Decline of Arctic Sea-Ice Thickness as Evidenced by Submarine Measurements

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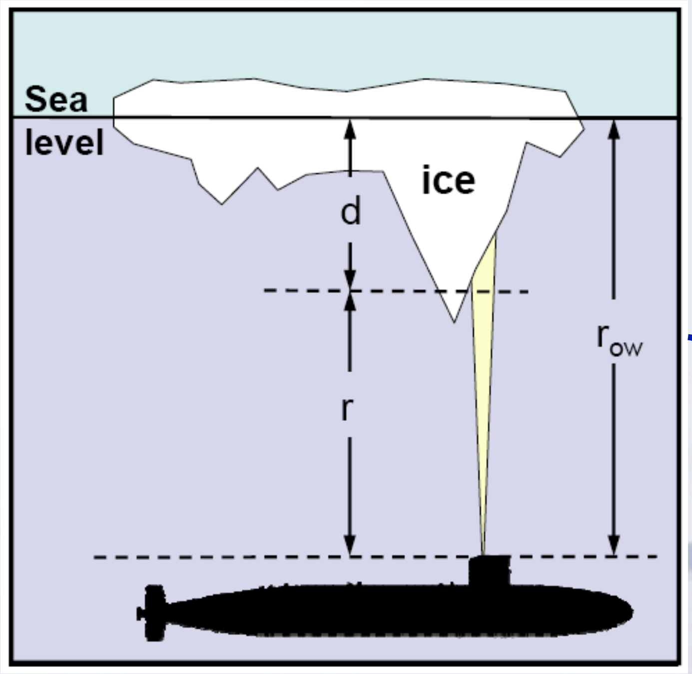
Overview

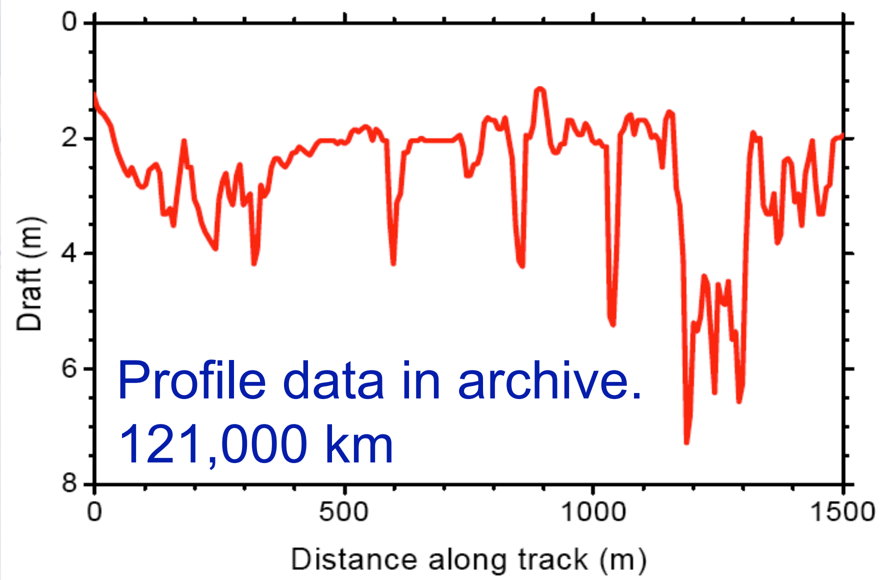
- scientific question of interest: has the average thickness of Arctic sea ice declined significantly over the past 30+ years?
- thickness can be deduced from measurements of draft (submerged portion of sea ice – 93% of ice thickness)
- draft measured using upward-looking sonars on submarines
- there have been previous analyses of these data, but ours differs because of the use of
  - new statistical model for correlation of measurements (incorporates so-called 'long-range' dependence)
  - multiple regression analysis to deduce space/time variations
  - newly archived data for submarine cruises from 1975 to 2001 (almost doubling the amount of available data)

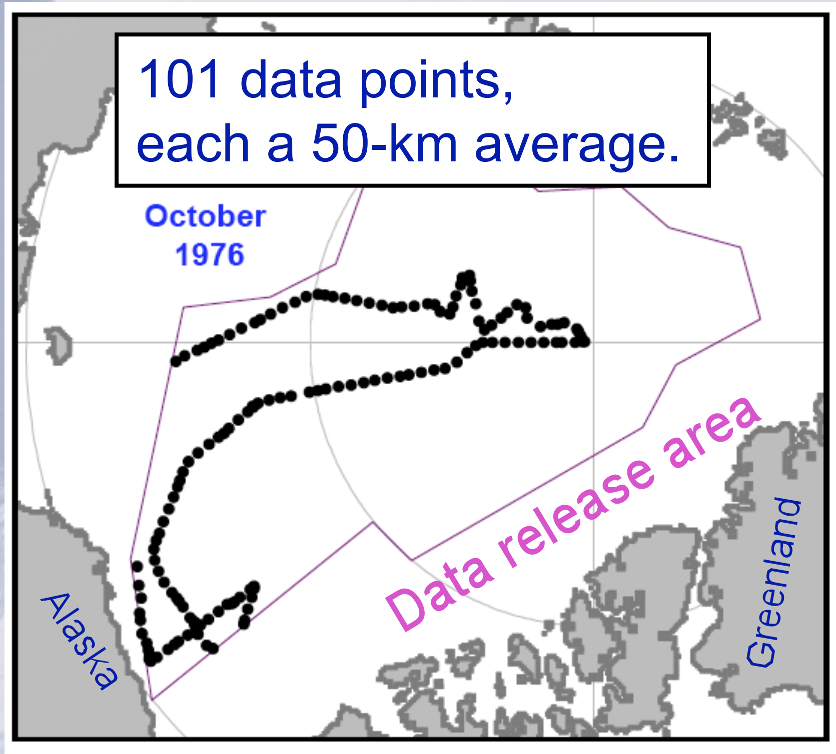
Outline of Talk

- show pictures giving an idea of how data were collected
- study single submarine track to develop model for correlation structure, contrasting properties of models based on first-order autoregressive (AR(1)) and fractionally differenced (FD) processes
- discuss embedding of model for single tracks into overall space/time model
- describe multiple regression model and rationale for fitting model using ordinary least squares rather than generalized least squares
- discuss conclusions that can be drawn from regression analysis

Ice Draft from Upward-Looking Sonar



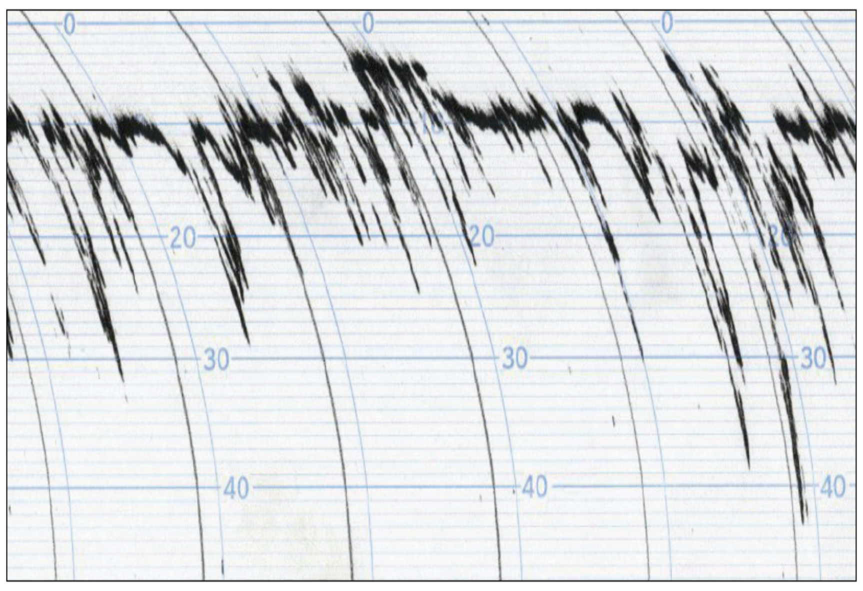


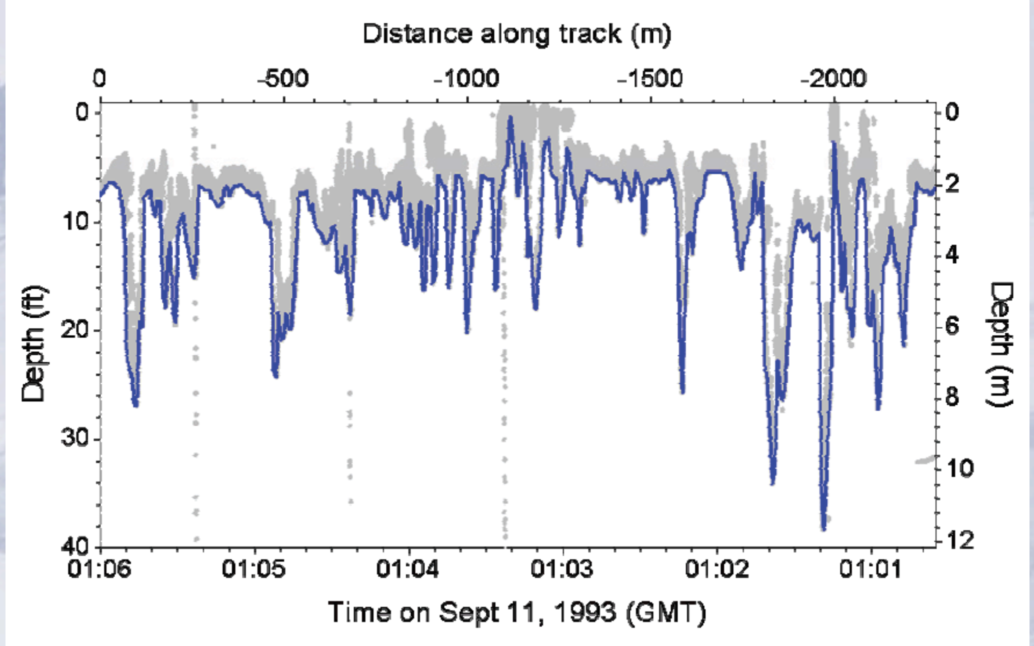


(Wensnahan et al., *EOS*, Jan., 2007)

Paper Charts (Analog)

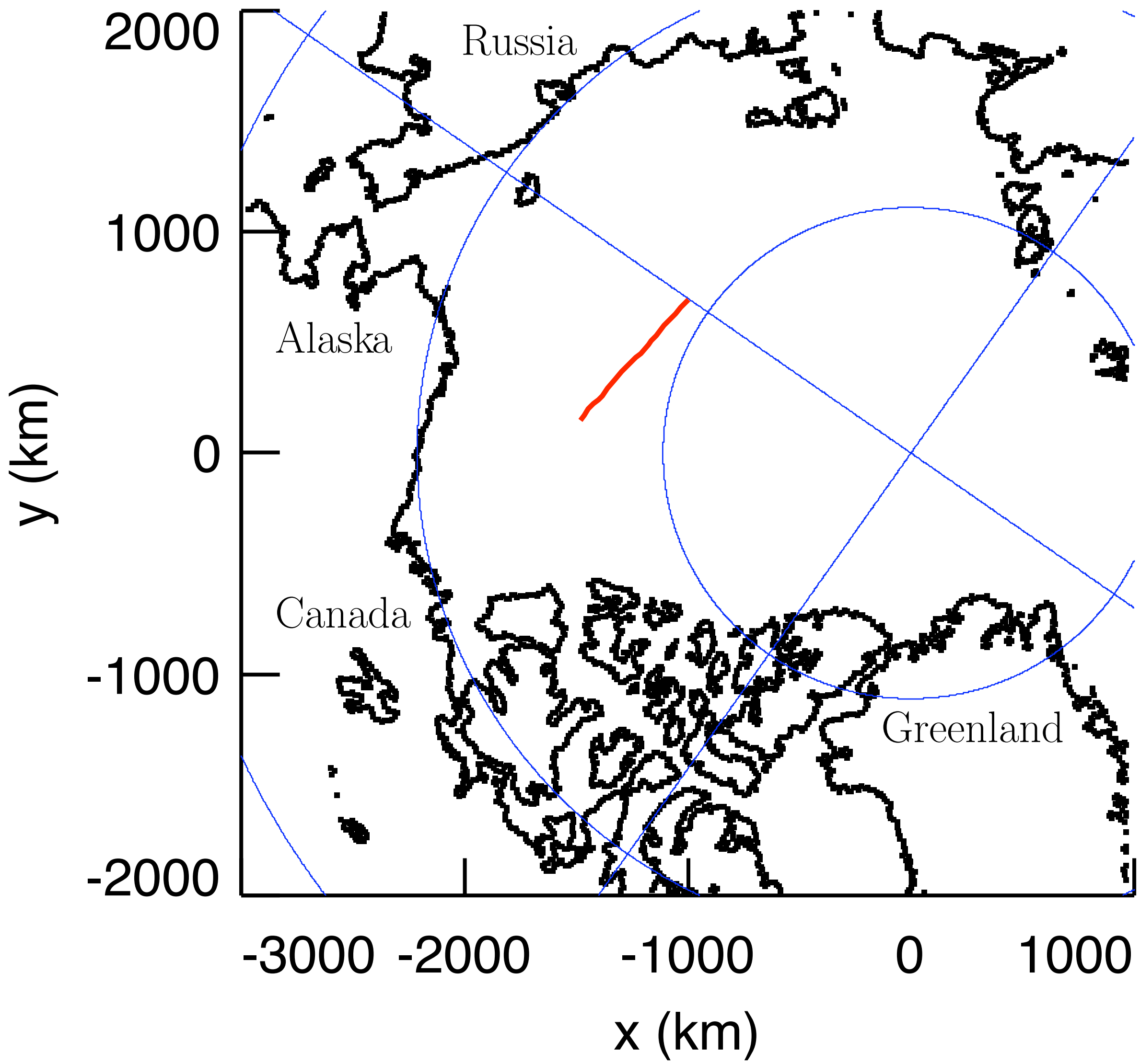
are scanned and
a digital trace extracted.





(Wensnahan & Rothrock, *GRL*, 2005; Wensnahan et al., *EOS*, 2007)

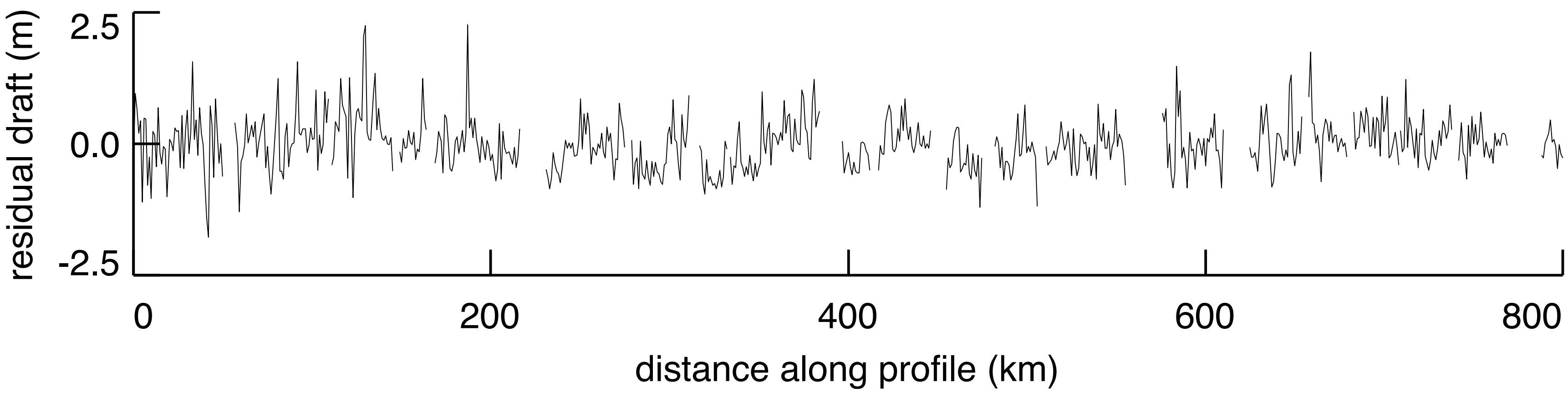
Map of Arctic Region with One Submarine Tract



Understanding Spatial Correlation

- as submarine moves below ice, returns from sonar provide measurements of ice draft averaged over 1 m patches, which are recorded at 1 m intervals
- when submarine is moving along a tract (straight line), use 1000 consecutive measurements to form 1 km averages $\overline{H}_{1,n}$, where n is the index for a particular average
- later on, will average 50 consecutive nonoverlapping $\overline{H}_{1,n}$'s to form 50 km averages – denote these as $\overline{H}_{50,\mathbf{x}_n,t}$, where $\mathbf{x}_n$ is the center of the tract and t is the associated time ($\mathbf{x}_n = [0,0] =$ North Pole, while $t \in [1975, 2001]$)
- need to understand spatial covariance properties of $\overline{H}_{50,\mathbf{x}_n,t}$, which we can begin to tackle by studying covariance of $\overline{H}_{1,n}$

Residual Draft Profile for One Submarine Track



- above comes from track shown on overhead 6 (longest in 1997)
- to study covariane, have subtracted off linear trend (for profiles less than 200 km or so, need only subtract sample mean)
- will now let $\overline{H}_{1,n}$ denote residual draft profile
- residuals approximately Gaussian (room for improvement?)
- note: lots of gaps in draft profile (632 averages over 803 km)

Statistical Modeling of Residual Draft Profiles

- simple model of independence for $\overline{H}_{1,n}$ along profile not viable (e.g., adjacent measurements $\overline{H}_{1,n}$ and $\overline{H}_{1,n+1}$ are correlated)
- assume $\overline{H}_{1,n}$ is a realization of a zero mean Gaussian stationary process (a 'time' series with distance replacing 'time')
- process fully characterized by its variance σ_1^2 and autocorrelation sequence $\rho_d \equiv E\{\overline{H}_{1,n}\overline{H}_{1,n+d}\}/\sigma_1^2$, where d is the distance between measurements (lag) expressed in km
- consider two simple parametric forms for ρ_d corresponding to
  - first-order autoregressive (AR(1)) process
  - fractionally differenced (FD) process

First-Order Autoregressive (AR(1)) Processes: I

- process satisfies $\overline{H}_{1,n} = \phi \overline{H}_{1,n-1} + \epsilon_n$, where $|\phi| < 1$, and ϵ_n's are IID Gaussian with mean 0 and variance σ_ϵ^2
- note that

$$\begin{aligned} \overline{H}_{1,n} &= \phi \left(\phi \overline{H}_{1,n-2} + \epsilon_{n-1} \right) + \epsilon_n \\ &= \phi^2 \overline{H}_{1,n-2} + \phi \epsilon_{n-1} + \epsilon_n \\ &= \phi^2 \left(\phi \overline{H}_{1,n-3} + \epsilon_{n-2} \right) + \phi \epsilon_{n-1} + \epsilon_n \\ &= \phi^3 \overline{H}_{1,n-3} + \phi^2 \epsilon_{n-2} + \phi \epsilon_{n-1} + \epsilon_n \\ &\vdots \\ &= \phi^J \overline{H}_{1,n-J} + \sum_{j=0}^{J-1} \phi^j \epsilon_{n-j} \longrightarrow \sum_{j=0}^{\infty} \phi^j \epsilon_{n-j} \end{aligned}$$

First-Order Autoregressive (AR(1)) Processes: II

- process thus can be expressed as

$$\overline{H}_{1,n} = \sum_{j=0}^{\infty} \psi_j \epsilon_{n-j} \text{ with } \psi_j = \phi^j$$

- have $\rho_d = \phi^{|d|}$, so model exhibits 'short-range' correlation: measurements that are close in distance are correlated, but correlation disappears rapidly (exponentially) with increasing distance
- related to a first-order stochastic differential equation with 'correlation time' dictated by ϕ
- widely used in climate research to model time series
- given gappy draft profiles, can estimate ϕ and σ_ϵ^2 using maximum likelihood (Jones, 1980), yielding $\hat{\phi} \doteq 0.36\ (\pm 0.04)$

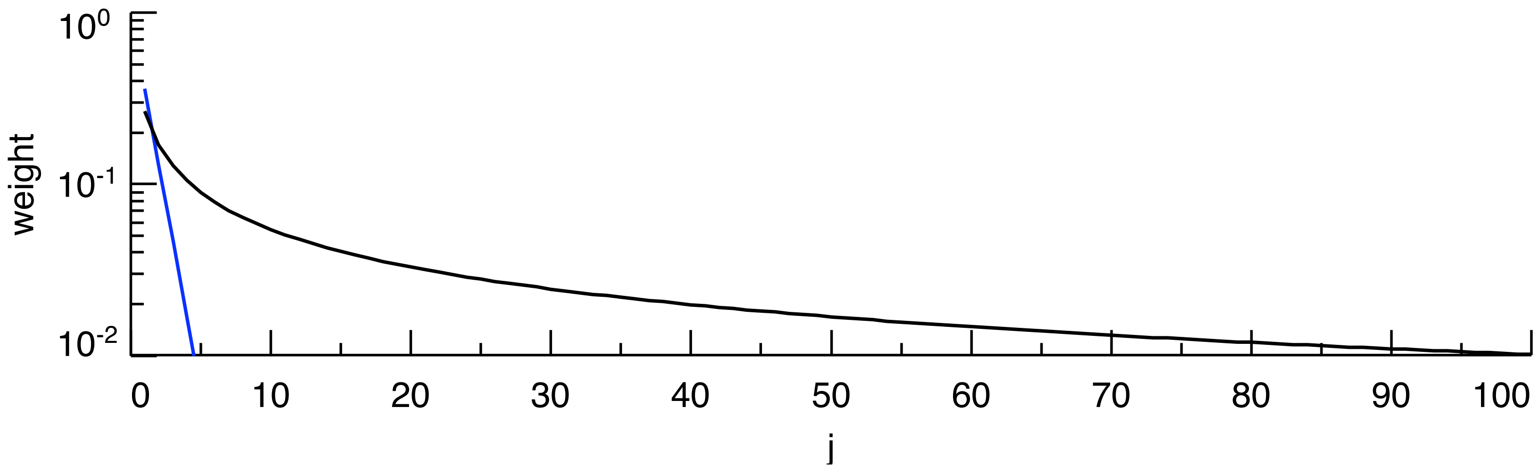
Fractionally Differences (FD) Processes

- process satisfies

$$\overline{H}_{1,n} = \sum_{j=0}^{\infty} \psi_j \epsilon_{n-j} \text{ with } \psi_j = \frac{\Gamma(j+\delta)}{\Gamma(j+1)\Gamma(\delta)},$$

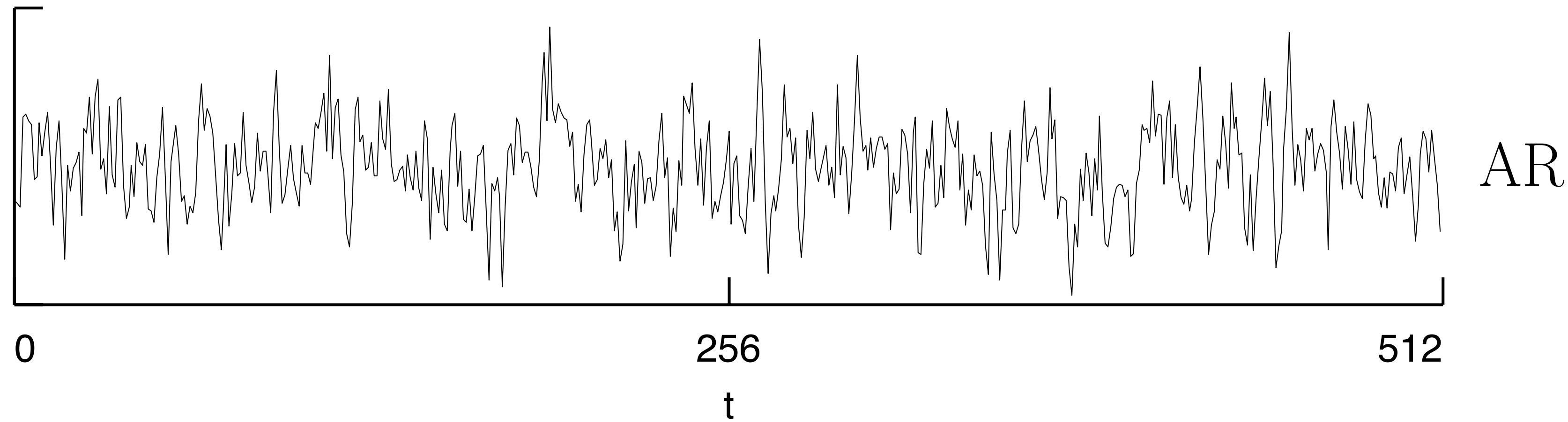
  where $|\delta| < 1/2$, and ϵ_n is as before
- have $\rho_d \approx C|d|^{2\delta-1}$ (much slower decay rate than for AR(1))
- related to average of many first-order stochastic differential equations with different correlation times
- popular model for 'long-range' (or 'long-memory') dependence
- given gappy profiles, can estimate δ and σ_ϵ^2 using maximum likelihood (Palma & Chan, 1997), yielding $\hat{\delta} \doteq 0.27$ (± 0.03)
- Q: how do these two models compare?

Qualitative Comparison I: ψ Weights



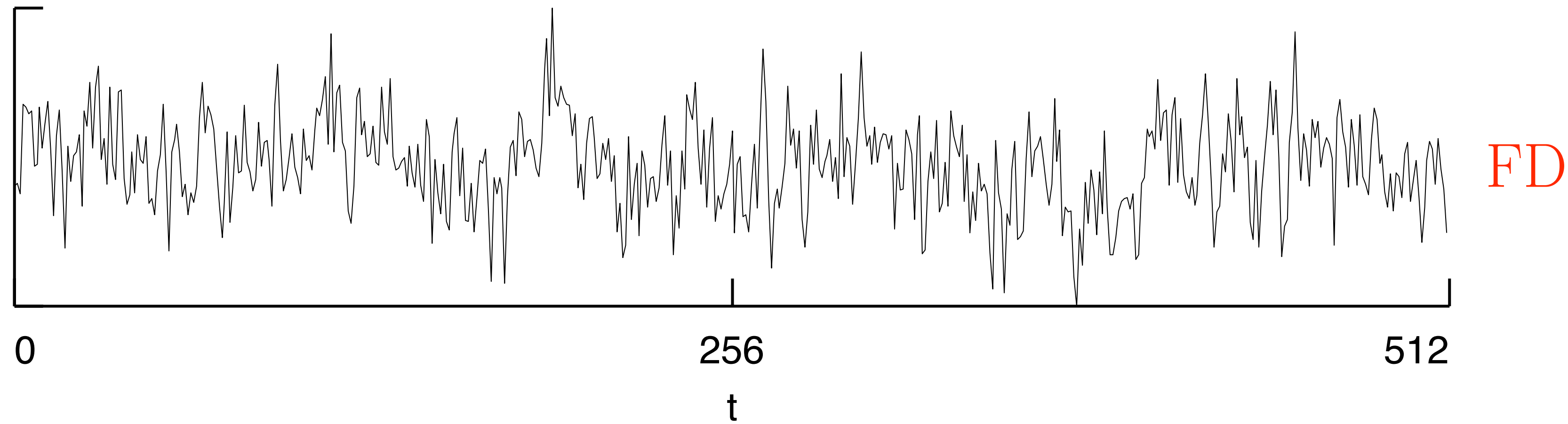
- weights ψ_j used to create AR with $\phi = 0.36$ (blue curve) and FD with $\delta = 0.27$ (black curve) processes from a weighted average of white noise

Qualitative Comparison II: Simulated Draft Profiles



- consider simulated AR ($\phi = 0.36$) & FD ($\delta = 0.27$) profiles
- 'exact' simulations formed using circulant embedding technique that maps same 1024 IID Gaussian deviates to both profiles (Davies and Harte, 1987; Craigmile, 2003)

Qualitative Comparison II: Simulated Draft Profiles



- consider simulated AR ($\phi = 0.36$) & FD ($\delta = 0.27$) profiles
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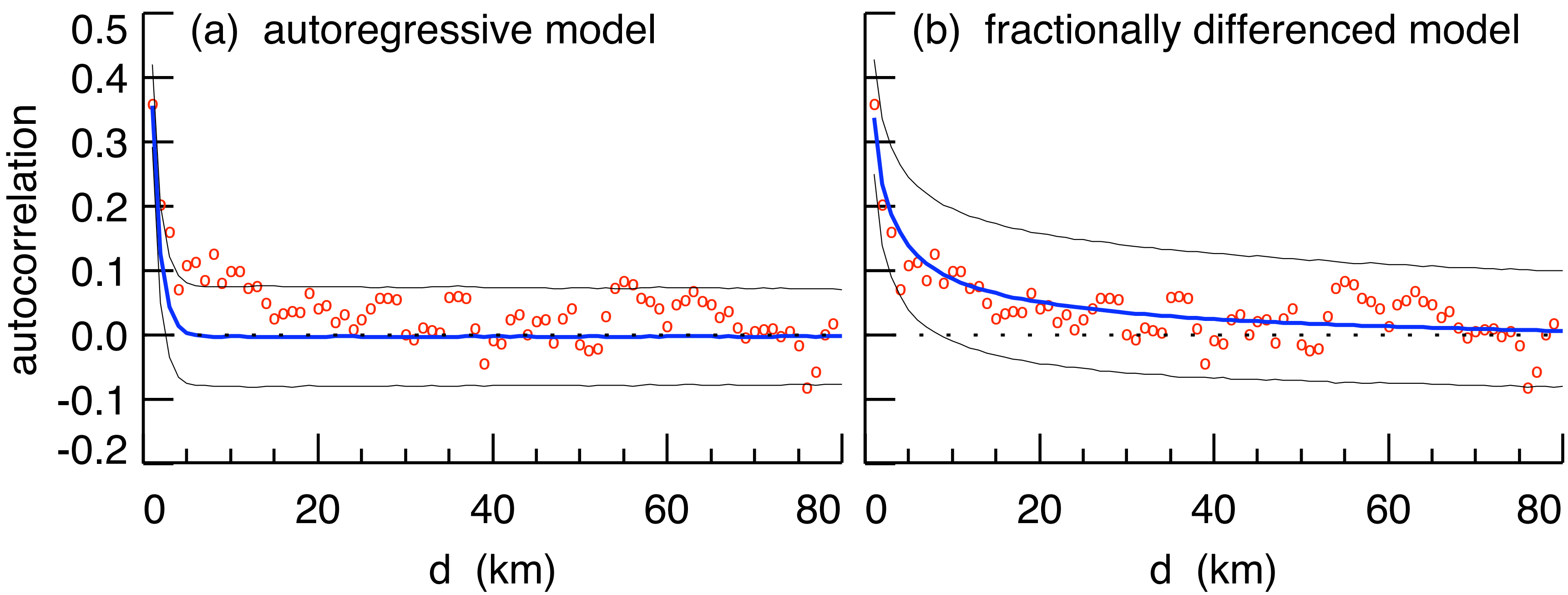
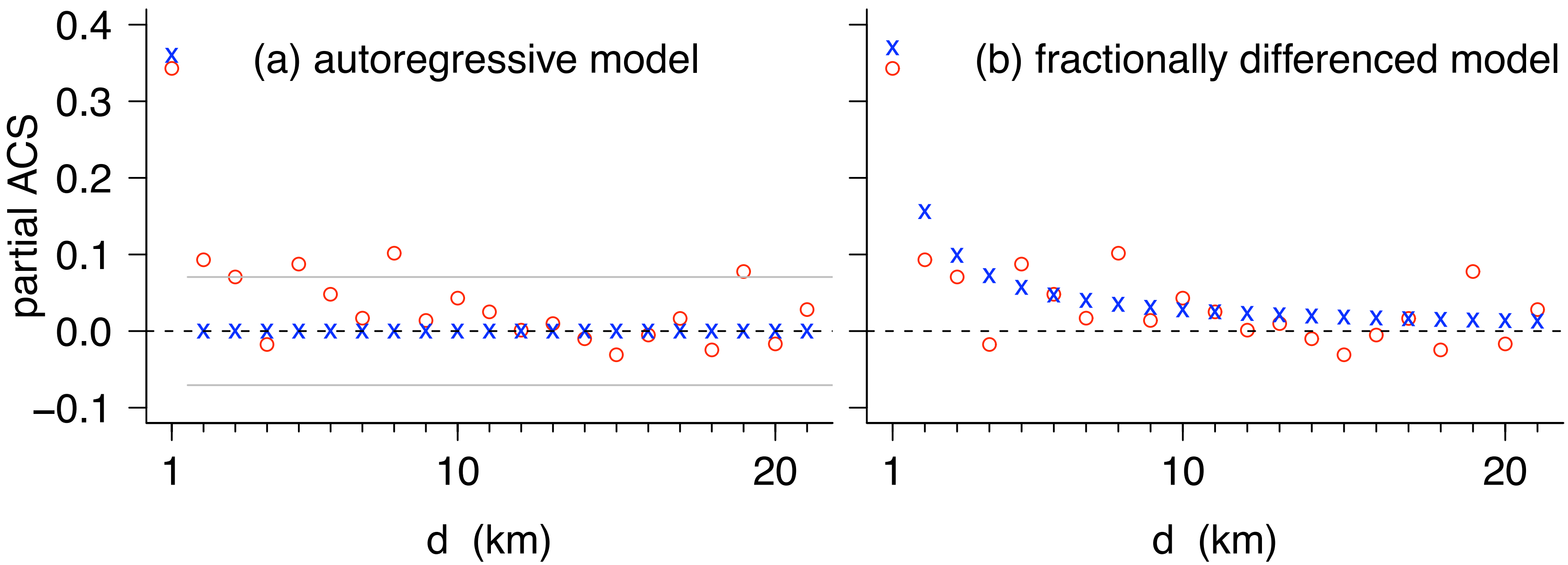
Third Comparison: Autocorrelation Sequences



- sample (circles) and theoretical sequences (middle curves)
- upper and lower curves are 95% pointwise confidence intervals for ρ_d assuming relevant model (AR or FD)

Fourth Comparison: Partial Autocorrelations



- sample (circles) and theoretical sequences (**x**'s)
- parallel gray lines in (a) are limits between which approximately 95% of samples $\hat{\phi}_{d,d}$ at lags $d \geq 2$ should fall under assumption that AR(1) model is correct

PACS-Based Portmanteau Test for AR(1) Process

- can test null hypothesis that $\overline{H}_{1,n}$ comes from a Gaussian AR(1) process versus nonspecific alternative hypothesis of non-white Gaussian stationary process
- test is variation on standard portmanteau test for white noise (using sample autocorrelation sequence) and is given by

$$T \equiv N \sum_{d=2}^{K+1} \hat{\phi}_{d,d}^2$$

- can reject null if T is 'too large' in comparison to upper percentage points of χ^2 distribution with K degrees of freedom
- $\hat{\alpha}$ (observed level of significance) is < 0.014 for both $K = 10$ and $K = 20$, so AR(1) hypothesis unlikely to be true

Fifth Comparison: Variance of Sample Means

- given $\overline{H}_{1,n}, n = 0, \ldots, N-1$, consider statistical properties of length L averages

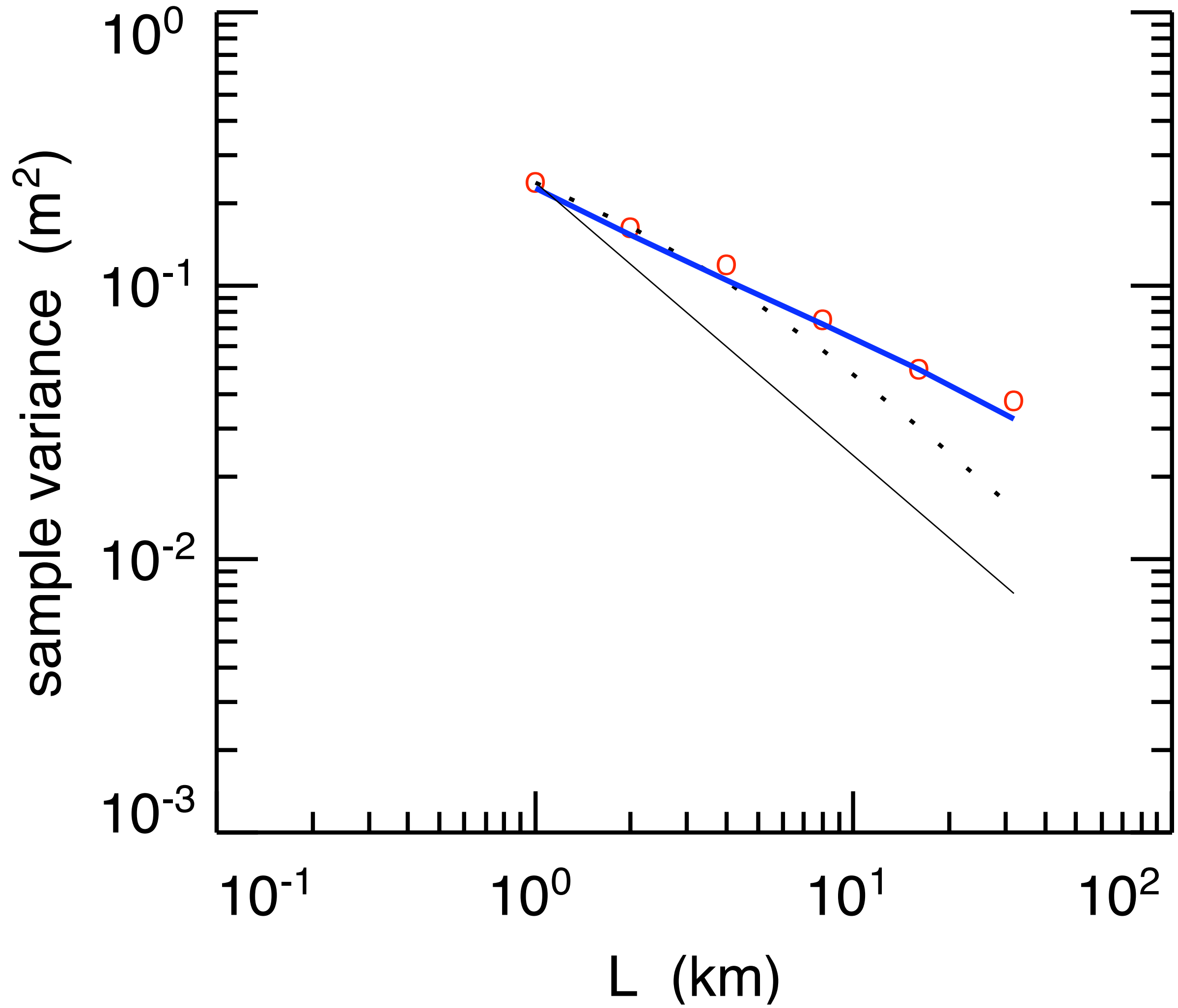
$$\overline{H}_{L,m} = \frac{1}{L}\sum_{l=0}^{L-1} \overline{H}_{1,mL+l}, \quad m = 0, 1, \ldots, \lfloor N/L \rfloor - 1$$

- let $\sigma_L^2 = \text{var}\,\{\overline{H}_{L,m}\}$
- for AR and FD models, have

$$\sigma_L^2 \approx \sigma_1^2 \times \frac{1+\phi}{1-\phi} \times L^{-1} \text{ and } \sigma_L^2 \approx \sigma_1^2 \times \frac{\Gamma(1-\delta)}{(2\delta+1)\Gamma(1+\delta)} \times L^{-1+2\delta}$$

- can compare sample estimates $\hat{\sigma}_L^2$ with $E\{\hat{\sigma}_L^2\}$ for various L

Sample $\hat{\sigma}_L^2$ (Circles) and Theoretical σ_L^2 versus L

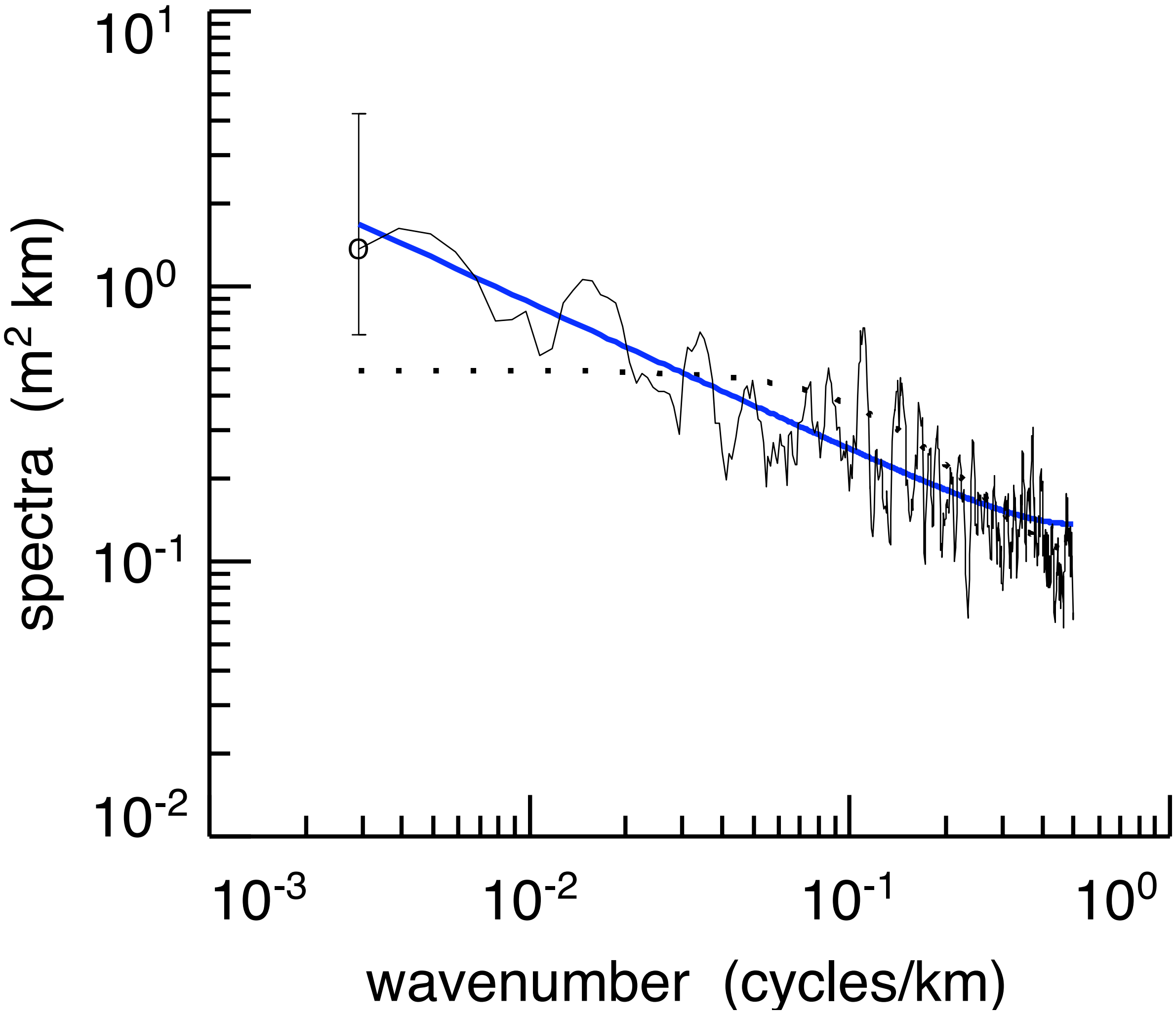


blue: FD

dotted: AR(1)

solid: white noise

Sixth Comparison: Sample and Theoretical Spectra

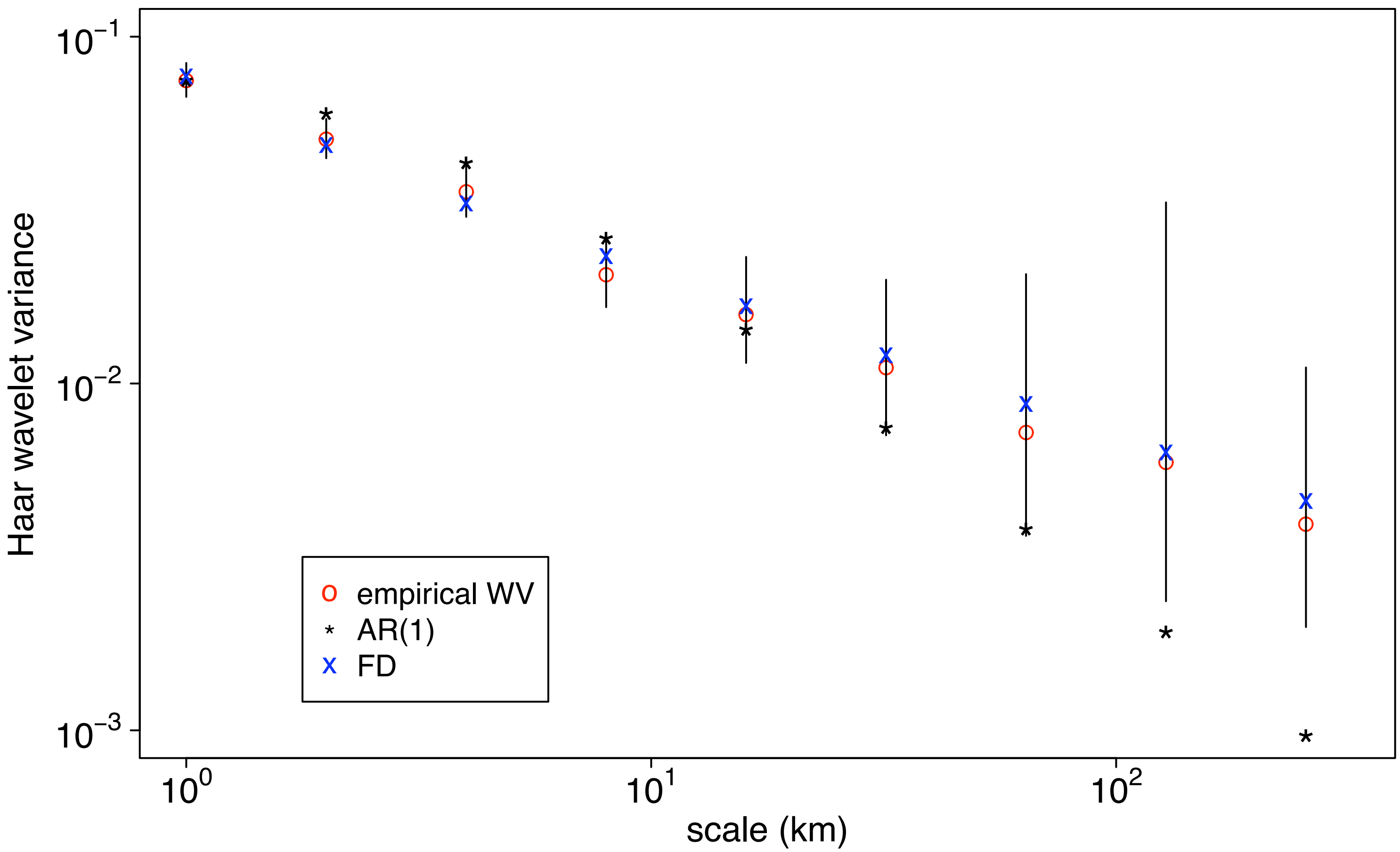


blue: FD

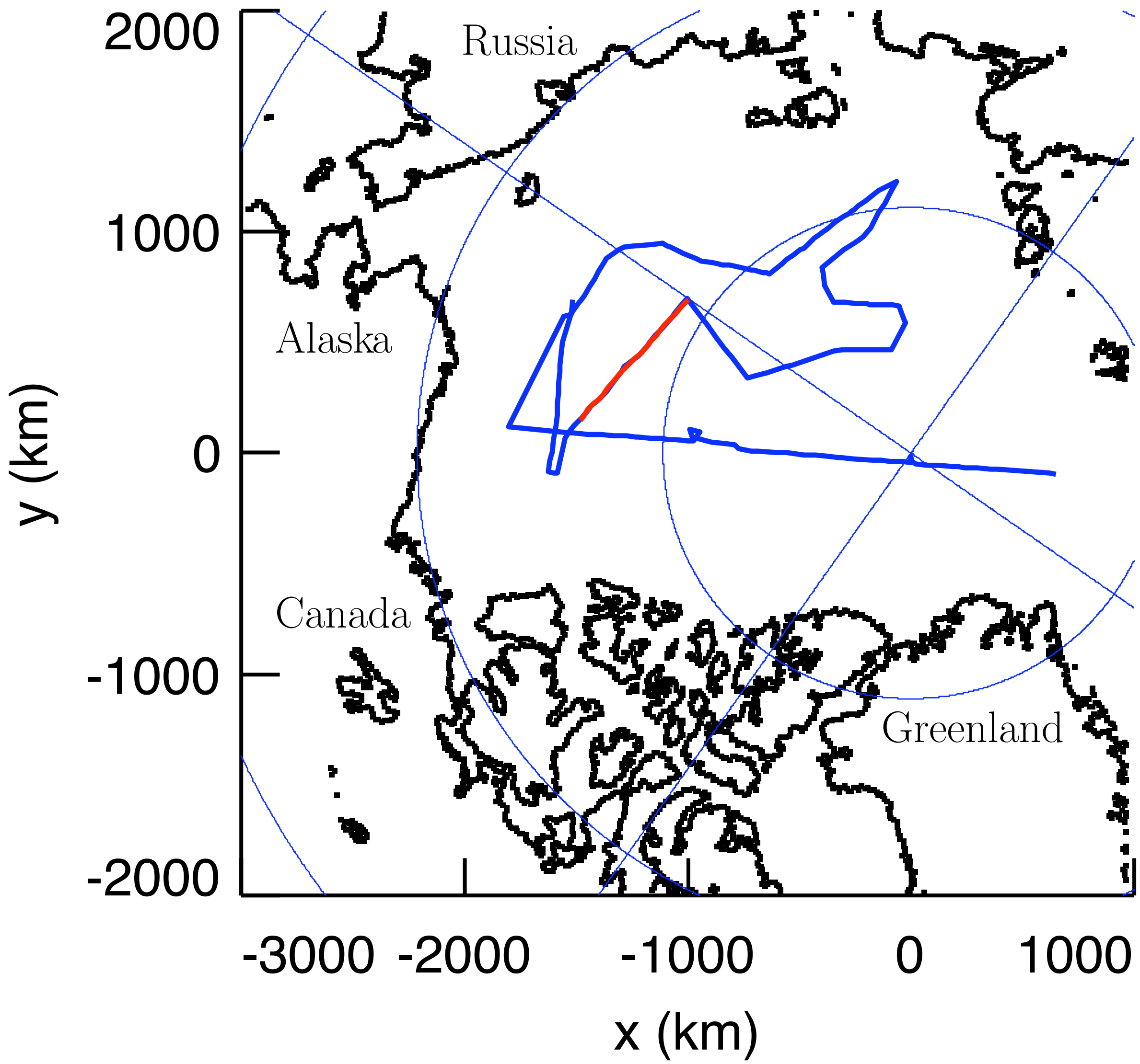
dotted: AR(1)

solid: sample

Seventh Comparison: Wavelet Variances



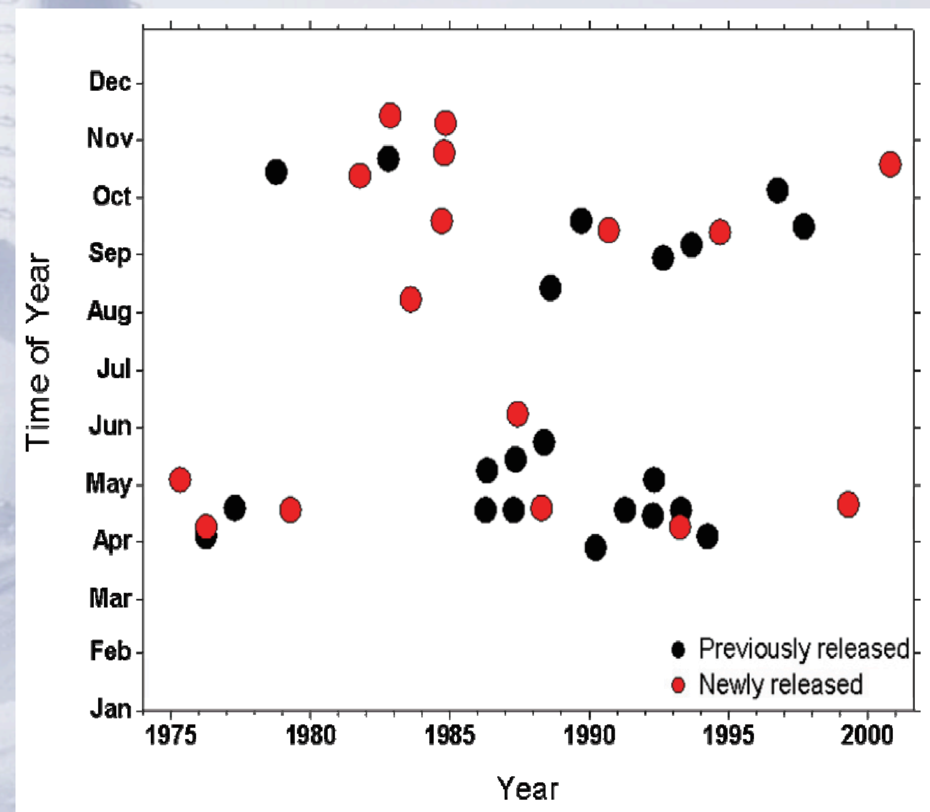
Map of Arctic Region with Tracks Taken in 1997

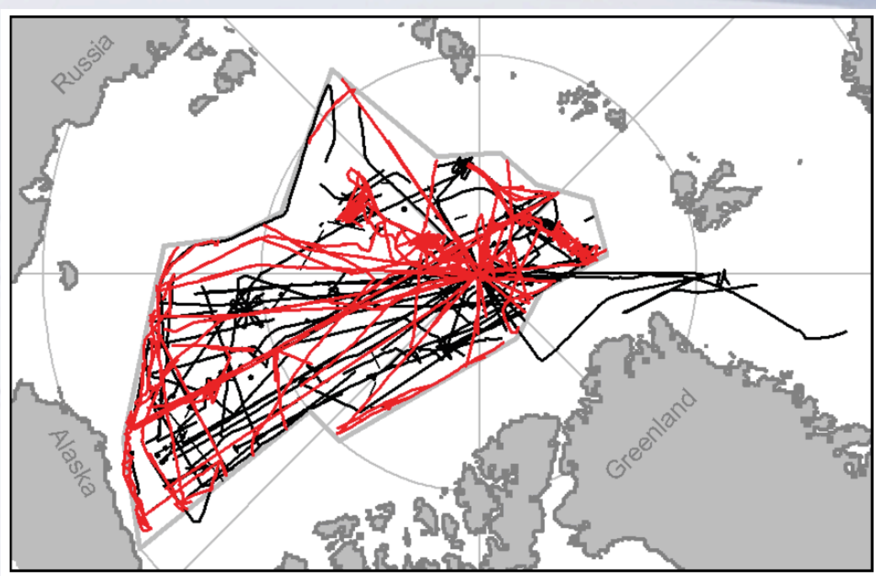


Submarine Cruises

- NSIDC has draft data from 121,000 km of cruise tracks
- New data (2006) in red added 81%.

Month vs Year





Data are archived at the National Snow and Ice Data Center
[*Google*: NSIDC]

Spatial Model for One Kilometer Averages

- analysis of additional profiles in 1997 and other years indicates FD model with $\delta = 0.27$ is good overall choice
- will now reindex $\overline{H}_{1,n}$ using a 2D vector $\mathbf{x}_n$ indicating the location of the 1 km average (needed for dealing with data from multiple tracks)
- can regard $\overline{H}_{1,\mathbf{x}_n}$ as samples from a stationary and isotropic two-dimensional (2D) random process with covariances given by

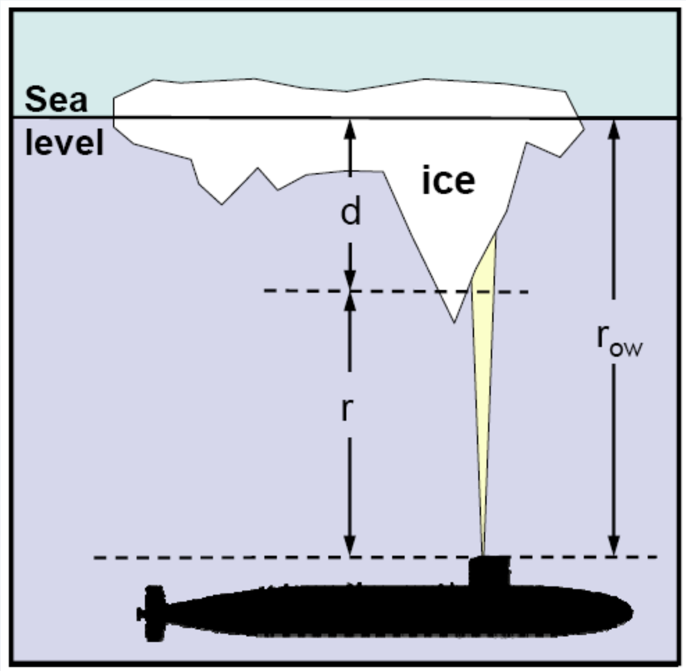
$$\mathrm{cov}\,\{\overline{H}_{1,\mathbf{x}_n}, \overline{H}_{1,\mathbf{x}_n+\mathbf{d}}\} \equiv \sigma_1^2 \times \frac{\Gamma(|\mathbf{d}| + \delta)\Gamma(1-\delta)}{\Gamma(|\mathbf{d}| + 1 - \delta)\Gamma(\delta)},$$

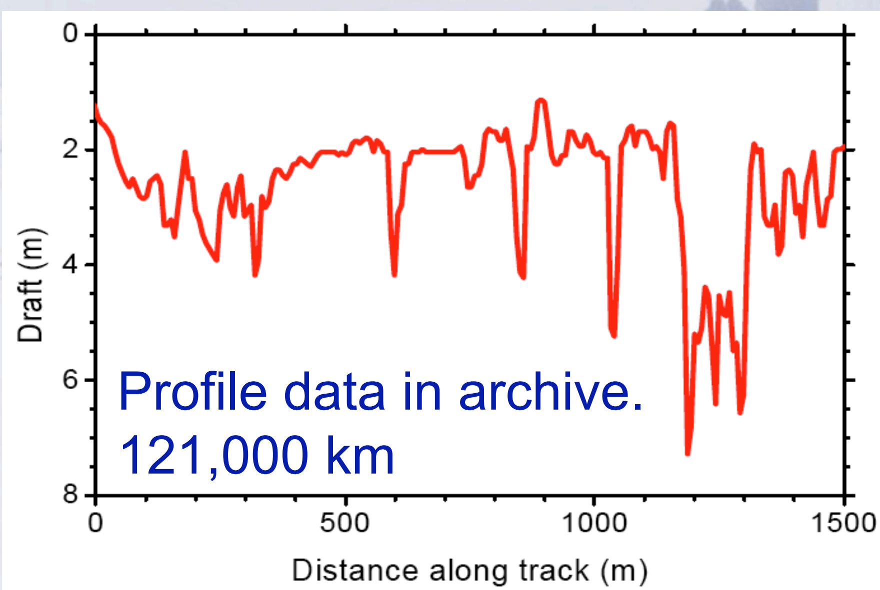
  where $\mathbf{d}$ is an arbitrary 2D vector, and $|\mathbf{d}|$ is its Euclidean norm
- 1D tracks through this 2D process yield an FD(δ) process

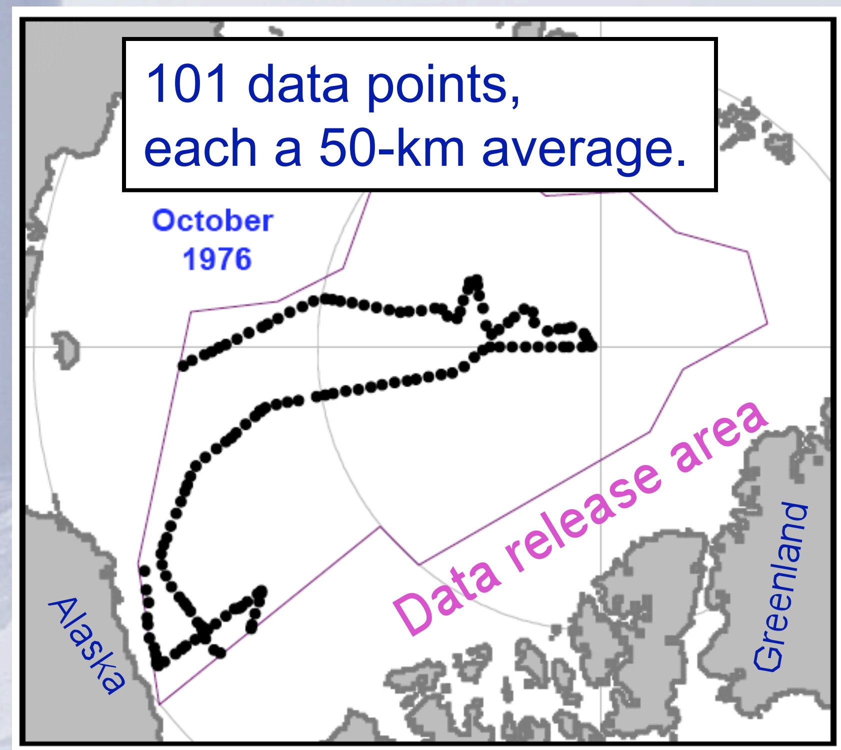
Fifty Kilometer Averages

- to reduce computational burden, combined 1 km averages into 50 km averages, yielding $\overline{H}_{50,\mathbf{x}_n}$
- assumed form for $\text{cov}\,\{\overline{H}_{1,\mathbf{x}_n}, \overline{H}_{1,\mathbf{x}_n+\mathbf{d}}\}$ can be used to deduce $\text{cov}\,\{\overline{H}_{50,\mathbf{x}_n}, \overline{H}_{50,\mathbf{x}_n+\mathbf{d}}\}$

Ice Draft from Upward-Looking Sonar







(Wensnahan et al., *EOS*, Jan., 2007)

Multiple Regression Model: I

- let $\overline{H}_{50,\mathbf{x}_n,t}$ represent average of 1 km measurements taken at location $\mathbf{x}_n$ and time t ($\mathbf{x}_n = [0,0] =$ Pole & $t \in [1975, 2001]$)
- let τ represent the time of year (i.e., $\tau = t \bmod 1$)
- assume simple model

$$\overline{H}_{50,\mathbf{x}_n,t} = C + I(t) + A(\tau) + S(\mathbf{x}_n) + \epsilon_{\mathbf{x}_n,t},$$

  where

  - C is a constant
  - $I(t)$ is the interannual variation
  - $A(\tau)$ is the annual cycle
  - $S(\mathbf{x}_n)$ is the spatial field
  - $\epsilon_{\mathbf{x}_n,t}$ is an error term dictated by FD model within a given season (different seasons/years assumed independent)

Multiple Regression Model: II

- assumed form for interannual variation $I(t)$ is

$$I(t) = I_1(t - 1988) + I_2(t - 1988)^2 + I_3(t - 1988)^3,$$

  where I_1, I_2 and I_3 are parameters to be estimated (experimented with other polynomials, but cubic is adequate)

- assumed form for annual cycle is

$$A(\tau) = A_s \sin(2\pi\tau) + A_c \cos(2\pi\tau) = A \cos(2\pi[\tau - \tau_{\max}]),$$

  where A_s and A_c are parameters to be estimated, from which A and $\tau_{\max}$ can be deduced (experimented with adding terms with frequencies at harmonics of annual cycle, but simple form is adequate)

Multiple Regression Model: III

- letting $\mathbf{x}_n = [x, y]^T$, assumed form for spatial field $S(\mathbf{x}_n)$ is

$$\begin{aligned} S(\mathbf{x}_n) = {} & S_{10}x + S_{01}y \\ & + S_{20}x^2 + S_{11}xy + S_{02}y^2 \\ & + S_{30}x^3 + S_{21}x^2y + S_{12}xy^2 + S_{03}y^3 \\ & + S_{40}x^4 + S_{31}x^3y + S_{22}x^2y^2 + S_{13}xy^3 + S_{04}y^4 \\ & + S_{50}x^5 + S_{41}x^4y + S_{32}x^3y^2 + S_{23}x^2y^3 + S_{14}xy^4 + S_{05}y^5, \end{aligned}$$

  where S_{ij}'s are parameters to be estimated (experimented with 6th order polynomials, but t-tests say 5th order is adequate)

- within a given season and year, error term $\epsilon_{\mathbf{x}_n,t}$ has a covariance structure dictated by FD model

- $\epsilon_{\mathbf{x}_n,t}$'s from different seasons or years are assumed to be independent (reasonable assumption, based upon ice physics)

Fitting Multiple Regression Model: I

- used ordinary least squares (OLS) to fit model, even though correlated errors recommends generalized least squares (GLS)
- to see rationale for ignoring usual recommendation, need to review relationship between OLS and GLS estimators
- write regression model as $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$, and let $\Sigma_{\boldsymbol{\epsilon}}$ denote covariance matrix for $\boldsymbol{\epsilon}$
- OLS estimator is $\widehat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$
- GLS estimator makes use of 'square root' of $\Sigma_{\boldsymbol{\epsilon}}$; i.e., matrix $\Sigma_{\boldsymbol{\epsilon}}^{1/2}$ such that $\Sigma_{\boldsymbol{\epsilon}}^{1/2}\Sigma_{\boldsymbol{\epsilon}}^{1/2} = \Sigma_{\boldsymbol{\epsilon}}$

Fitting Multiple Regression Model: II

- use inverse $\Sigma_{\boldsymbol{\epsilon}}^{-1/2}$ of $\Sigma_{\boldsymbol{\epsilon}}^{1/2}$ to transform regression model:

$$\Sigma_{\boldsymbol{\epsilon}}^{-1/2}\mathbf{Y} = \Sigma_{\boldsymbol{\epsilon}}^{-1/2}\mathbf{X}\boldsymbol{\beta} + \Sigma_{\boldsymbol{\epsilon}}^{-1/2}\boldsymbol{\epsilon},$$

  which we can rewrite as

$$\widetilde{\mathbf{Y}} = \widetilde{\mathbf{X}}\boldsymbol{\beta} + \widetilde{\boldsymbol{\epsilon}},$$

  where covariance matrix for $\widetilde{\boldsymbol{\epsilon}}$ can be expressed as $\sigma_{\widetilde{\boldsymbol{\epsilon}}}^2\, I_N$ (here I_N is $N \times N$ identity matrix, and N is length of vector $\mathbf{Y}$)

- OLS estimator $\widetilde{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ in transformed model is GLS estimator for original model:

$$\widetilde{\boldsymbol{\beta}} = (\widetilde{\mathbf{X}}^T\widetilde{\mathbf{X}})^{-1}\widetilde{\mathbf{X}}^T\widetilde{\mathbf{Y}} = (\mathbf{X}^T\Sigma_{\boldsymbol{\epsilon}}^{-1}\mathbf{X})^{-1}\mathbf{X}^T\Sigma_{\boldsymbol{\epsilon}}^{-1}\mathbf{Y}$$

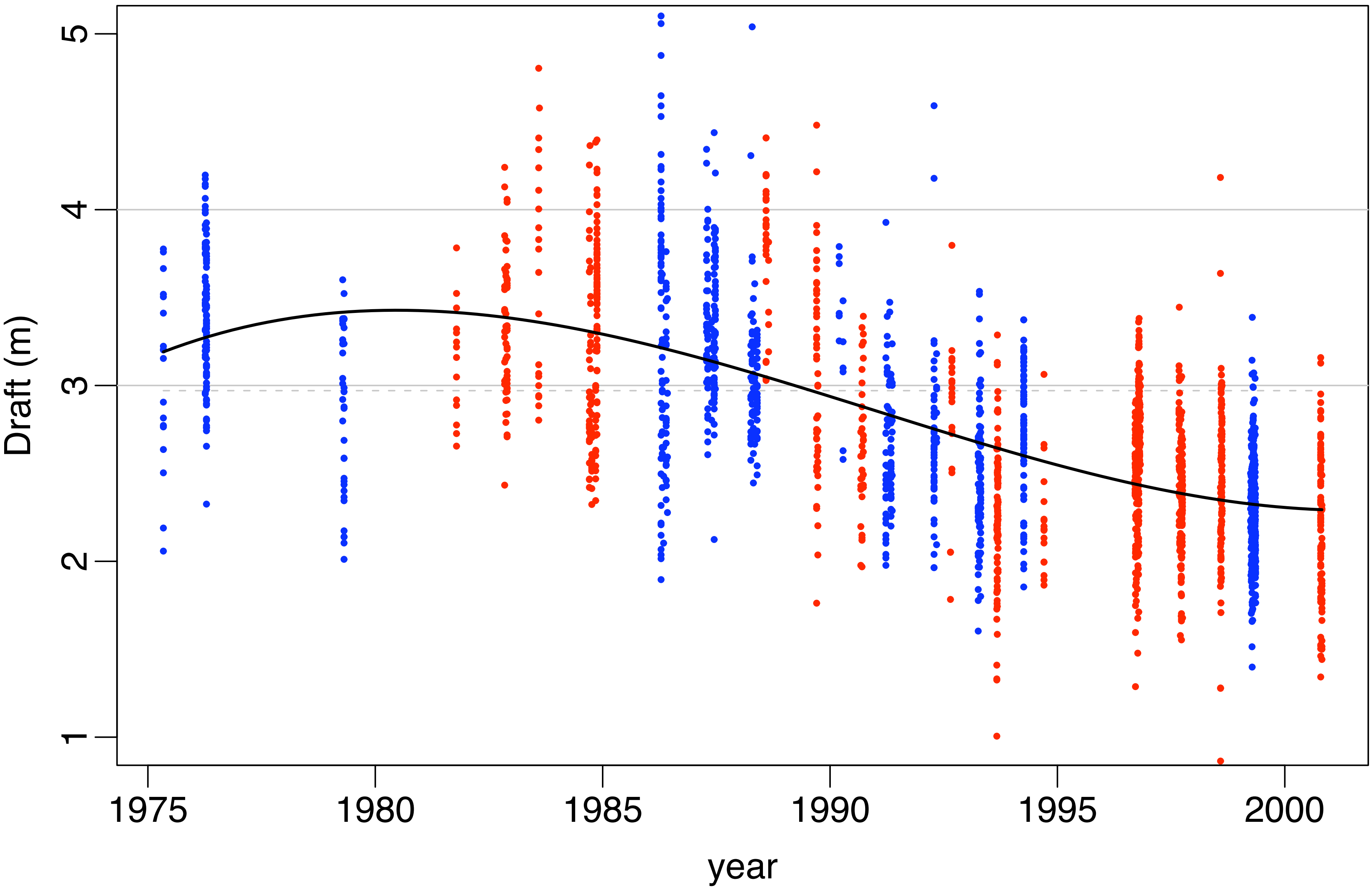
Fitting Multiple Regression Model: III

- in general, $\mathbf{Y}^T\mathbf{Y} \neq \widetilde{\mathbf{Y}}^T\widetilde{\mathbf{Y}} = \mathbf{Y}^T\Sigma_{\boldsymbol{\epsilon}}^{-1}\mathbf{Y}$; i.e., total sums of squares in original and transformed spaces are not equal
- portion of sum of squares explained by transformed model cannot be related directly to sum of squares for original model
- through study of measurement process, have estimate of variance of measurement errors in observations
- want to relate this variance estimate to sum of squares due to error in regression model; i.e., can unexplained variability be chaulked up to just measurement errors?
- cannot state an 'error budget' using transformed model
- standard deviation of OLS-estimated parameters only 5% greater on the average than those for GLS

Interannual Variation $I(t)$: I

- interannual variation $I(t)$ is a cubic polynomial
- residuals shown after addition to mean draft and fitted $I(t)$ (blue for January to June data, red for rest of year)
- change from 1981 to 2000 is -1.13 m
- steepest decline (-0.08 m/yr) occurred in 1991
- no recovery by 2000
- much fuller data set strengthens previous results (Rothrock *et al.*, 1999, and Tucker *et al.*, 2001)

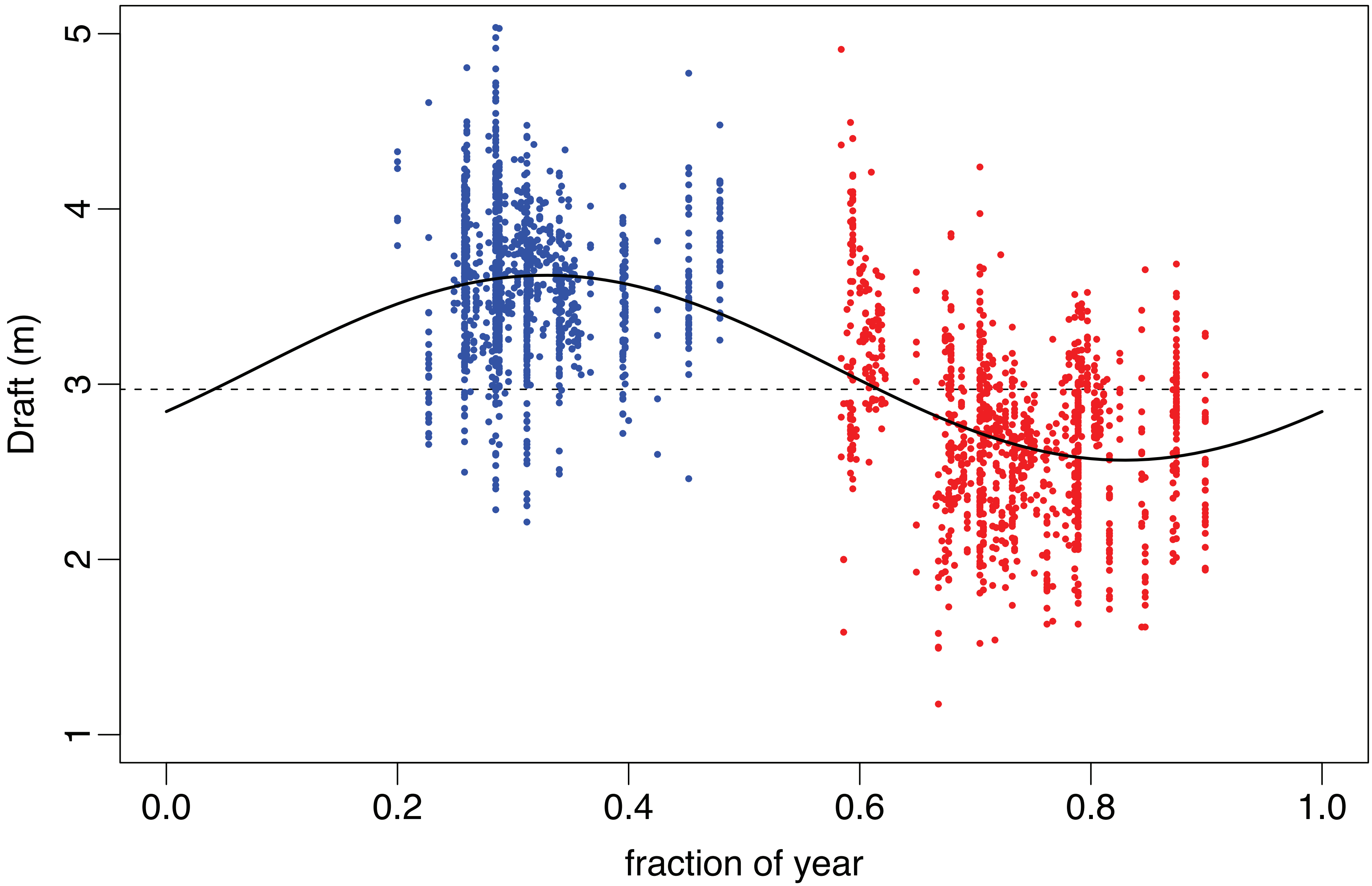
Interannual Variation $I(t)$: II



Annual Cycle $A(\tau)$: I

- annual cycle $A(\tau)$ is a sinusoid with a period of a year
- residuals shown after addition to mean draft and fitted $A(\tau)$ (blue for January to June data, red for rest of year)
- maximum (minimum) occurs on 30 April (30 October)
- peak-to-trough amplitude is 1.06 m, which is much larger than what would be expected from thermodynamic annual cycle of thickness of multiyear ice (≈ 0.43 m)
- sea-ice models predict asymmetric annual cycles, suggesting the need for harmonics, but data do not support this need (possibly due to preferential sampling during certain parts of the year)

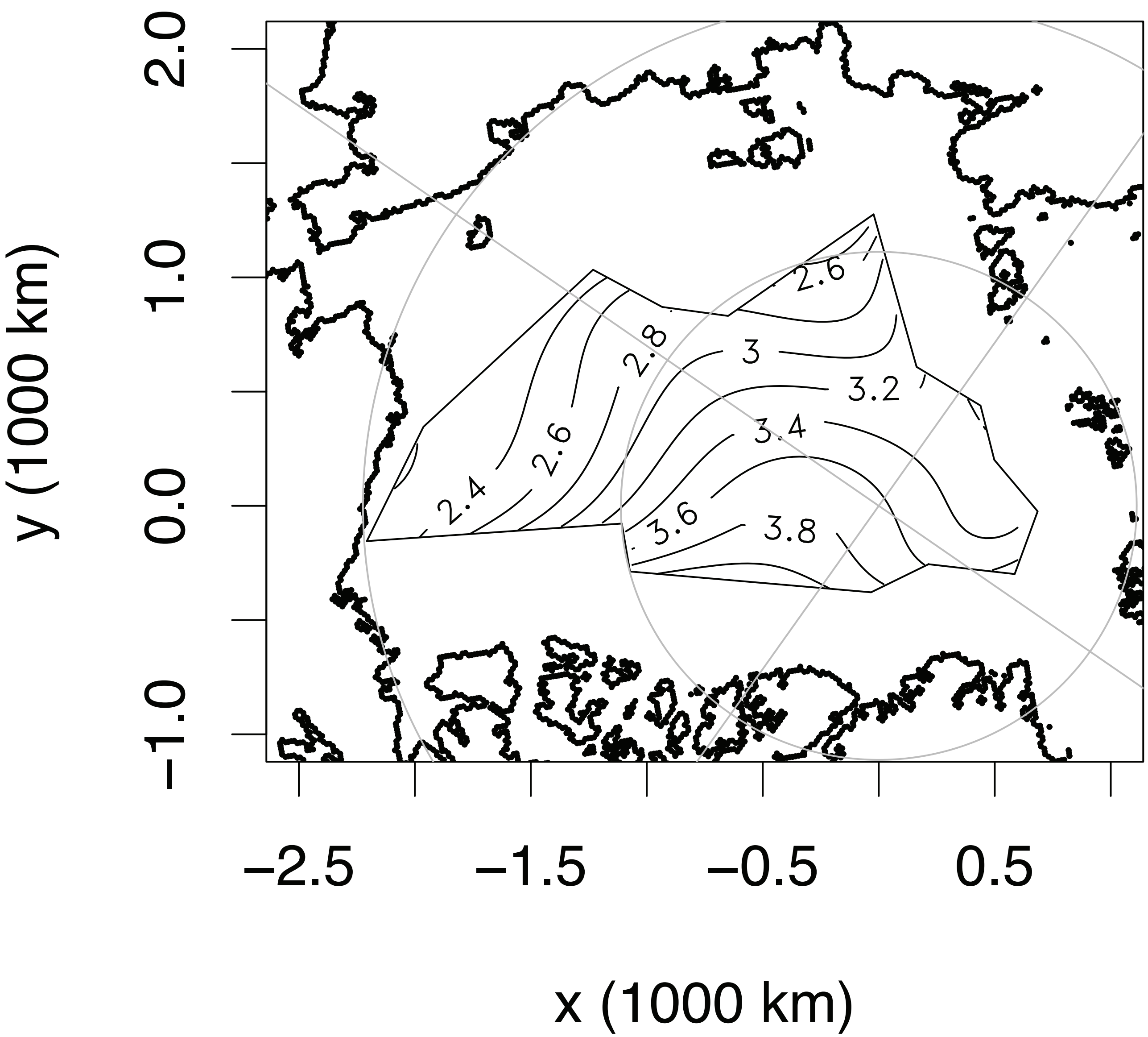
Annual Cycle $A(\tau)$: II



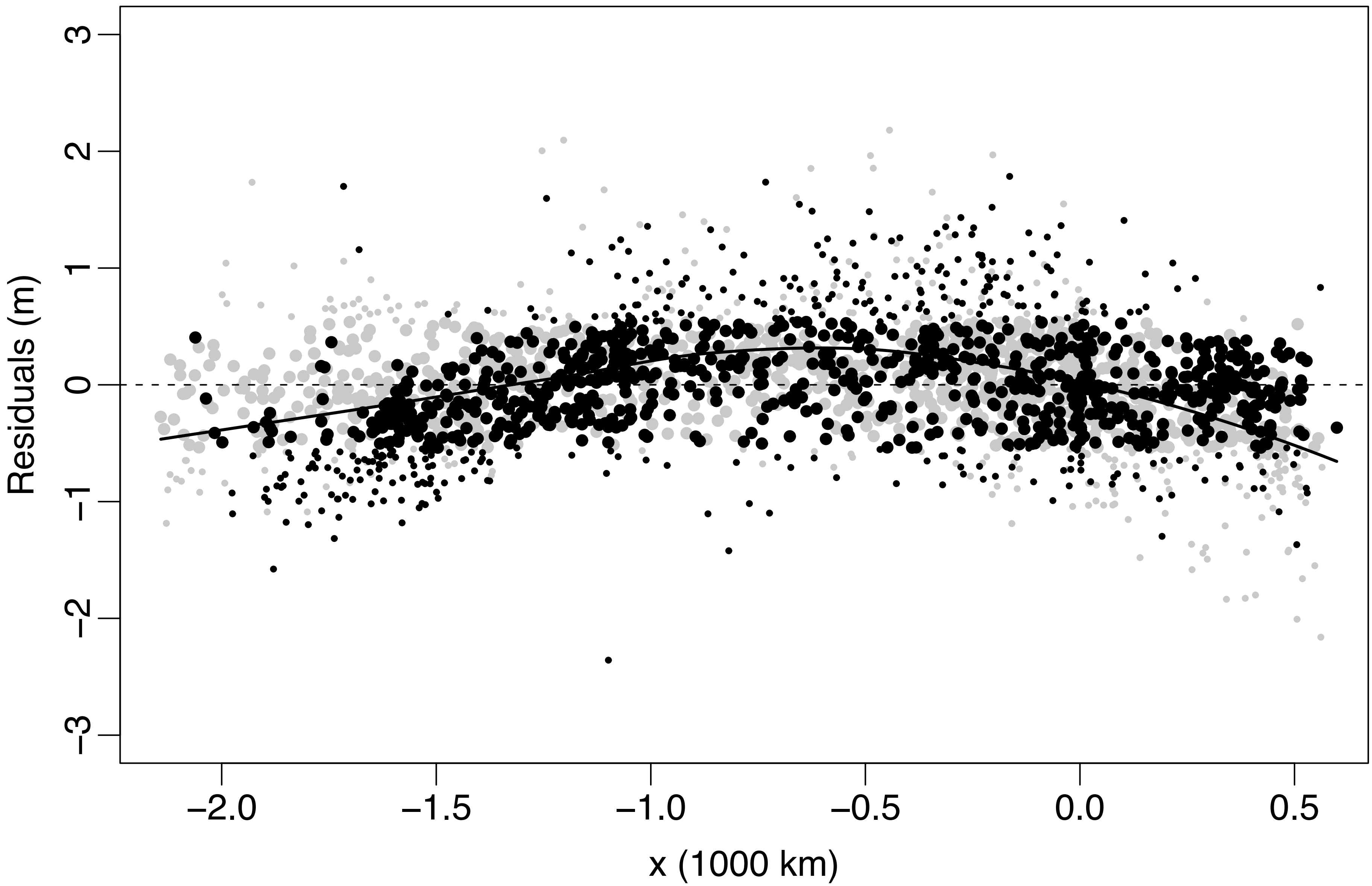
Spatial Field $S(\mathbf{x}_n)$: I

- spatial field $S(\mathbf{x}_n)$ is a fifth order polynomial in x & y
- draft varies from 2.2 m near Alaska to just over 4 m near Ellesmere Island
- need for polynomial of higher order than linear indicated by examination of residuals versus x – obvious structure remaining in linear fit (black for summer/fall, grey for winter/spring)
- corresponding plot of residuals versus y for linear model doesn't have same obvious structure

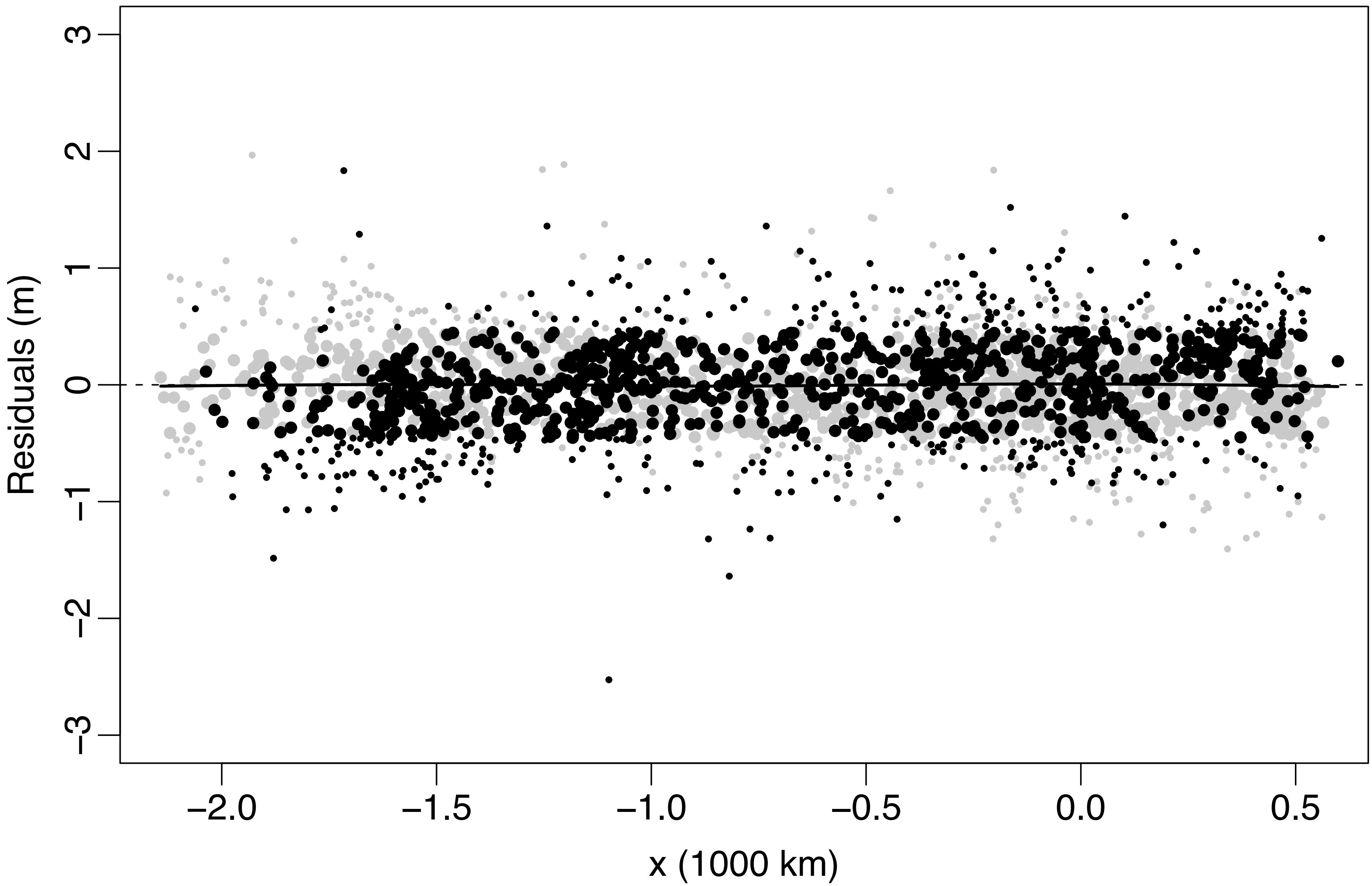
Spatial Field $S(\mathbf{x}_n)$: II



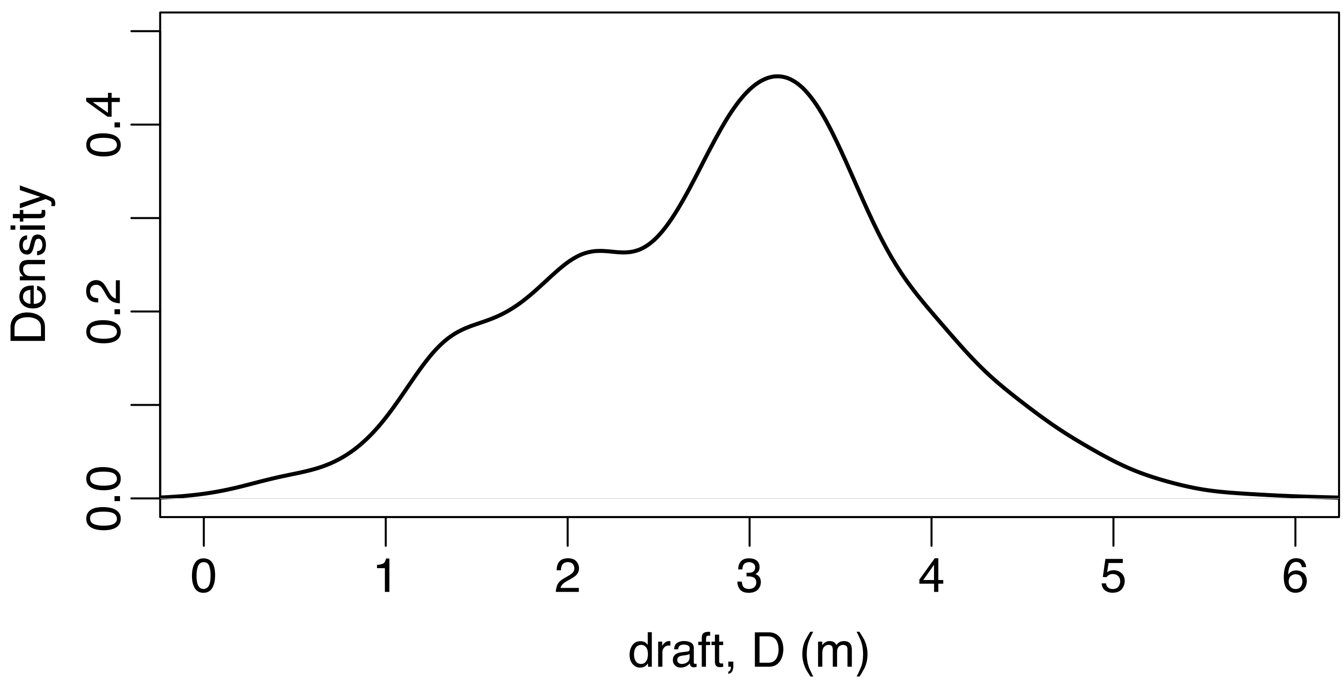
Spatial Field $S(\mathbf{x}_n)$: III



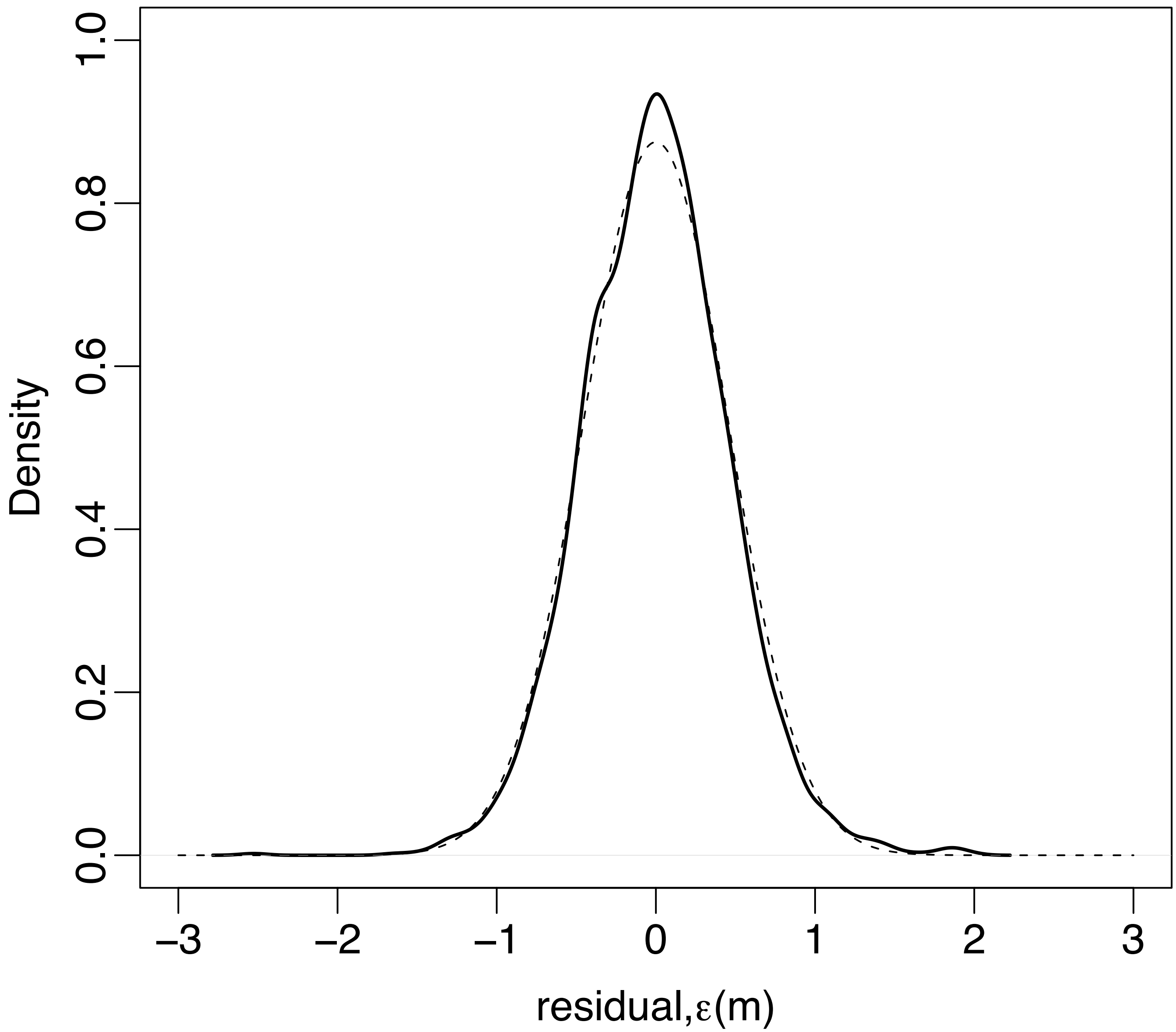
Spatial Field $S(\mathbf{x}_n)$: IV



PDF of Observations (SD = 0.99 m)



PDF of Residuals (SD = 0.46 m)



Concluding Remarks

- multiple regression model explains 79% of variance in data (standard deviation is 0.98 m)
- unexplained variance has standard deviation of 0.46 m
- estimated standard deviation of measurement errors is 0.25 m
- improvements ('polishing the cannon ball'):
  - relax assumption of a constant spatial field across time
  - estimate δ from spatial data, not from profiles

Main References

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