michel Serres has written brilliantly about Lucretius, not entirely from our point of view. Here is his poetic description of the aspect of Being in Lucretius, Lucretius without the Swerve:[iv]

> Without the declination, there are only ...the chains of order. ... the new is only the repetition of the old. ... The order of reasons is repetitive, and the train of thought that comes from it, infinitely iterative, is but a science of death. ... the law is the plague; the reason is the fall; the repeated cause is death; the repetitive is redundance. And identity is death. Everything falls to zero, a complete lack of information, the nothingness of knowledge, non-existence. *The Same is Non-Being.* [Serres 1982 pp. 99-100]

We would rather say, *the Same is Non-Becoming*. Identity is death. The apparent flow of the parallel atomic tracks is really static, instantiating unchanging law. The aspect of becoming enters only with the Swerve – weaving a horizontal thread across the primordial atomic warp. The charm of the Swerve is its irrationality – it breaks the law (*quod fati foedera rumpat*, II 254) – and its random temporality (*incerto tempore incertisque locis*, II 218-19).[v] But it remains a Swerve founded on atomic Beings and their unchanging drizzle of death.

Ilya Prigogine and Isabelle Stengers have illuminated the history of dynamics, the mathematics of physical change, with respect to these issues.[vi] They show that the world of Leibnitzian metaphysics and physics was a far more respectable endeavor than it is

usually credited with being, in comparison to that of Newton, and that a Leibnitzian monadic dynamic system has as its physical translation every integrable system, every physical system whose equations of movement can be integrated. Prigogine says:

> A problem put in the canonic language of dynamics is presented in the form of a set of differential equations that describes the following situation for every point: at every instant, a set of forces derived from a function of the global state (such as the Hamiltonian, the sum of kinetic and potential energies) modifies the state of the system. Therefore this function as well is modified: from it, a moment later, a new set of forces will be derived. To resolve a dynamic problem is, ideally, to integrate these differential equations and to obtain the set of trajectories taken by the points of the system.
>
> It is evident that the complexity of the equations to be integrated varies according to the more or less judicious choice of the canonic variables that describe the system.[vii]

Prigogine points out that every integrable system can be represented as a system in which all the energy is kinetic, in which the potential energy redefined in terms of its variables is cancelled out. This is a monadic system without any interaction among the monadic units, a Leibnitzian system, a clockwork world of reversible processes – a predictable system.

However, the class of integrable systems is very restricted (the theorem of Liouville). Moreover, modern science has had to learn to deal with unpredictable systems of irreversible processes, in which interactions do play a role, starting with the relatively simple problem of three balls in a box. So the ideal of the predictable, integrable system fails as a model for the world of physical change.

It is the Lucretian world that, by adding turbulence into laminar flow in its grand metaphor, better models systems of irreversible processes, non-integrable systems that incorporate non-eliminable interactions. If the downward drizzle of atoms is the metaphor for law without change or interaction, Serres' dead "nothingness of knowledge," Sartre's hell in *Huis clos*, a Leibnitzian integrable dynamic system, laminar flow – then, as the velocity of the laminar flow increases, vortices arise spontaneously, as it were, irreversible processes of turbulence deriving catastrophically from infinitesimal changes in initial conditions. Prigogine says, "The fall [of Lucretian atoms] is nothing but the universal without a memory whose every instant is the integral repetition of the preceding instant." But turbulence evolves as a kind of creative disorder which breaks the law of the integrable Fall, but is not random, not stochastic; a self-organizing system in which "correlations can appear among distant events; local deviations echo throughout the system."[viii]

This is a world of Becoming. The Heraclitean flux/flow/plasma of Becoming correlates with the Lucretian Swerve; while the Lucretian flow, the drizzle down, correlates with the integrable system of Being. Life and death both properly enter the picture with the

Swerve; without the Swerve, there is only a changeless world of Being in which neither life nor death finds a place.

We do have to be careful about these metaphors. The mathematics of dynamic systems models systems of physical objects in physical motion. Musical change, and metaphysical change, are not precisely thus. Turbulence is not free will, laminar flow is not determinism or slavery or totalitarianism (or even bad art). But the kind of globally predictable (and reversible) system in which entities do not have to influence each other on the fly and nothing really changes, still stands in contrast to a system of interacting agents which evolves irreversibly and almost capriciously to a complex organization of its own.

One could easily paint a picture of artistic endeavor as one whose nature is bound up with the Swerve. Without the frame, there cannot be a swerve: there is a need for both frame and swerve. Composers, even the most anarchic composers, know this: to play with structure or to break it or flee it, you need to invent or find structure to play with/break/flee. Can mathematics describe both frame and swerve by "adding one dimension n+1" a la Deleuze and Guattari? – a meta-structure of frame+swerve? Would this capture the reality of the swerve in art? We must remember that a discourse of frame+swerve remains founded on the frame of Being, and on attempts to translate the untamable Swerve into a captured line in a larger frame.

We may ask, how important is it, and under what circumstances, for a model of music to sing alongside the wildness of music, its turbulent flow and untamed swerve? Historically, we see attention to Being in music theory, nailing down simple invariant structures such as scales, chords, and pitch class sets, making taxonomies, natural histories – what are the kinds, the types? Taxonomy is simple science, but perhaps music's temporal and fugitive nature makes it more attractive to parse it in such clumps. Perhaps music's flickering flame is most in need of a frame, and has the least to fear from being framed. It will always already have escaped its frame.

No one would suppose that a "formal analysis" that identifies sections of a piece according to criteria such as themes and harmonies, with respect to some conventional historical paradigm (such as sonata form), there they are, the sections, clunk clunk clunk – no one would suppose that this adequately describes the music. It certainly does not sing alongside the music. If music were like such a description, no one would want to listen to it. We can say the same about descriptions which list chord identifications, or theme identifications (and chord types and theme types). We could say the same about the notes in a written score. The entities of such a description are at best merely the fumets of the fleeing beast of music. It is the relations set up between the entities and how these relations play as the music moves along in time that better sketch the flight of the beast.

Not all music theory aims to analyze a given piece of music, or to provide a theoretical framework within which it would be possible to set up a plausible model of some piece of music as it is experienced in time. Analysis is a recent addition to music theory. The question, "What is it made of?" (notes, scales, chords, relations, and so on) has been prompted primarily by a need to understand music *in a way which will help a musician*

create more of it. Compositional theory and pedagogical theory are the historical mainstreams. This kind of theory aims at praxis: how to *do* it.

A good analytical theory would need to model the dynamics of musical experience, which is a very difficult undertaking. Compositional theory only needs to focus on whatever is of most practical use to musicians who are performing, composing, or improvising music. The wildfire of the musical swerve and flow is part of the act of musical creation, but compositional *theory* may choose to focus on a relatively simple box of *tools* for the creator to use.

We have not seen much attention focused on frame+swerve descriptions in musical analyses that are mathematically oriented. We do see some analyses that pay attention to the swerve within a context of mathematical structure, but they tend to formalize the frame and not the swerve. This may in fact be the best way to do it. Iannis Xenakis talked about musical structures outside of time versus structures in time: *hors-temps* vs. *en-temps.* Benjamin Boretz has talked about the coordinate roles of syntax and structure: syntax is the set of entities and relations relevant to a piece of music, and the musical structure is how these entities and relations play among themselves as musical time goes on. The musical piece becomes a sort of playful path in time through a field of temporally invariant relations. Both kinds of relations can be modeled formally, but it is clear that syntactical relations, outside of time, are better at staying still to be pinned down in a formal description. Formalizing the dynamic, temporal, phenomenal, playful musical path through these relations is possible, but harder, and runs up against the refractory anti-essence of the swerve.

Music theory in general, historically, has tended to focus on syntactical relations of this sort, with little attention paid to what music does with the relations. One of the first theorists to harmonize the syntactical and the structural in a way that emphasizes the dynamic/temporal/structural is Heinrich Schenker. His work has become in many ways, at least in the USA, a model and a standard for any music-analytical endeavor. Schenker is interesting metaphysically. His essentialism is really a mysticism which is probably related to or derived from Jewish Hasidic mysticism and the Kabbala. He was raised in a professional Jewish family in nineteenth-century Polish Galicia, the center of such thinking at the time. Schenker as Tzaddik, the Genius: the prophet-leader with a privileged relation to God, and with a special revelation, who passes this on to his followers. This predilection to monistic emanationism, to a secular Plotinism, transfers structurally to the generative theory of hierarchical levels which is Schenker's innovative music theory. As shown by the work of a group including Michael Kassler, Steven Smoliar, James Snell, myself, Fred Lerdahl and Ray Jackendoff, Mario Baroni, Bernard Bel, and others, a theory like Schenker's is formalizable in two stages, the first of which is a phrase-structure grammar (or set of recursive functions, or any equivalent formulation in logic or group theory); the second is a kind of transformational grammar subsequent to the output of the phrase-structure grammar. In general, this is a very successful way to model much musical experience. The phrase-structure grammar induces a partitioning (if not a temporal segmentation per se) on the elements of the experience, the partitions at various levels are consistently related to each other generatively through a small set of inter-level relations, and any musical relations not amenable to the resulting abstract order

are manipulated by the transformational component. Finally, some more or less explicit or sophisticated modeling relation connects the abstract model as a whole with the musical experience as a temporal whole, meticulously through their relational details.

In this kind of theory, the syntax is not merely a box of relations which are animated in time as the music passes through them to produce a structure. Yes, there is a syntax, but it includes on the one hand decisions as to useful entityhood – taxonomy – and the relations among these canonic "things," and on the other hand, the inter-level relations governing the generation of the structure in levels. And as the musical experience progresses through time, it is the particular Schenkerian model of this particular piece of music which can be thought of as animated in time. The Schenkerian analysis is a fabric of relations, and every moment of the music lights up patterns of swatches in this fabric, so that the progress of the piece in time resembles an abstract sound and light show. In reply to Occam's famous criticism of Plotinism, we can say, we find that we really do need all these hierarchical levels of entities, the whole structure of them, in order to model music as it is experienced. Flatter models do not work as well for this music.

Of course Schenker is Parmenidean: "*semper idem, sed non eodem modo*." What makes Schenker Heraclitean? There is the mysticism: the hidden harmony surpasses the apparent harmony, nature likes to conceal itself. There is its temporality: Schenker's theory is a *linear* one, growing out of concern for the overall structure of melodies and the counterpoint that emerges among them. Counterpoint is at the root of Schenker's thinking. There is also a temporal teleology deeply inherent in Schenker's thinking: as the music progresses in time, the fabric of relations highlights exactly the ways in which the music is complete and incomplete, and the musical relations are thought of as moving strongly toward completion. Also Heraclitean is the irreducible, productive tension of irreconcilables: the linear, contrapuntal basis of the theory is in constant, productive tension with its inherent tendency to hierarchical chunkings; without this tension, the theory would probably congeal unattractively. Finally, there is Schenker's sense that, after all, music is not a craft, but a high calling, and not for everyone: "Only the Genius is connected with God, not the people. For this reason it is necessary to strip the masses of their halo."[ix] Such an ecstatic mysticism introduces the unpredictable Swerve at the root of Schenker's view of the world, and of music.

Schenker severely restricted the scope of application of his theory (Bach through Brahms). Others have shown clearly that such grammars work well for quite different kinds of music, as Bernard Bel has done for North Indian art music. But there is at least a question as to whether there is a question (as Milton Babbitt might put it) of whether this kind of theoretical structure fits all kinds of music well. In particular, it has been argued that atonal and serial music is flat, not hierarchically leveled. Babbitt himself thinks of serial music in this way, in explicit contrast to tonal music: in Benjamin Boretz's phrase, serial music is order-determinate rather than content-determinate. This view has necessitated the growth of atonal and serial theory along these lines.

Joe Dubiel has perceptively described Milton Babbitt's music as the animation of lists, rather than the animation of a fabric of relations. The things in the lists can be large and complex, such as types of partitions, or the "compositional mosaics" theorized so well by

composers such as Robert Morris and Andrew Mead, but the order-determinate syntax, in the large and in the small, remains one of completion of lists ordering an aggregate universe of things. One is tempted to say, lists of things at some level. An ordering of the pitch-class aggregate is one kind of thing, and an ordering of all possible partition types within a combinatorial matrix is another, larger kind of thing. Or, lists in some direction. There can be a fabric, an n-dimensional fabric, whose threads are lists, lists up and lists across, as in mosaics. It need not be dull or simple! However, there are no consistent theoretical relations that have emerged for this serial music which formalize inter-level relations of a kind that would allow setting up a grammar of hierarchical levels for it.[x]

Most of the American music theory of serialism has derived from compositional theory, theory by composers, such as Babbitt and Morris, thinking about what might be useful theory for their compositional activity. Although this can be used for analysis, such analysis tends to uncover the "precompositional" structures that were, or could have been, used by the composer. It will be interesting to see whether some theories for serial music emerge that are more analytical in genesis and orientation, and what the structure of such theories may be.

However, it is primarily this compositional theory which has driven the development of mathematical music theory in the USA, deriving first from Milton Babbitt, who introduced concepts of group theory in the 1950s which were extended and built on by theorists such as David Lewin, Robert Morris, and myself. This tradition is indirectly responsible at least to some degree also for more recent applications of group theory and other mathematics in scale theory (John Clough's extended family of friendly researchers) and the neo-Riemannian theory which derives from David Lewin's work (David Lewin, Richard Cohn, Julian Hook, and many others). An astonishingly similar tradition of compositional theory arose independently in Romania from the composer Anatol Vieru, which nourished also Dan Vuza's highly mathematical applications of group theory to structures of pitch and time. And Iannis Xenakis went his own way in France, with an unquenchable intellectual curiosity about intimate applications of mathematics to music. Group theory is an important component of all these theories.

It was, of course, Allen Forte who in the USA pioneered the analytical application of concepts from mathematics, first with a taxonomy of pc-set types (the concept arose also in Babbittian serial theory), and following up with some relations (such as abstract inclusion and similarity relations) meant for analytical use. Forte's "set theory" (as it is somewhat misleadingly known, because it deals with sets of pitch classes) has had its own ramifications and influence. In particular, Forte's own analyses of individual pieces of music have led many others to do likewise, and Forte's initial idea of similarity relations (as distinct from equivalence relations) among pitch-class sets has seen a flourishing theoretical industry grow around it, after seminal articles by Morris, Rahn, and Lewin appeared in 1980.

I hope it is fair to characterize Forte-analytical theory as of the syntax/structure kind, where the syntax is of the box of tools kind. There is in general no theory of how to put the tools together to model the flow or swerve, and, given an analytical structure of theoretical entities, no explicit theory of the modeling involved, how the theoretical

entities map on to the phenomenal ones in the experienced music. For example, even the "segmentation" – more properly, any covering – of the total set of notes in the piece into component pitch-class sets is un-theorized. Any individual analysis is free to make its own case for coherence emerging in the temporal structure of its particular piece of music, using the tools in the box in any way it pleases.

Now is the time to talk about David Lewin. David, who passed away this year, was a colleague whose work and example have played a major part in building the American music-theory community. He was also the most mathematical of music theorists in the USA in the twentieth century. He was always careful to present his mathematics not so much as an application of mathematics to music theory, but as valuable music-theoretical thinking whose most appropriate formulation happened to be mathematical. This approach went a long way towards leading our community to accept the use of mathematics in music theory. There was always a clear music-theoretical payoff for using the math.

David Lewin contributed in many areas, two of which are especially worth mentioning here: his ground-breaking article on phenomenology in music theory, called "Music Theory, Phenomenology, and Modes of Perception,"[xi] and his theory of transformational networks.[xii]

Lewin problematizes the notion of perception, while also theorizing it and to some extent, formalizing this theory. He insists on the poetic, creative, performative nature of every musical activity. He says: "...since music is something you *do*, and not just something you *perceive* (or understand), a theory of music cannot be developed fully from a theory of musical perception...."[xiii] Not only is any musical activity active and poetic, but so are musical perceptions and music analysis: David says, "To the degree that analytic records of musical perceptions are poems, ski tracks tracing the poetic deeds that were the perceptions themselves, then critics – if not analysts – must concern themselves with the poetic resources at hand, that is, the sorts of poetic spaces analysts inhabit and the varieties of poetic media through which they move in executing their deeds."[xiv] Of course, mathematics is one such poetic medium.

One recognizes the effects of this stance in Lewin's transformational networks, in which each relation, including elementary relations such as intervals, is construed rather as an act, a performed transformation. In his book on transformational networks, Lewin says:

> In contrast [to thinking about relations among elements], the transformational attitude is much less Cartesian. Given locations s and t in our space, this attitude does not ask for some observed measure of extension between reified "points"; rather it asks: "If I am *at* s and wish to get to t, what characteristic gesture... should I perform in order to arrive there?" The question generalizes...: "If I want to change Gestalt 1 into Gestalt 2...., what sorts of admissible transformations in my space will do the best job?"....This attitude is ... the attitude of someone inside the music, as idealized dancer and/or singer. No external observer (analyst, listener) is needed.[xv]

This is an anti-essentialist and Heraclitean stance: no atoms of musical data or perception, no transcendent essence of music. Music is just what we make of it, as we make it. The rhythm of this process assumes a central and foundational importance.

Lewin's formal model of perception is based on Husserl. An individual perception (as modeled) is a list containing an event, a context, a list of other perceptions and their relations to the current perception, and a list of statements in some language(s) about this current event (such as music-theoretical statements). Lewin is at some pains to point out explicitly the various ways in which his model accommodates self-referential recursion, using a Lisp-like meta-model, and how the mutually referring recursive loops can be resolved either by a pre-parsing compiler or by an external interrupt (a musical event in time as time goes on).

Is this model fully consistent with Lewin's overall Heraclitean, even Nietzschean, ethos? The model atomizes and particularizes experience, when the experience may be less explicitly assertive and more continuous, more sinuous and fluid, than a set of indexed perception-atoms can model well. Moreover, Lewin points out that there is no logical need to include the objective individual "event" in the model of the individual perception. Lewin says: "The role of EV in my model corresponds ... with Miller's analysis of Husserl's "determinable-X"....a feature... which determines the purported object of the act *in abstraction from its (purported) properties*....something like the meaning of an indexical...."[xvi] Lewin's "context" component of a perception-as-modeled is also built on such non-perceived event-things, so there is also no logical need to include context as Lewin has described it. So the model of an atomic perception would condense to a list containing a list of other relevant perceptions and their relations to the current perception, and a list of statements in some language(s). The relations of other perceptions to the current perception would include information previously relegated to a context of objective things, without the need for objective things. But how do we express the relations of perceptions to each other? Surely, the expression must be in some music-theoretical or other language. And what would we find a need to say, within the theory, about the perceived music that would not relate perceptions to each other? So the whole list of lists modeling the individual perception condenses at last into a list of the relations perceptions bear to each other, with an overall theoretical language for expressing those relations – in fact, a syntax.

I would add some way of indexing any given perception as a "current" one: the moving finger indicating a temporal experience. Presumably, the relations among perceptions are taken as acts, transformations by an agent from one perception to another, along the lines of Lewin's later statements already quoted here. Once we have a set of sets of such perceptions, in each set exactly one perception indexed as current, we can act to yank the whole ensemble out of time and consider it as a whole, including the various positions of the current-perception index in each perceptual pattern. This model hashes experience into a set of slices, as noted earlier, which may not be always the best way to do it, although one can amuse oneself thinking about possible analogies with offset Poincare time sections in n-dimensional phase space.

Lewin's theory of transformational networks builds a formal meta-model of structure on his phenomenological foundations. Lewin meticulously develops the formalism required for his networks, from node-arrow systems to transformation graphs to transformation networks, defining isomorphism among node-arrow systems and among transformation graphs, and isography among transformation networks (IFF their underlying transformation graphs are isomorphic); also homomorphisms among all these and the usual concepts of connection, and so on. The book brilliantly illustrates each mathematical distinction with a wide range of music-analytical examples, some of which, like neo-Riemannian theory, have themselves become new subfields of music theory. Lewin has convinced American music theory to take this underlying framework of transformation networks very seriously, so that it has become a new paradigm for expressing musical thinking.

Lewin's transformational networks are, formally, graphs whose arrows are labeled in some semigroup, and nodes in some set acted on by the semigroup, in such a way that the resulting diagrams "commute" in the sense of category theory. Indeed, if the semigroup whose elements label the arrows has an identity, the resulting monoid-graph is kin to a category.[xvii]

What kind of kin? In fact, the necessary identities are provided by Lewin's definition of the "node-arrow system" underlying transformation graphs. However, categories allow more than one arrow between a given pair of nodes. The way Lewin defines node-arrow system and transformation graph allows only one arrow, or label on an arrow, from one node to another. This precludes multivalent musical interpretations of relations (or transformations) between ordered musical objects (nodes). For example, when the semigroup supplying the labelling is the group of transpositions and inversions, and the objects are pitch classes, any ordered pair of objects could have two distinct labels; for example, <2 5>, the arrow from 2 to 5, could be labelled either T3 or T7I, but according to Lewin's definitions, not both. Yet we often want to assert multivalent relations (or transformations) among musical objects. So if there is no overwhelming musical reason not to amend Lewin's definitions to allow multiple arrows, or multiple labels on a given ordered-pair arrow if we follow Lewin's definition of node-arrow system, there is some musical reason for doing so, added to the mathematical reason for doing so.

The standard way for allowing multiple arrows is to define arrows independently of objects (rather than as ordered pairs of objects) and to define also two functions from arrows to objects, one function yielding the tail or source of each arrow, the other function yielding its head or sink.

This change would bring Lewin-networks much closer to categories. The question would be, how usefully would category theory in general "transfer" to Lewin-net theory, in the sense that group theory "transfers" to GIS theory, as shown by Oren Kolman.[xviii] Lewin's further constructs, such as isography, would then also find an interpretation in category theory. One possibility is to interpret a Lewin-net as the free category generated by a graph,[xix] or it could be viewed simply as a commutative diagram within a category.

As already mentioned, a Lewin network can be interpreted as showing actions transforming musical perceptions into each other. The network does not itself model time; the network floats freely over musical time. Lewin does develop the formalism of partial ordering implicit in the directed-graph basis of the networks, a hierarchy of possible partial orderings of which the strongest is, of course, linear. Lewin points out that such formal orderings need not reflect temporal orderings.[xx] In fact, any Lewin-net itself (if labelled in a group) models a kind of instantaneity outside time, in that any change to any part of one refigures the rest of it already.

The Lewin network is a communicative tool, or poetic medium. The analyst can also use the display of the network in some space – a page – to communicate diacritically, as the display is independent of the network itself. The idea of the network does not itself prescribe the choice of included node-entities (perceptions) and arrows drawn among them, nor, in general, the relation of the transformations shown to the in-time experience of the music.[xxi]

The newest currents in mathematical-musical space are coming from some people who are primarily mathematicians, such as Guerino Mazzola and Thomas Noll, who, I am sure, will express themselves better than I can express them.[xxii] The overarching role of category theory and, for Mazzola, Grothendieck topologies, at least does seem to conform to the anti-essentialist Heraclitean mode, pretty radically at a mathematical level. It remains to be seen how well music theory as a discipline will adjust to these strong Boreal winds from the heights.

I close with a sonnet by Shakespeare.

Music to hear, why hear'st thou music sadly?
Sweets with sweets war not, joy delights in joy:
Why lov'st thou that which thou receiv'st not gladly,
Or else receiv'st with pleasure thine annoy?
If the true concord of well-tuned sounds,
By unions married, do offend thine ear,
They do but sweetly chide thee, who confounds
In singleness the parts that thou should'st bear.
Mark how one string, sweet husband to another,
Strikes each in each by mutual ordering;
Resembling sire and child and happy mother,
Who, all in one, one pleasing note do sing:
Whose speechless song, being many, seeming one,
Sings this to thee: 'Thou single wilt prove none.'[xxiii]

This paper was delivered as the keynote talk at a conference at IRCAM in Paris, October 15-17 2003, titled “Autour de la Set Theory.” The event was sponsored by a number of agencies, including IRCAM and SFAM, the French music theory society, and *Perspectives of New Music*. The other speakers were Jonathan Dunsby, Luigi Verdi, Moreno Andreatta and Stephan Schaub, Allen Forte, Xavier Hascher, Andrew Mead, Joseph Dubiel, Robert Morris, Marilyn Nonken, Andre Riotte, Paul Nauert, Jason Eckardt, Jean-Jacques Nattiez, Celestin Deliege, Guerino Mazzola, and Thomas Noll. Discussions were lively, and included substantial participation from a number of people in the audience.

Endnotes

[i] From Parmenides Fragment VIII, my translation from the Greek text in Taran.

[ii] Heraclitus. Translations, and even the Greek texts in the various editions of the fragments in Greek, differ considerably. The best overall (of the ones I have run across) is probably Marcel Conche, *Heraclite: Fragments* (Paris: Presses Universitaires de France, 1986); this is a very scholarly edition of the Greek with good French translations and with intelligent commentary. Charles Kahn, *The Art and Thought of Heraclitus* (Cambridge: Cambridge University Press, 1979) is also quite scholarly. In Yves Battistini, *Trois Presocratiques* (Paris: Gallimard, 1968), is a lyrical, enthusiastic French translation for popular consumption, which is very readable but in my opinion not always entirely justifiable in its interpretations. Battistini emphasizes the mystical, wild-man side of Heraclitus (the opposite of Conche). I have resorted to a reading of the best English translation (Kahn), and readings of the French versions of Conche and Battistini, and have ended by offering here my translation from the Greek texts found in one or another of these sources, none of which seems entirely satisfactory. The rest of this already long footnote documents how each of the translations of fragments in this paper were arrived at.

1. My translation from Greek text in Conche 43.
2. My translation from Greek in Kahn LXXX.
3. My translation from Greek in Kahn LXXXVI.
4. My translation from the Greek and French, following Conche 13, who makes a good point against the standard interpretation.
5. My translation from Greek text in Conche 12, but not following his French version.
6. My translation from the Greek text in Kahn LXXXII.
7. My translation from Greek text in Conche 125.
8. My translation from the Greek, following Conche 87.
9. My translation from the Greek and French in Conche 134.
10. My translation from the Greek in Conche 69.

11. My translation from Greek (Conche 118, Kahn CIII).

[iii] The "ligne de fuite," line of escape, does not always work well partly because it is not properly construed from the "pointe de fuite," the vanishing point which is a centralizer par excellence, the point from or to which all relates.

[iv] Michel Serres, "Lucretius: Science and Religion," in *Hermes: Literature, Science, Philosophy* (Baltimore: Johns Hopkins, 1982), pp. 99-100. This appeared originally as "Conditions Culturelles. Violence et contrat: Science et religion," in *La Naissance de la physique dans le texte de Lucrece: Fleuves et turbulences* (Paris: Minuit, 1977).

[v] Lucretius, *De Rerum Naturae.*

[vi] Ilya Prigogine and Isabelle Stengers, "Postface: Dynamics form Leibniz to Lucretius." In *Hermes: Literature, Science, Philosophy*, Op. Cit, pp. 137-155. Subsequent discussion of dynamics here is indebted to this article. See also their *From Being to Becoming* (San Francisco : W.H. Freeman, c1980), and *The End of Certainty* (NY: The Free Press, 1996).

[vii] Prigogine, Postface, p. 143.

[viii] Prigogine, Postface, pp. 153-154.

[ix] Heinrich Schenker, *Free Composition*, translated by Ernst Oster. NY: Longman, 1978: p 159.

[x] An interesting exception is the hybrid compositional theory of Ciro Scotto. "A Hybrid Compositional System: Pitch-Class Composition with Tonal Syntax," *Perspectives of New Music* vol. 38, no. 1 (Winter 2000): 169-222.

[xi] David Lewin, "Music Theory, Phenomenology, and Modes of Perception," in *Music Perception* vol. 3, number 4, Summer 1986: 327-392.

[xii] This appeared initially as "Transformational Techniques in Atonal and Other Music Theories" in *Perspectives of New Music* Vol. 21, numbers 1-2 (Fall-Winter 1982/Spring-Summer 1983): 312-71, and subsequently was extended and illustrated with many examples in his influential book, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987) (abbr. GMIT).

[xiii] Lewin, "Phenomenology," p. 377.

[xiv] Lewin, "Phenomenology," p. 382.

[xv] Lewin, GMIT p. 159.

[xvi] Lewin, "Phenomenology," p. 336.

[xvii] See Saunders Mac Lane, *Categories for the Working Mathematician* (New York: Springer-Verlag, 1970). There are also parallels in the theory of abstract machines.

[xviii] Oren Kolman, "Transfer Principles for Generalized Interval Systems," *Perspectives of New Music* 42, no.1 (2004).

[xix] Mac Lane, *Categories* p. 48ff

[xx] Lewin, GMIT pp. 212 ff.

[xxi] A Lewin network would embody phenomenal and temporal information only to the extent that its nodes were labeled in a set of objects which incorporated such information, such as the perceptions modeled in Lewin's article on modes of perception. In GMIT, the examples usually show one-dimensional abstract node-objects which do not incorporate time information, such as pitch classes, pc sets, tonal chord-functions, and so on. The formalities of relating multi-dimensional perception-objects in a Lewin network are complex, involving at least direct product semigroups.

[xxii] Guerino Mazzola, *The Topos of Music* (Basel: Birkhauser Verlag, 2002). See also Michael Leyton*, A Generative Theory of Shape* (Berlin: Springer Verlag, 2001), and related to this and to Mazzola, John Rahn, "Chloe's Friends: A Symposium on Music and Mathematics," *Perspectives of New Music* vol. 41, no. 2 (2003). Oren Kolman's recent work is also notable here: see "Transfer Principles for Generalized Interval Systems," *Perspectives of New Music* 42, no.1 (2004).

[xxiii] William Shakespeare, Sonnet VIII.

Abstract
John Rahn, "The Swerve and the Flow"

An historical dialog between philosophies of Being and those of Becoming also plays out in the history of music theory. This paper argues that artistic endeavor is bound up with Becoming and the Lucretian Swerve. Music theory has most often been essentialist, focusing on the frame and not the swerve. This is justifiable in compositional theory, but any analytical theory that sings alongside music needs to embody the flow and the swerve of artistic thought and experience. Schenker is paradigmatic in this respect, in spite of (and because of) his neo-Plotinic mysticism. Other analytical theories attempt to model musical experience by playing with the syntax/structure model in ways that are often less subtle. American serial theory deriving from Milton Babbitt has been primarily compositional, rather than analytical. Allen Forte has elaborated an atonal analytical theory. David Lewin has formally articulated an important new analytical theory, both using and reacting against phenomenology in favor of a theory of acts and agents, and formally defining networks of transformational acts. The category theory implicit in Lewin's networks has recently been foregrounded by Guerino Mazzola.