

# ***Ab Initio XAS Debye-Waller Factors Beyond the Harmonic Approximation***

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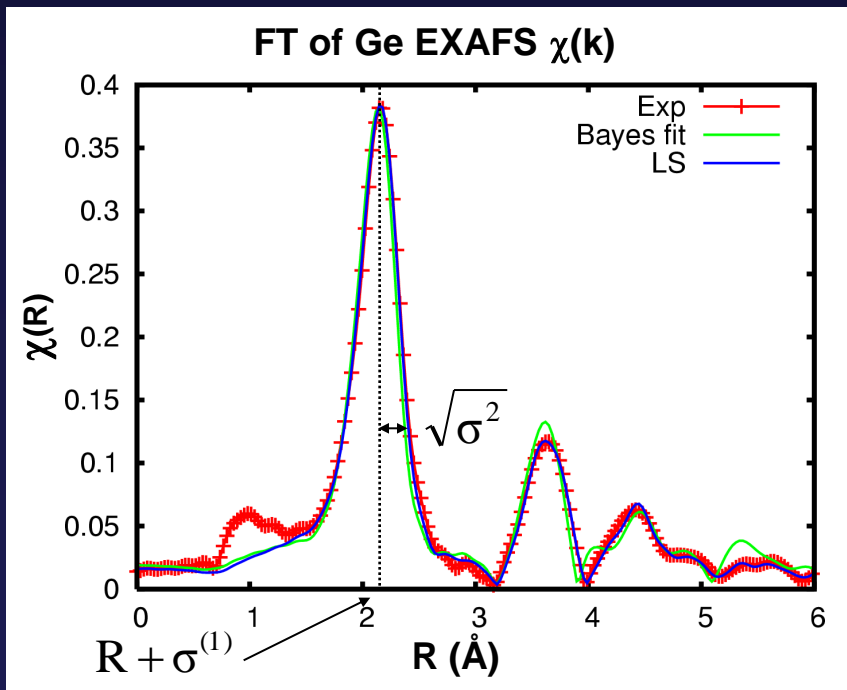
# Introduction

XAFS Debye-Waller (DW) Factor  $\langle e^{i2kr} \rangle \equiv e^{2ikR_0} e^{-W(T)}$

- Originate from thermal and configurational disorder
- Crucial for a quantitative treatment of XAS

Average commonly expressed in terms of the cumulant expansion:

$$W(T) = - \sum_{n=1}^{\infty} \frac{(2ik)^n}{n!} \sigma^{(n)}(T)$$



J. Kas et al. (2007)

First cumulants:

$$\left\{ \begin{array}{l} \sigma^{(1)} = \langle r - R_0 \rangle \\ \sigma^{(2)} = \langle (r - \bar{r})^2 \rangle \equiv \sigma^2(T) \\ \sigma^{(3)} = \langle (r - \bar{r})^3 \rangle \end{array} \right.$$

# Lanczos Recursion Algorithm

XAFS DW Factor for path  $R$ :

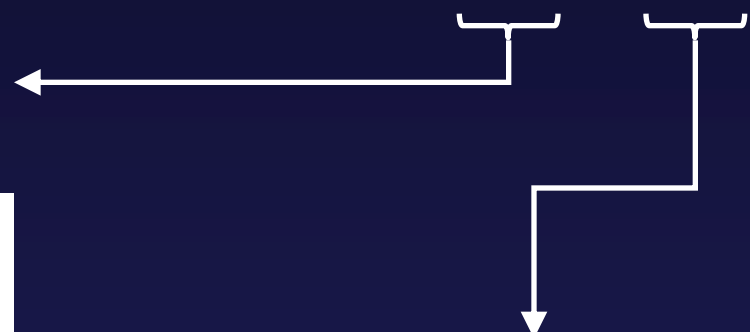
$$\sigma_R^2(T) = \frac{\hbar}{2\mu_R} \int_0^\infty \frac{1}{\omega} \coth\left(\frac{\beta\hbar\omega}{2}\right) \rho_R(\omega) d\omega$$

- Originally developed by Poiarkova and Rehr
- Extended by Krappé and Rossner

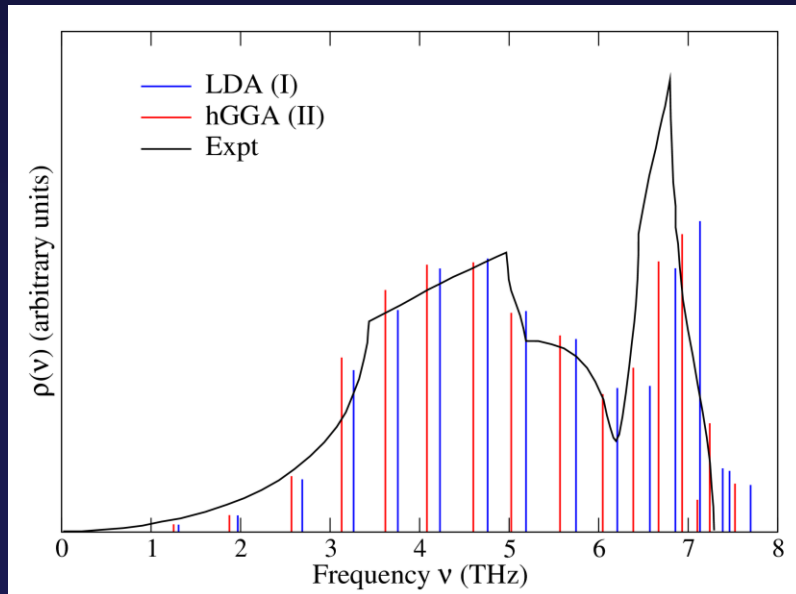
VDOS expressed as imaginary part of the phonon propagator:

$$\rho_R(\omega) = -\frac{2\omega}{\pi} \text{Im} \left\langle 0 \left| \frac{1}{\omega^2 - \mathbf{D} + i\varepsilon} \right| 0 \right\rangle$$

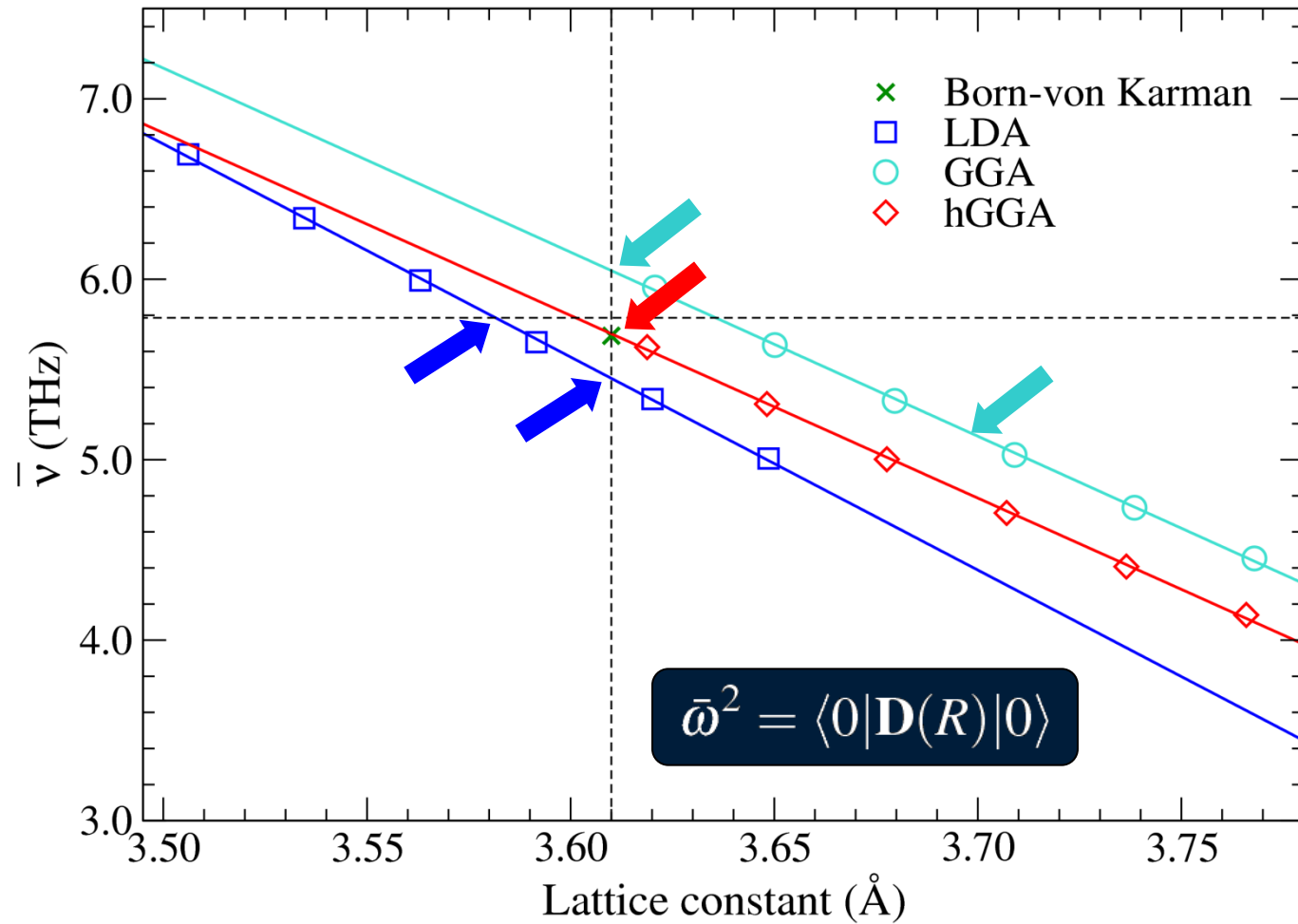
Seed state: Displacement along path  
Total VDOS of Cu



Dynamical Matrix: Calculated using the Density Functional Theory (DFT) of Response implemented in ABINIT (Gonze et al., 2002)



# Anharmonicity Effects on the Einstein Freq. of Cu



BvK: Nicklow *et al.* (1967)

# Prescriptions for the Calculation of the Dynamical Matrix

## I. Calculate $D$ with the LDA at the theoretical estimate of $a(T)$

- Fully *ab initio*
- Provides lattice expansion directly
- Computationally demanding

$$F(a, T) = E(a) + 3k_B T \int_0^\infty d\omega \ln \left[ 2 \sinh \left( \frac{\beta \hbar \omega}{2} \right) \right] \rho_a(\omega)$$

$$D_{jl\alpha, j'l'\beta} = A_{jl\alpha, j'l'\beta} + B_{jl\alpha, j'l'\beta} \Delta a$$

## II. Calculate $D$ with the hGGA at the experimental estimate of $a(T)$

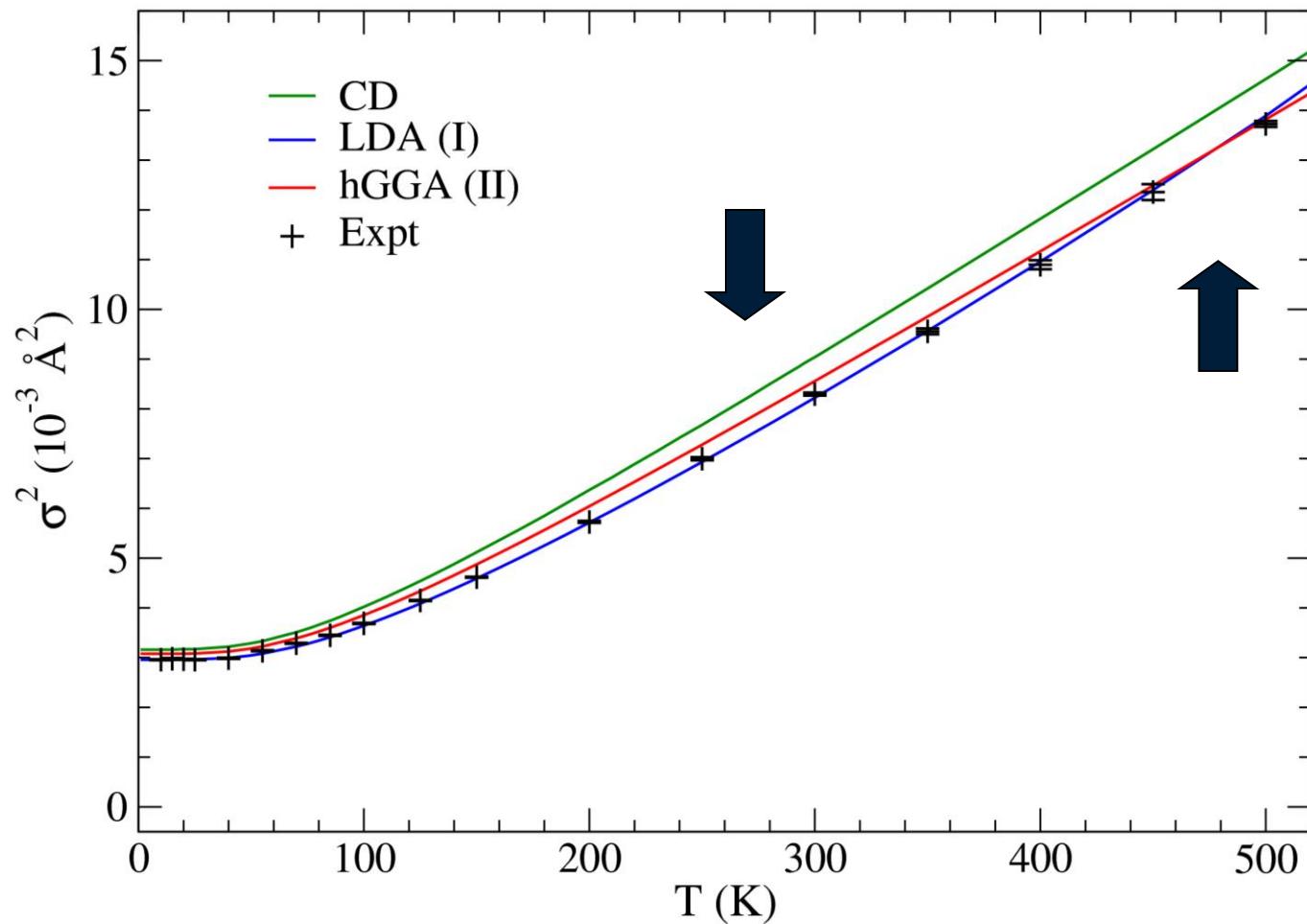
- Uses a new functional (hGGA): 50% LDA + 50% GGA
- Requires experimental input
- Lattice expansion from Correlated Einstein model
- Less computationally demanding

## Debye-Waller factors (in $10^{-3} \text{ \AA}^2$ ) for Pt and GaAs

	n	CD	LDA(I)	hGGA(II)	Expt
<b>Pt</b> (190 K)	1	3.55	3.23	3.23	3.22 ± 0.05
	2	4.38	4.64	4.78	4.7 ± 0.3
	3	4.50	4.44	4.49	4.3 ± 0.4
	4	4.56	4.60	4.66	4.5 ± 0.4
<b>Pt</b> (300 K)	1	5.41	4.98	4.90	4.83 ± 0.05
	2	6.69	7.23	7.34	6.8 ± 0.5
	3	6.91	6.92	6.89	6.7 ± 0.6
	4	7.01	7.17	7.16	7.0 ± 0.6
<b>GaAs</b> (295 K)	1	5.17	3.97	3.86	4.2 ± 0.1
	2	7.75	12.70	12.01	11.7 ± 0.14
	3	7.69	14.91	14.01	
<b>GaAs</b> (295 K)	1	5.15	3.96	3.86	4.2 ± 0.1
	2	7.20	10.80	10.19	9.6 ± 0.11
	3	7.68	14.83	14.00	

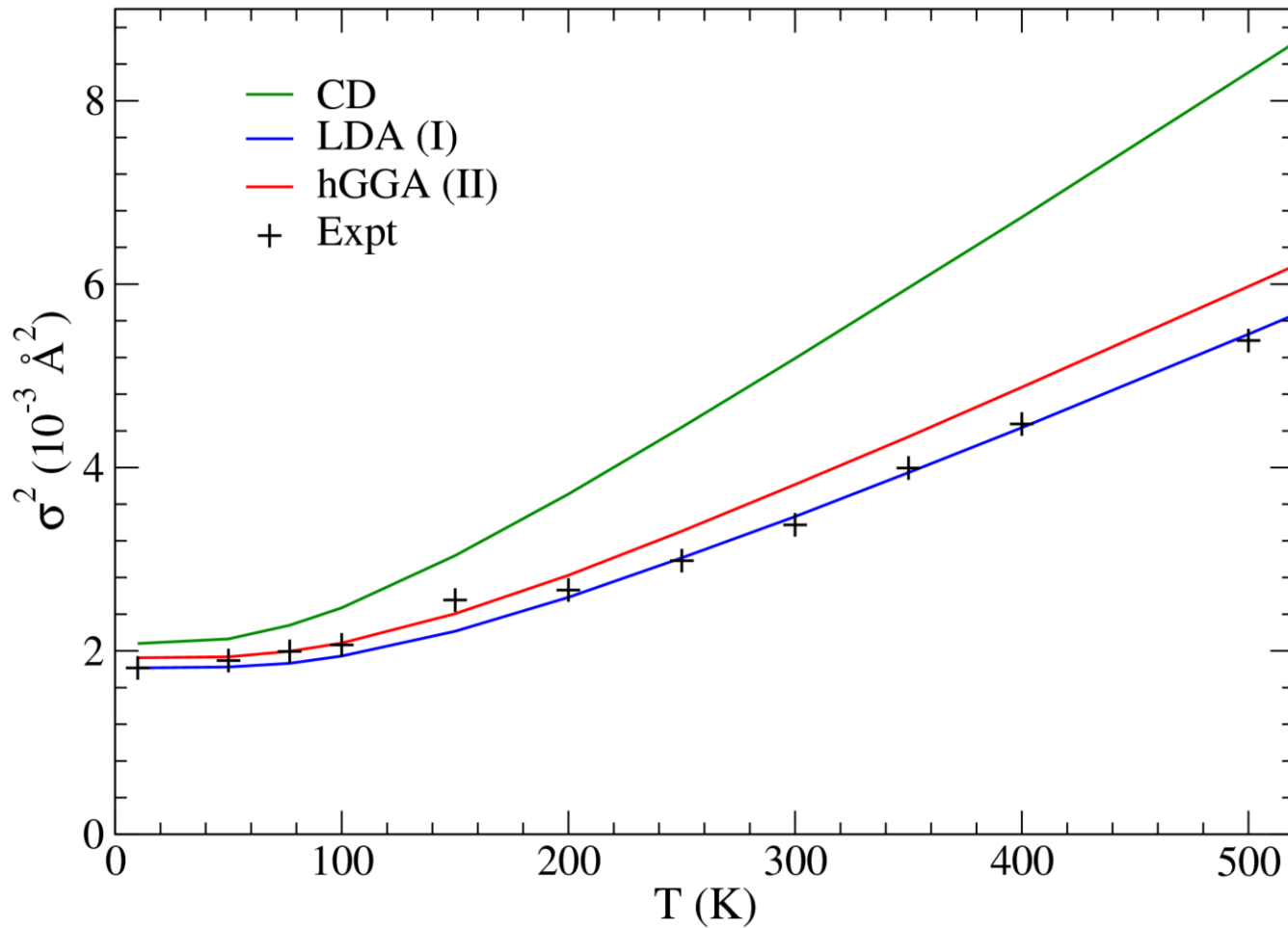
Expt: Stern *et al.* (1980)

## XAFS Debye-Waller Factor of Cu



Expt: Fornasini *et al.* (2004)

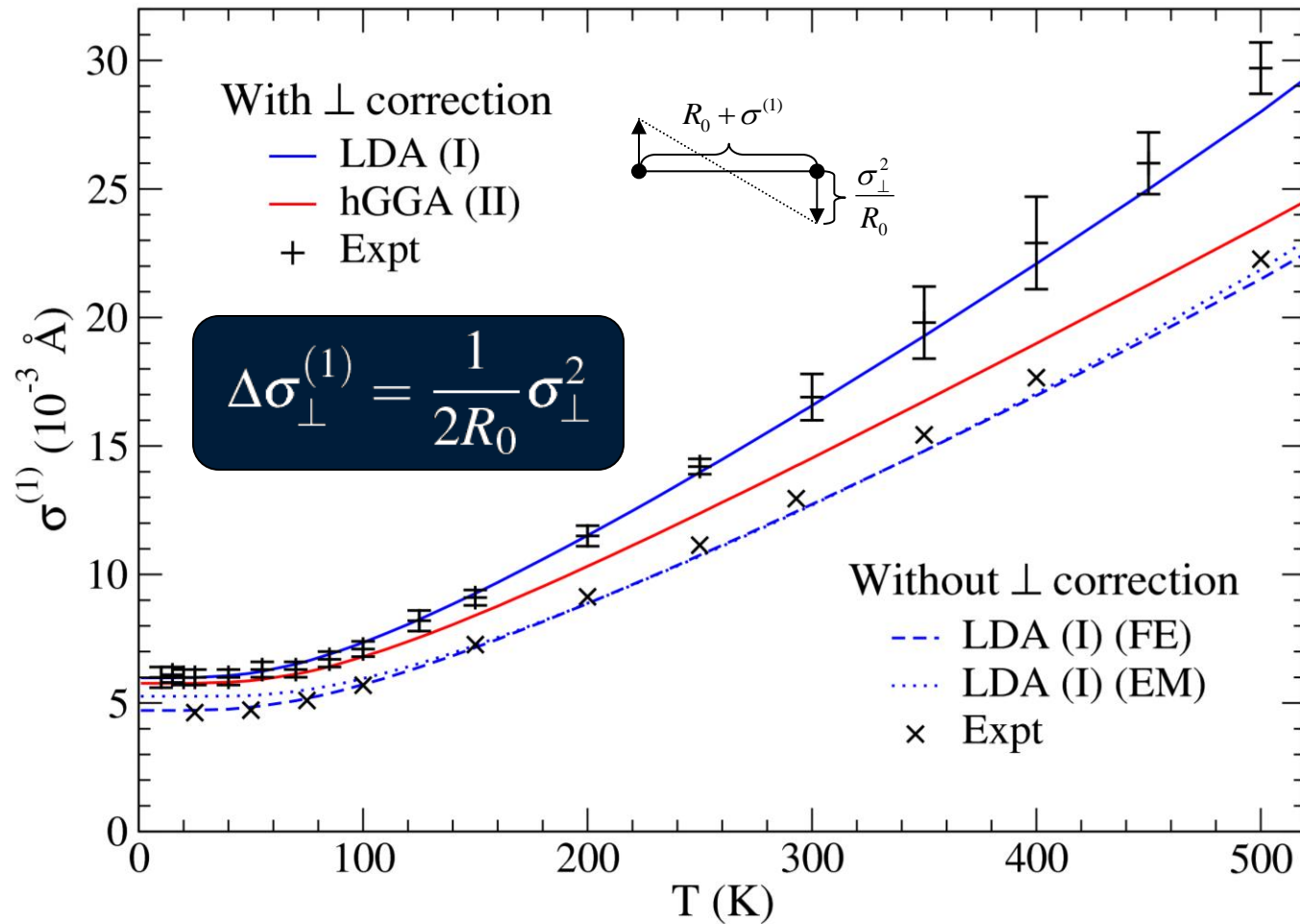
# XAFS Debye-Waller Factor of Ge



Expt: Dalba *et al.* (1999)

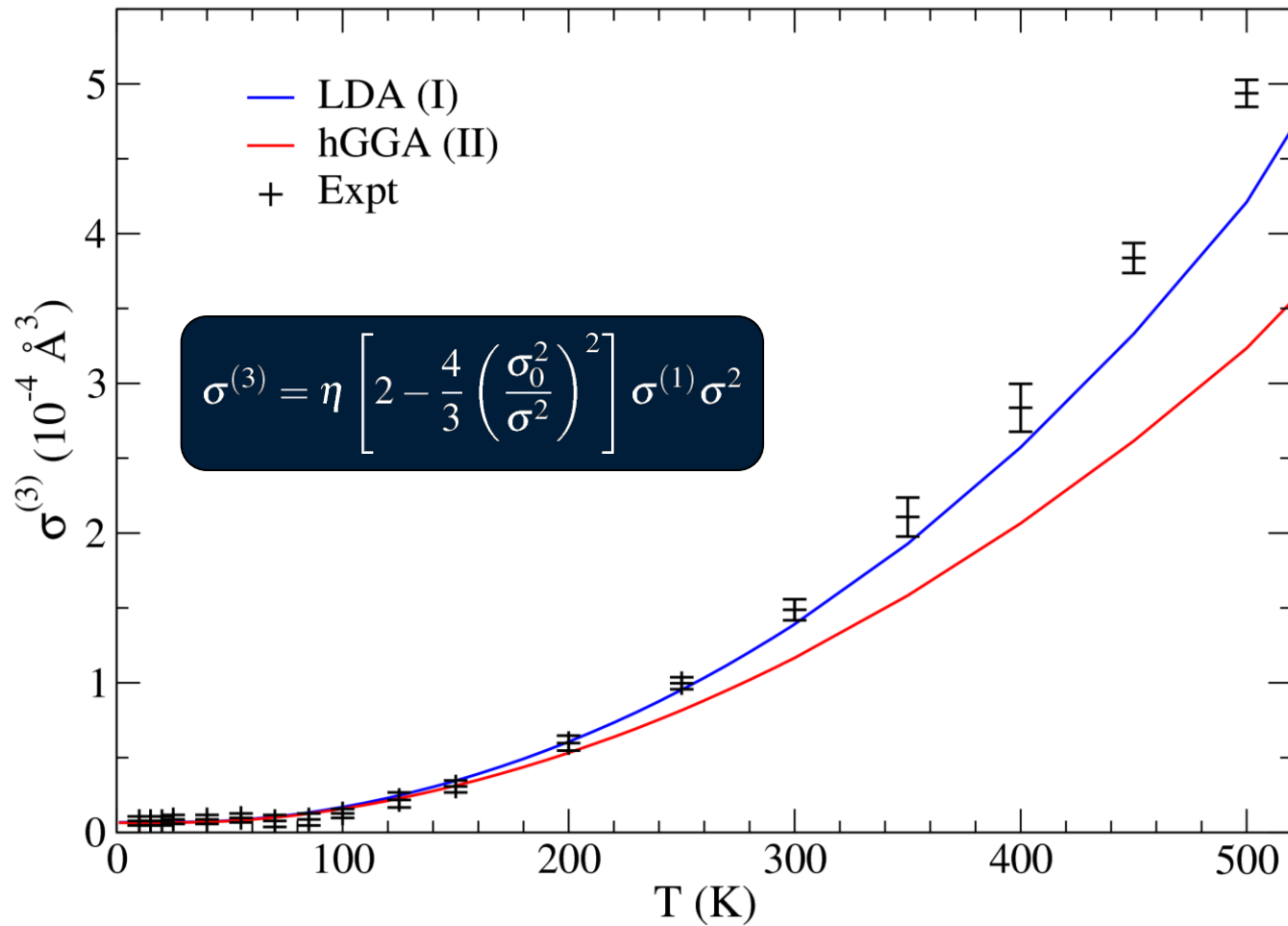


# First XAFS Cumulant of Cu



Expt: Fornasini *et al.* (2004), AIP Handbook (1972)

# Third XAFS Cumulant of Cu



Expt: Fornasini *et al.* (2004)

## Conclusions

- Developed two prescriptions for the calculation of XAFS cumulants
- Can determine lattice expansion from:
  - Free energy minimization
  - Correlated Einstein model
- New Lanczos seed states provide:
  - Total VDOS
  - Perpendicular correction to first cumulant
  - Crystallographic Debye-Waller factors
- Developed new functional (hGGA): 50% LDA + 50% GGA

## Future Prospects

- Implementation for arbitrary crystals
- Better form of hGGA functional (“Diminished functionals”, *cf. Perdew et al.*)
- “Black Box” implementation for FEFF XAFS
- Tables of Debye-Waller factors for most common substances