

# CSSS/POLS 510

## Lab Session 2: Introduction to Maximum Likelihood Estimation

### 0.

Clear memory

Set seed (123456)

Install MASS and simcf packages

### 1. Simulating heteroskedastic normal data

Set the number of observations to 1500

Set a parameter vector for the mean (assume 2 covariates plus the constant)

Set a parameter vector for the variance (assume heteroskedasticity)

Generate the constant and the covariates, length 1500 for each (draw from a uniform distribution)

Create the systematic component for the mean

Create the systematic component for the variance (the same covariates affect  $\mu$  and  $\sigma$ )

Generate the response variable

Save the data to a data frame

Plot the data

## 2. Fitting a model using the simulated data

Assume we don't know the true value of the parameters and fit a model using least squares (use the `lm()` function and regress the response variable on the two covariates)

Calculate and print the AIC

## 3. Calculating predicted values

### Scenario 1: Vary covariate 1

Create a data frame with a set of hypothetical scenarios for covariate 1 while keeping covariate 2 at its mean

Calculate the predicted values using the `predict()` function

Plot the predicted values

### Scenario 2: Vary covariate 2

Create a data frame with a set of hypothetical scenarios for covariate 2 while keeping covariate 1 at its mean

Calculate the predicted values using the `predict()` function

Plot the predicted values

## 4. Fitting the heteroskedastic normal model using ML

Create the input matrices (the two covariates)

Write a likelihood function for the heteroskedastic normal model

Find the MLEs using the `optim()` function

Extract the point estimates

Compute the standard errors

Compare with the least squares estimates

Find the log likelihood at its maximum

Compute the AIC

Simulate the results by drawing from the model's predictive distribution

Separate the simulated betas from the simulated gammas

## **5. Simulating predicted values and confidence intervals**

### **Scenario 1: Vary covariate 1**

Create a data frame with a set of hypothetical scenarios for covariate 1 while keeping covariate 2 at its mean

Simulate the predicted values and the confidence intervals using `simcf`

Plot the results

### **Scenario 2: Vary covariate 2**

Create a data frame with a set of hypothetical scenarios for covariate 2 while keeping covariate 1 at its mean

Simulate the predicted values and the confidence intervals using `simcf`

Plot the results