Mapping low-mode internal tides near Hawaii using TOPEX/POSEIDON altimeter data

Brian D. Dushaw
Applied Physics Laboratory, College of Ocean and Fishery Sciences,
University of Washington, Seattle, Washington

Abstract. Objective maps of M_2 internal-tide variability are derived from TOPEX/POSEIDON altimeter data obtained near the Hawaiian Ridge. An estimate of the wavenumber spectrum shows that the radiation is dominated by a few spectral peaks at wavenumbers consistent with the theoretically expected value for mode-1 internal tides. The data are not always consistent with the traveling wave model. Because the spacing of the altimeter tracks is larger than the internal-tide wavelength, the objective map underestimates the energy between the altimeter tracks. With this caveat, the maps are used to determine that 2.6 GW of power is radiated from a 1700 km section of the Hawaiian Ridge. Time series of hydrographic data obtained at the HOT site north of Oahu, Hawaii show that changes in mode-1 phase speed, caused by variations in stratification, have only a small effect on the propagation of the mode-1 internal tides.

Introduction

In the past several years, ocean acoustic tomography and satellite altimetry have detected large-scale, low-mode internal-tide waves that remain coherent over distances of at least two thousand kilometers [Dushaw et al. 1995; Ray and Mitchum 1996; Ray and Mitchum 1997; Kantha and Tierney 1997; Dushaw and Worcester 1998; Ray and Cartwright 2001; Cummins et al. 2001]. For acoustic tomography, the observations were possible because the line-integrating nature of tomography provided the spatial filtering that allowed very specific wavenumber and mode components of the internal-wave fields to be observed. For TOPEX/POSEIDON altimetry, the internal tides were detected using the temporal coherence of the tidal signals and the oscillations of elevation along the tracks that were consistent with the mode-1 wavelength. The temporal sampling at any given point along the altimeter tracks is only once every 9.9156 days

In support of these observations, numerical models have recently been used to calculate the generation of internal tides at topographic features such as the Hawaiian and Aleutian Ridges. Using a primitive-equation ocean model forced by the barotropic tides, Merrifield et al. [2000] modeled the internal-tide field emanating from the Hawaiian Ridge. The sea surface heights derived from the model were surprisingly similar to those determined from the along-track altimeter data. Cummins et al. [2001] modeled internal-tide generation at the Aleutian Ridge and also found a favorable comparison between the model results and the along-track altimeter data. While interesting in their own right, the observations and modeling of the internal tides are important because they provide estimates of the conversion of barotropic to baroclinic tidal energy, and this energy may become available for ocean mixing [e.g., Munk 1997].

This paper discusses several aspects of the analyses of the TOPEX/POSEIDON altimeter data that have been overlooked in previous discussions. In particular, techniques for objective mapping and spectral estimation of internal-tide radiation are applied to the altimeter data obtained near the Hawaiian Ridge. Such techniques combine all the altimeter data simultaneously, including data from both ascending and descending tracks, to obtain a best estimate for the entire tidal field. This approach offers a test of the extent to which the tidal variations observed on various altimeter tracks are consistent with the view that the tidal field near Hawaii consists of coherent waves. The objective maps may offer a better way of testing the models against the data insofar as they reconcile the data of both ascending and descending tracks. It is not *a priori* obvious that data from separate tracks are consistent, although with some exceptions this is found to be generally true near Hawaii.

Incoherence of internal tides is sometimes presumed to be caused by ambient ocean currents and variations in stratification. The latter effect is examined here using the Hawaiian Ocean Time-Series (HOT) hydrographic data [Karl and Lukas 1996].

Internal tides from altimeter data

The tidal analyses of the altimeter data by Ray and Mitchum [1996; 1997] was limited to the variability along the satellite tracks because they were concerned with establishing the basic properties of the data. The observed harmonic oscillations have amplitudes of 1-2 cm of sea-surface height. The internal tide has sufficient coherence in both time and space that its signals appear in the altimeter data, although because of the poor temporal sampling any incoherent elements in the internal-tide field would not be resolved [Kantha and Tierney 1997]. More recently, Ray and Cartwright [2001] performed a two-dimensional mapping of the altimeter data by fitting monochromatic plane waves to subdomains of the altimeter data. Ray and Cartwright struggled with the size of the subdomain in which to fit the plane wave model; they chose an area that spanned 3 to 4 different altimeter tracks. This least squares approach can over- or underestimate the tidal energy depending on the nature of the noise or signal. In addition, it may be misleading to do a simple least squares analysis based on a simple plane wave on a small subset of data, if the data are not consistent with that model. However, the results from the objective map described here provide justification for it. Given the apparent spatial coherence of the tide along the altimeter tracks near Hawaii, it seems plausible that a single objective map for the internal-tide field could be derived. Such an objective map can test the coherence between ascending and descending tracks; the temporal and along-track coherence is obvious.

The data consist of the tidal amplitude and phase at points separated by about 5.8 km along the altimeter tracks. The harmonic constants for each point consist of a combination of barotropic, load, and internal tides. As described by Ray and Mitchum [1996; 1997], however, the

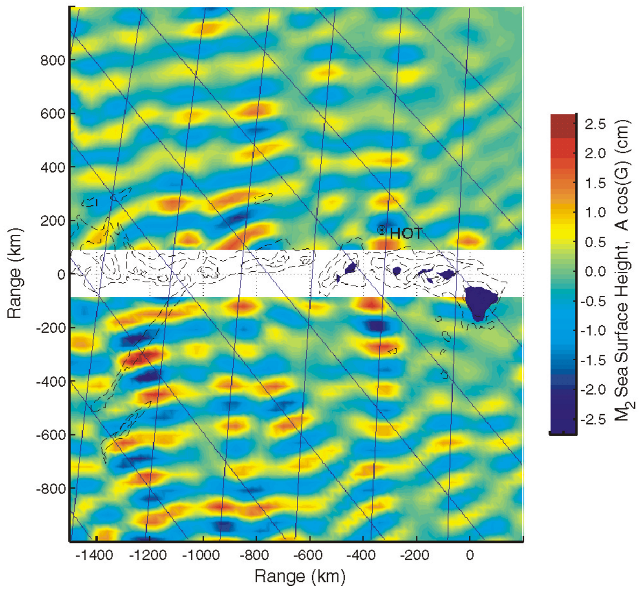

Figure 1. The map of the M_2 internal-tide field observed by the TOPEX/POSEIDON satellite altimeter near the Hawaiian Ridge. As described by Ray and Mitchum (1997) the data show that the internal tide propagates away from the Hawaiian Ridge. This map shows that the altimeter data are consistent with wave crests that have a large zonal coherence. The vertices of the altimeter track diamonds show the direction of north.

barotropic and load tides are large scale, so the internal-tide signals along each track can be isolated using a high-pass filter. The data were filtered using a 300-km running mean of the along-track data; the values for A cos(G) and A sin(G), where A is amplitude and *G* is phase, were filtered separately. Ray and Mitchum [1996; 1997] filtered the amplitude and phase, a procedure which does not completely separate the barotropic and baroclinic components. The uncertainties in the amplitude of the sea surface height estimated from the tidal analysis were 0.7-1.2 cm [R. Ray, personal communication 2000], but these values are evidently too large to be appropriate for the tidal variability.

An objective map of the tidal field was calculated using the filtered altimeter data (Figure 1). The *a priori* wavenumber spectrum assumed for the map consisted of a fairly broad peak centered on the wavenumbers expected theoretically for mode 1 (Figure 2a). It was assumed that the wavenumbers present in the internal-tide field were generally away from the Hawaiian Ridge, and that the tidal field was sufficiently complicated to warrant a range of wavenumber angles. Wavenumber spectra estimated directly from the along-track altimetry [Ray and Mitchum 1997] are not expected to be reliable for selecting an *a priori* spectrum because of the weak signal-to-noise ratio and sampling

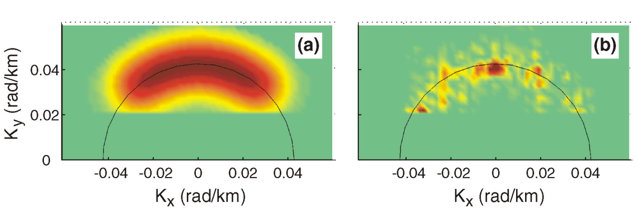

Figure 2. (a) The *a priori* wavenumber spectrum, and (b) the spectrum estimated from the objective map for the north side of the Hawaiian Ridge. *Kx* and *Ky* refer to wavenumbers parallel and perpendicular to the Ridge. The values of the periodograms range from 0 cm^2 (green) to 0.0034 cm^2 (red).

properties of the altimetry. The physical width of spectral peak for internal tides is unknown. Therefore, the *ad hoc* wavenumber spectrum shown in Figure 2a was assumed. The wavenumbers used for the map had a spacing of 0.0018 rad/km. The resulting map was consistent with an uncertainty in sea surface elevation of about 0.4 cm.

The objective map has wave crests that are coherent over O(1000 km) distances. This result is consistent with the long coherence lengths suggested by Dushaw et al. [1995] [or Hendry 1977]. A wavenumber spectrum is naturally obtained from the amplitudes of the wavenumbers used for the map. The spectrum for the north side of the Ridge shows that the radiation is dominated by a few wavenumbers mainly perpendicular to the Ridge (Figure 2b), which is consistent with the appearance of the map itself. If the internal-tide field was incoherent or stochastic, the spectrum would not have just a few distinct peaks.

An objective map was also obtained using a model that included wavenumbers for the second internal-wave mode. An order of magnitude less variance was found for the mode-2 wavenumbers than for the mode-1 wavenumbers. In addition, no clearly identifiable spectral peaks were found for the second mode. The objective map approach does not find an obvious signal of the second mode. The lack of a signal for the second mode may be because this mode is prone to incoherency, and so difficult to observe by altimetry. In addition, given the same amplitude for subsurface displacement, the second mode will have a surface expression about three times less than the first mode.

Although a "good" fit to the tidal waves on the along-track data is generally obtained (e.g., Figure 3), there is evidence that the data are not always consistent with traveling internal waves. The differences between tidal harmonic constants at the crossover points of ascending and descending paths have a mean and rms of 0.018±0.69 cm. If the tidal field consisted of phase-locked internal waves, the harmonic constants at the crossover points should be nearly identical, even though the temporal sampling of ascending and descending tracks were separated by about 5 days. Because the rms of the crossover differences is significantly larger than the rms misfit obtained by the objective map (0.4 cm), the observed tidal variations are not always consistent with phase-locked internal waves. In addition, Figure 3 shows an example of data that are not consistent with traveling internal waves. While the sine component of the wave in this example

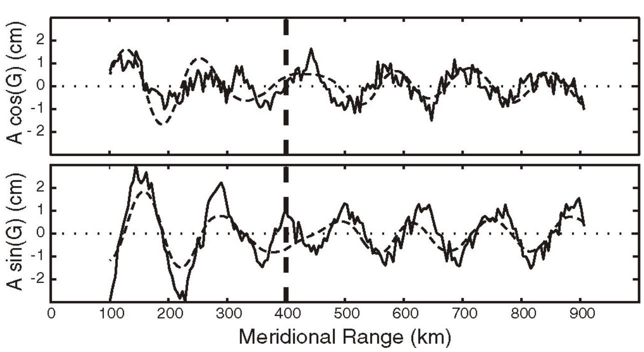

Figure 3. A comparison between the data (solid line) and the fit of the objective map (dashed line) for a descending track north of the Hawaiian Ridge. The vertical dashed line indicates a point at 164.4°W, 26.9°N (-1100 km, 400 km in Figure 1) crossed by an ascending track. At this point, where the harmonic constants for the ascending track are both nearly zero, the tracks have inconsistent values, so the misfit between the model and data is large.

has an amplitude of 3 cm, the cosine component of the wave has an amplitude of 1 cm. This result would normally suggest a partial standing wave, but standing internal waves cannot reasonably exist in these circumstances [but see Dushaw and Worcester 1998]. Such inconsistencies likely result from the extreme temporal under sampling of the tides by the altimeter. There is no obvious way to reconcile these difficulties in the data, other than to begin to make some strong assumptions about the tidal variability, e.g., to assume that the amplitude for the internal tides is the maximum value observed in a given area. The inconsistencies suggest that estimates of energy flux from these data, which depend on the square of the tidal amplitudes, are likely to be ambiguous.

M_2, Mode-1 Tidal Energy Flux

The mapped tidal displacements were used to calculate the energy density and energy flux on either side of the Hawaiian Ridge (Figure 4). The potential energy of the mode-1 waves can be calculated from their surface displacements, if the stratification is known. The total energy can then be calculated because the ratio of kinetic to potential energy for these waves is $(\omega^2 + f^2)/(\omega^2 - f^2)$, where ω and f are the tidal and inertial frequencies. The energy flux is the energy density times the group velocity (2.8 m/s), and the direction of energy flux is determined from the gradient of the mapped elevations (i.e., the energy flux is perpendicular to the wave crests and in the direction of propagation). Profiles of buoyancy frequency were obtained from the World Ocean Atlas [Antonov et al. 1998] in order to calculate the properties of the modes. A nominal surface displacement of +2 cm for mode 1 corresponds to an internal displacement of about -11 m at mode maximum at about 1500 m depth. A mode-1 internal tide with this displacement has a time-averaged energy density of about 1080 Jm^{-2} and energy flux of 3020 Wm^{-1} for the north side of the Ridge (25°N), and energy density and flux 760 Jm^{-2} and 2200 Wm^{-1} on the south side of the Ridge (20°N). The values are sensitive to the stratification used for the calculations, and they depend on the

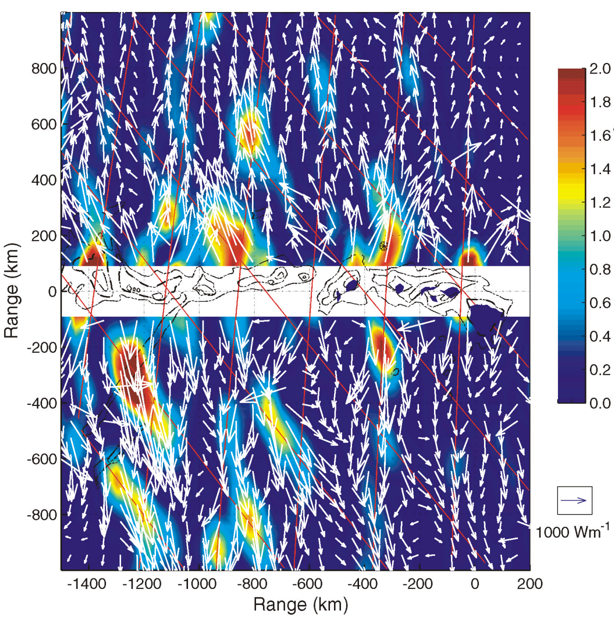

Figure 4. The energy density and energy flux derived from the tidal map of Figure 1. The areas of large energy density lie on the altimeter tracks; the objective map suppresses energy in regions where there is no data.

inertial frequency.

The largest energy flux obtained was about 2.5 kWm^{-1}, which occurred in a few energetic areas on both sides of the Ridge. This value is comparable to the value 2.4 kWm^{-1} obtained by Ray and Cartwright [2001]. The average values of energy flux on the north and south sides of the Ridge were 800±200 and 650±120 Wm^{-1}, respectively. The uncertainties were determined from the rms of flux at various ranges from the Ridge. The net energy carried by the internal tide calculated from these maps for the 1700 km stretch of the Hawaiian Ridge between the Gardner Pinnacles and Hawaii was about 2.6±0.5 GW. This value is likely to be a significant underestimate, although it is consistent with the data. The inverse solution, which is a biased estimator, underestimates the energy of the radiation between the altimeter tracks. In addition, inconsistencies in the data also cause the energy to be underestimated. In the example of Figure 3, the data suggest the surface amplitude of the internal tide may be about 3 cm (6.8 kWm^{-1} energy flux), while in balancing the available data the objective map determined an amplitude of only about 1.5 cm (1.7 kWm^{-1} energy flux).

Greater estimates for the energy radiated by the internal tides may be obtained if the peaks in *a priori* spectrum used for the objective map are assumed be narrower. For example, halving the spectral width in $|\mathbf{k}|$ gave a map that had noticeably larger amplitudes between the altimeter tracks, but only 8% more energy. A model with such stronger constraints is more similar to the model assumed by Ray and Cartwright; such a model also requires greater faith in one's knowledge of the tidal field. The main point here, however, is to highlight some of the ambiguities or difficulties in determining energy flux from the altimeter data. As stressed by Ray and Cartwright [2001] the limitations in the altimeter observations of internal tides suggest that internal-tide energy fluxes determined from these data represent only lower bounds in many cases.

Time variability of mode phase speeds

Variations in stratification are sometimes assumed to adversely affect the spatial and temporal coherence of low-mode internal tides [anonymous reviewers, personal communication 1995-2001]. The HOT hydrographic data [Karl and Lukas 1996] show that, at least near Hawaii, the effects of changes in stratification on the phase speed of the first mode are not expected to dramatically affect the coherence of the outgoing waves.

Stratification was calculated using CTD data obtained during 67 monthly visits to the HOT site (noted on Figure 1) over an 8 year time interval. About 1200 CTD casts, or 10-20 casts per HOT cruise, were used. Most of these casts extend to 1000-m depth, with at least one cast to 4500 m during each cruise. The hydrographic casts were used to calculate monthly-averaged profiles of buoyancy frequency. Parameters of internal-wave modes (e.g., phase speed) were then calculated using the average buoyancy frequency and a bottom depth of 4500 m.

For the M_2 frequency, the mean and rms phase speeds of modes 1 and 2 were 3.26±0.08 and 1.71±0.04 m/s. The phase speeds derived from these hydrographic data fluctuated rapidly from month to month; seasonal or annual variations were not obvious. Much of the variation in phase speed likely results from the inability of the CTD data to accurately determine a mean buoyancy frequency profile because of internal wave or other noise. The spatial extent of the variations is unknown, although mesoscale variability is generally assumed to have O(100-km) spatial scales.

At 2000-km range, the distance from Hawaii (or the Aleutians) that mode-1 internal-tide signals were observed by acoustic tomography [Dushaw et al. 1995], the internal tide is 6 days old. At this range, 0.08 m/s rms variations in phase speed are equivalent to 40-km rms shifts in internal-tide wave crests, if the changes in phase speed are independent of distance from Hawaii. Since the mode-1 wavelength at M_2 frequency is calculated to be 146±3 km from the HOT data, the observed variations in buoyancy, assumed to occur over a large area, cause O(1/4) wavelength shifts in phase at 2000-km range with monthly time scales. The values for the phase shifts are obviously exaggerated by this simple calculation, but even such small variations in phase were not apparent in the tomography data [Dushaw et al. 1995]. It appears, therefore, that the incoherence caused by variations in stratification is not as substantial as is sometimes assumed. A larger effect may result from the changes in stratification near the topography where the internal tides are generated [Mitchum and Chiswell 2000], since such changes will affect the initial phase of the waves. The possible effects of ocean currents, e.g., mesoscale processes, are not considered here.

Conclusions

An objective map of the TOPEX/POSEIDON altimeter data showed that the internal tides on either side of the Hawaiian Ridge were consistent with waves that have crests extending considerable distances along the Hawaiian Ridge. An estimate of the wavenumber spectrum showed that the field was dominated by only a few distinct wavenumbers. The objective map approach offers a way to construct tidal maps that are consistent with all available data for comparison to model outputs or calculation of tidal energetics. A careful construction of the model used to fit the data, e.g., a judicious selection of *a priori* wavenumber spectra, may result in more realistic estimation of the tidal fields. The altimeter data were sometimes inconsistent with harmonic "constants," or with traveling internal waves, probably because the data were under sampled in time and space. The variations in stratification, as determined by the HOT time series of hydrographic data, were used to show that the propagation characteristics of mode-1 internal tides were not significantly affected by changes in stratification. This result is consistent with the spatial and temporal coherence of the internal tides seen by the altimeter.

At the M_2 frequency, mode-1 internal tides determined from the objective map carried 2.6±0.5 GW of energy from the Hawaiian Ridge, which is considerably less than other estimates of this energy. Ray and Cartwright [2001] estimated an energy value of 6 GW. A crude earlier calculation of Ray and Mitchum [1997] gave a value of 15 GW. The modeling results of Kang, et al. [2000] (5.4 GW, a 2-layer model) and Merrifield, et al. [2001] (9 GW, including all modes) for the Hawaiian Ridge were more consistent with the Ray and Cartwright estimate. The energy found from the objective map may be too small because the mapping domain did not include the entire Hawaiian Ridge. In addition, objective maps underestimate energy, particularly when employing data that are under sampled and not altogether consistent. The Hawaiian Ocean Mixing Experiment (HOME) [Pinkel et al. 2000] is presently underway to more accurately assess the tidal radiation and dissipation around the Hawaiian Ridge and their role in ocean mixing.

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B. D. Dushaw, Applied Physics Laboratory, University of Washington, Seattle, Washington, 98105, USA; dushaw@apl.washington.edu

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