Arctic Sea-Ice Thickness: Evidence of Decline from a Multiple Regression Analysis Incorporating Long-Range Dependence

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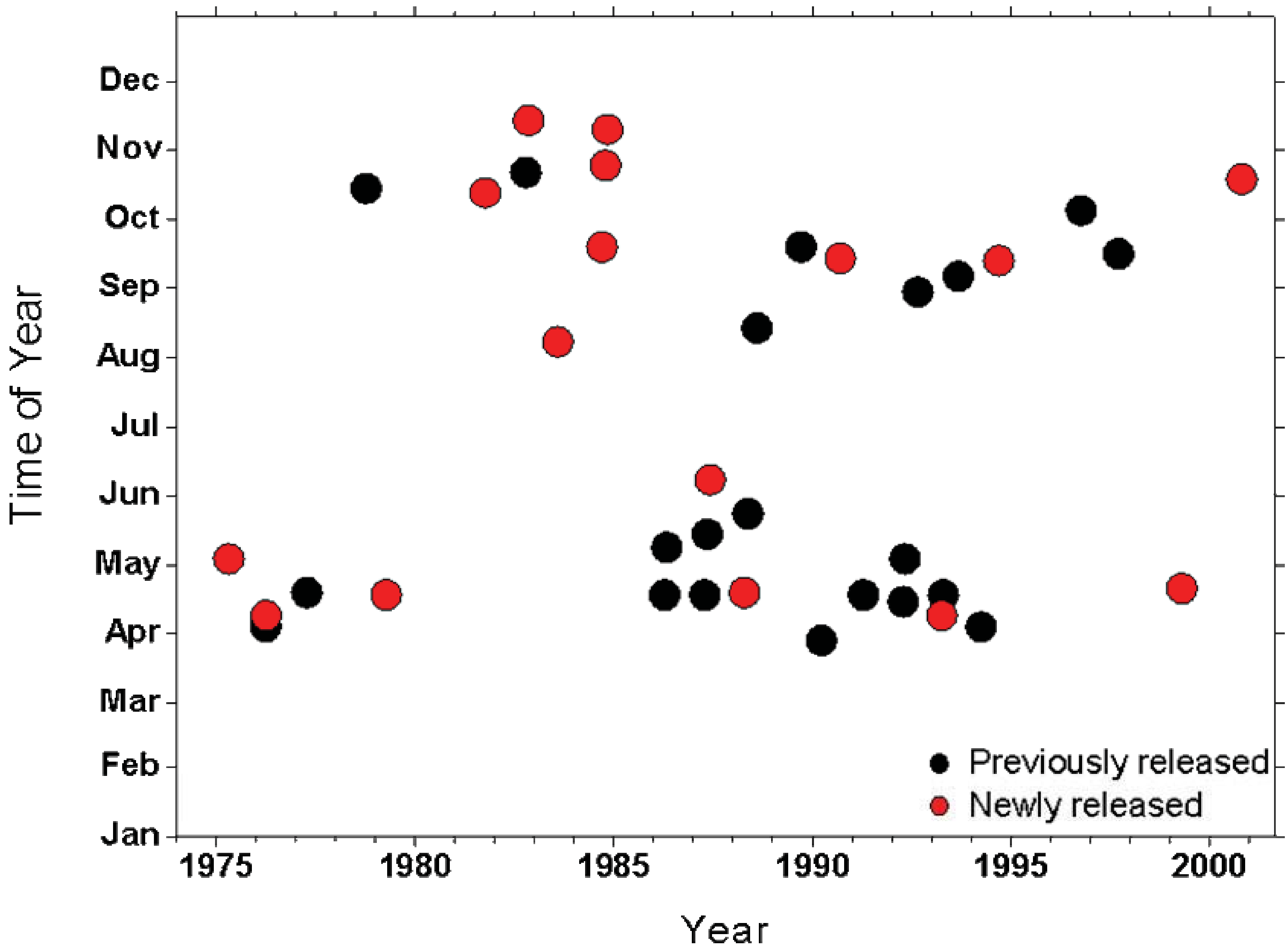
overheads for talk available at

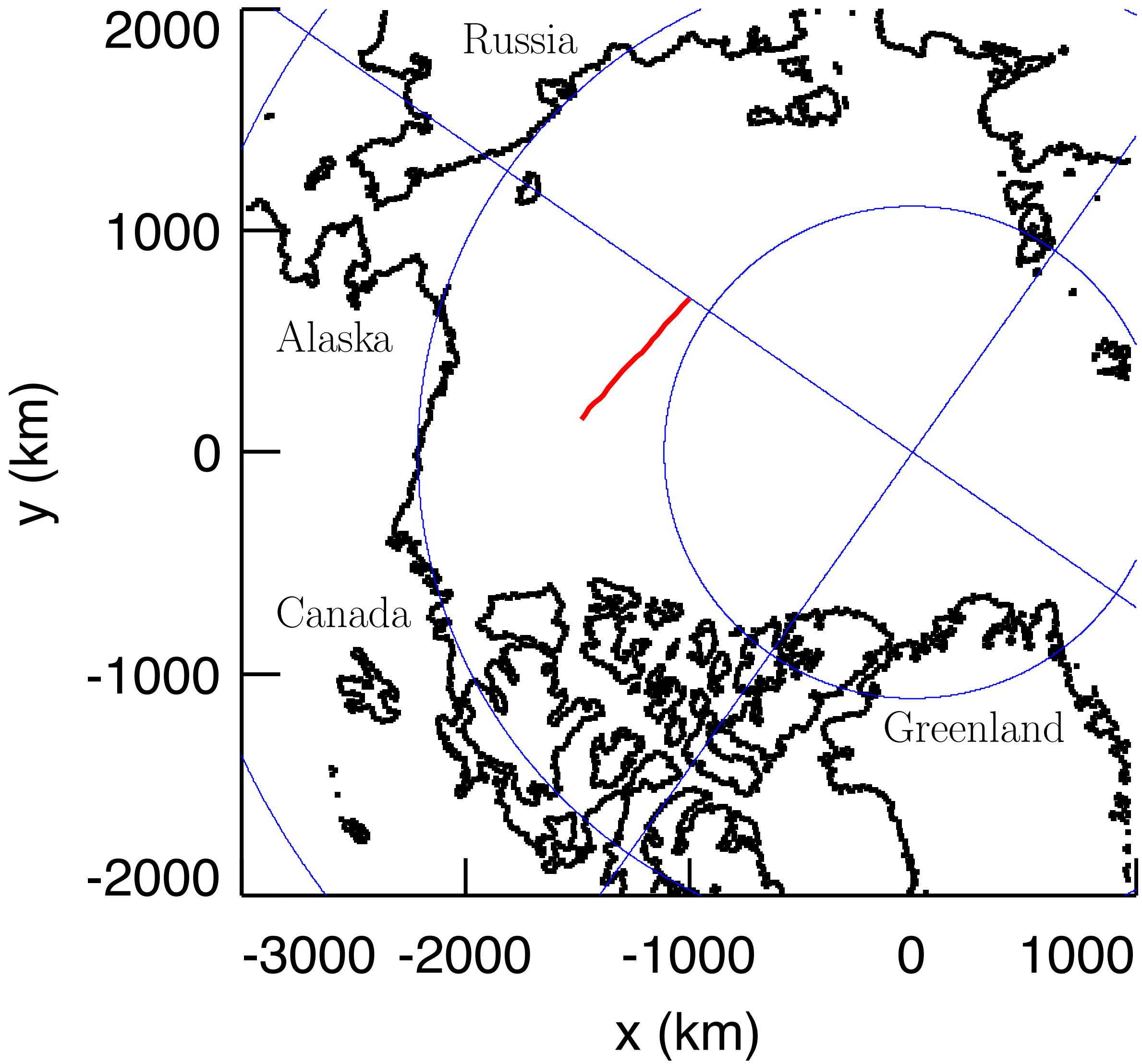
`http://faculty.washington.edu/dbp/talks.html`

Background

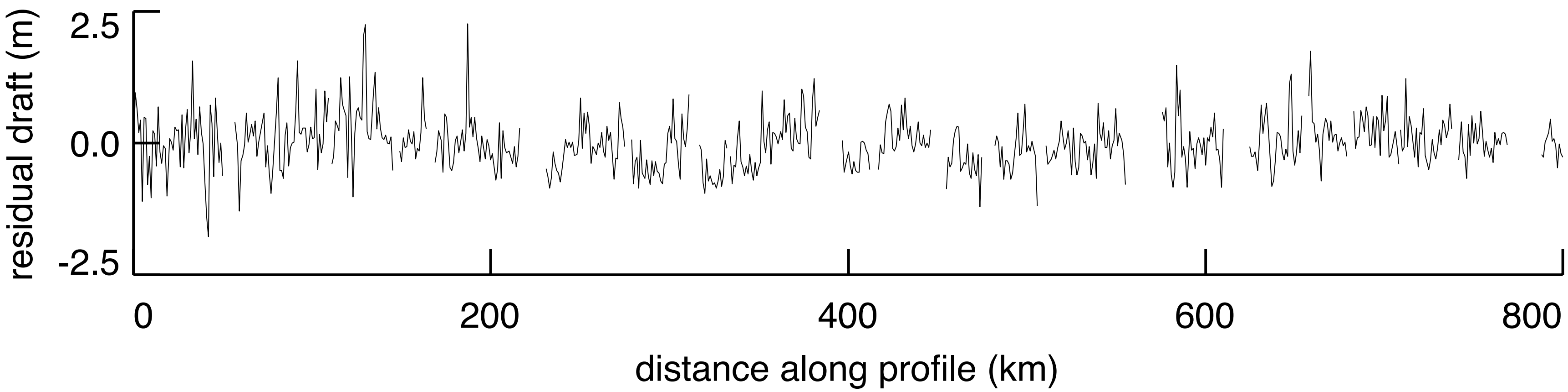
- scientific question of interest: has the average thickness of Arctic sea ice declined significantly over the past 30 years?
- thickness can be deduced from measurements of draft (submerged portion of sea ice)
- draft measured using upward-looking sonars on submarines
- our analysis differs from previous ones by use of
  - a new statistical model for draft measurements
  - newly archived data for submarine cruises from 1975 to 2001 (almost doubling the amount of available data)


Time of Year
Dec
Nov
Oct
Sep
Aug
Jul
Jun
May
Apr
Mar
Feb
Jan
1975
1980
1985
1990
1995
2000
Year
Previously released
Newly released

Map of Arctic Region with One Submarine Track



Residual Draft Profile along One Submarine Track



- above comes from longest track from 1997 cruises
- draft measured every meter & used to form 1 km averages
- linear trend subtracted to form residual draft profile $\overline{H}_{1,n}$ (for profiles less than 200 km or so, need only subtract sample mean)
- residuals approximately Gaussian (room for improvement?)
- note: lots of gaps in draft profile (632 averages over 803 km)

Statistical Modeling of Draft Profiles

- simple model of independence for $\overline{H}_{1,n}$ along profile not viable (e.g., adjacent measurements $\overline{H}_{1,n}$ and $\overline{H}_{1,n+1}$ are correlated)
- assume $\overline{H}_{1,n}$ is a realization of a zero mean Gaussian stationary process (a 'time' series with distance replacing 'time')
- process fully characterized by its variance σ_1^2 and autocorrelation sequence $\rho_d \equiv E\{\overline{H}_{1,n}\overline{H}_{1,n+d}\}/\sigma_1^2$
- consider two simple parametric forms for ρ_d corresponding to
  - first-order autoregressive (AR(1)) process
  - fractionally differenced (FD) process

First-Order Autoregressive (AR(1)) Processes

- process satisfies

$$\overline{H}_{1,n} = \phi \overline{H}_{1,n-1} + \epsilon_n = \sum_{j=0}^{\infty} \psi_j \epsilon_{n-j} \text{ with } \psi_j = \phi^j,$$

  where $|\phi| < 1$, and ϵ_n is Gaussian white noise with variance σ_ϵ^2
- have $\rho_d = \phi^{|d|}$
- related to a first-order stochastic differential equation with 'correlation time' dictated by ϕ
- widely used in climate research to model time series
- given gappy draft profiles, can estimate ϕ and σ_ϵ^2 using maximum likelihood (Jones, 1980), yielding $\hat{\phi} \doteq 0.36$ (± 0.04)

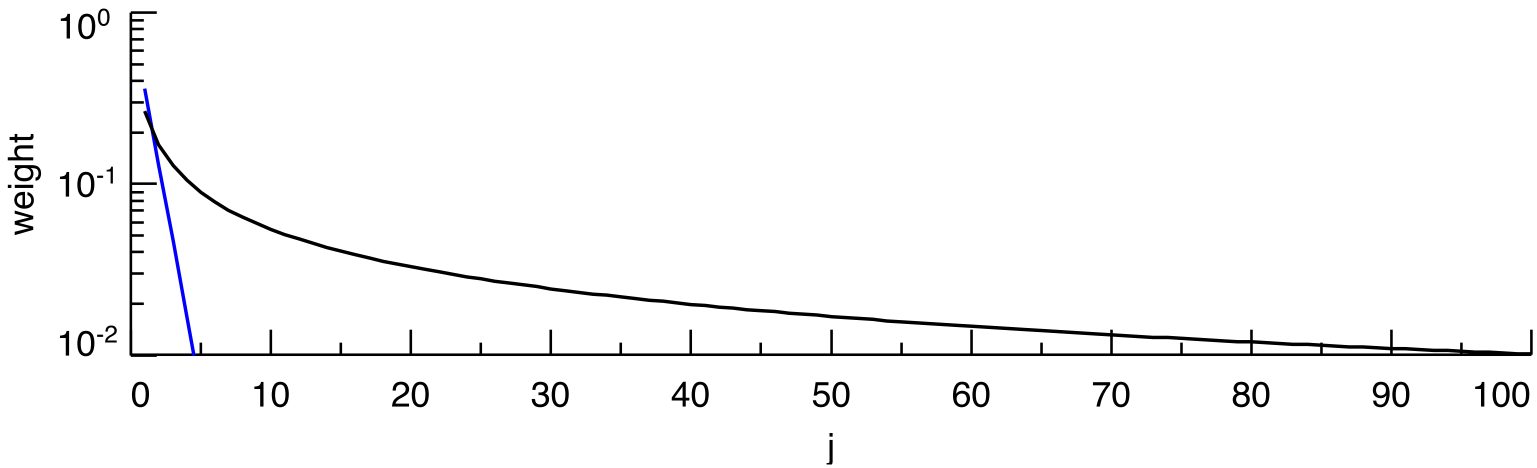
Fractionally Differences (FD) Processes

- process satisfies

$$\overline{H}_{1,n} = \sum_{j=0}^{\infty} \psi_j \epsilon_{n-j} \text{ with } \psi_j = \frac{\Gamma(j+\delta)}{\Gamma(j+1)\Gamma(\delta)},$$

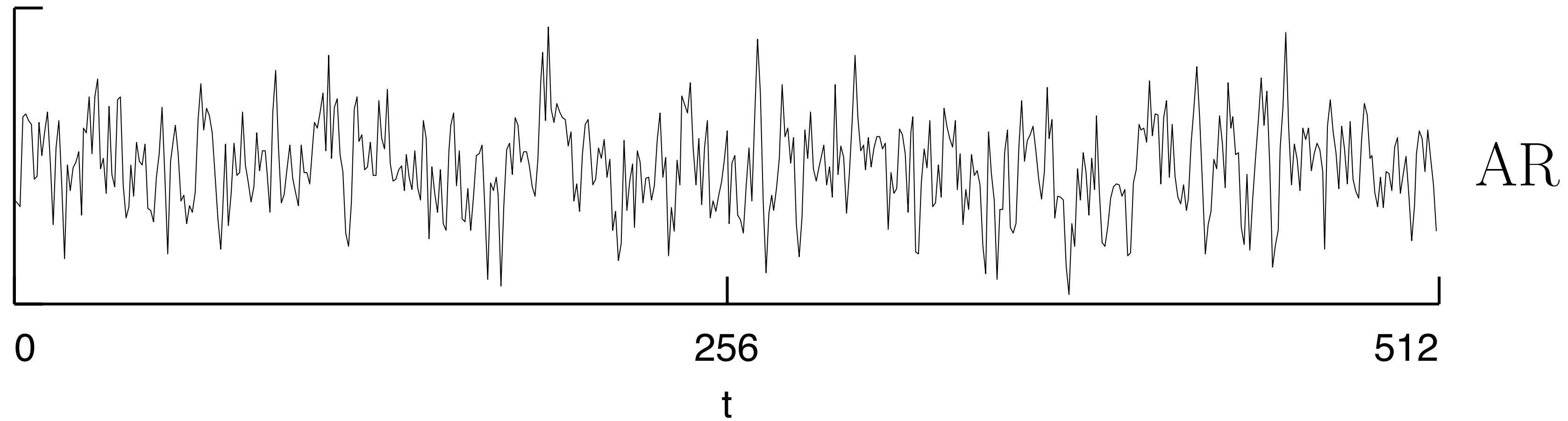
  where $|\delta| < 1/2$, and ϵ_n is as before
- have $\rho_d \approx C|d|^{2\delta-1}$ (much slower decay rate than for AR(1))
- related to average of many first-order stochastic differential equations with different correlation times
- popular model for 'long-range' (or 'long-memory') dependence
- given gappy profiles, can estimate δ and σ_ϵ^2 using maximum likelihood (Palma & Chan, 1997), yielding $\hat{\delta} \doteq 0.27$ (± 0.03)
- Q: how do these two models compare?

Qualitative Comparison I: ψ Weights



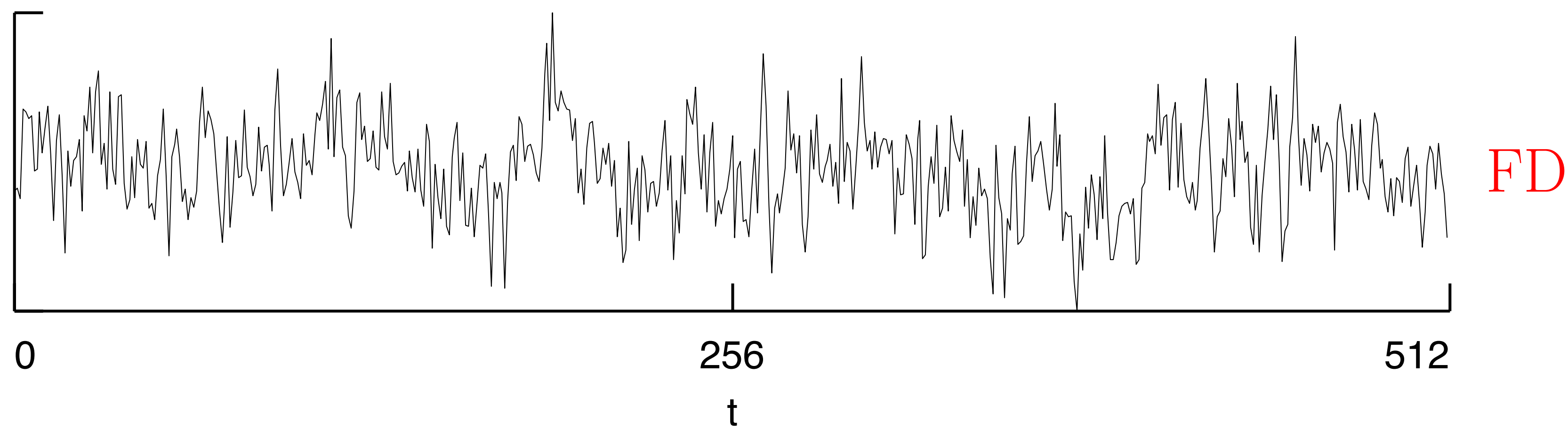
- weights ψ_j used to create AR with $\phi = 0.36$ (blue curve) and FD with $\delta = 0.27$ (black curve) processes from a weighted average of white noise

Qualitative Comparison II: Simulated Draft Profiles



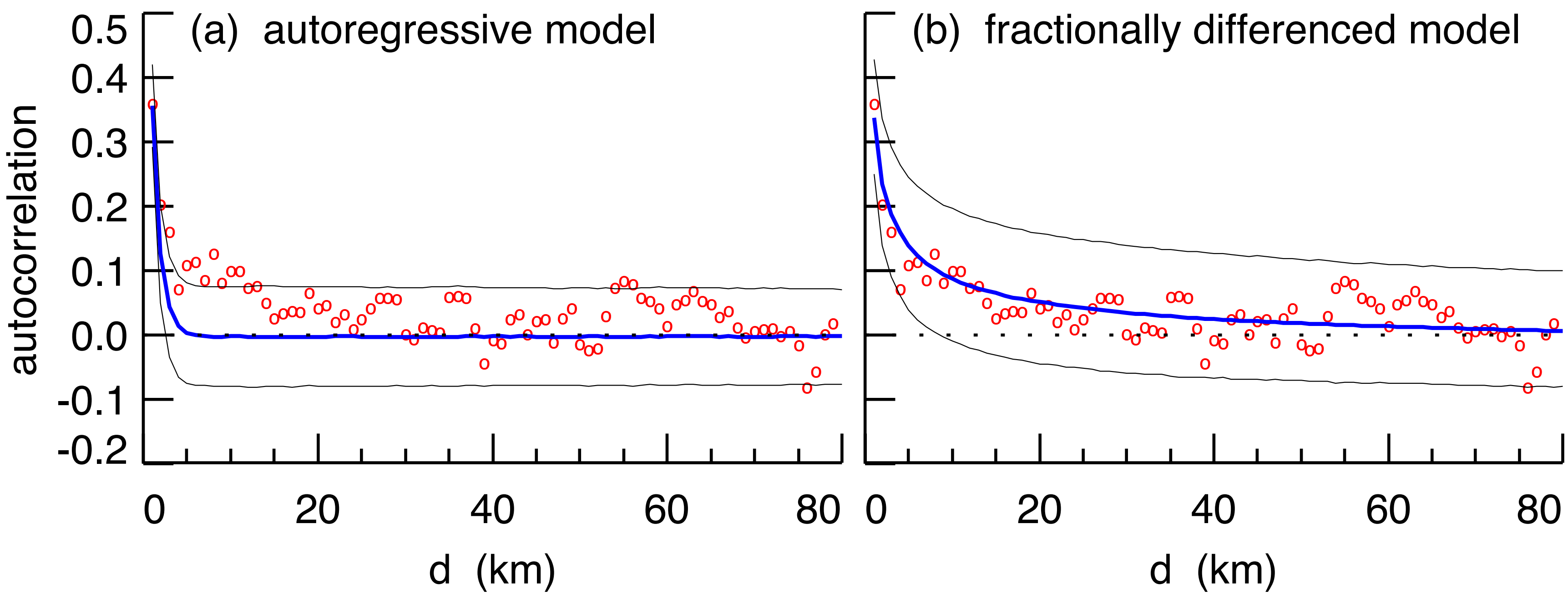
- consider simulated AR ($\phi = 0.36$) & FD ($\delta = 0.27$) profiles
- simulations formed using circulant embedding technique that maps same 1024 IID Gaussian deviates to both profiles

Qualitative Comparison II: Simulated Draft Profiles



- consider simulated AR ($\phi = 0.36$) & FD ($\delta = 0.27$) profiles
- simulations formed using circulant embedding technique that maps same 1024 IID Gaussian deviates to both profiles

Third Comparison: Autocorrelation Sequences



- sample (circles) and theoretical sequences (middle curves)
- upper and lower curves are 95% pointwise confidence intervals for ρ_d assuming relevant model (AR or FD)

Fourth Comparison: Variance of Sample Means

- given $\overline{H}_{1,n}, n = 0, \ldots, N-1$, consider statistical properties of length L averages

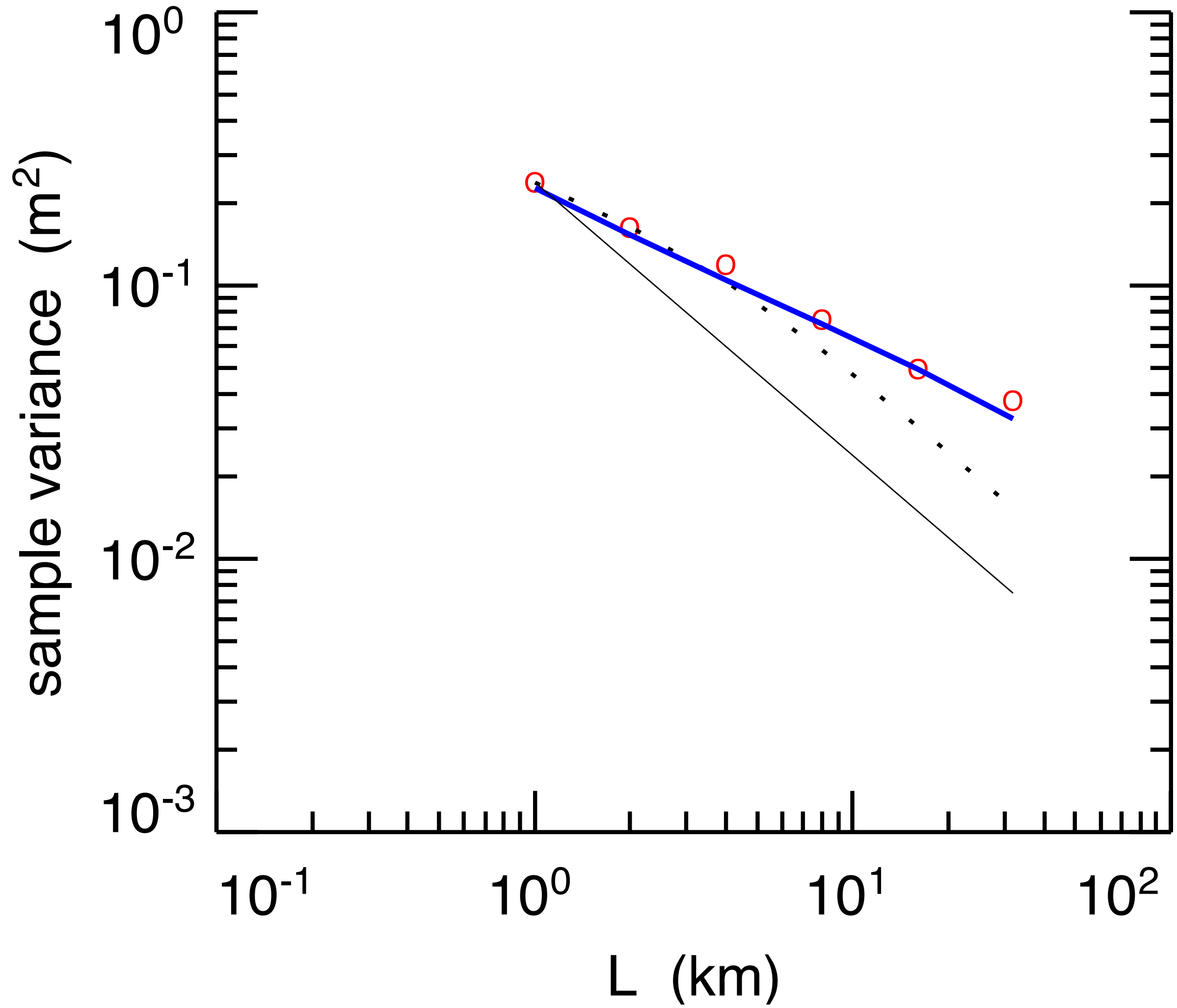
$$\overline{H}_{L,m} = \frac{1}{L}\sum_{l=0}^{L-1} \overline{H}_{1,mL+l}, \;\; m = 0, 1, \ldots, \lfloor N/L \rfloor - 1$$

- let $\sigma_L^2 = \operatorname{var}\{\overline{H}_{L,m}\}$
- for AR and FD models, have

$$\sigma_L^2 \approx \sigma_1^2 \times \frac{1+\phi}{1-\phi} \times L^{-1} \text{ and } \sigma_L^2 \approx \sigma_1^2 \times \frac{\Gamma(1-\delta)}{(2\delta+1)\Gamma(1+\delta)} \times L^{-1+2\delta}$$

- can compare sample estimates $\hat{\sigma}_L^2$ with $E\{\hat{\sigma}_L^2\}$ for various L

Sample $\hat{\sigma}_L^2$ (Circles) and Theoretical σ_L^2 versus L

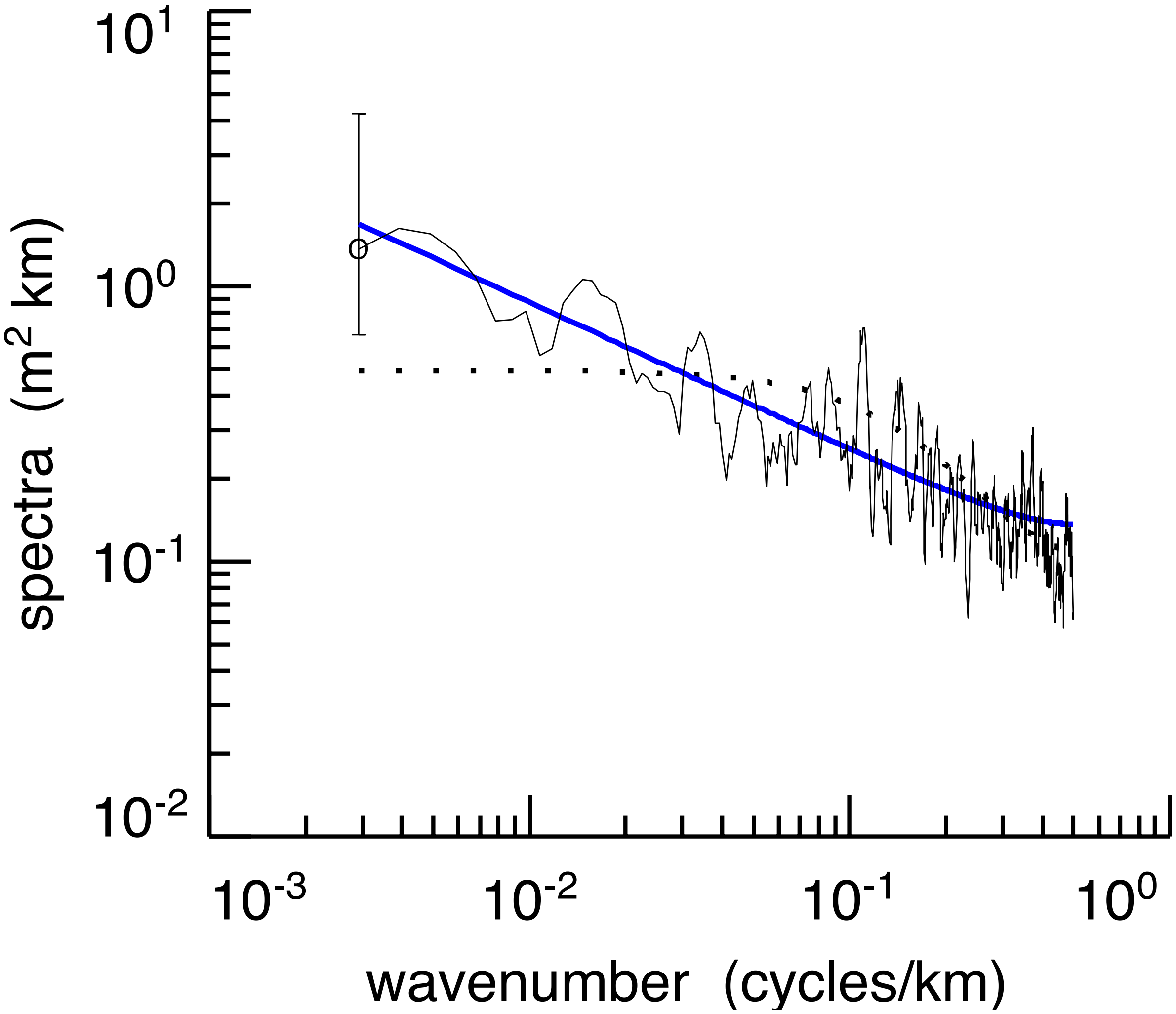


blue: FD

dotted: AR(1)

solid: white noise

Fifth Comparison: Sample and Theoretical Spectra

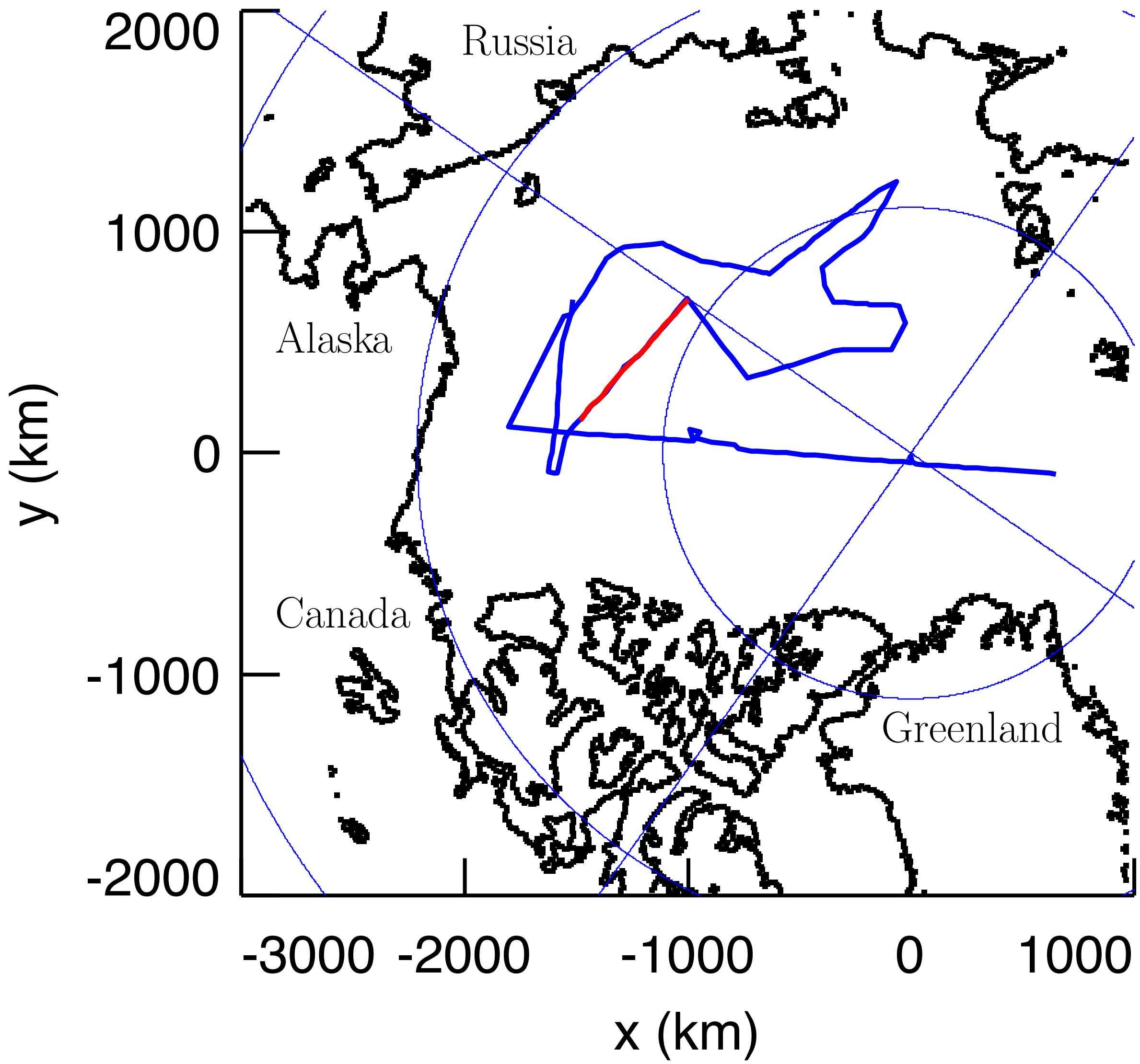


blue: FD

dotted: AR(1)

solid: sample

Map of Arctic Region with Tracks Taken in 1997



Spatial Model for One Kilometer Averages

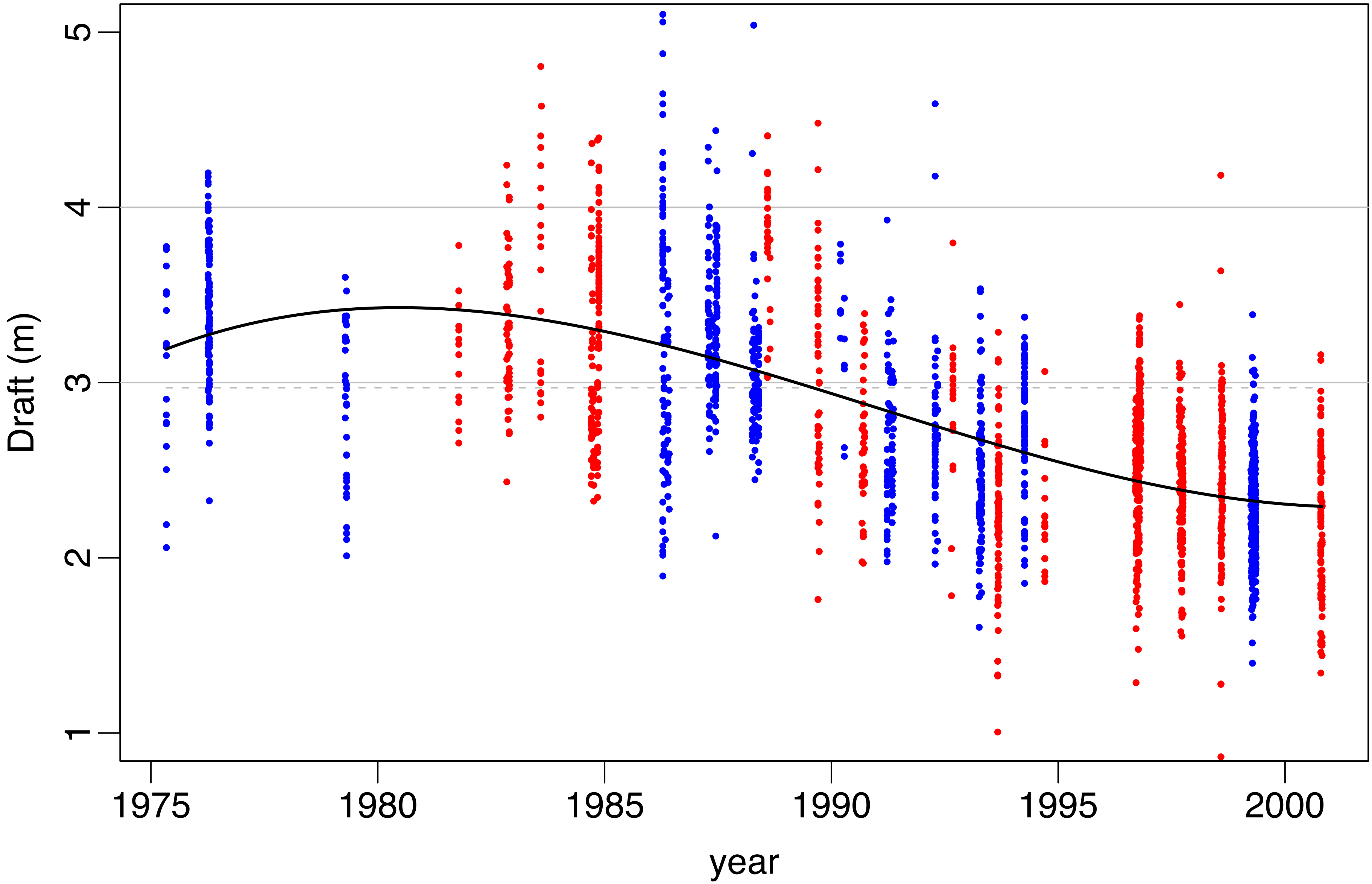
- analysis of additional profiles in 1997 and other years indicates FD model with $\delta = 0.27$ is generally viable
- can regard $\overline{H}_{1,n}$ as samples from a stationary and isotropic two-dimensional (2D) random process with covariances given by

$$\text{cov}\,\{\overline{H}_{1,\mathbf{x}_n}, \overline{H}_{1,\mathbf{x}_n+\mathbf{d}}\} \equiv \sigma_1^2 \times \frac{\Gamma(|\mathbf{d}| + \delta)\Gamma(1-\delta)}{\Gamma(|\mathbf{d}| + 1 - \delta)\Gamma(\delta)},$$

  where $\mathbf{x}_n$ is a 2D vector indicating the location of the 1 km average, and $\mathbf{d}$ is an arbitrary 2D vector

Multiple Regression Model

- let $\overline{H}_{1,\mathbf{x}_n,t}$ represent average of 1 km measurements taken at location $\mathbf{x}_n$ and time t ($\mathbf{x}_n = [0, 0]$ = Pole & $t \in [1975, 2001]$)
- let τ represent the time of year (i.e., $\tau = t \bmod 1$)
- assume model $\overline{H}_{1,\mathbf{x}_n,t} = C + I(t) + A(\tau) + S(\mathbf{x}_n) + \epsilon_{\mathbf{x}_n,t}$, where
  - C is a constant
  - $I(t)$ is the interannual variation (cubic polynomial)
  - $A(\tau)$ is the annual cycle (cosine, unknown amplitude & phase)
  - $S(\mathbf{x}_n)$ is the spatial field (fifth order polynomial)
  - $\epsilon_{\mathbf{x}_n,t}$ is an error term dictated by FD model within a given season (different seasons/years assumed independent)
- used generalized least squares to fit model, with order of polynomials dictated by t-tests



Draft (m)
5
4
3
2
1
1975
1980
1985
1990
1995
2000
year

Interannual Variation $I(t)$

- residuals shown about fitted $I(t)$ (blue for January to June data, red for rest of year)
- change from 1981 to 2000 is -1.13 m
- steepest decline (-0.08 m/yr) occurred in 1991
- no recovery by 2000
- much fuller data set strengthens previous results (Rothrock *et al.*, 1999, and Tucker *et al.*, 2001)

Concluding Remarks

- multiple regression model explains 79% of variance in data (standard deviation is 0.98 m)
- unexplained variance has standard deviation of 0.46 m
- estimated standard deviation of measurement errors is 0.25 m
- improvements ('polishing the cannon ball'):
  - relax assumption of a constant spatial field across time
  - estimate δ from spatial data, not from profiles

Main References (1–3) & Additional Ones (4–7)

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7. W. B. Tucker, J. W. Weatherly, D. T. Eppler, L. D. Farmer and D. L. Bentley (2001), 'Evidence for Rapid Thinning of Sea Ice in the Western Arctic Ocean at the End of the 1980s,' *Geophysical Research Letters*, **28**, pp. 2851–2854.