

# Real-time Modelling of Tsunami Data

Applied Physics Laboratory  
Department of Statistics  
University of Washington  
Seattle, Washington, USA

<http://faculty.washington.edu/dbp>

collaborative effort with NOAA Center for Tsunami Research

## Background - I

- even before disastrous Sumatra tsunami in December 2004, destructive potential of earthquake-generated tsunamis was well-known
- due to rate at which a tsunami advances across the ocean, possible to lessen its effect on some coastal communities through advance warnings
- in USA, warning centers in Alaska and Hawaii are responsible for issuing timely bulletins about impending tsunamis
- seismometers give first indication of a potential tsunami event
  - starting time of earthquake event
  - location of epicenter
  - initial estimate of magnitude

## Background - II

- seismic information alone is not enough to accurately forecast potential impact of tsunamis
- led to development of Deep-ocean Assessment and Reporting of Tsunamis (DART<sup>®</sup>) buoys

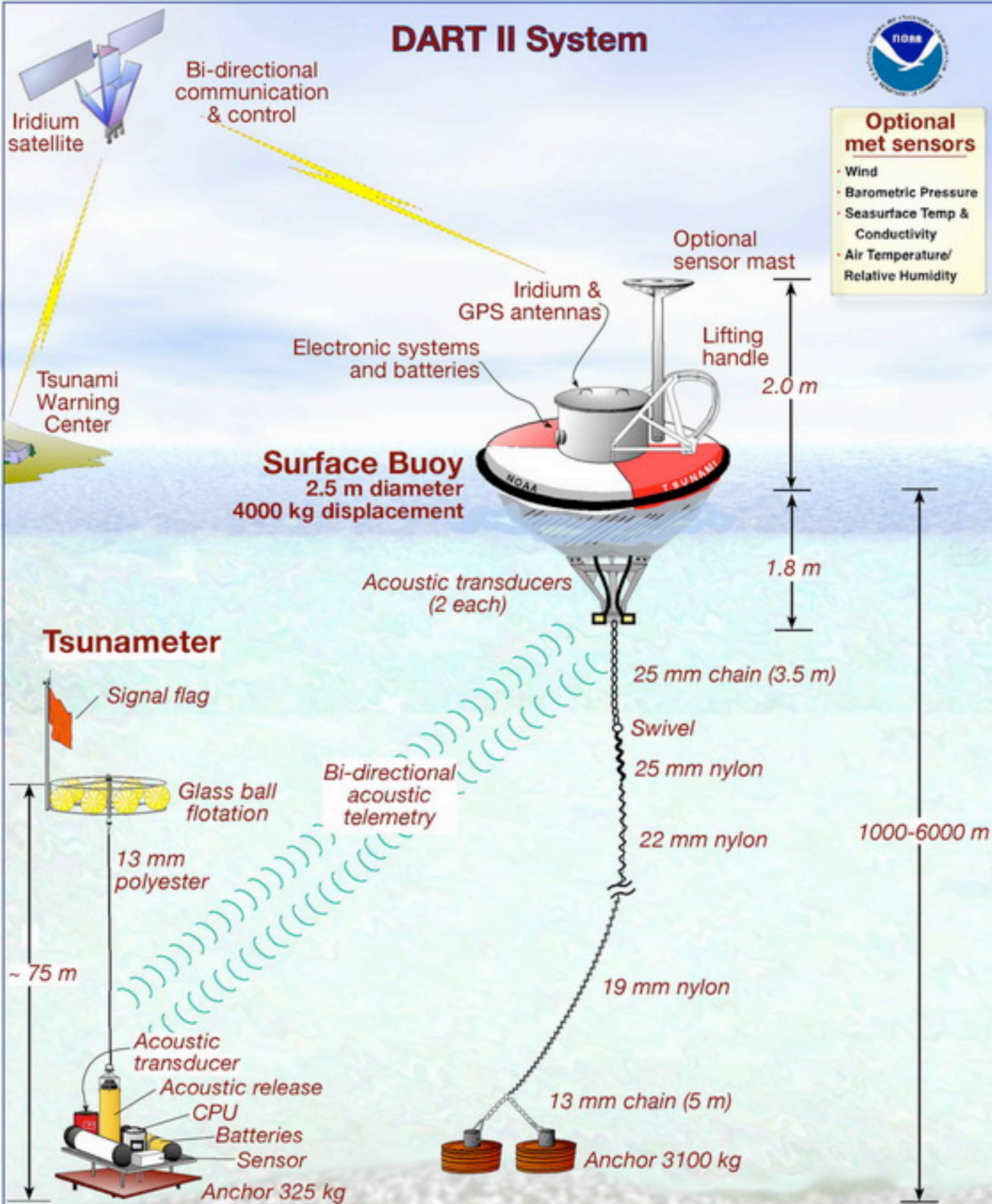


# DART II System



## Optional met sensors

- Wind
- Barometric Pressure
- Seasurface Temp & Conductivity
- Air Temperature/ Relative Humidity



## Background - III

- prior to December 2004, there were six DART<sup>®</sup> buoys deployed in Pacific
- since then, many more have been added
- mindful of Percival's First Law

‘Real-time computer demonstrations are doomed to fail!’

let's look at network of buoys and recent data by going to

<http://www.ndbc.noaa.gov/dart.shtml>

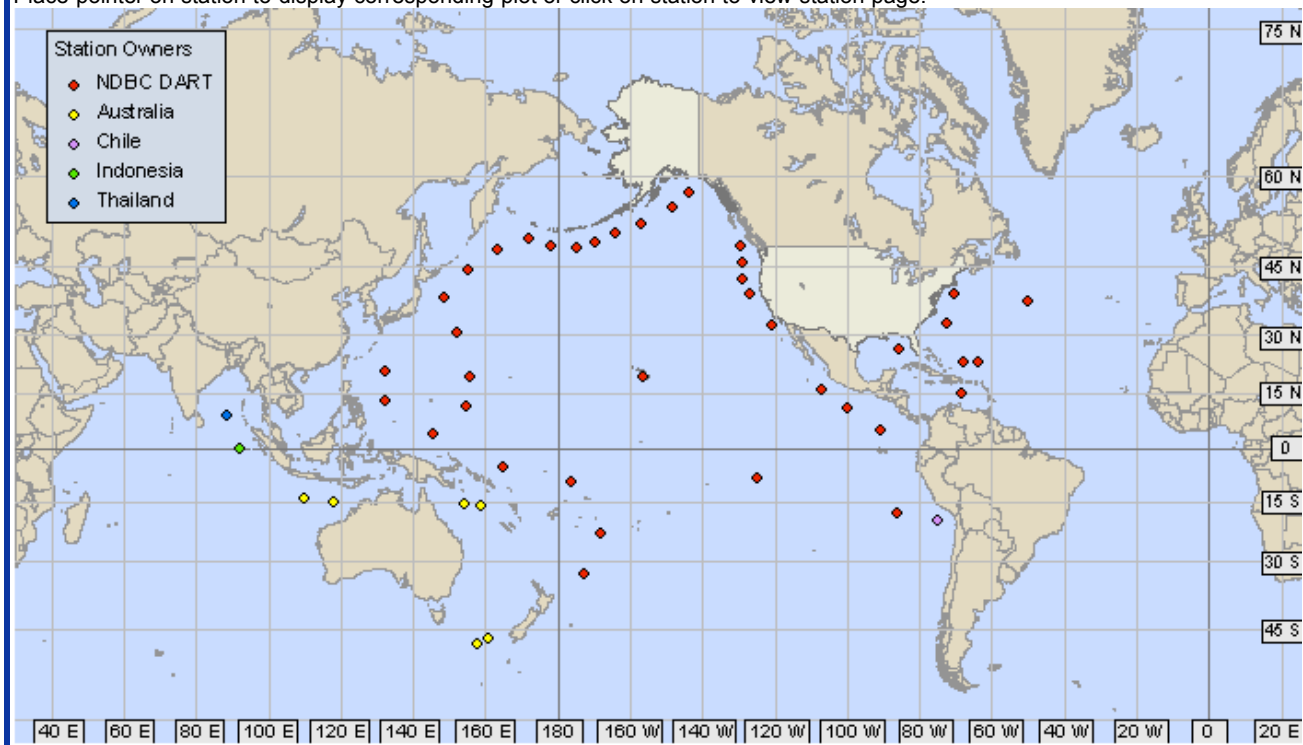
and movie of Nov. 2006 Kuril Island event downloadable from

<http://nctr.pmel.noaa.gov/kuril20061115.html>

Place pointer on station to display corresponding plot or click on station to view station page.

Station Owners

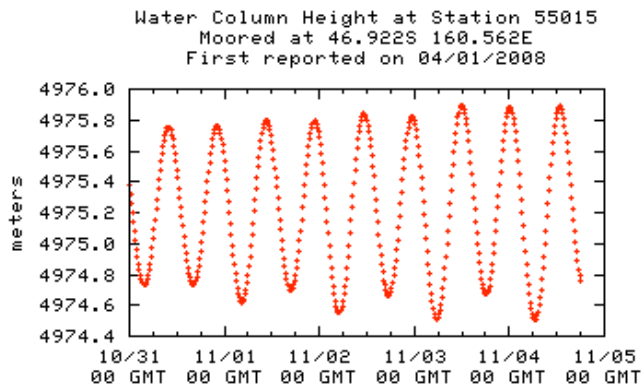
- ◆ NDBC DART
- ◆ Australia
- ◆ Chile
- ◆ Indonesia
- ◆ Thailand



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40 E 60 E 80 E 100 E 120 E 140 E 160 E 180 160 W 140 W 120 W 100 W 80 W 60 W 40 W 20 W 0 20 E



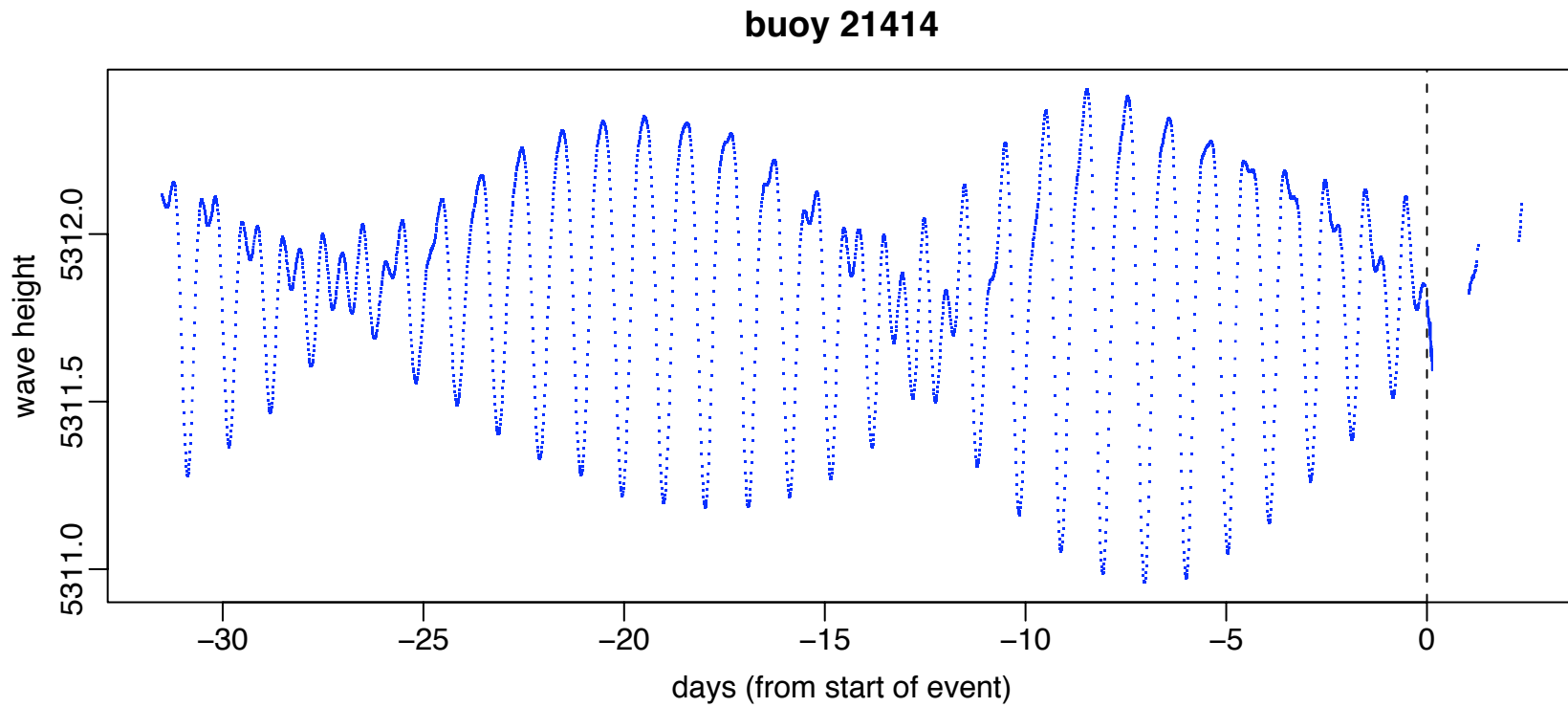
## Format of Data

- buoy records one pressure measurement every 15 sec (represents an average over 15 sec)
- **15-min stream**: when nothing is going on, buoy transmits one 15-sec measurement every 15 min
- **15-sec stream**: when triggered by a seismic event, transmits all recorded measurements
- **1-min stream**: during most of tsunami event, transmits averages of four consecutive 15-sec measurements

## Short-term Inundation Forecast for Tsunamis (SIFT)

- SIFT is a computer application that uses DART<sup>®</sup> data and precomputed geophysically-based predictions to assess magnitude of tsunami while tsunami event is evolving
- SIFT currently used at warning centers in Alaska and Hawaii, but is still under development at NOAA Center for Tsunami Research
- statistical issues involved in processing data within SIFT
  - signal extraction: need to remove tidal component before precomputed predictions can be used
  - assessment of uncertainty: need error bars on estimates of magnitude of tsunami
  - variable selection: help operators select predictors

# Buoy 21414 Data for Nov. 2006 Kuril Islands Event



## Approaches for Signal Extraction (Detiding)

- three approaches, each with certain strengths and weaknesses
  1. empirical orthogonal functions (Tolkova, 2009)
  2. localized polynomial fits
  3. two-stage approach, the second of which involves Kalman filter/smoothing
- will concentrate on third approach

## Model for Data with Tides

- assume measured data at time  $t$  can be expressed as

$$x(t) = \mu(t) + \sum_{l=1}^L c_l(t) + \epsilon(t)$$

where

- $\mu(t) + \sum_l c_l(t)$  represents tides;
- $\mu(t)$  is ‘unpredictable’ tidal component (has slowly varying mean level);
- $c_l(t) = C_l \cos(2\pi f_l t + \phi_l)$  is  $l$ th part of ‘predictable’ tidal component (sinusoid with amplitude  $A_l$ , frequency  $f_l$  and phase  $\phi_l$ ); and
- $\epsilon(t)$  is detided data

## Model for 15-Min Stream Prior to Tsunami: I

- let  $t = 0$  denote starting time (in days) of tsunami events
- can write  $C_l \cos(2\pi f_l t + \phi_l) = A_l \cos(2\pi f_l t) + B_l \sin(2\pi f_l t)$ , so assume data  $x(t)$  at times  $t < 0$  can be expressed as

$$x(t) = \mu + \sum_{l=1}^L A_l \cos(2\pi f_l t) + B_l \sin(2\pi f_l t) + \epsilon_t$$

where

- $\mu$  is an unknown overall mean level;
  - $f_l$  is a known tidal frequency;
  - $A_l$  and  $B_l$  are unknown amplitudes; and
  - $\epsilon_t$  is a residual term
- use linear least squares to estimate  $\mu$ ,  $A_l$  and  $B_l$  – denote these estimates as  $\hat{\mu}$ ,  $\hat{A}_l$  and  $\hat{B}_l$

## Model for 15-Min Stream Prior to Tsunami: II

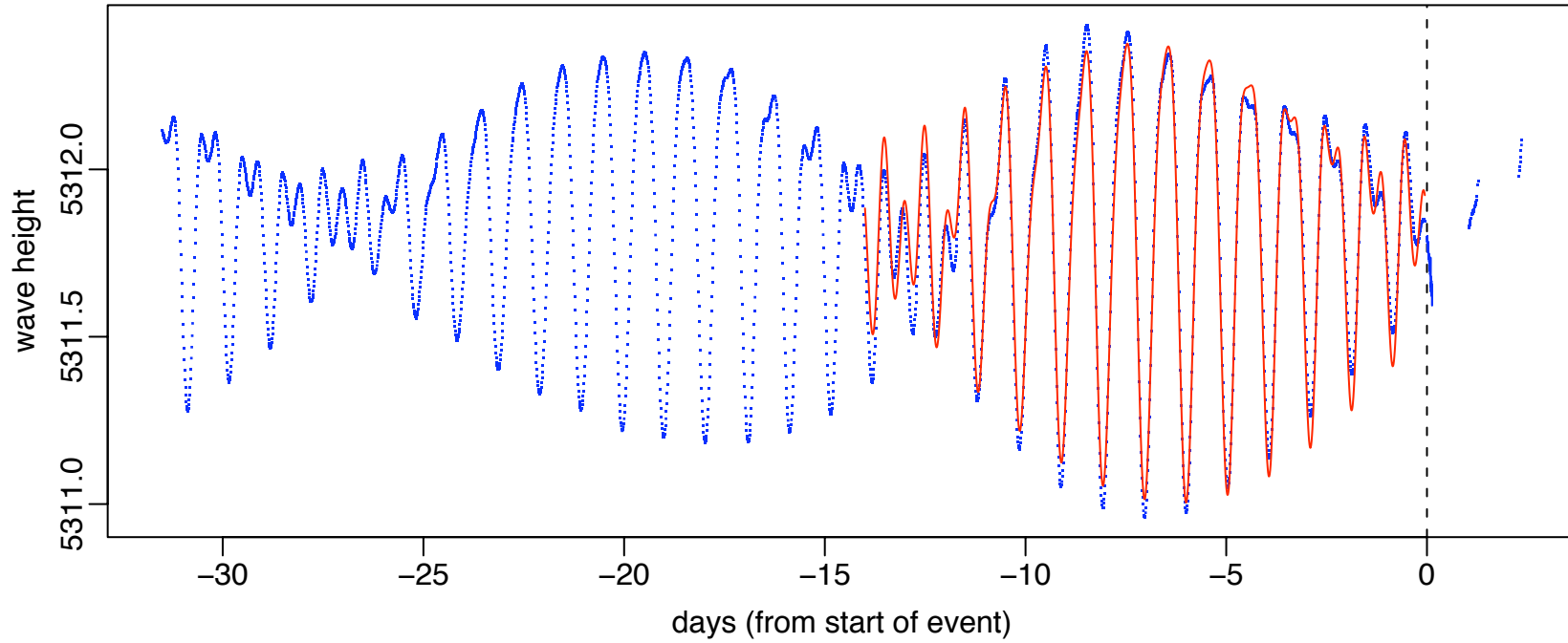
- if using 14 days of prior data, let  $L = 4$ , and set  $f_1, f_2, f_3$  and  $f_4$  to so-called M2, S2, O1 and K1 frequencies (other rules apply if use, e.g., 1 day or 29 days of prior data)
- estimate tidal component at time  $t$  using

$$\hat{x}(t) = \hat{\mu} + \sum_{l=1}^L \hat{A}_l \cos(2\pi f_l t) + \hat{B}_l \sin(2\pi f_l t)$$

- for  $t < 0$ , can compare estimated tide to actual data
- for  $t > 0$ , can detide data from 1 min and/or 15 sec streams by subtracting off  $\hat{x}(t)$

# 14 Day Model (M2, S2, O1 and K1)

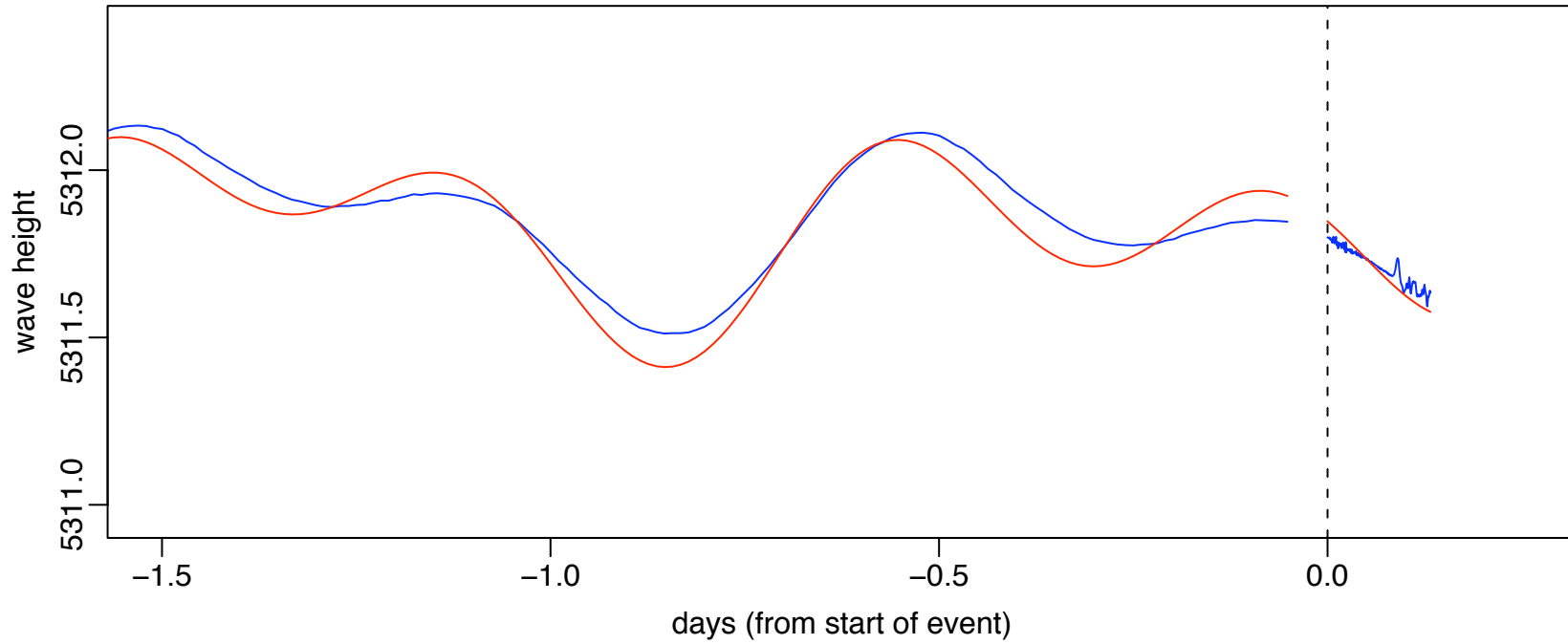
buoy 21414





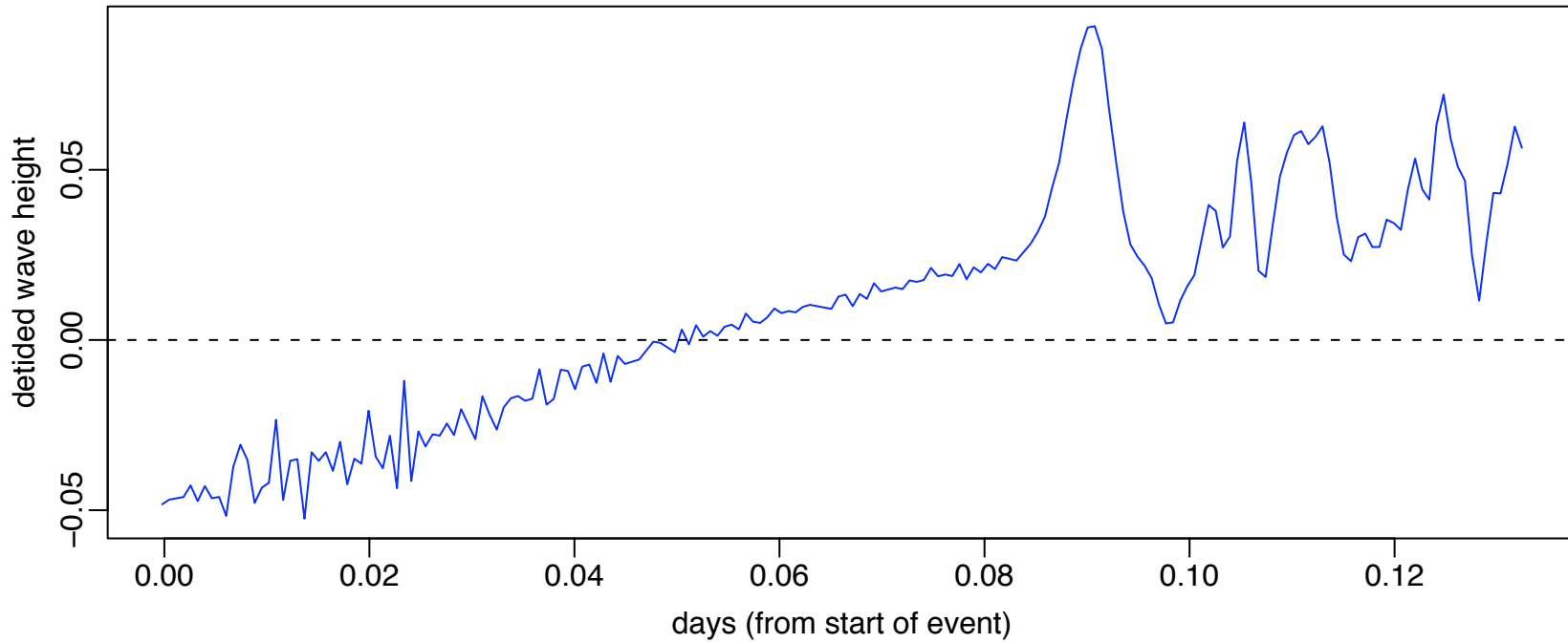
# 14 Day Model and Extrapolation

buoy 21414



# Data Detided using 14 Day Model

**buoy 21414**



## Second-Stage Detiding via Kalman Filter/Smoothen

- after first-stage detiding, can write

$$x(t) - \hat{x}(t) = \mu(t) + \epsilon(t),$$

where  $\mu(t)$  is ‘unpredictable’ tidal component

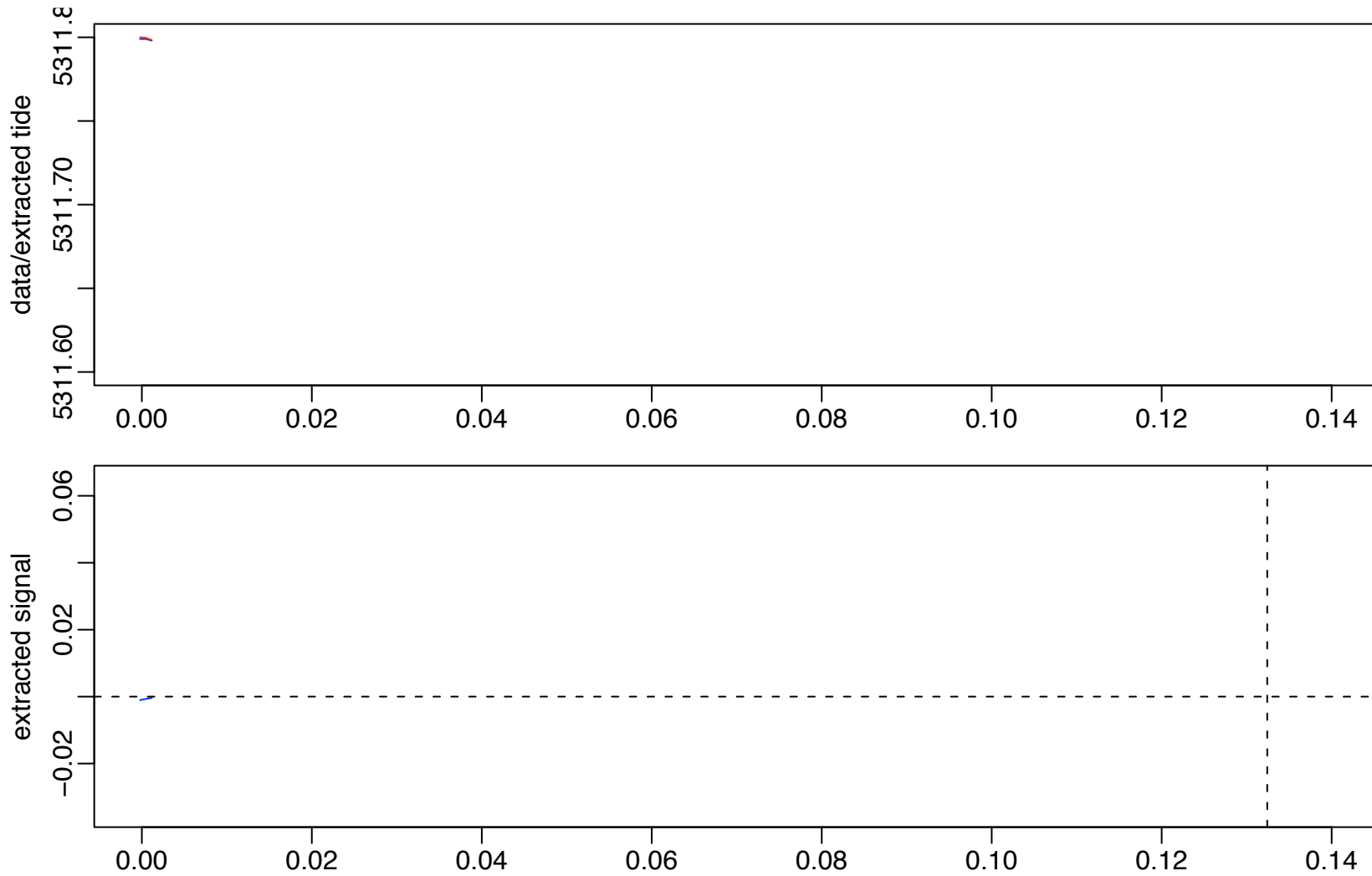
- extract  $\mu(t)$  by formulating a state-space model, which allows use of Kalman filter/smoothen:

$$\mu(t) = \mu(t-1) + v(t)$$

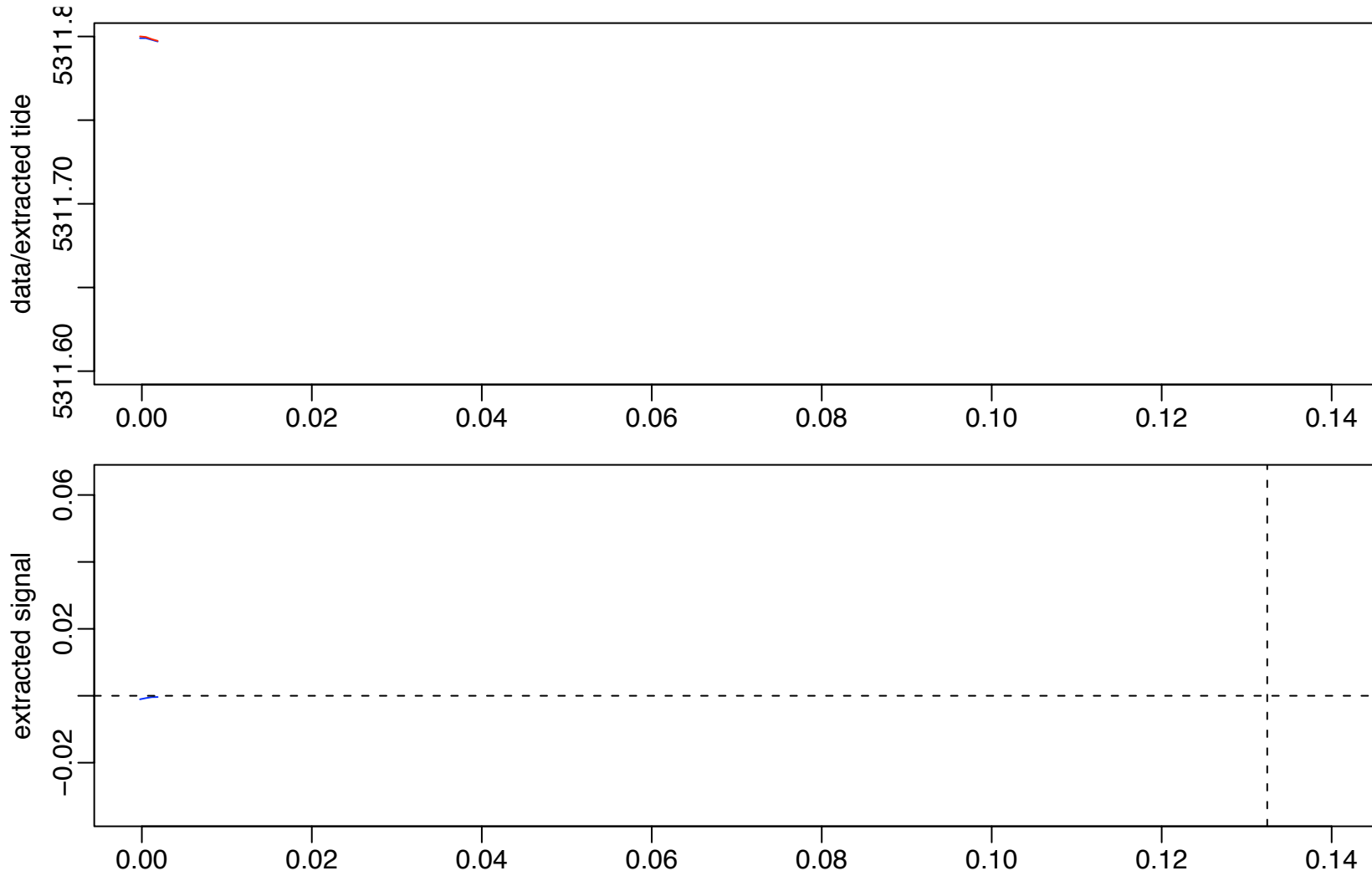
$$v(t) = v(t-1) + \zeta(t)$$

where  $\zeta(t)$  is Gaussian white noise with mean zero and variance  $\sigma_\zeta^2$  (can be set using historical data)

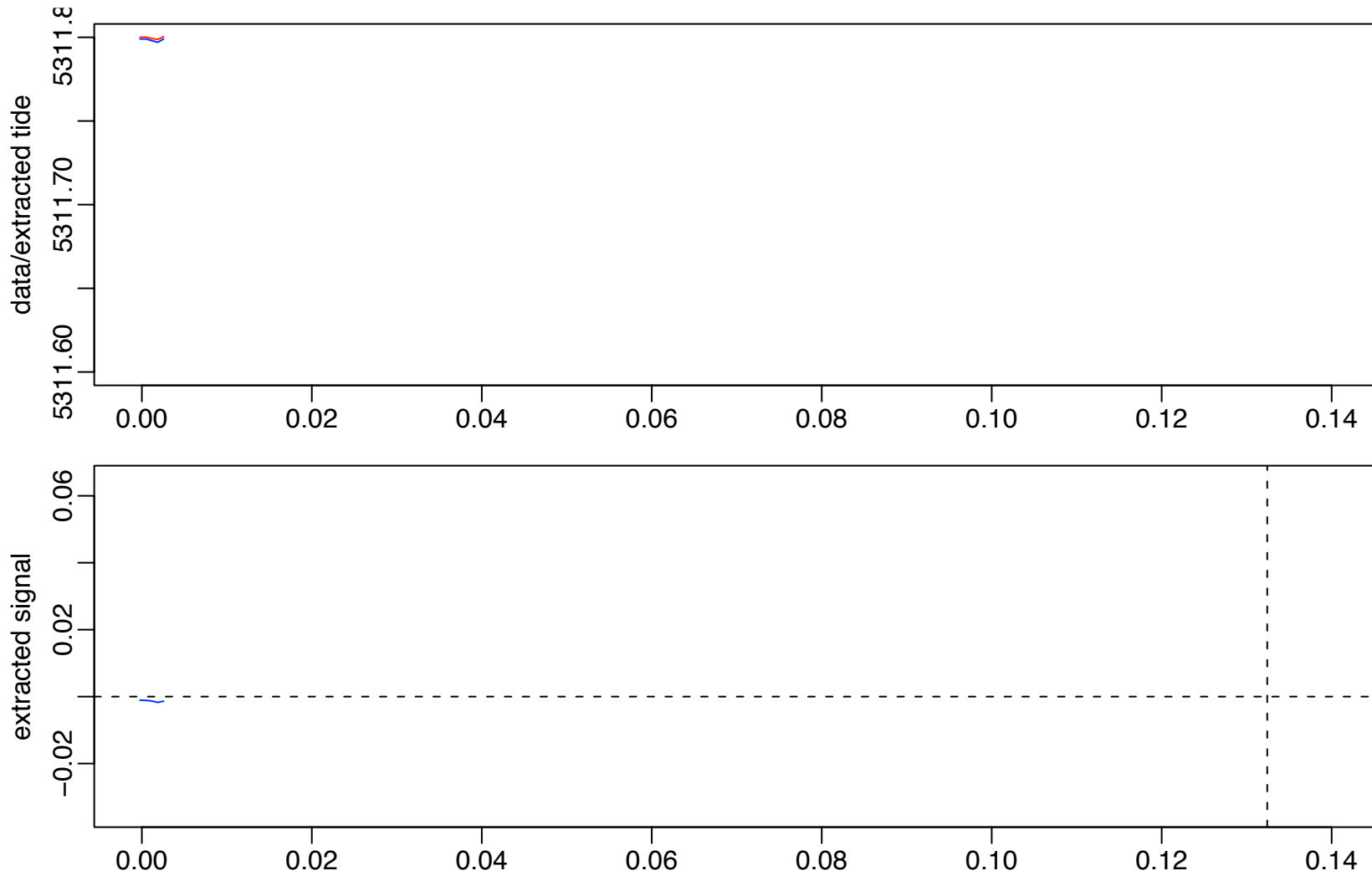
# Kalman Filtering/Smoothing – 14 Day Model



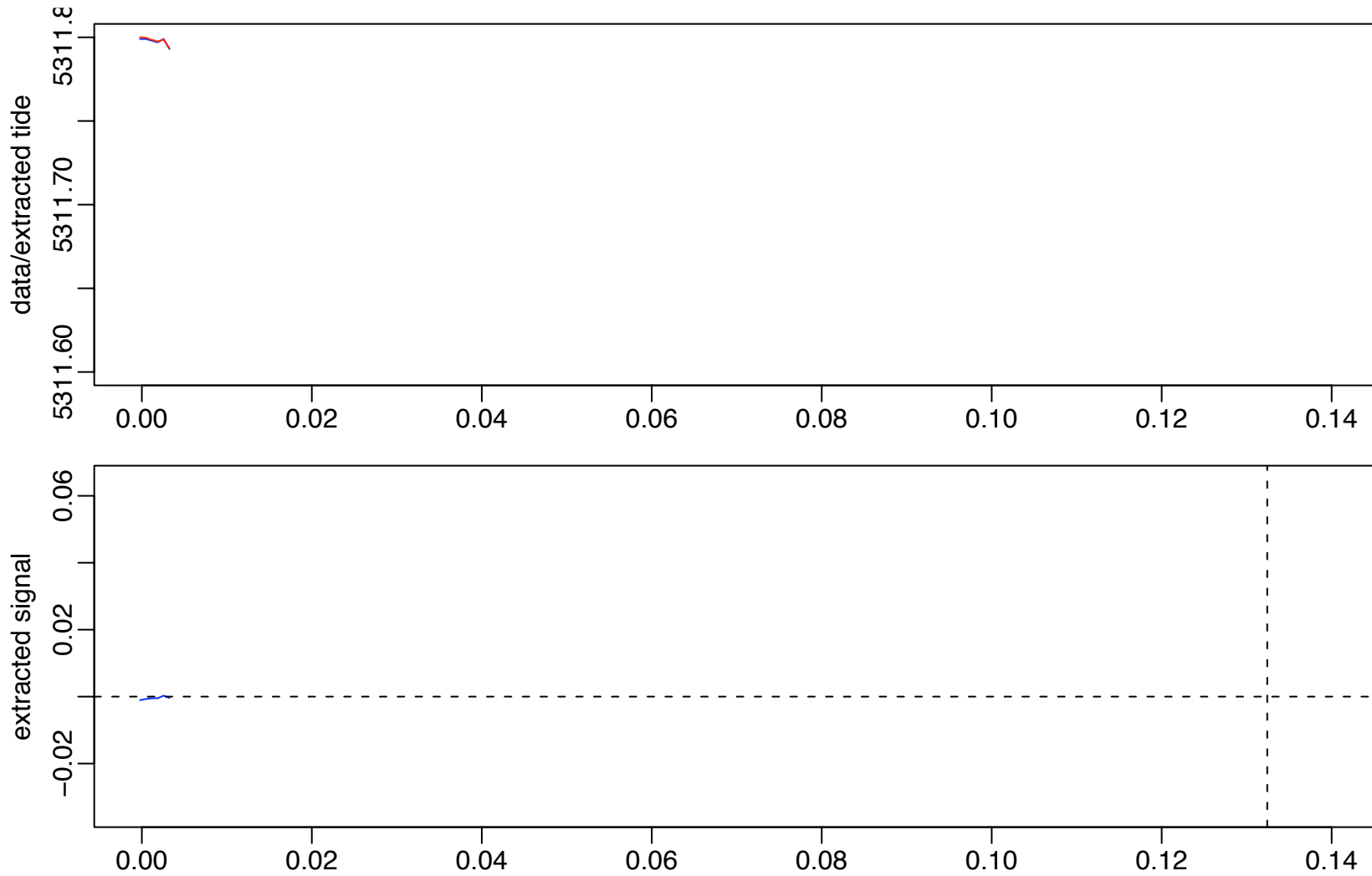
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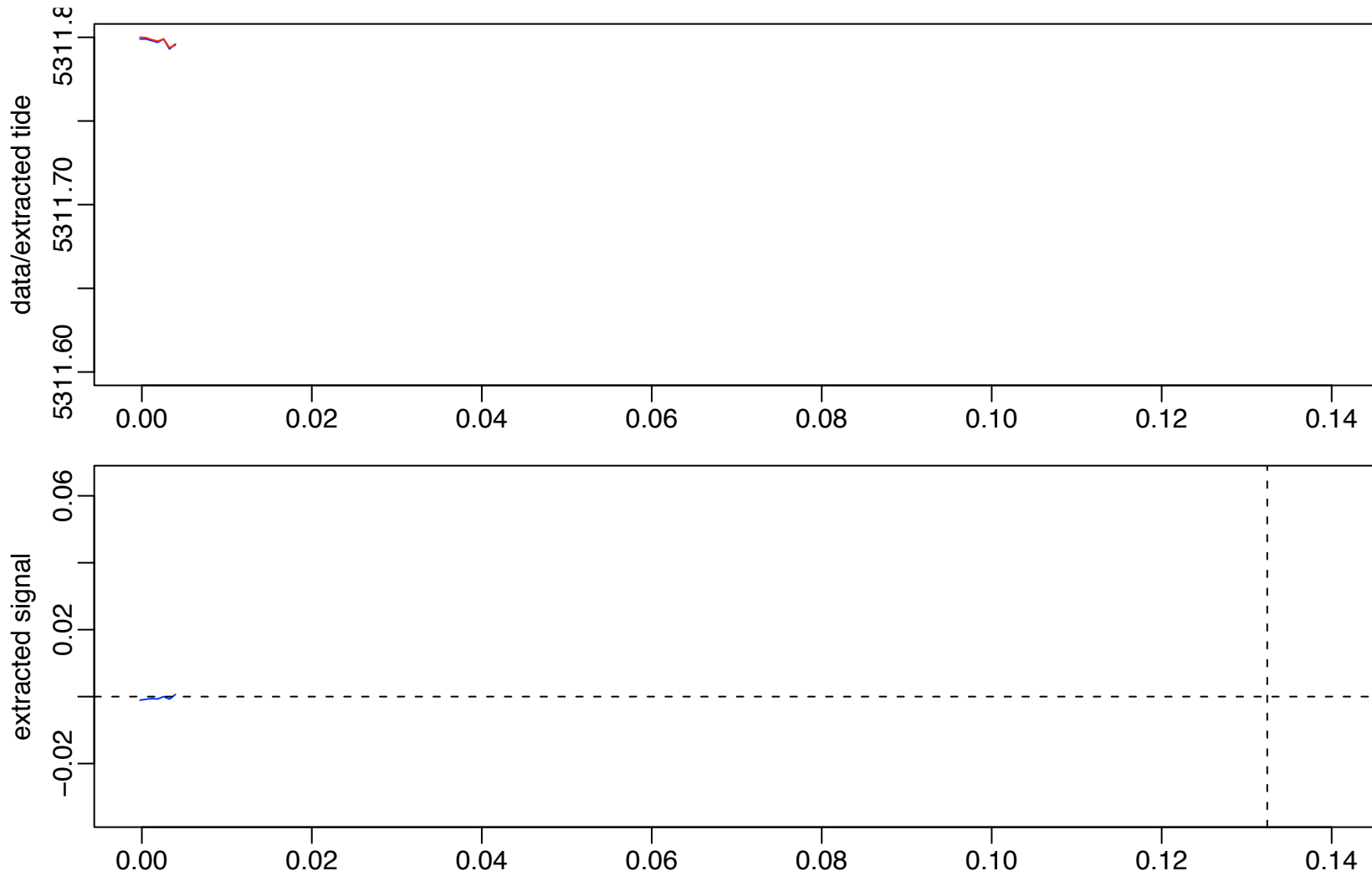
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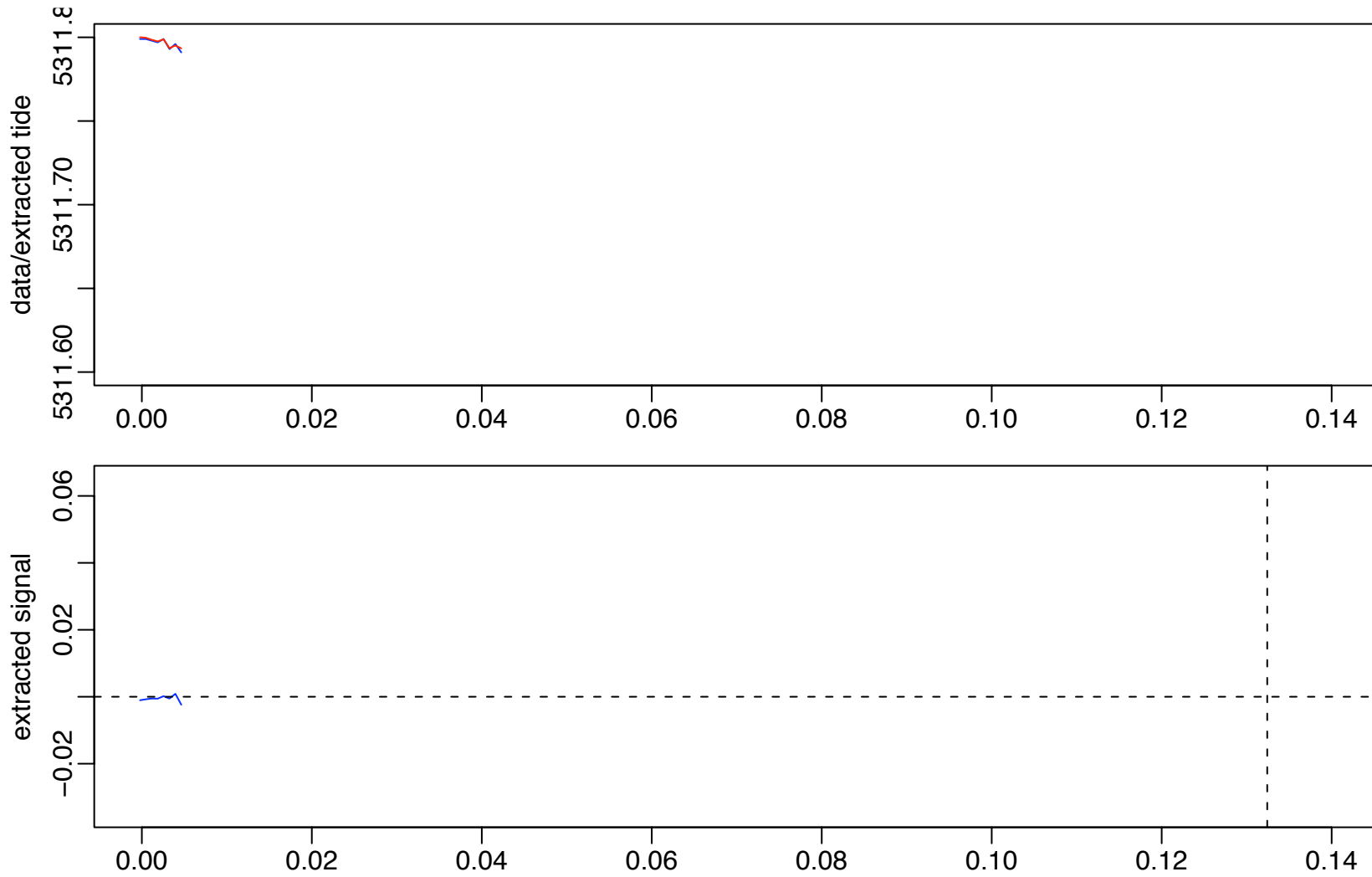


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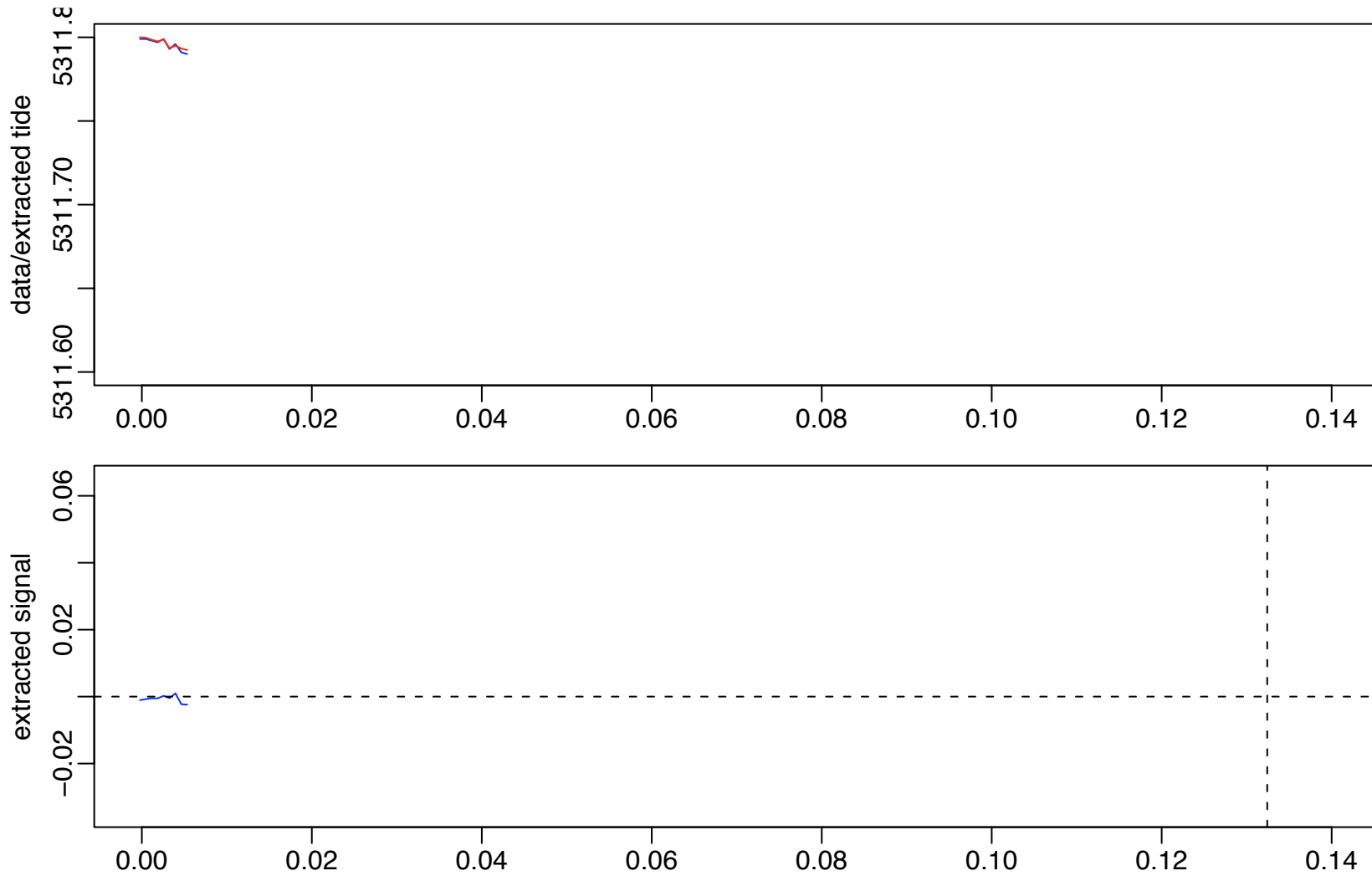




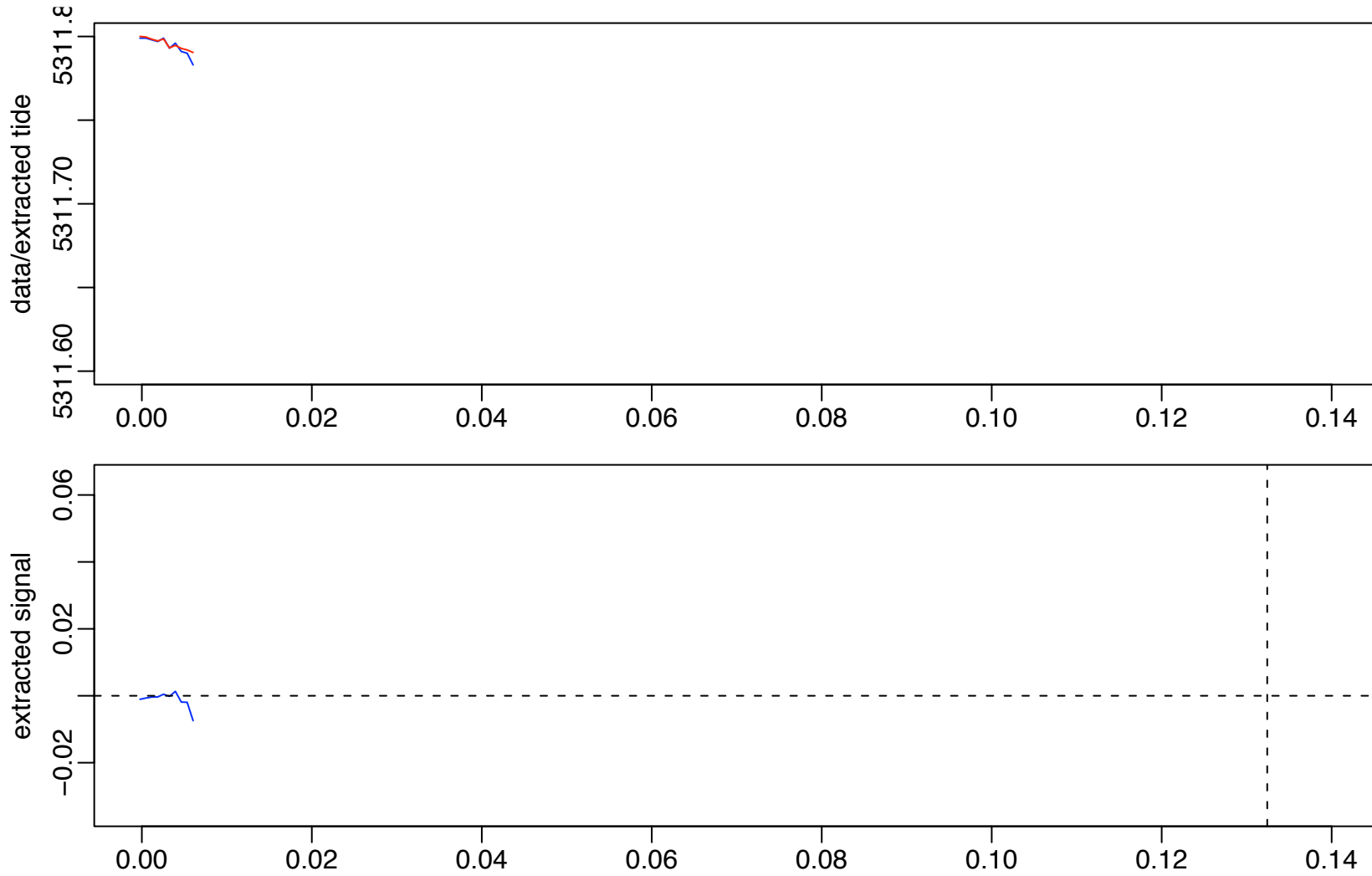
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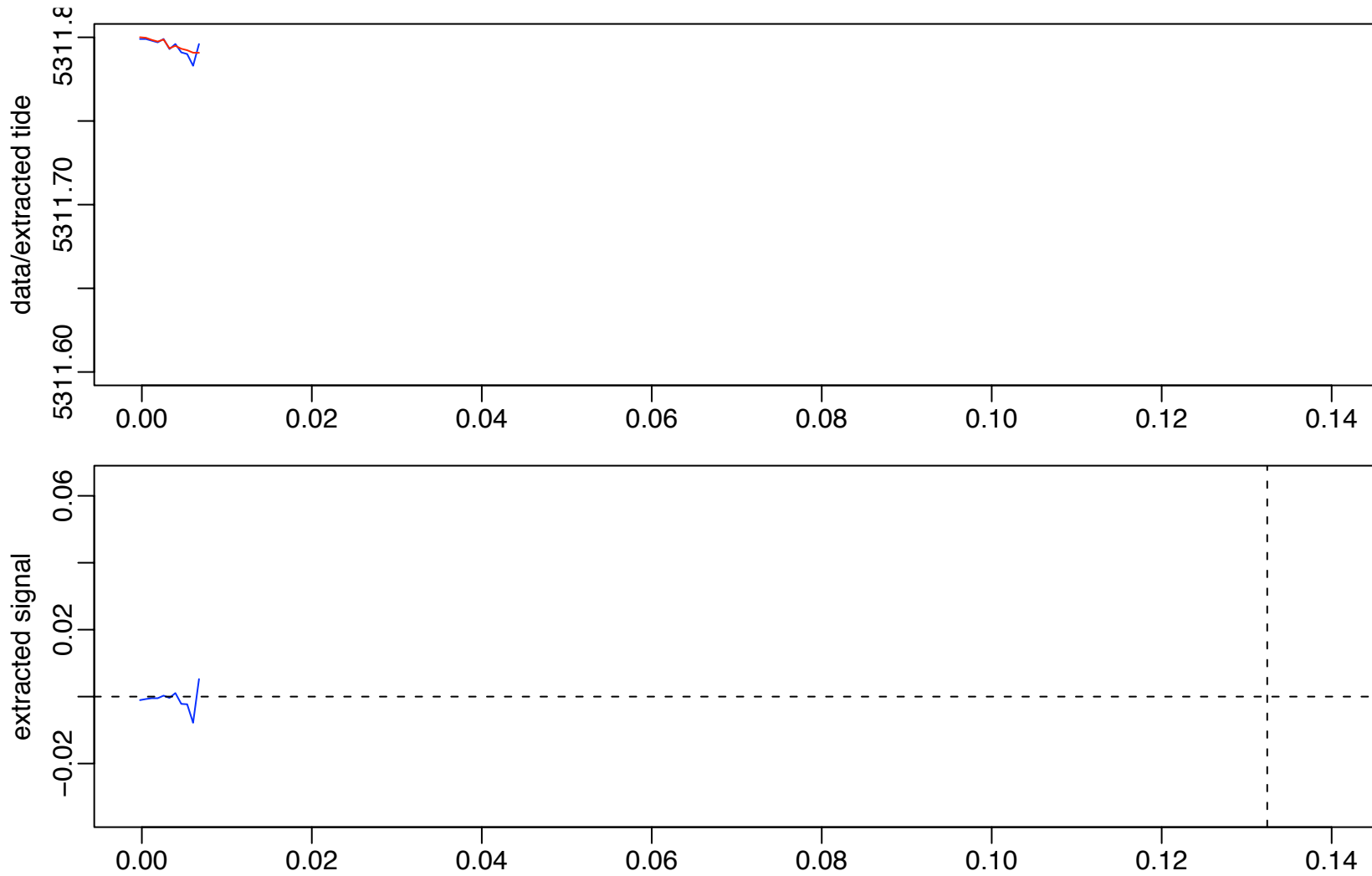
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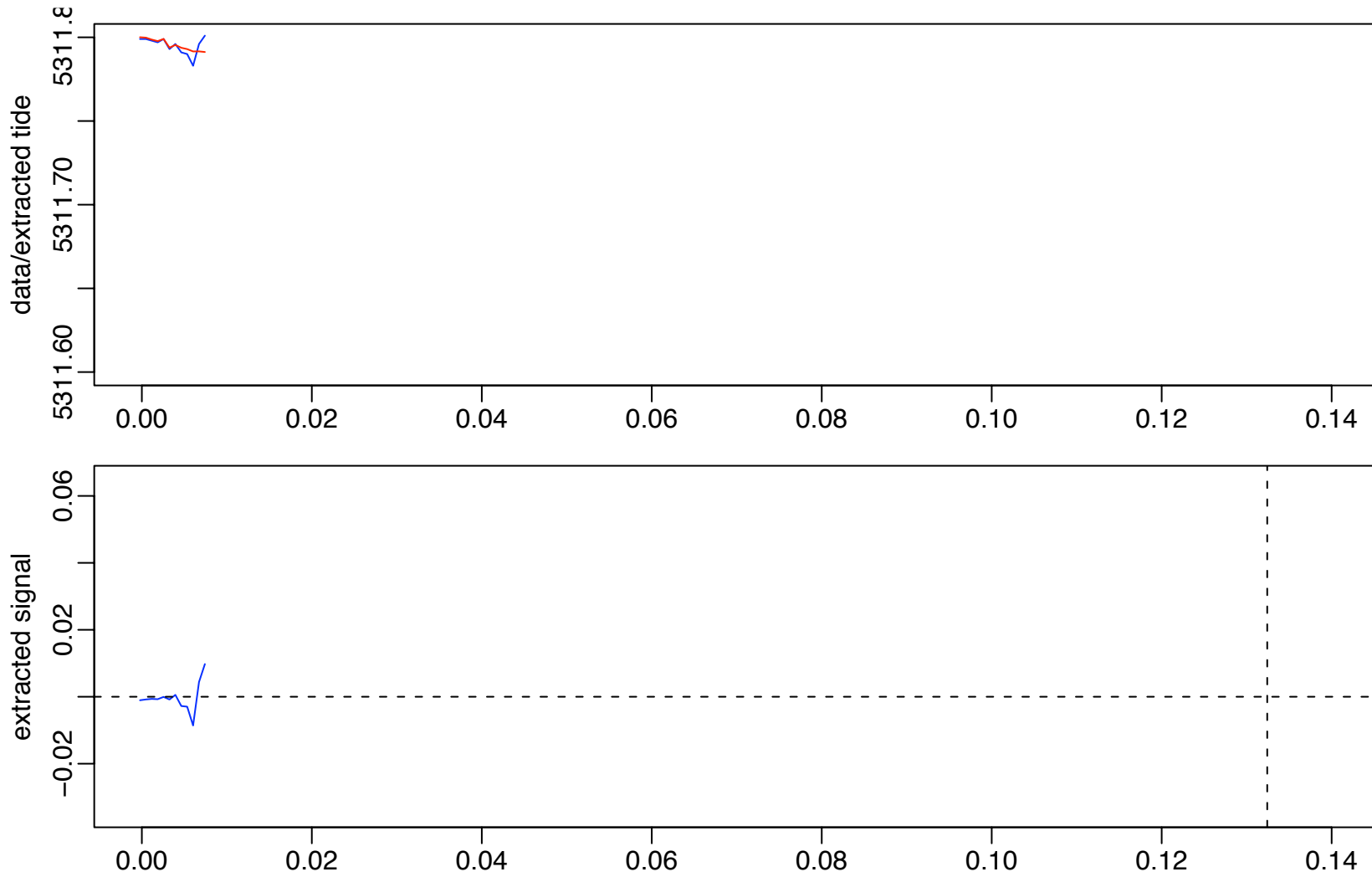
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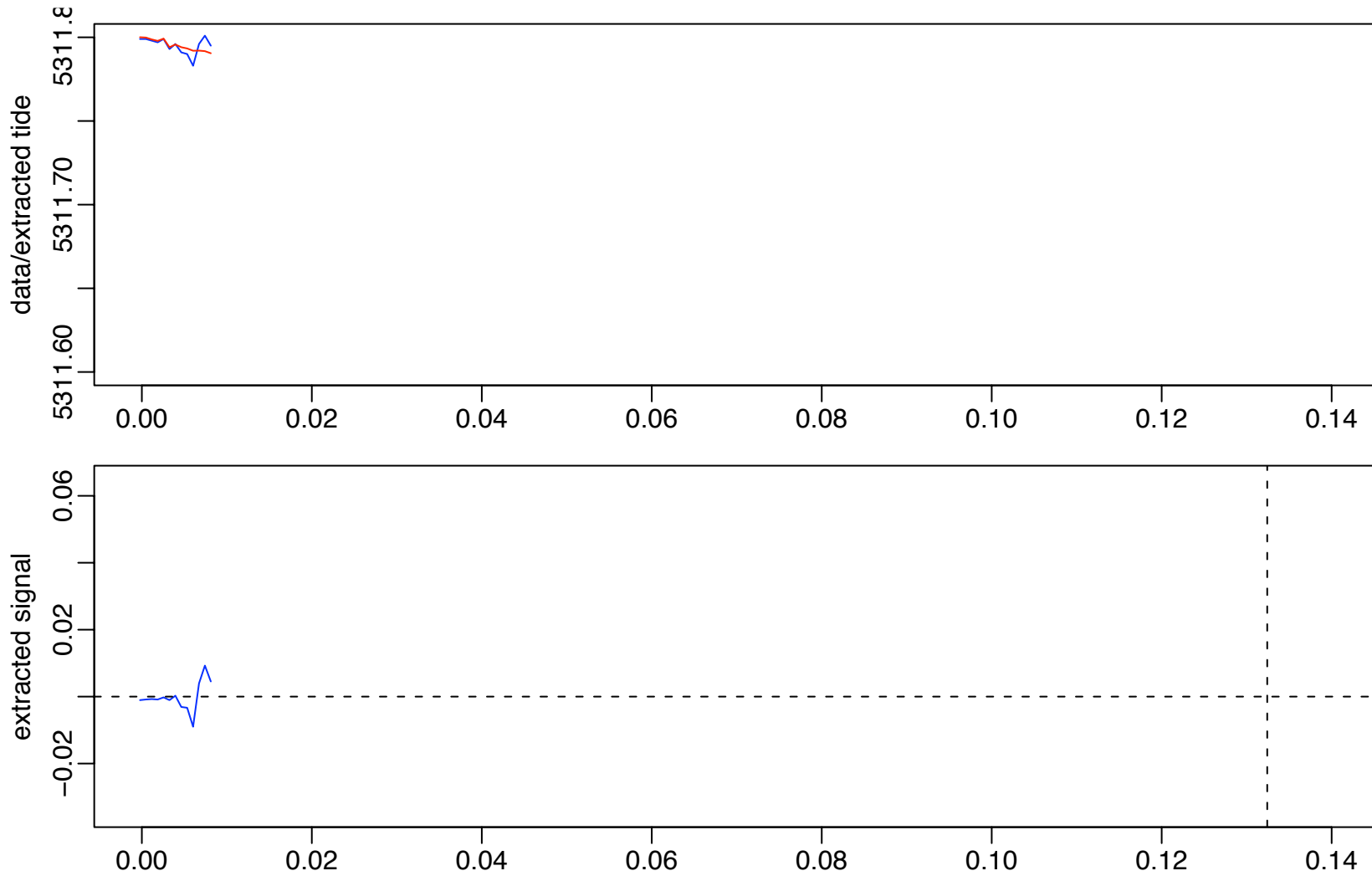
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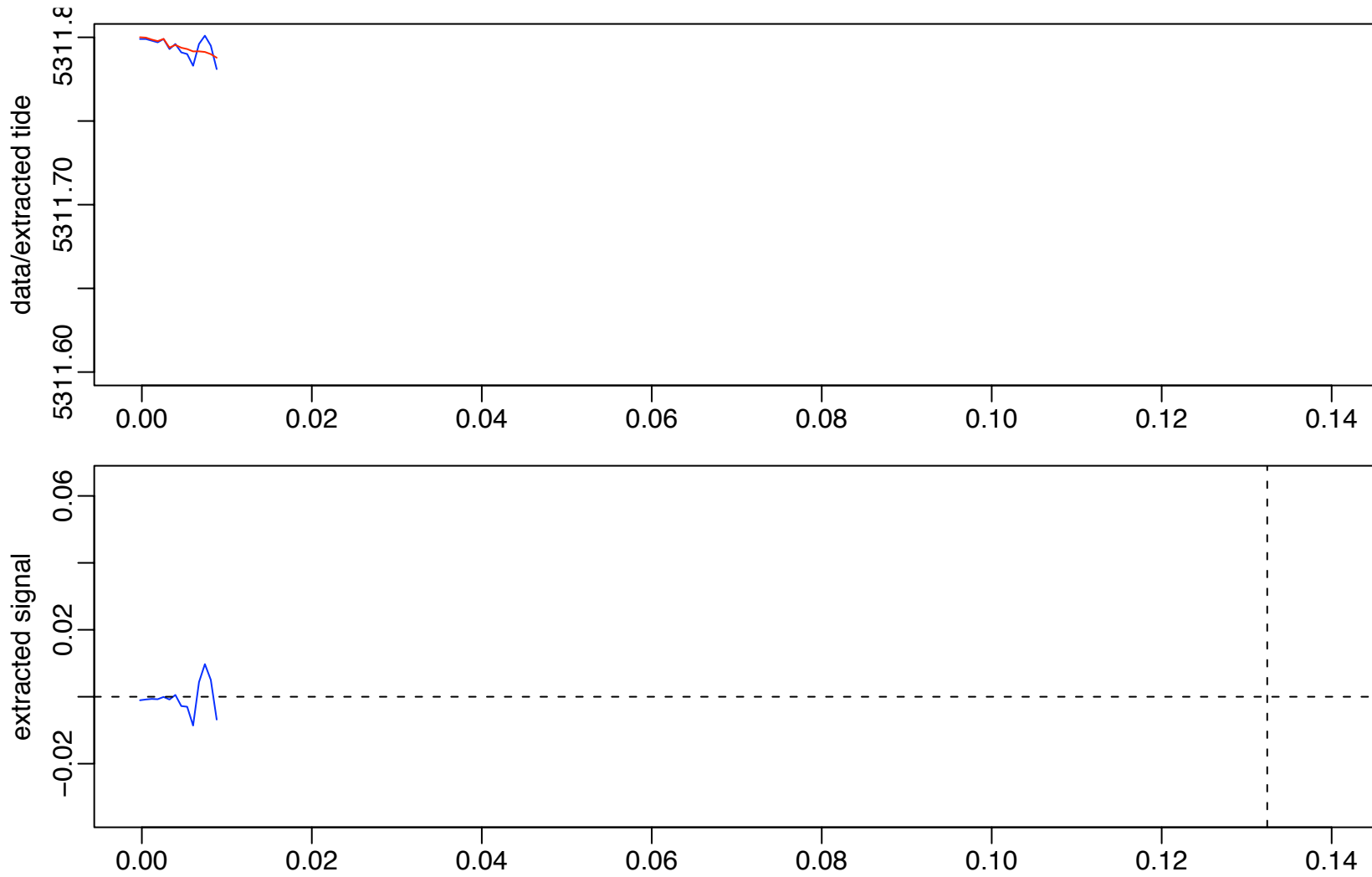
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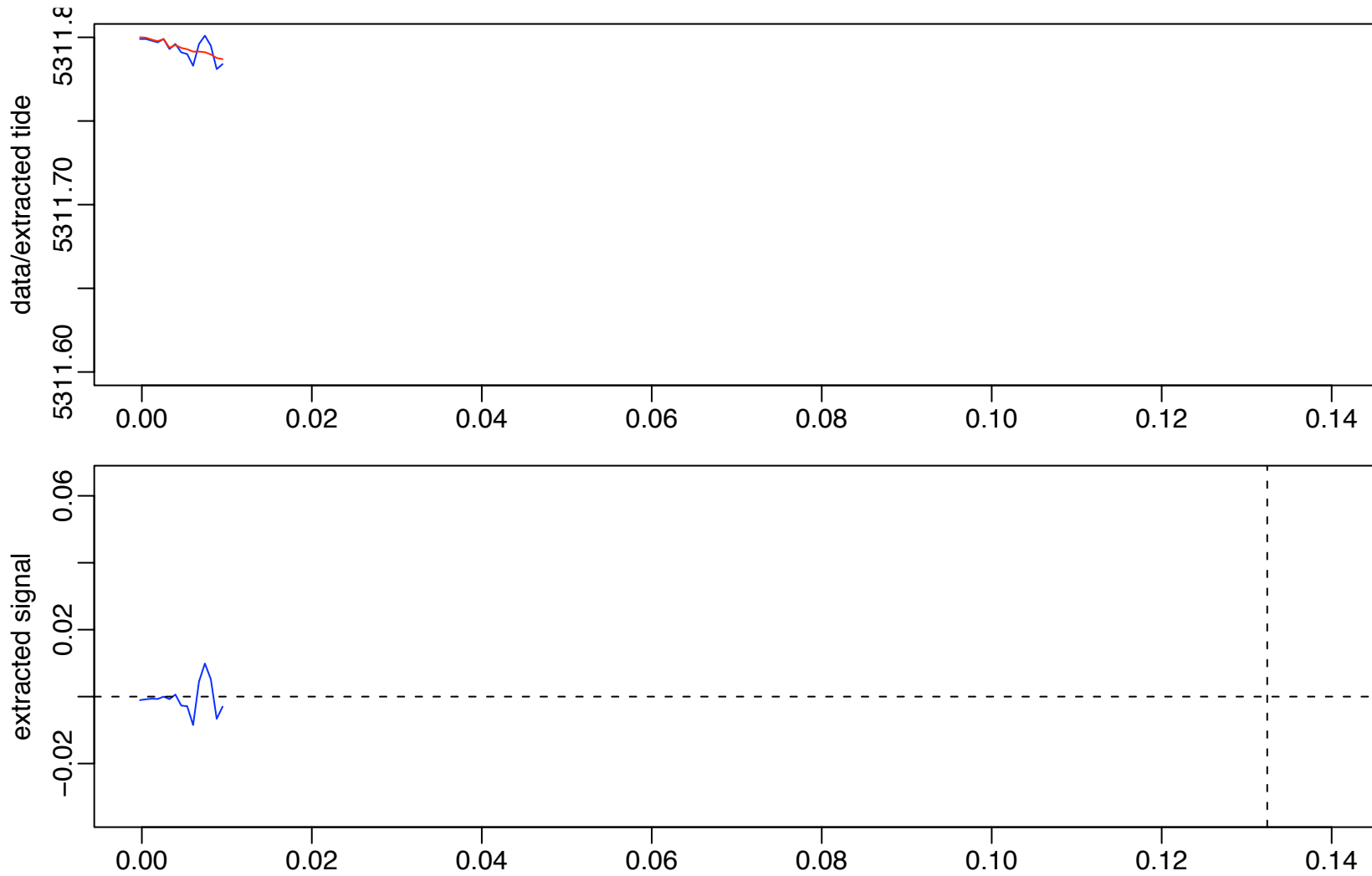
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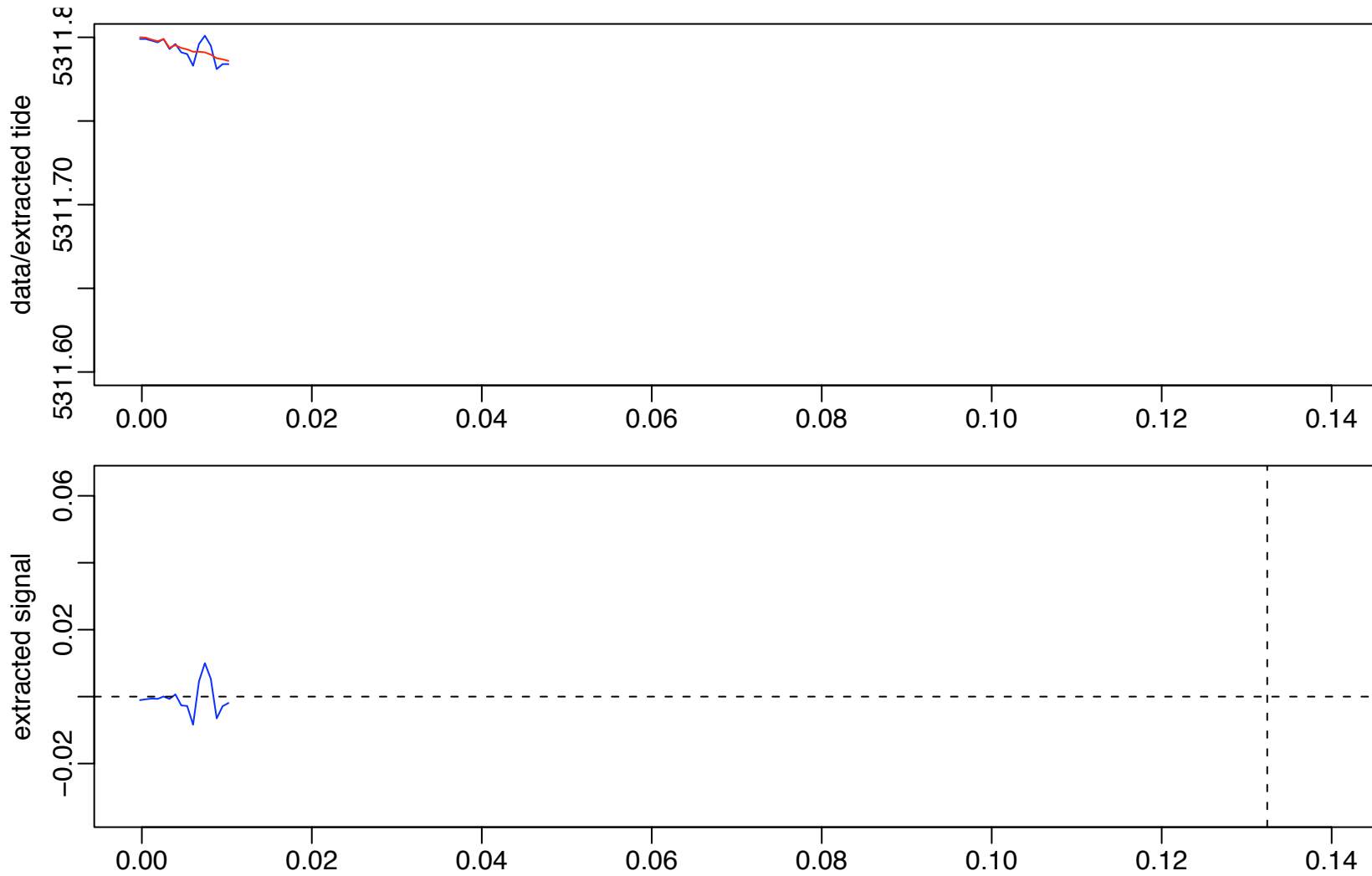


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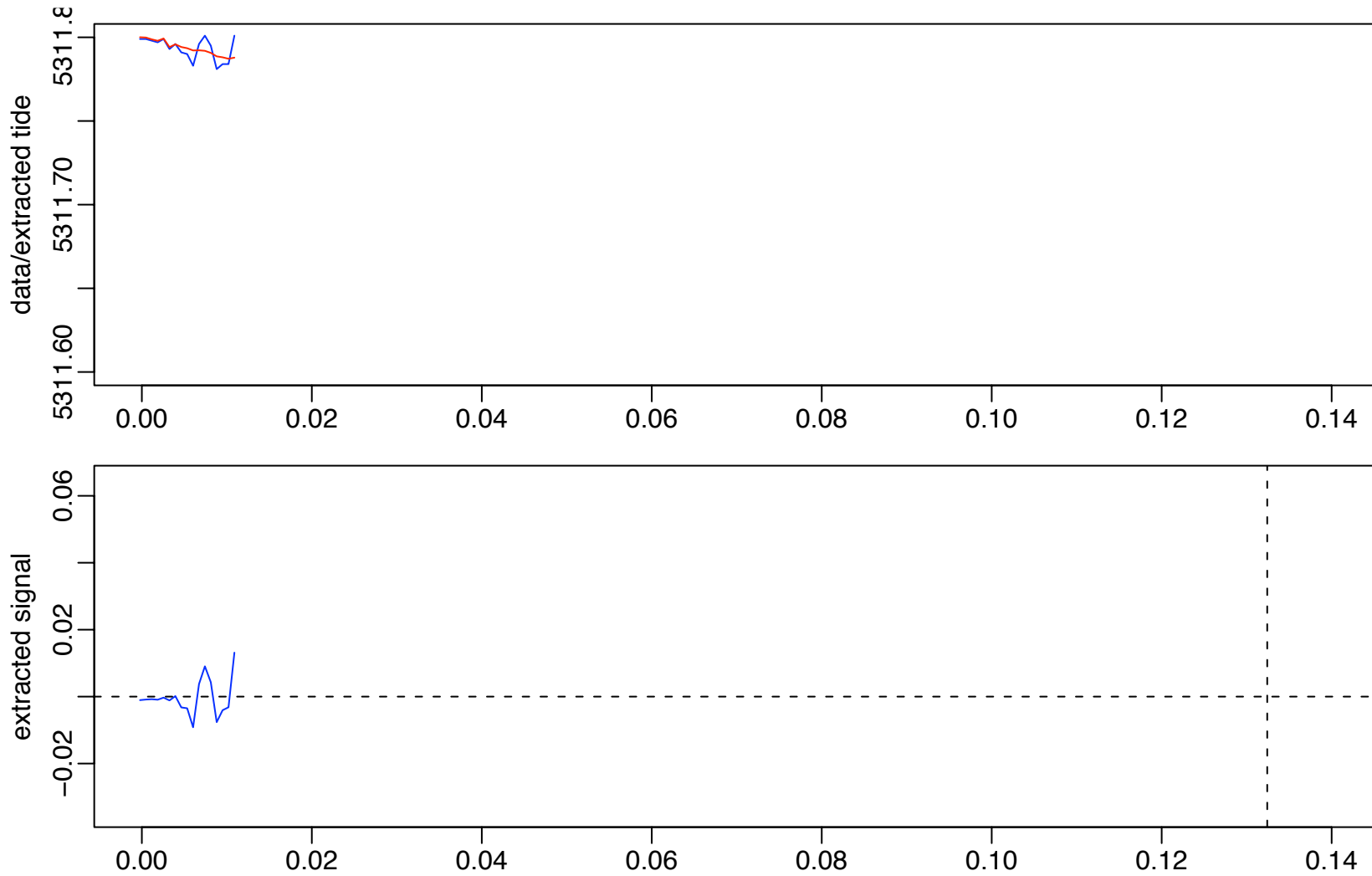




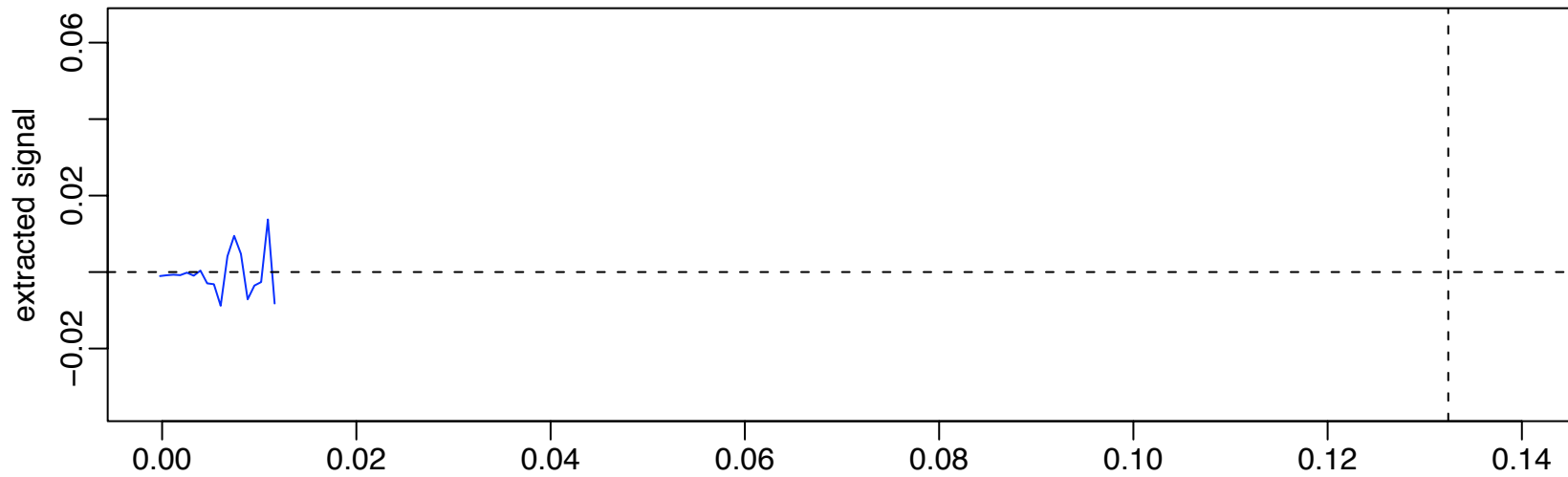
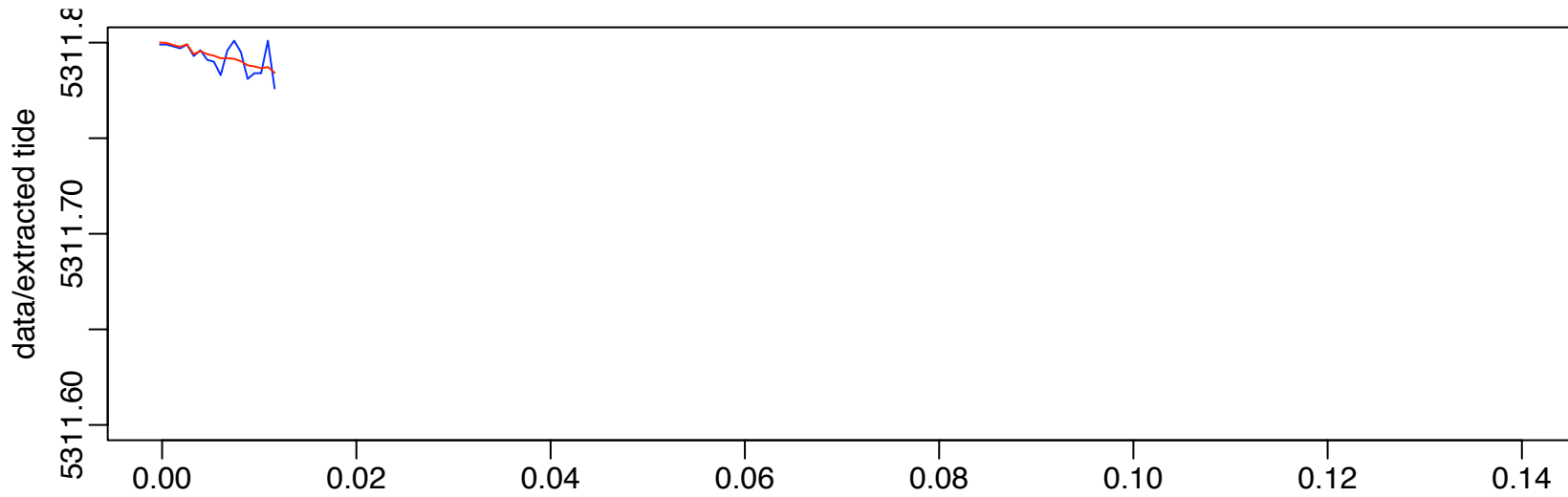
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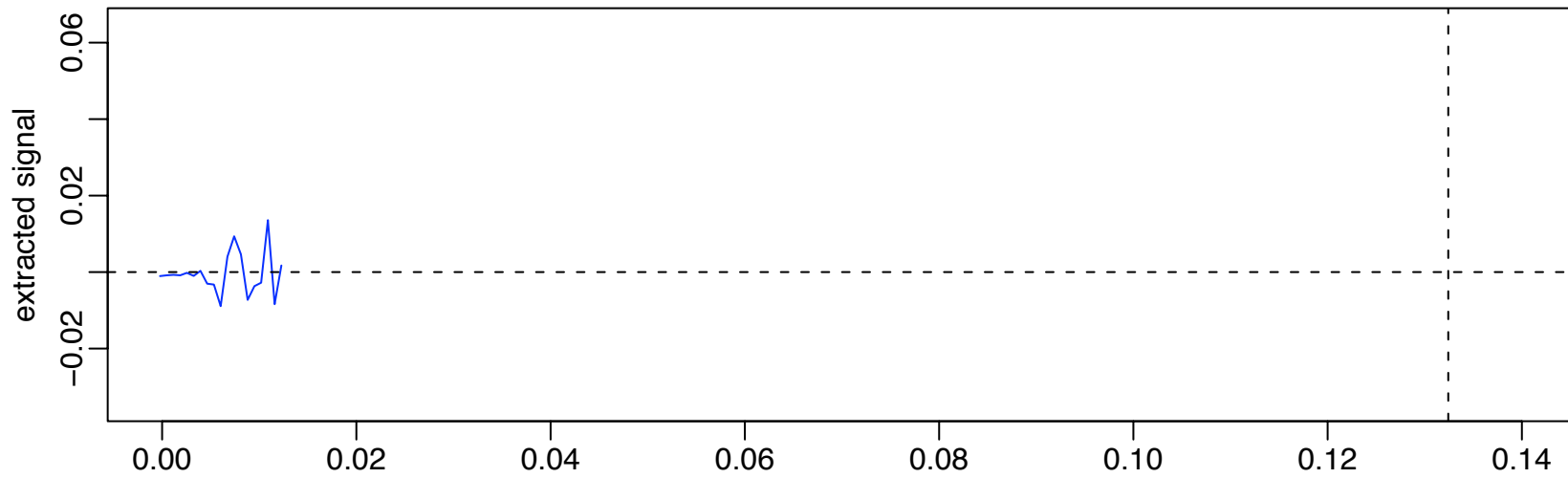
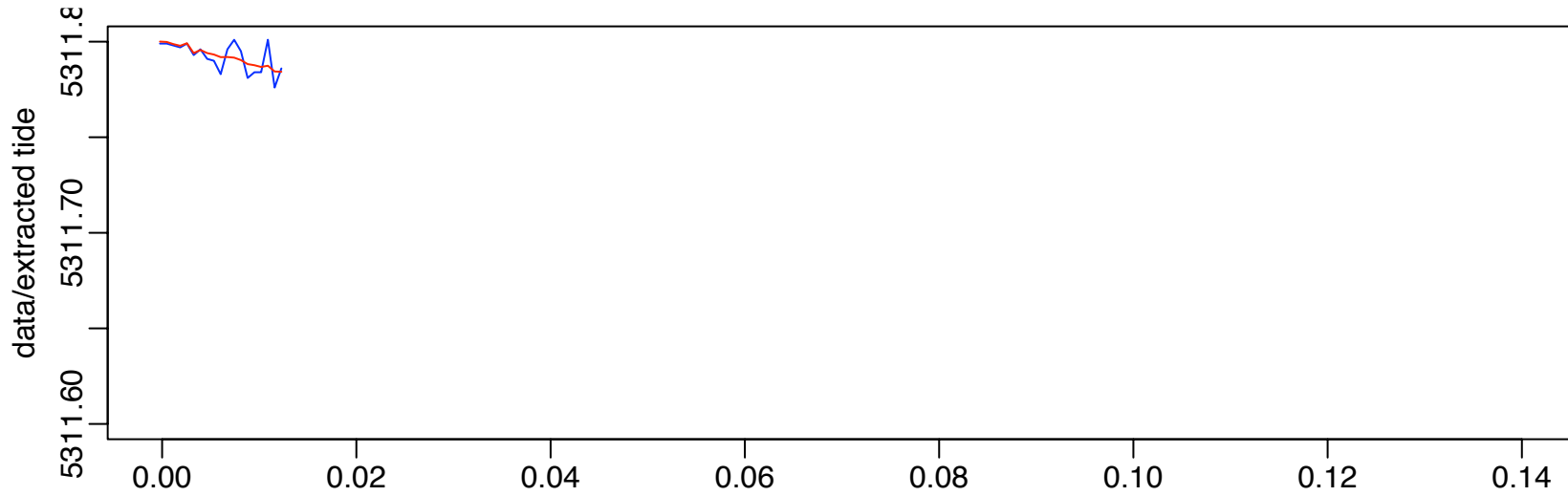
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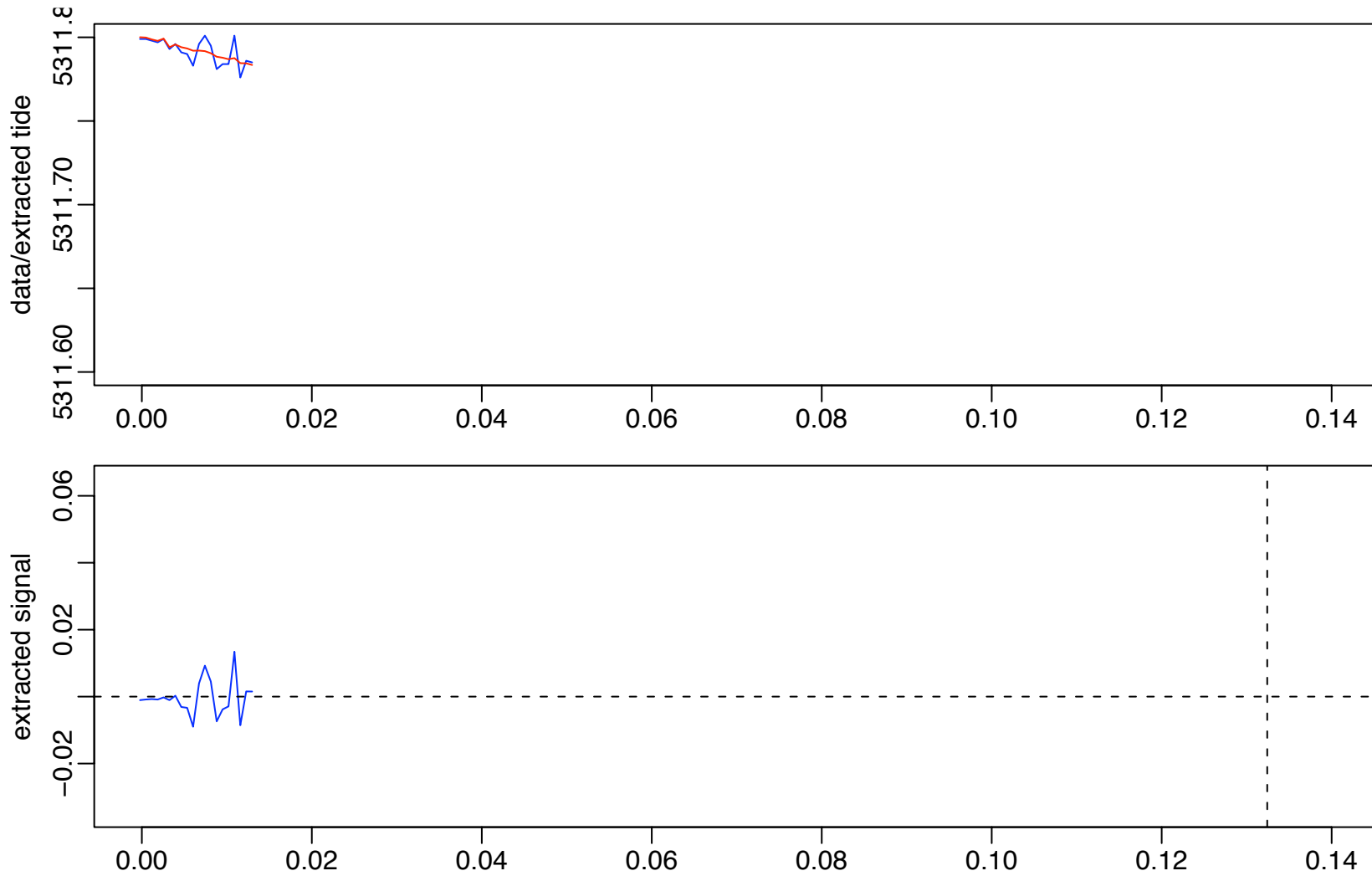
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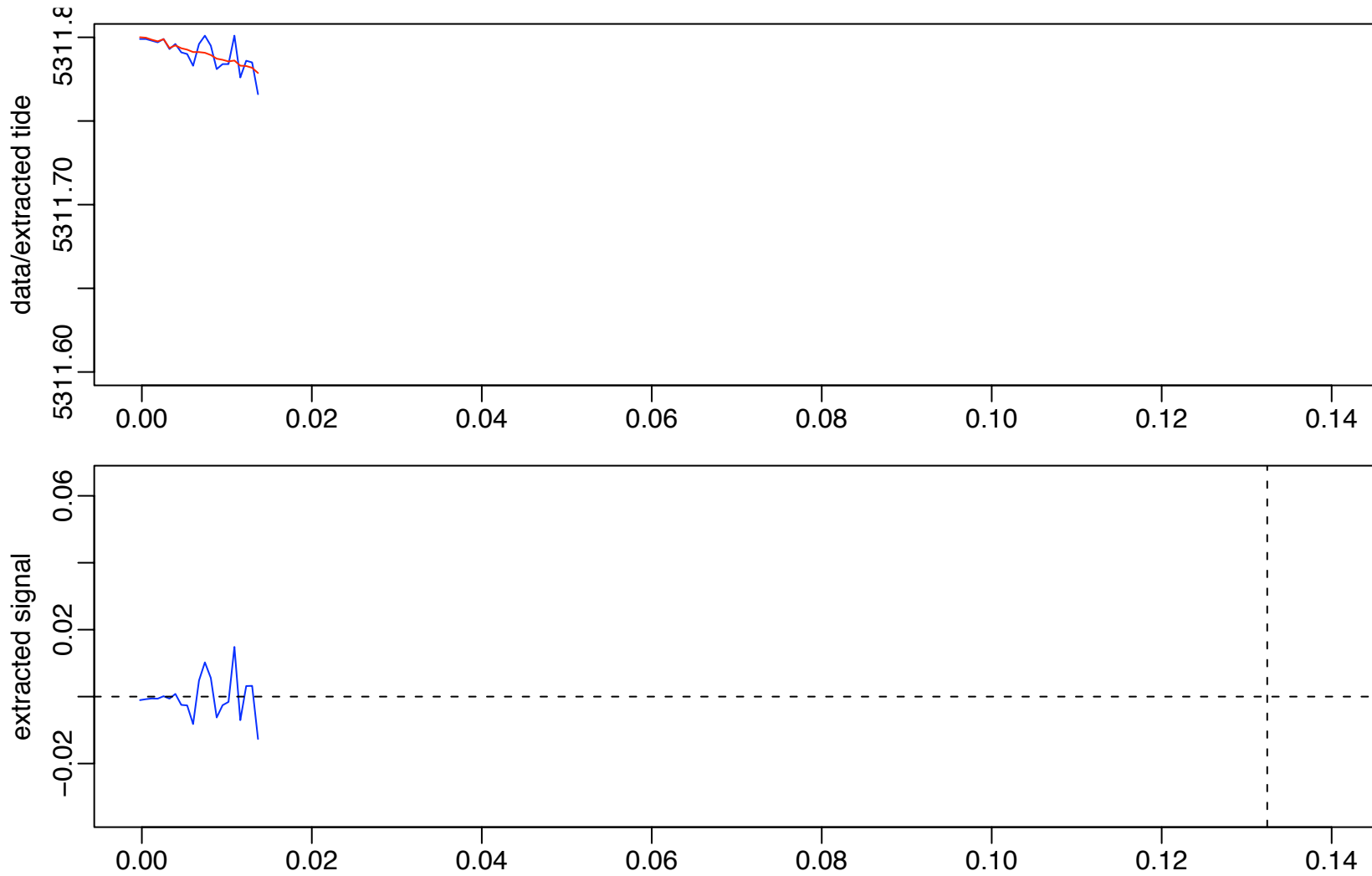
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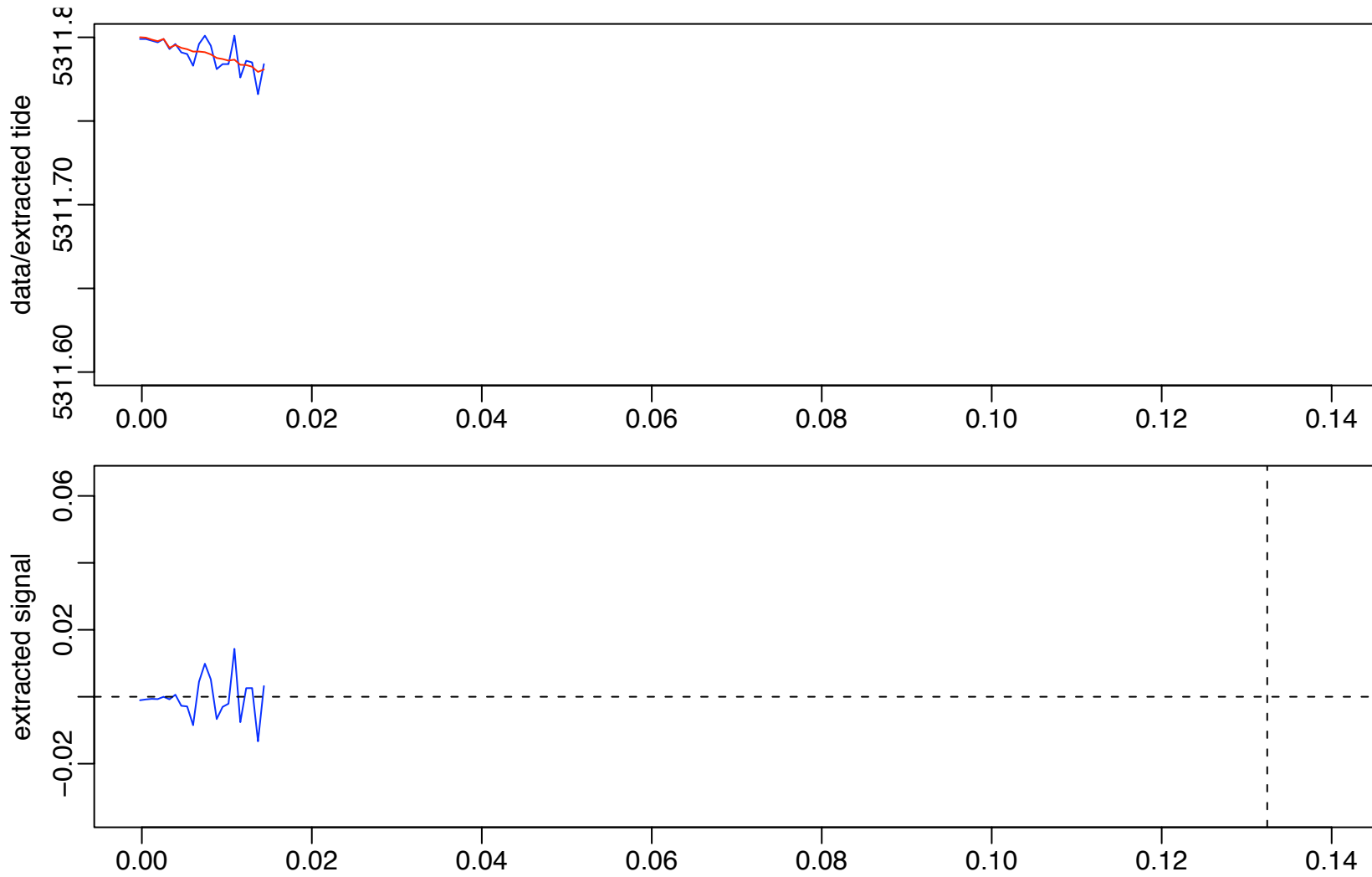
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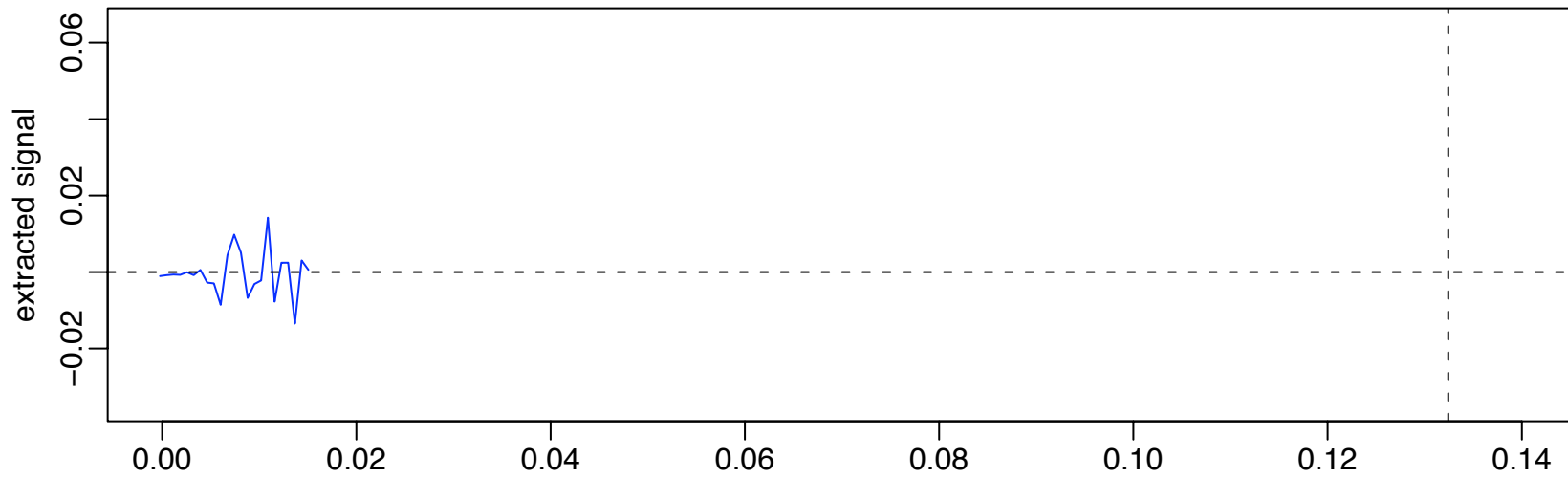
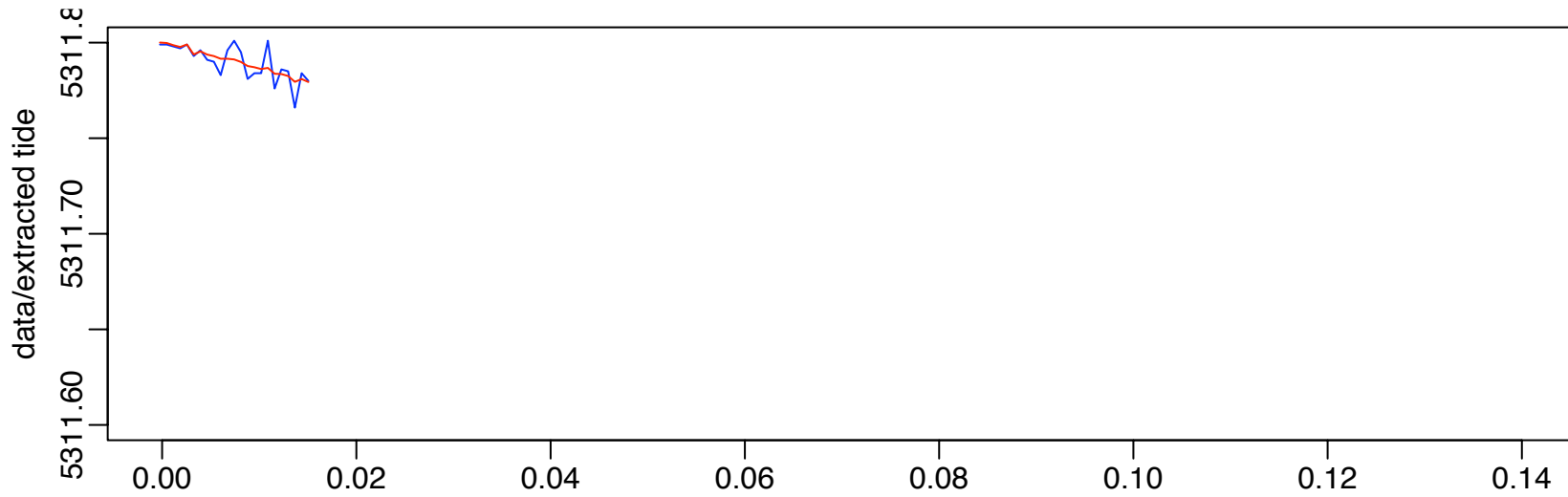
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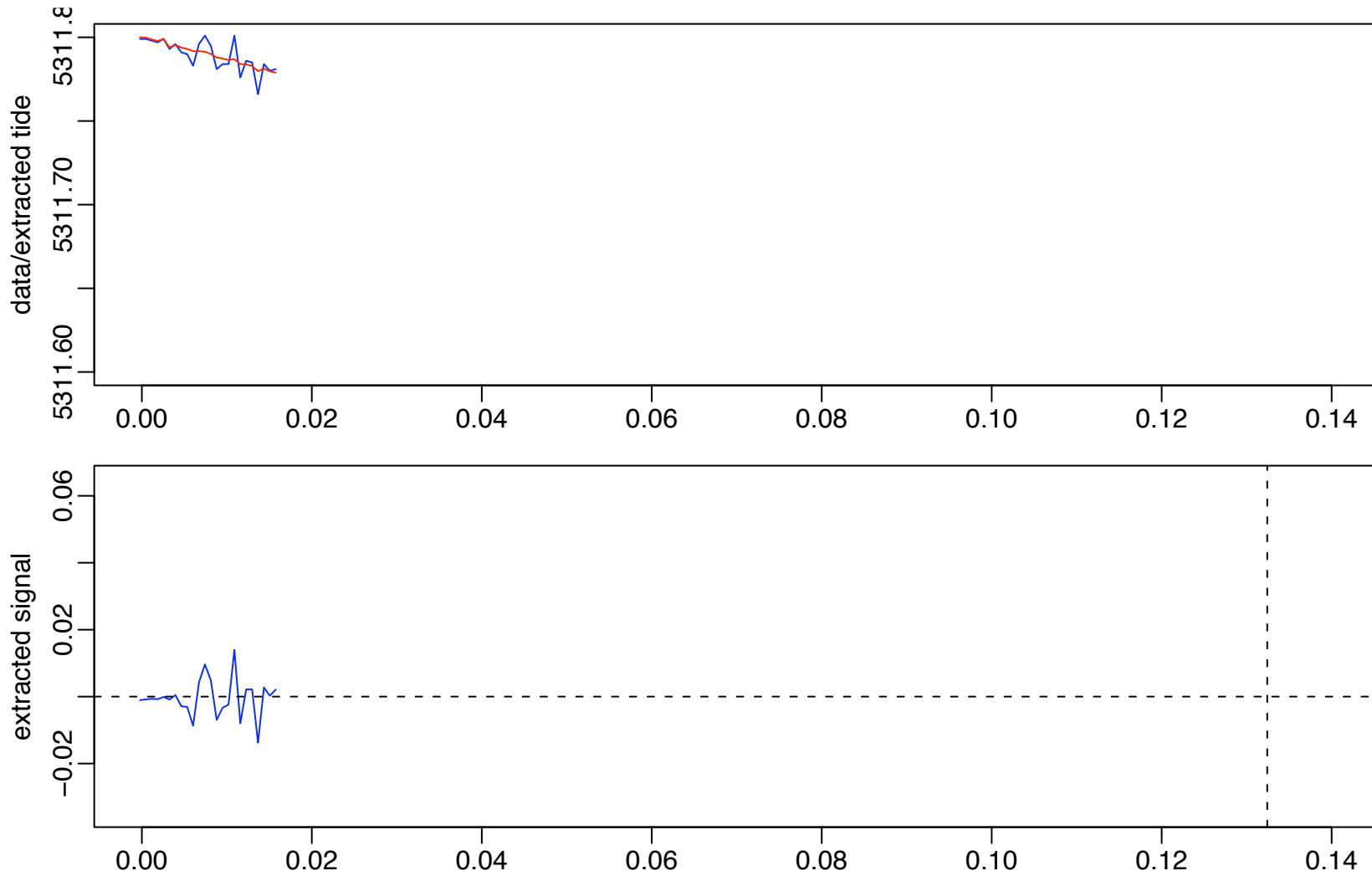


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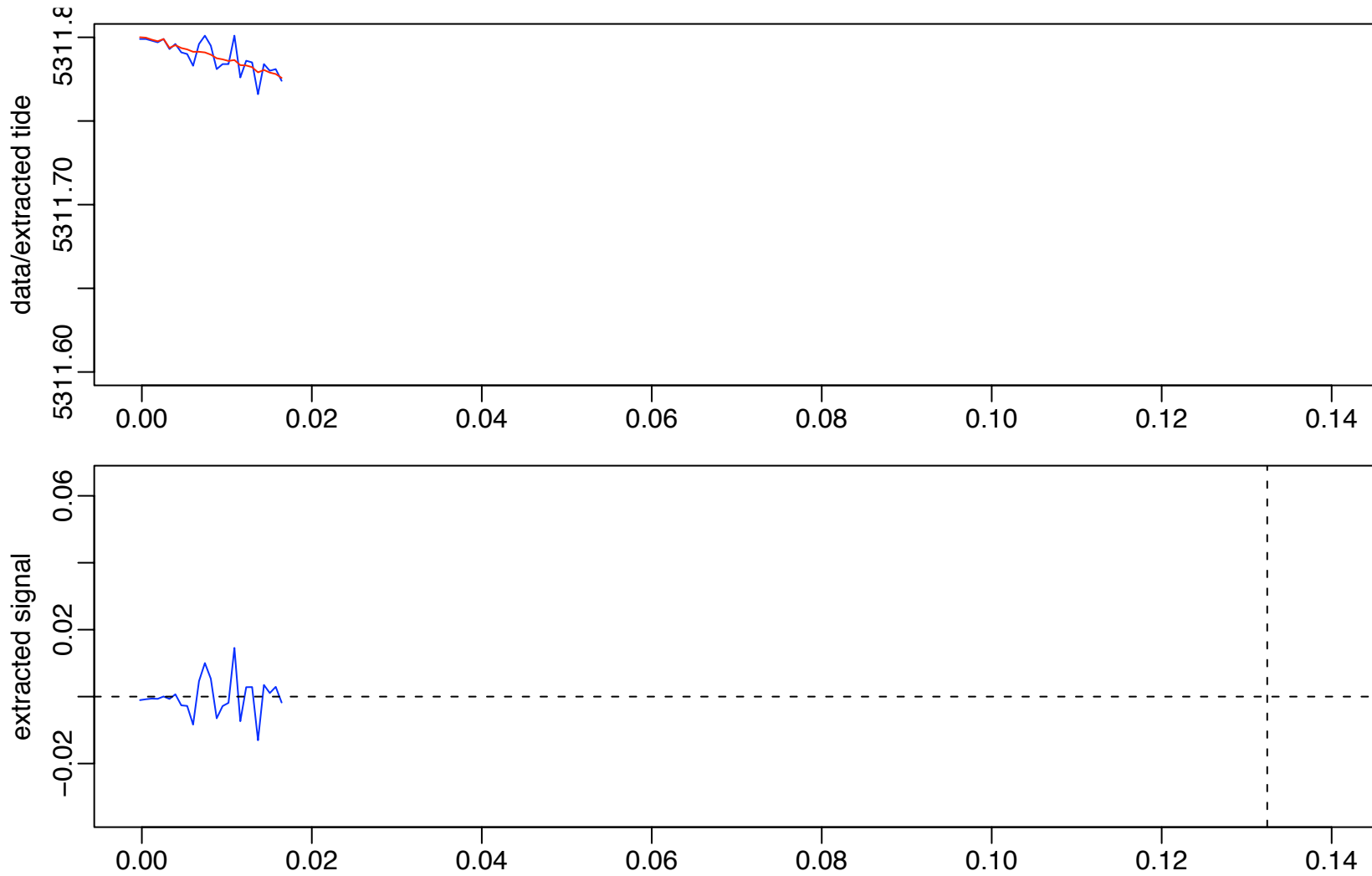




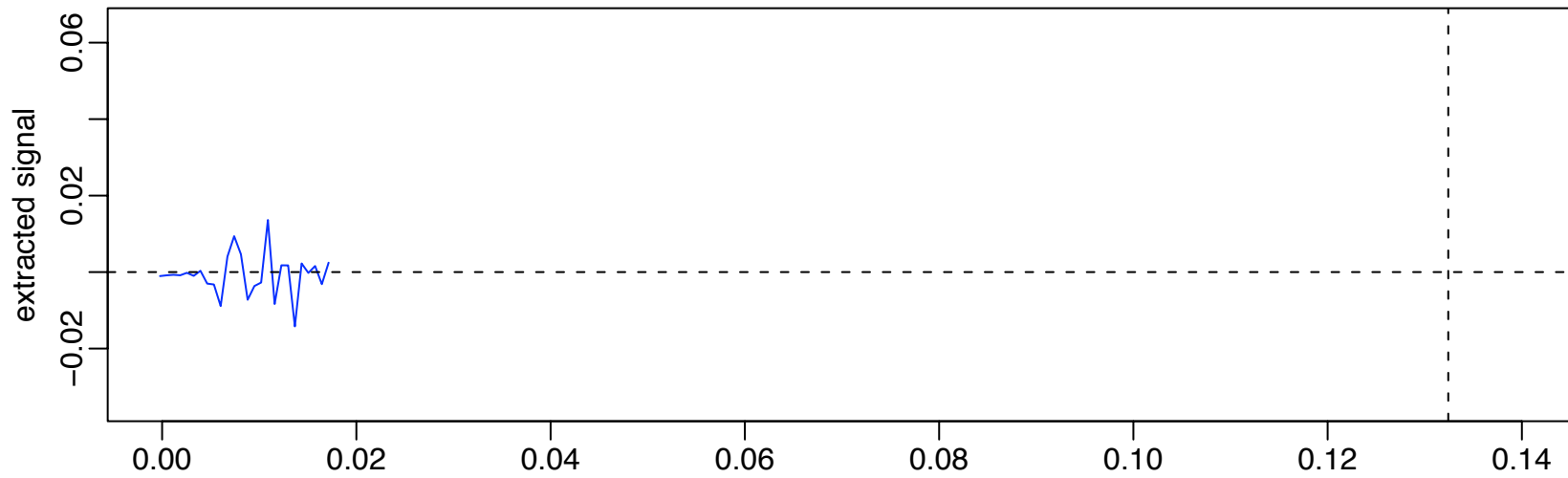
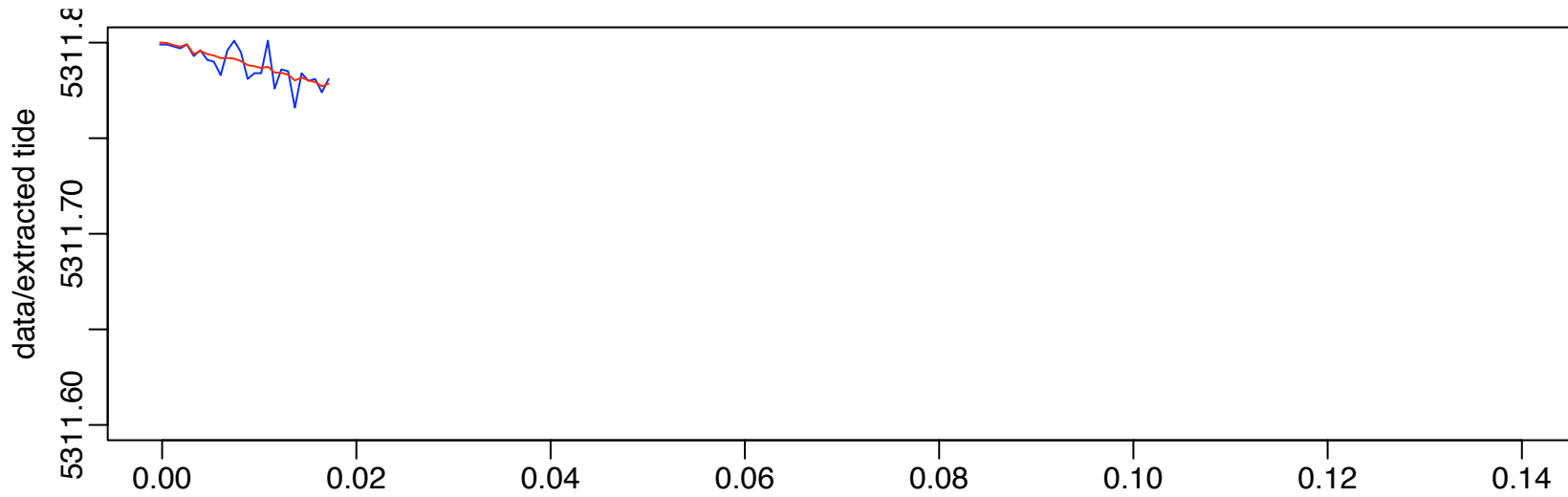
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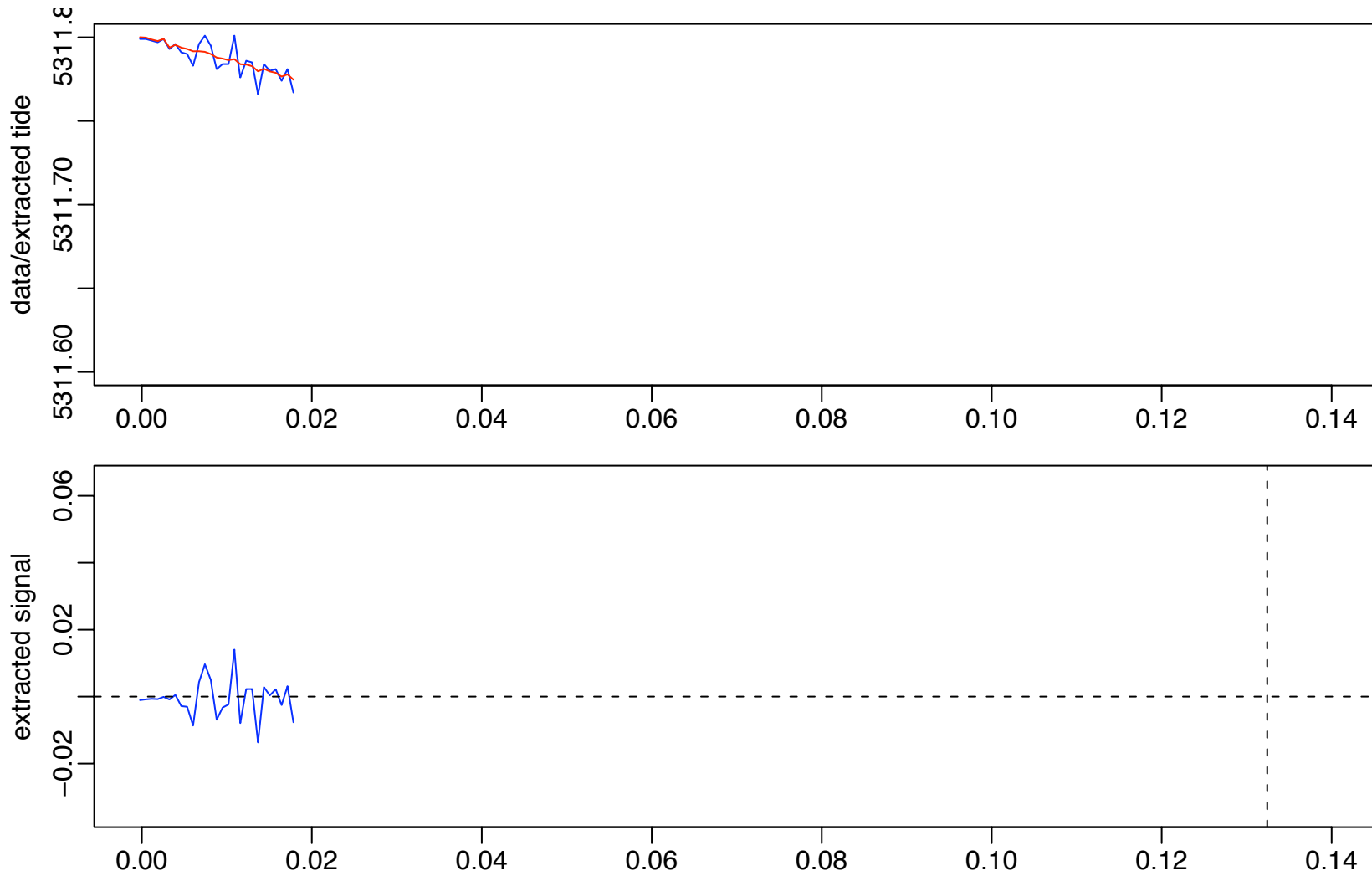
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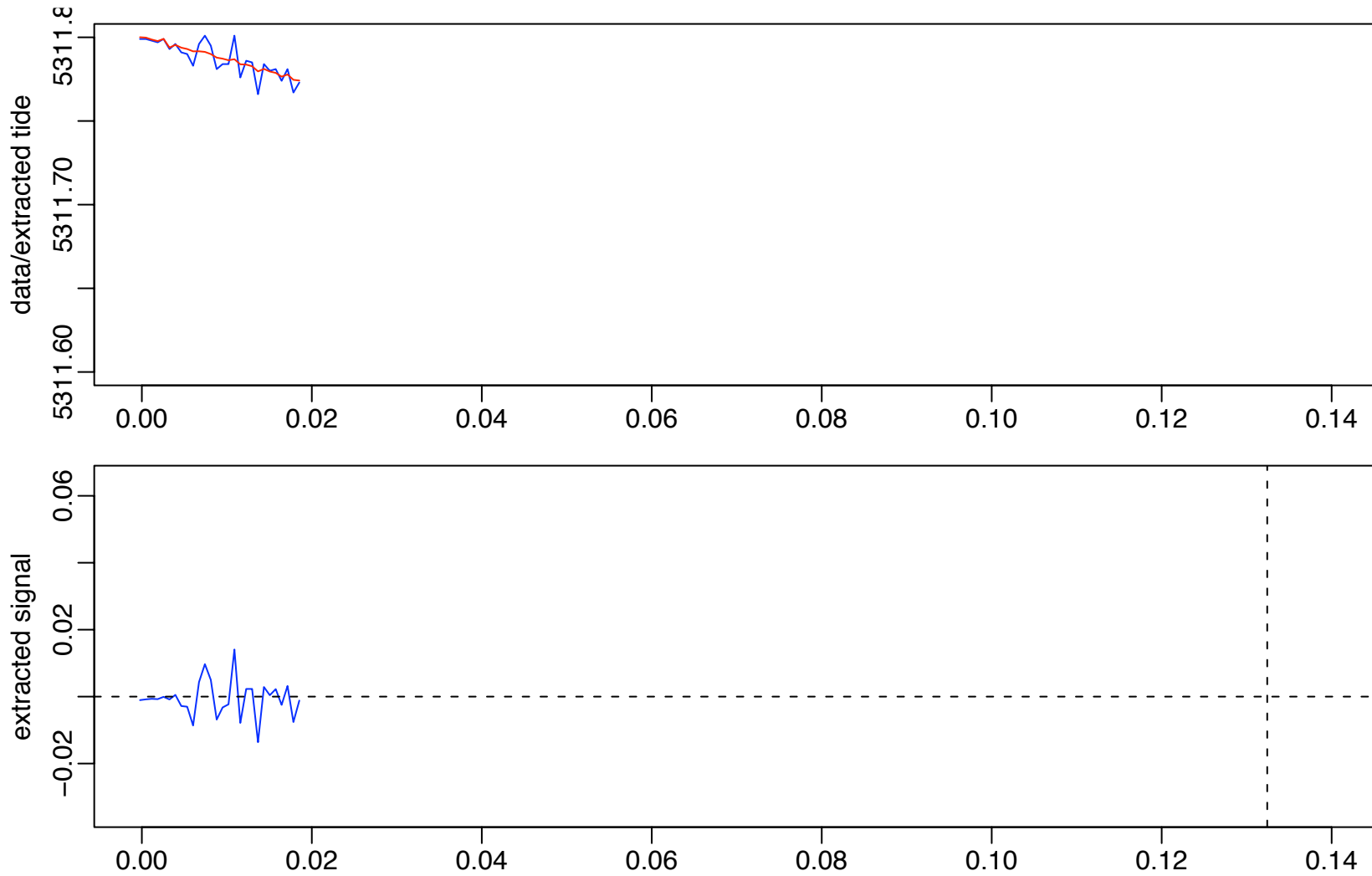
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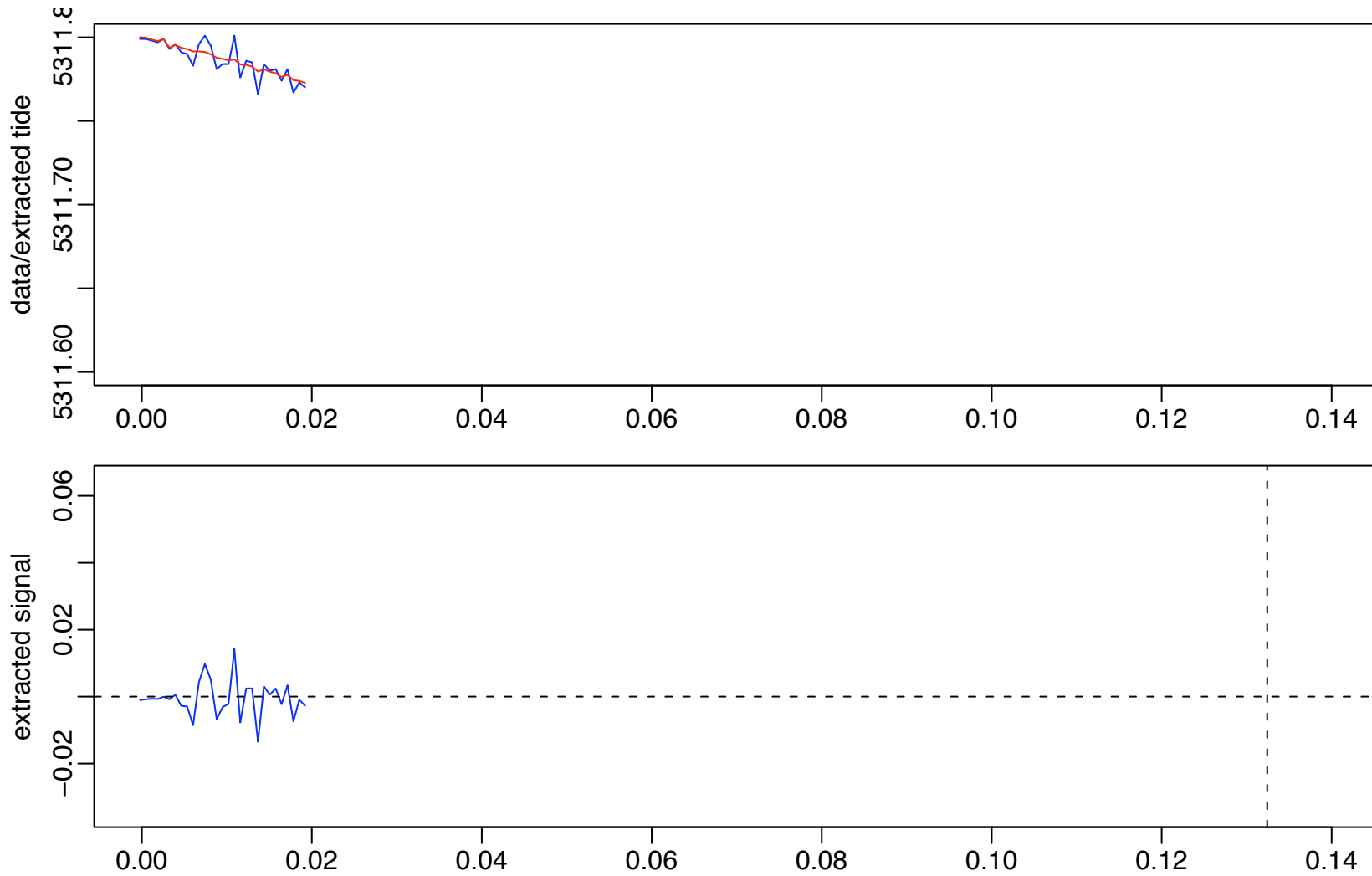
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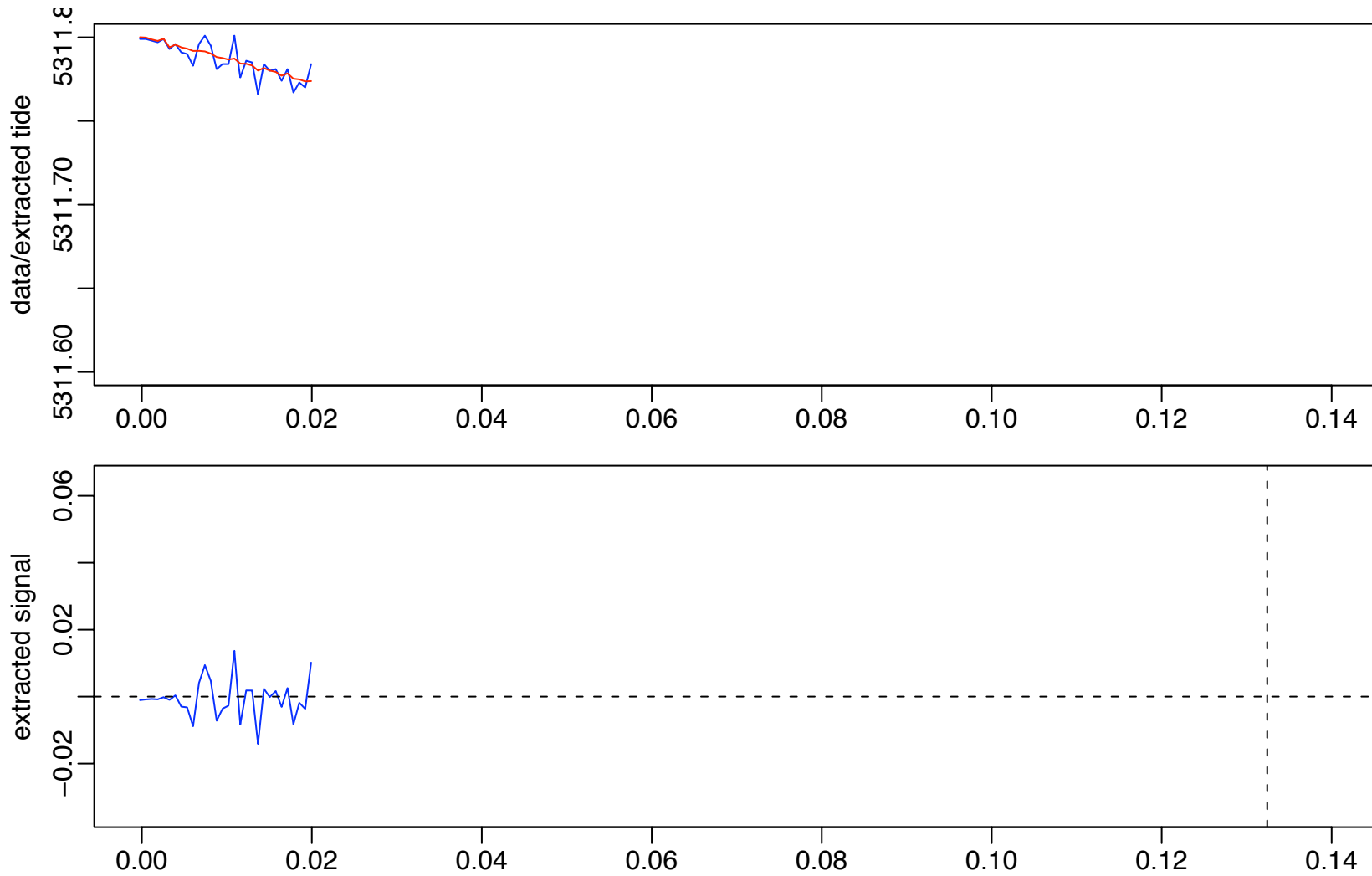
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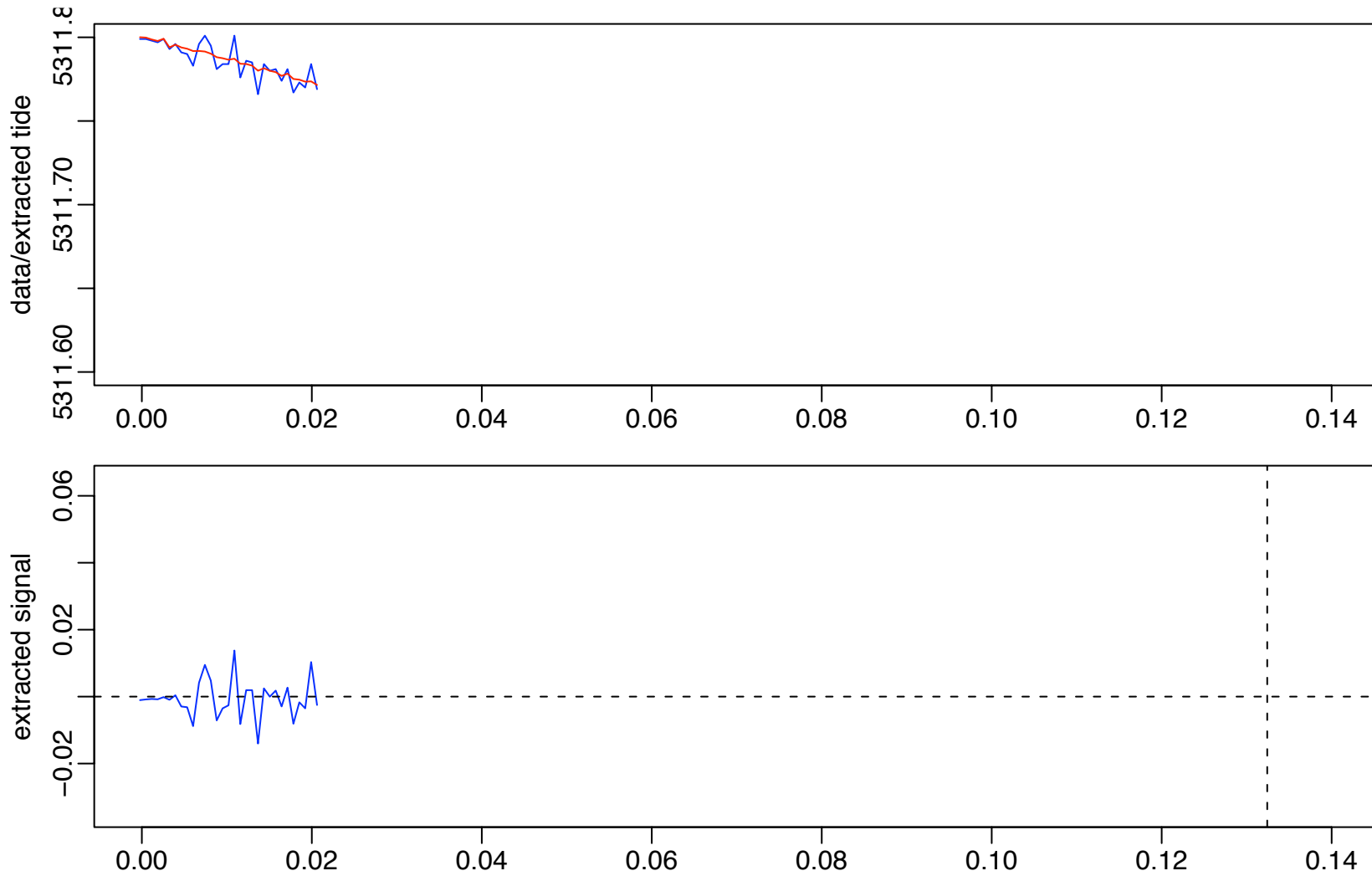
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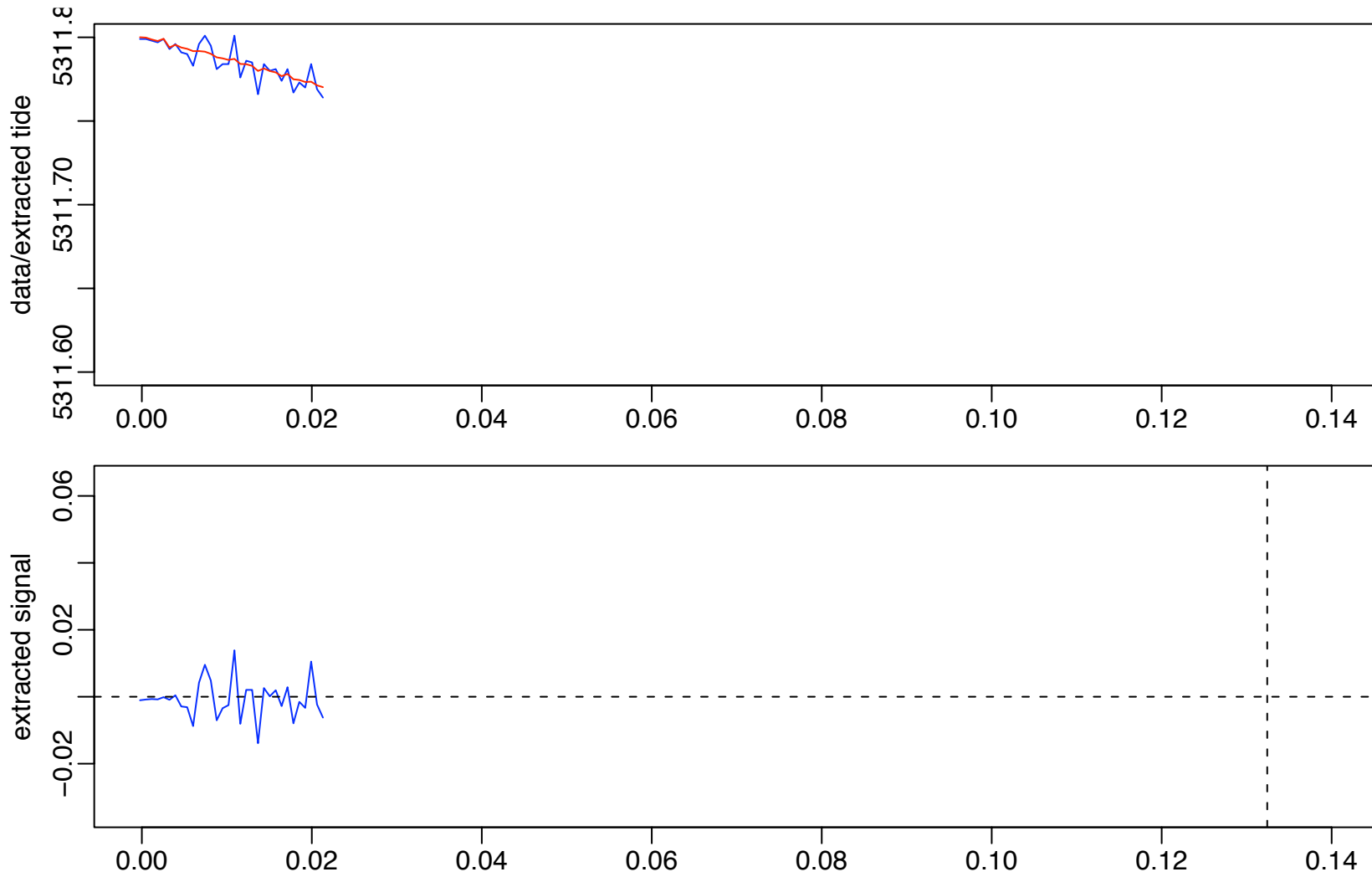


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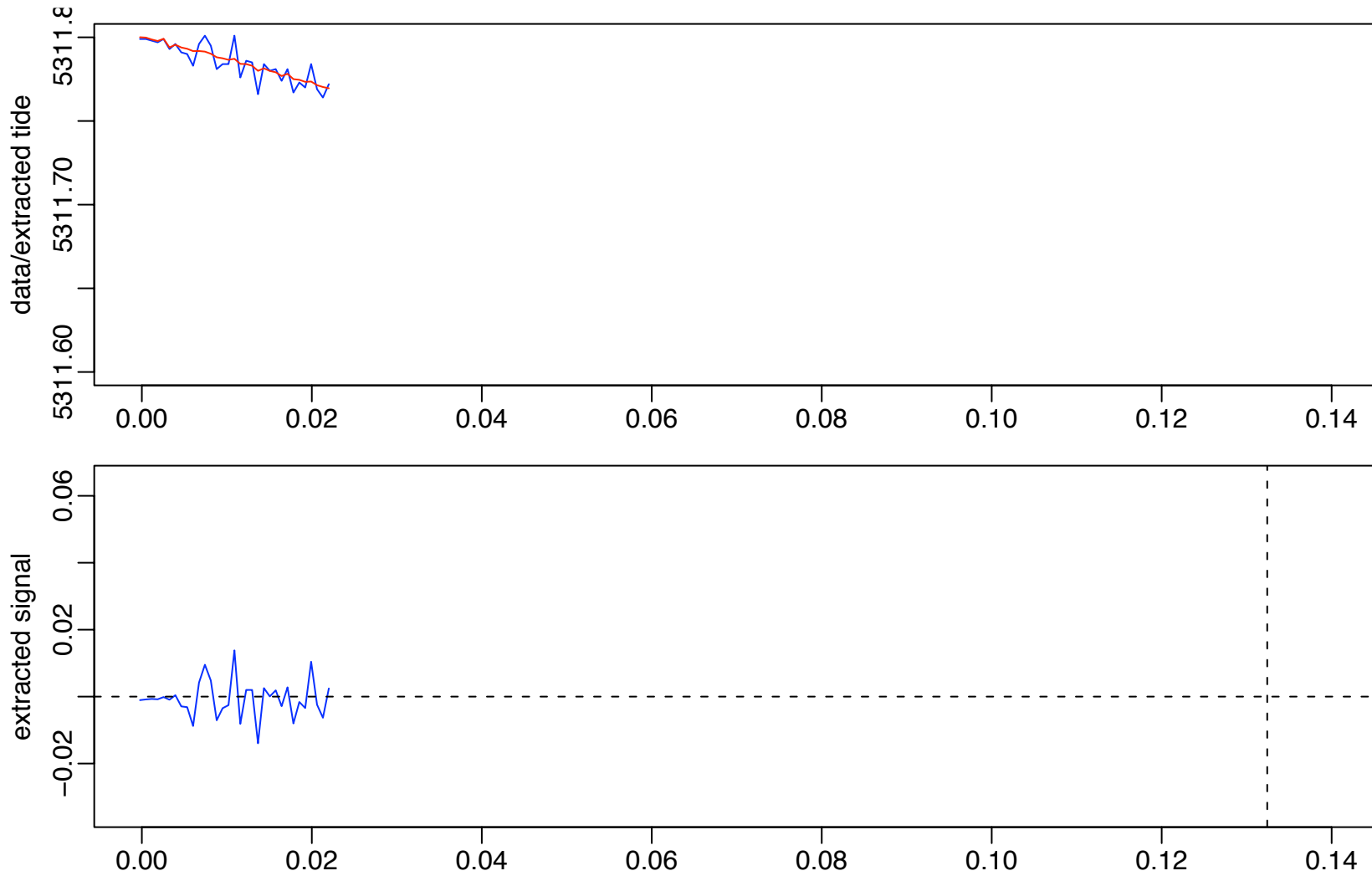




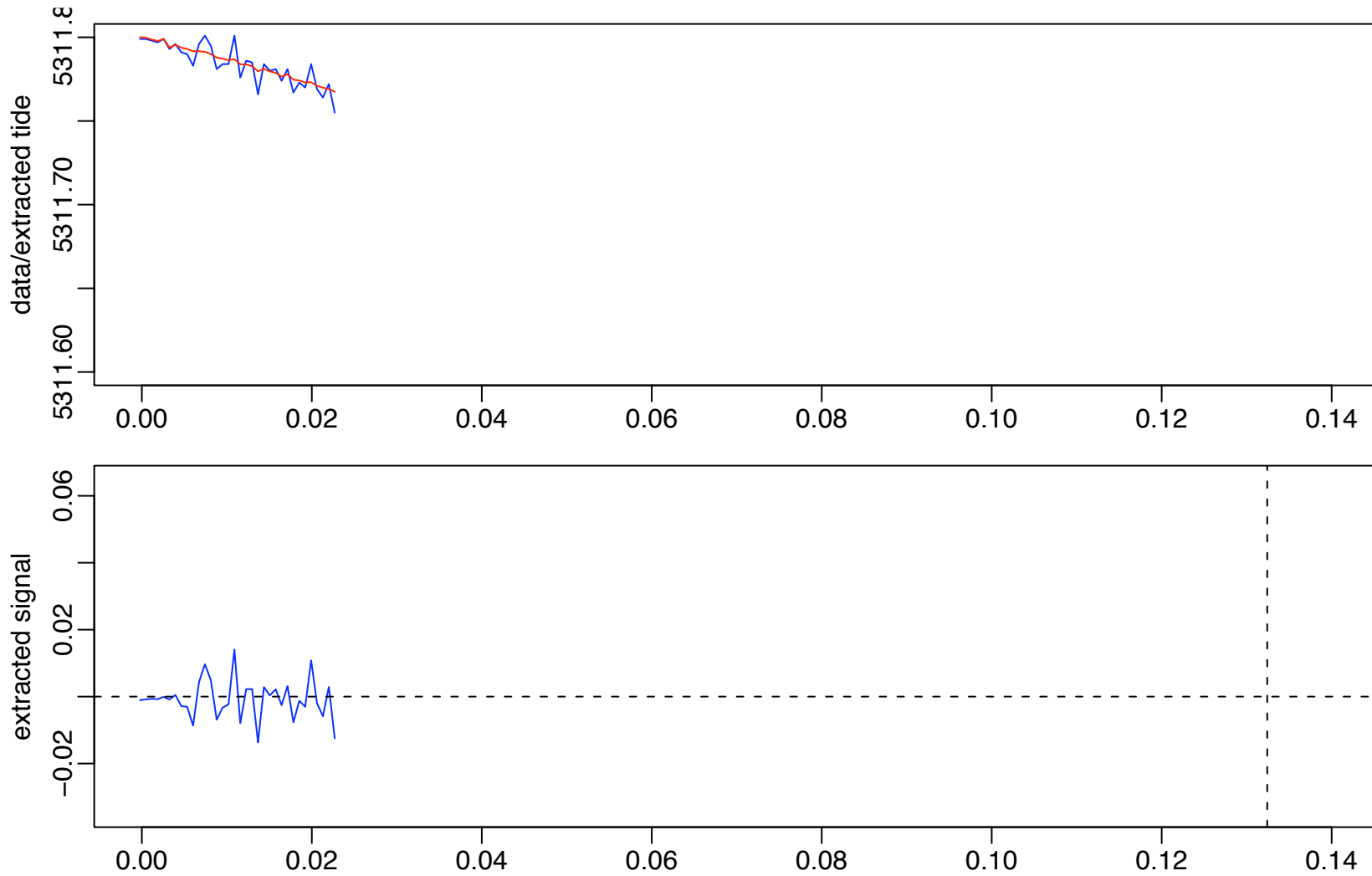
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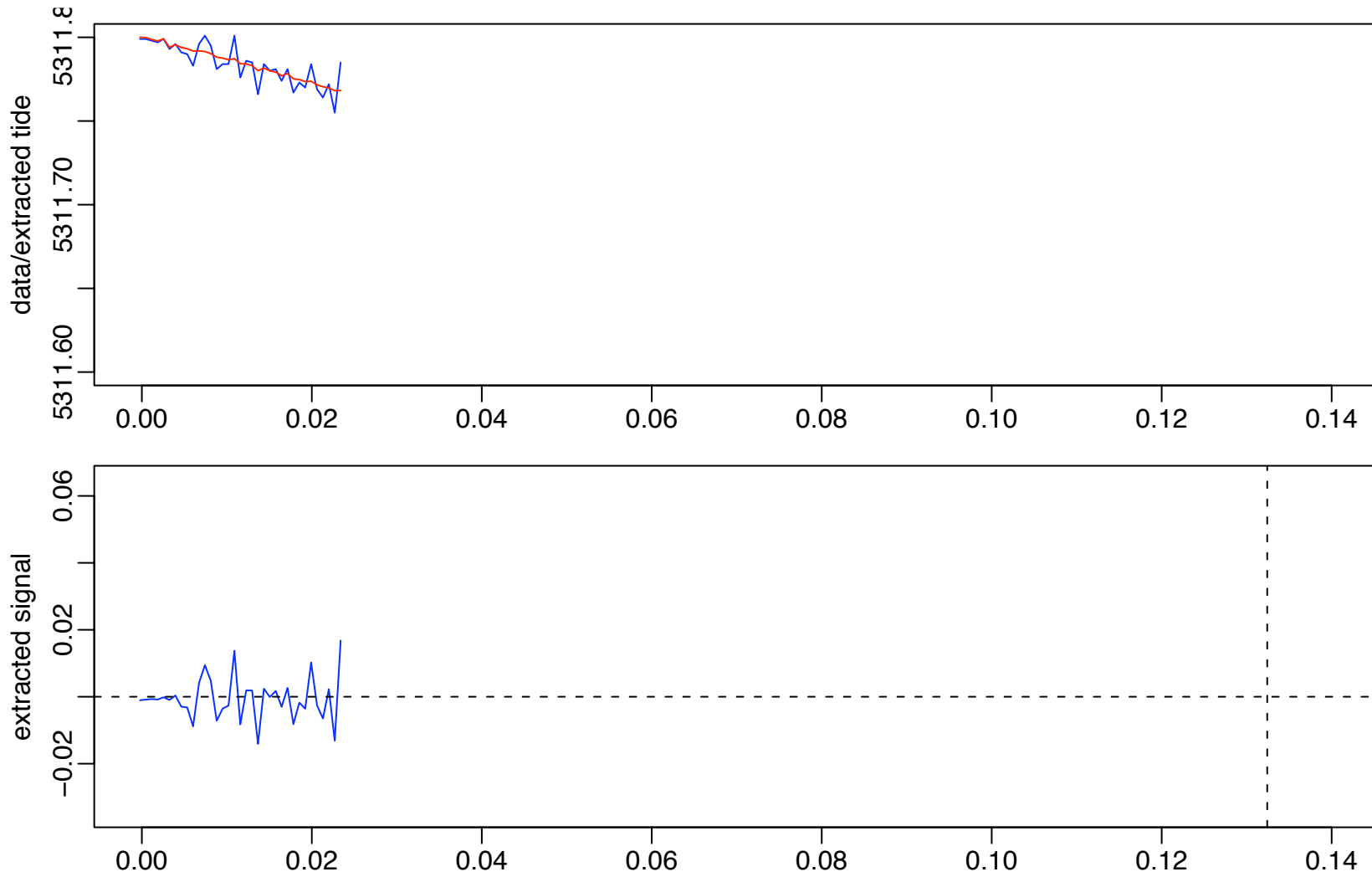
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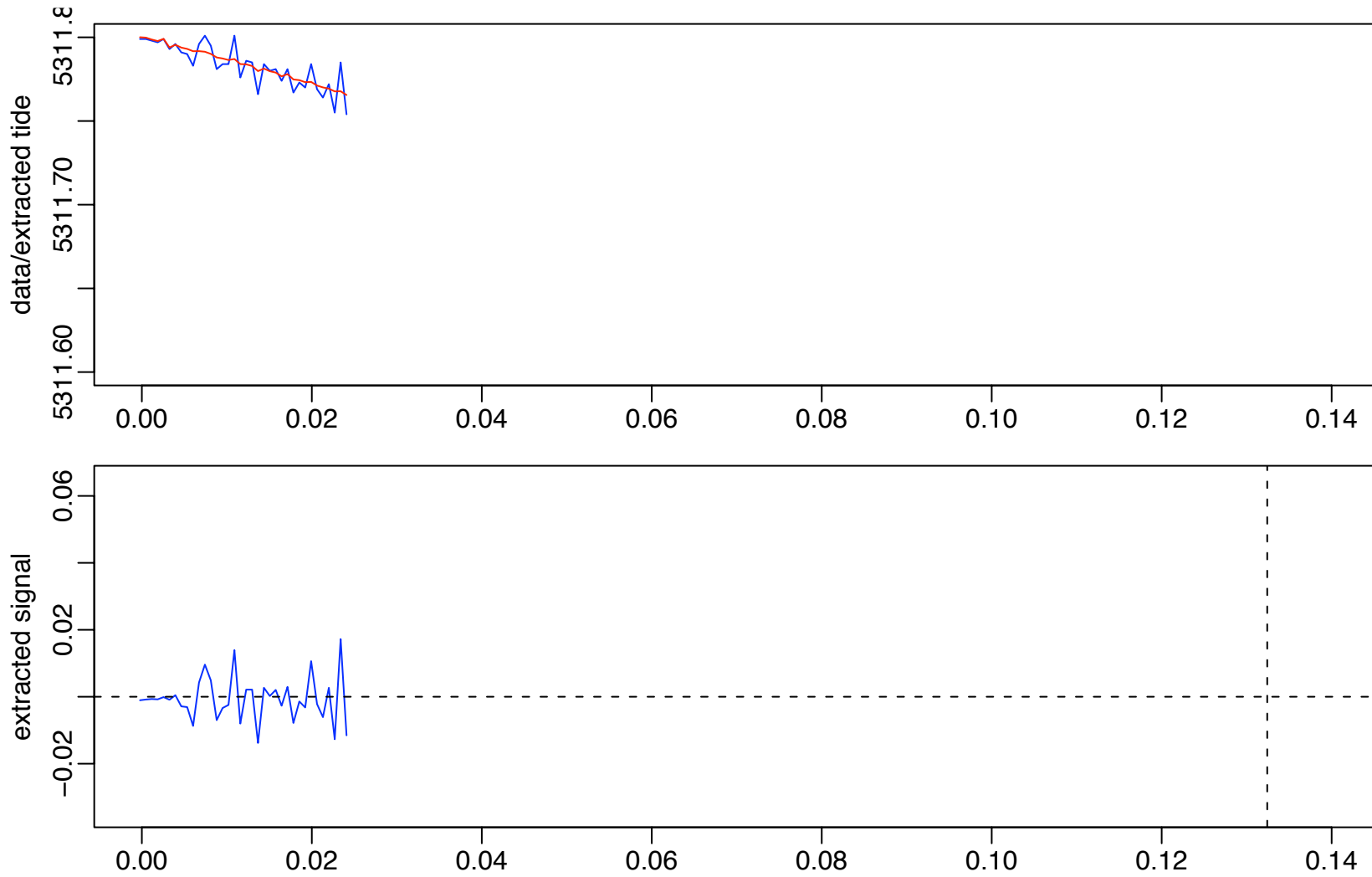
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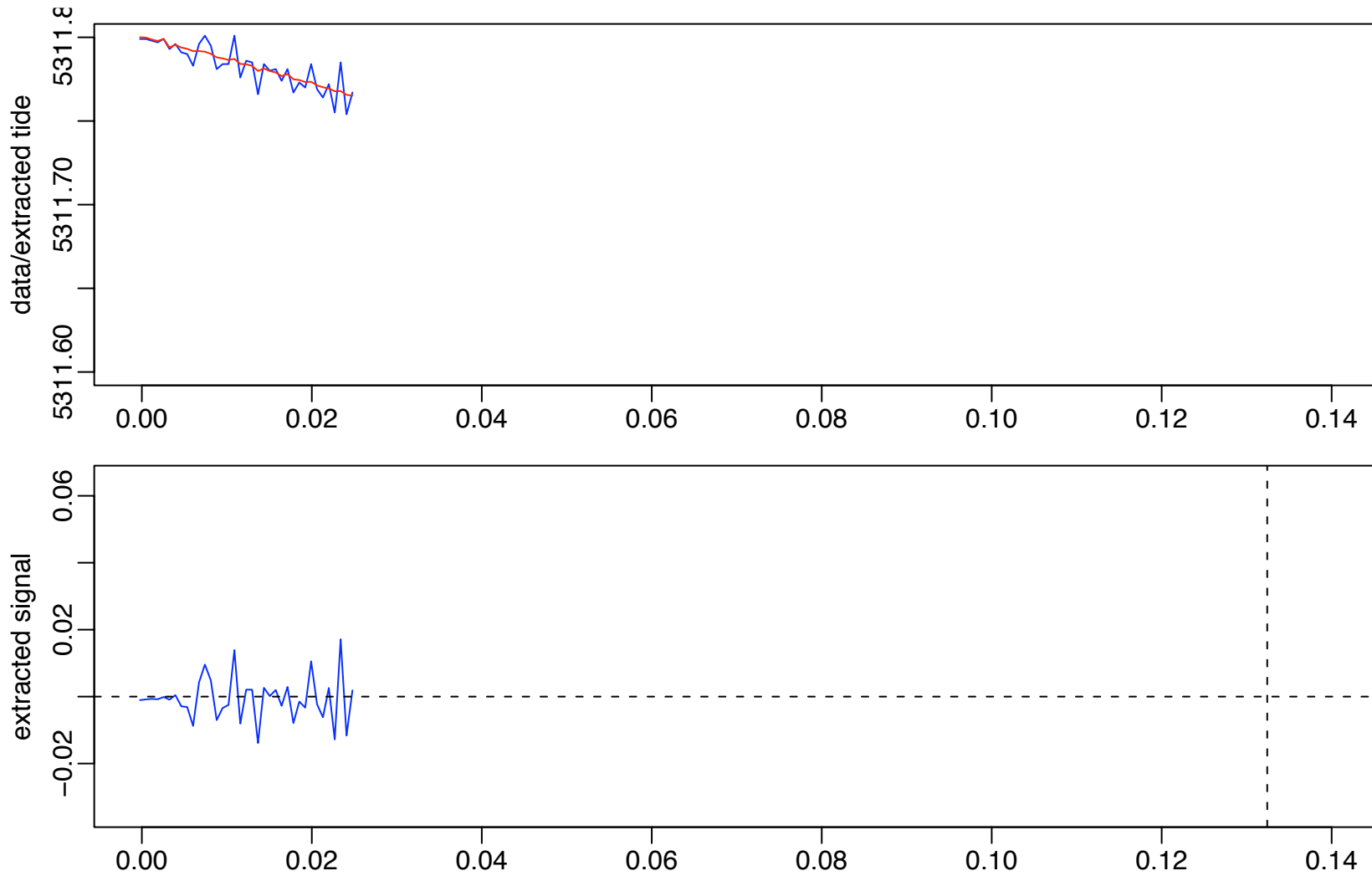
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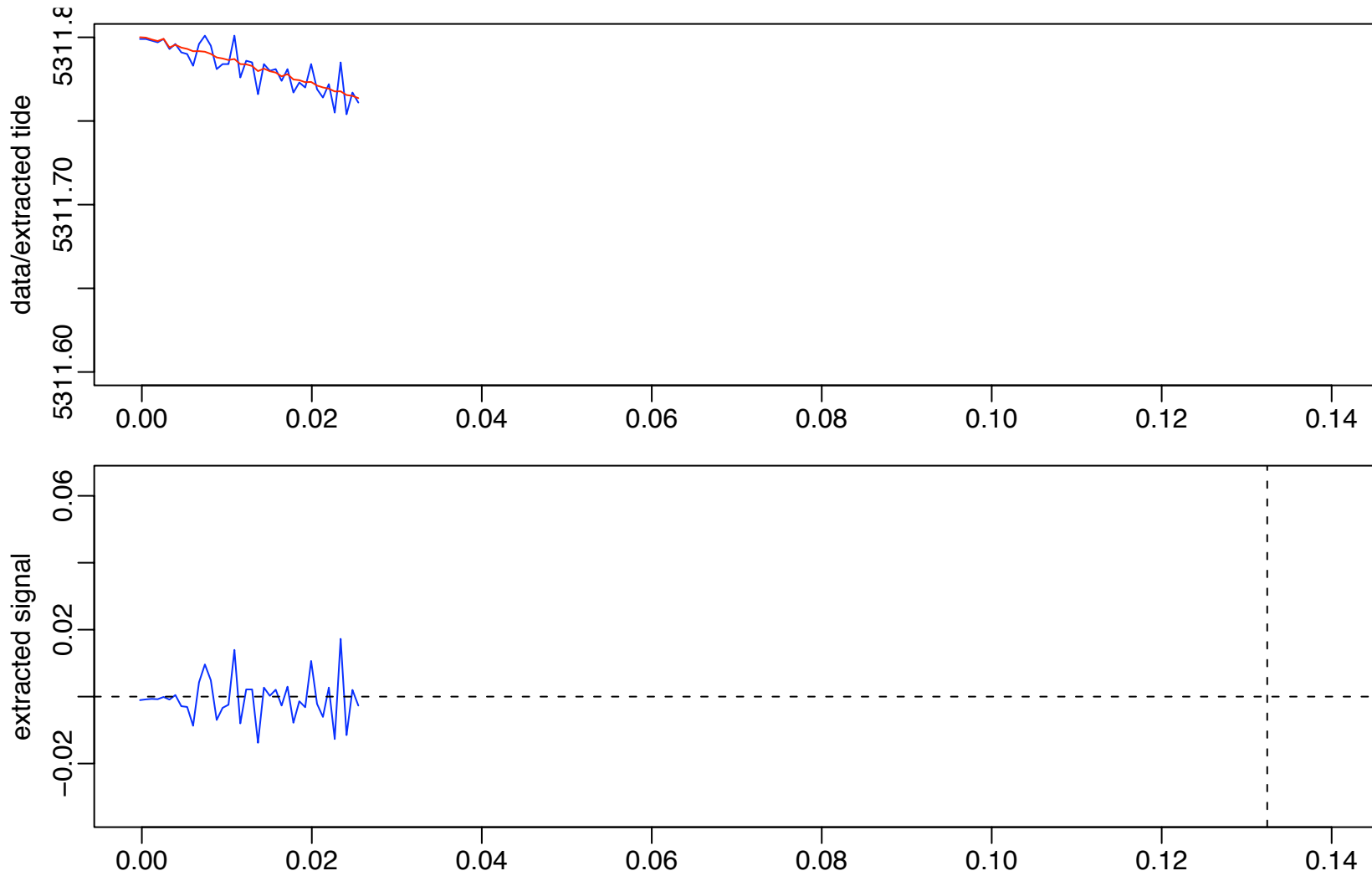
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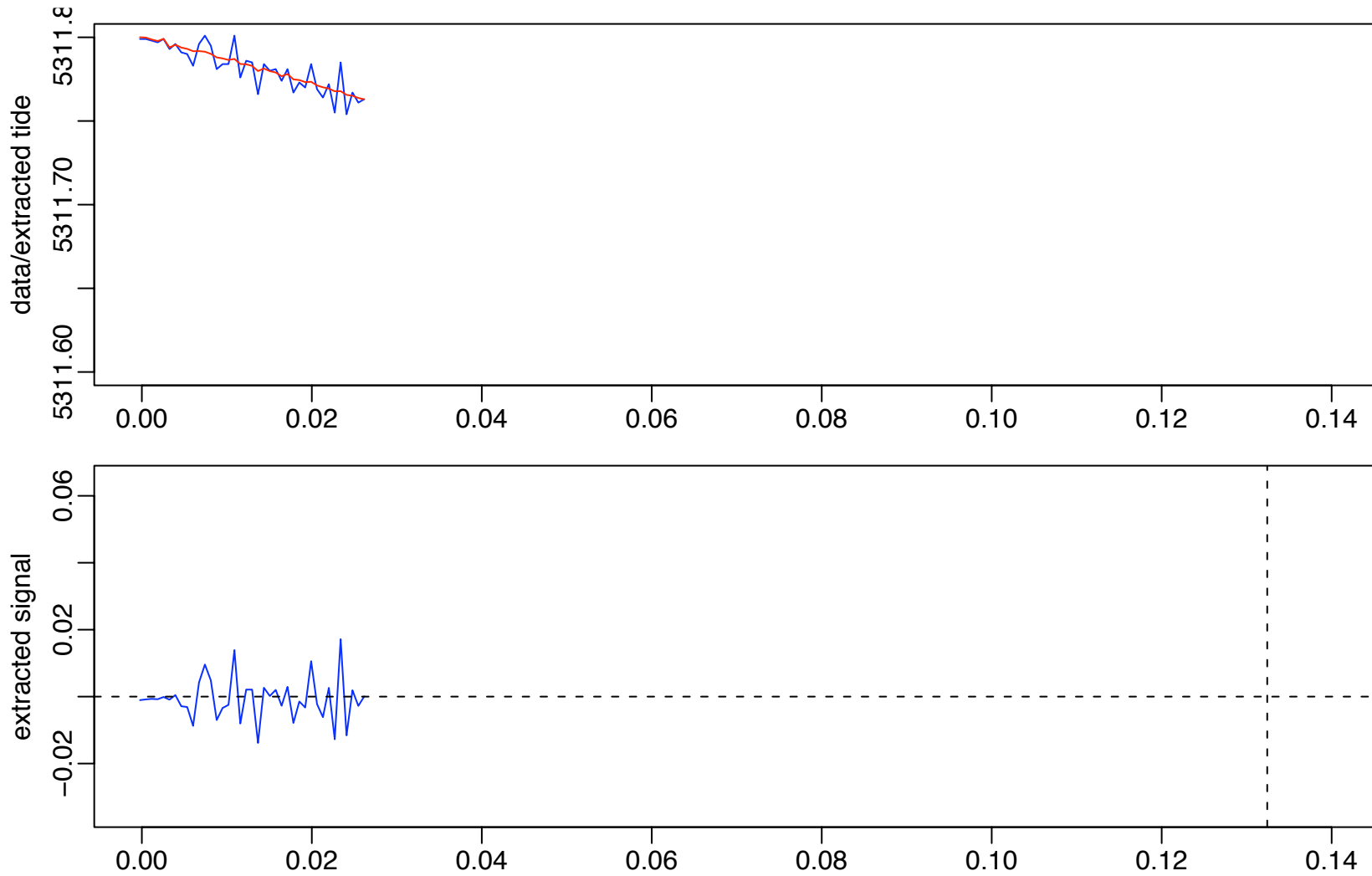
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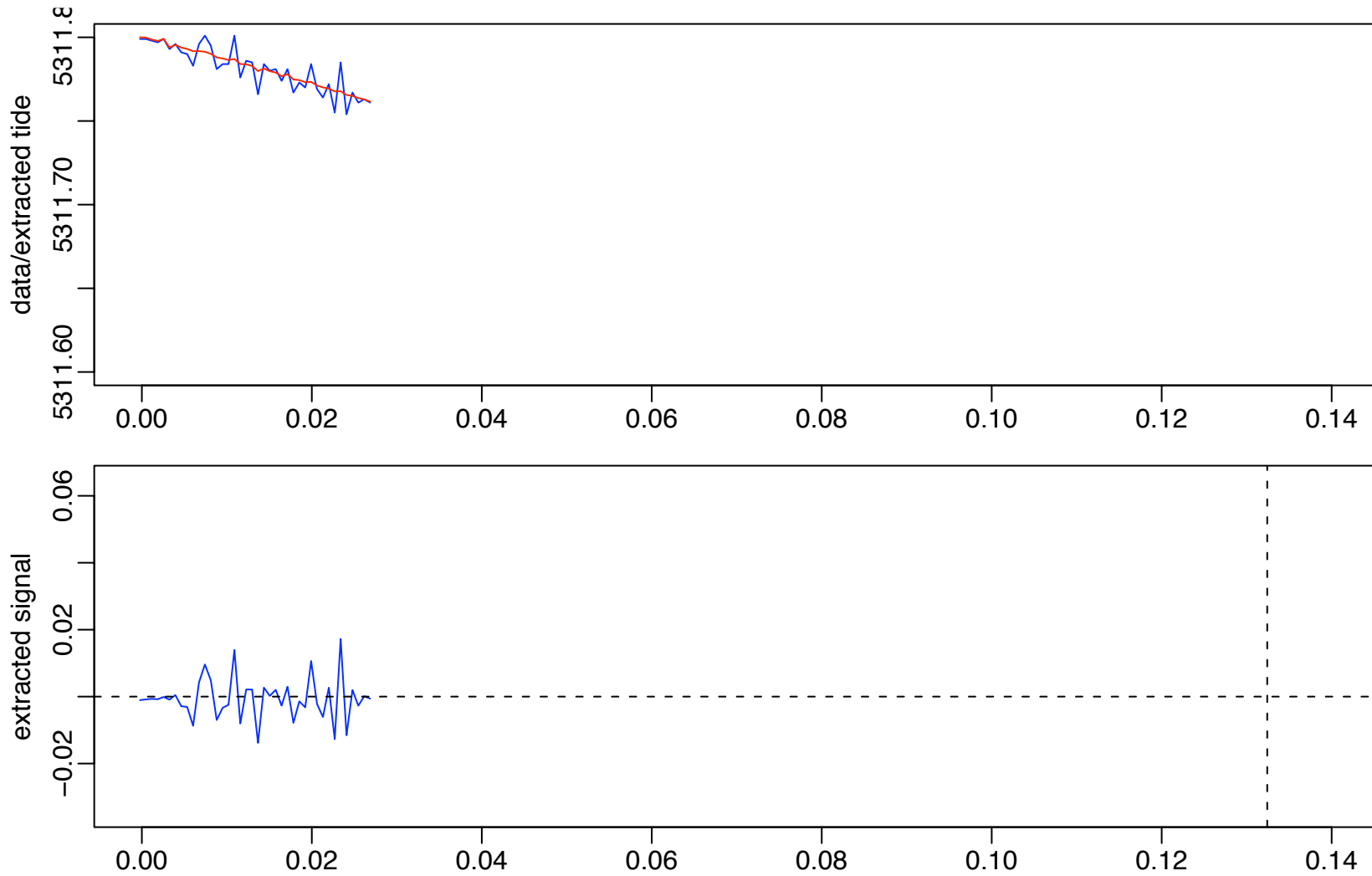


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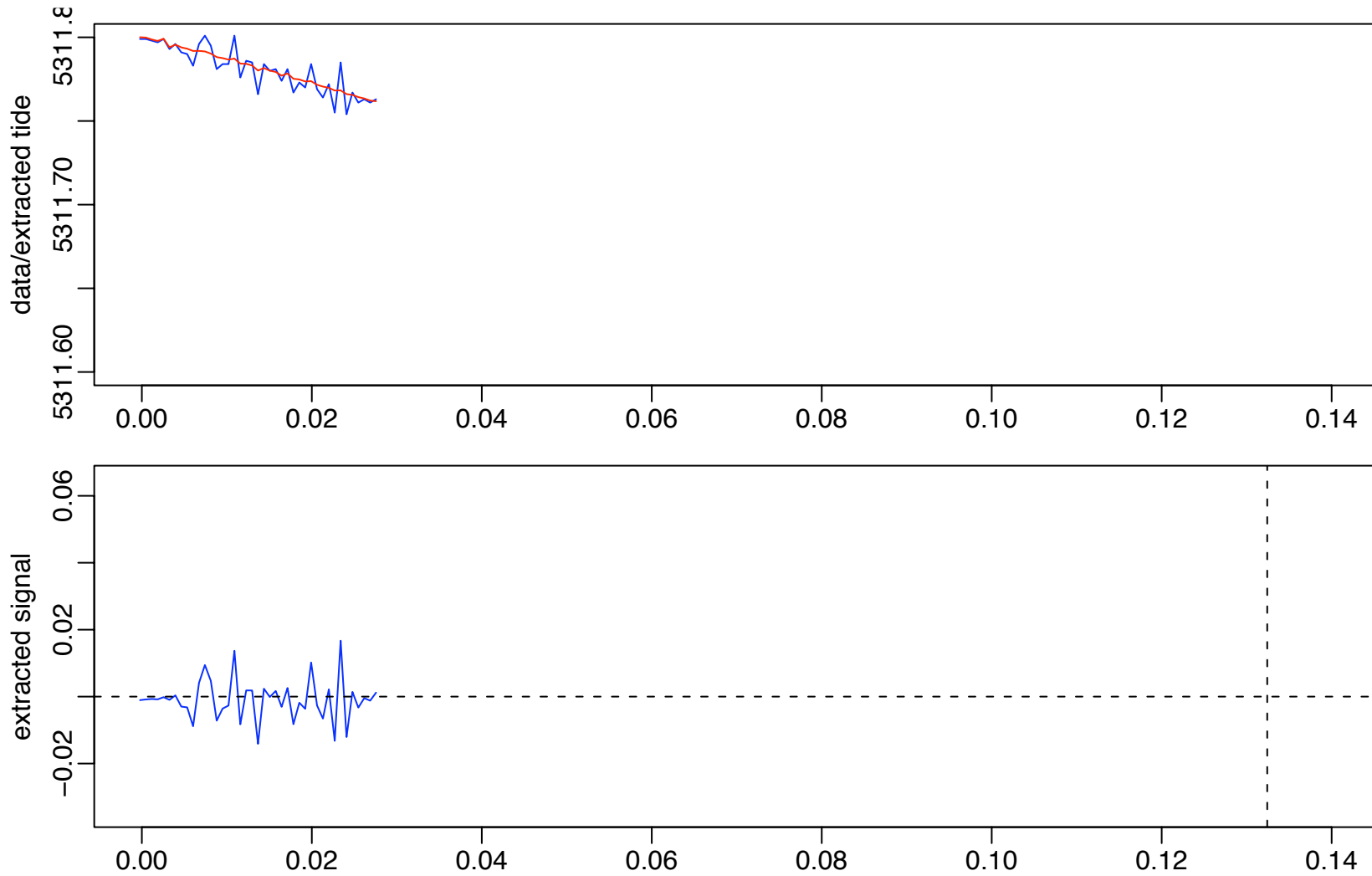




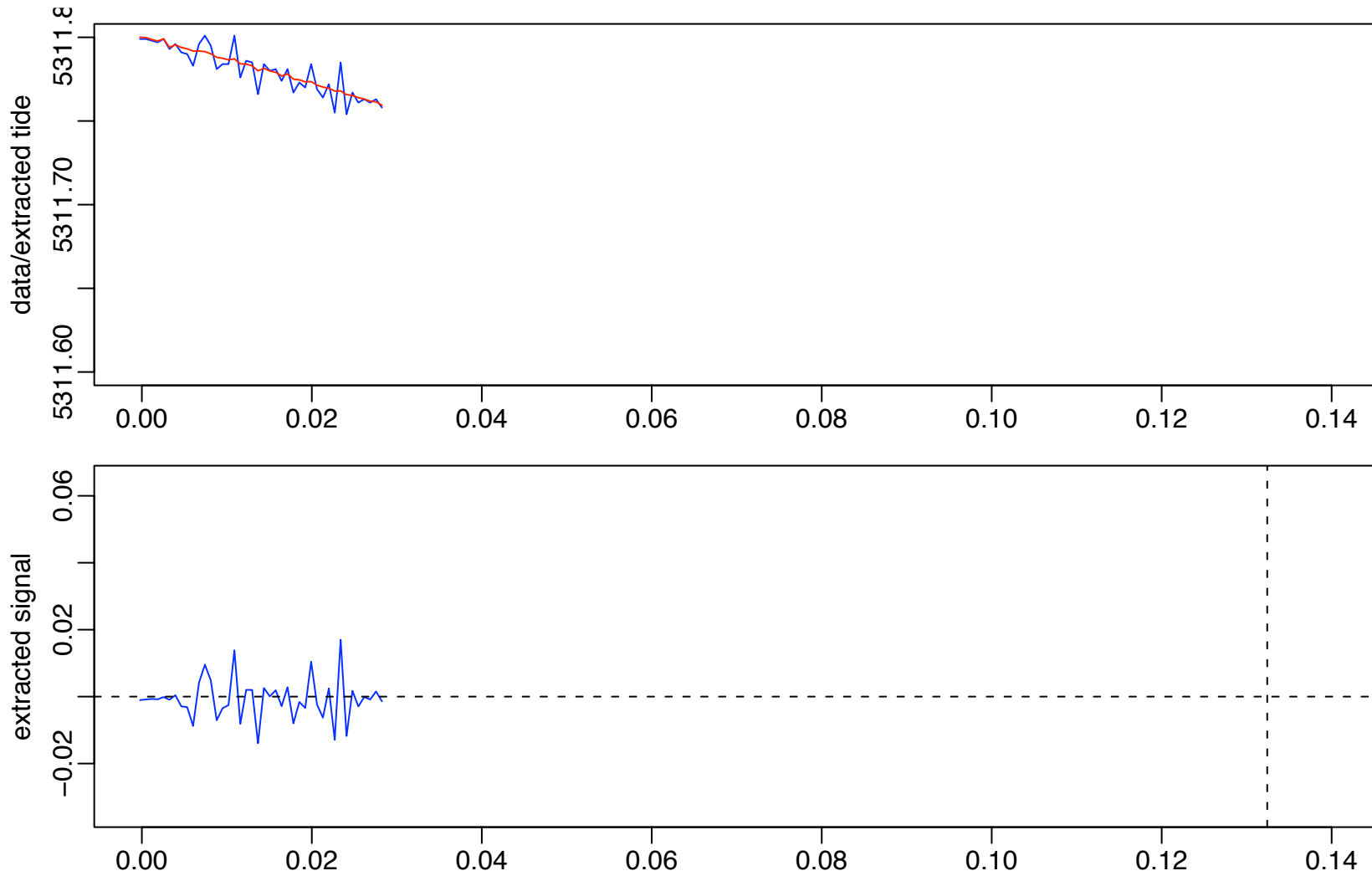
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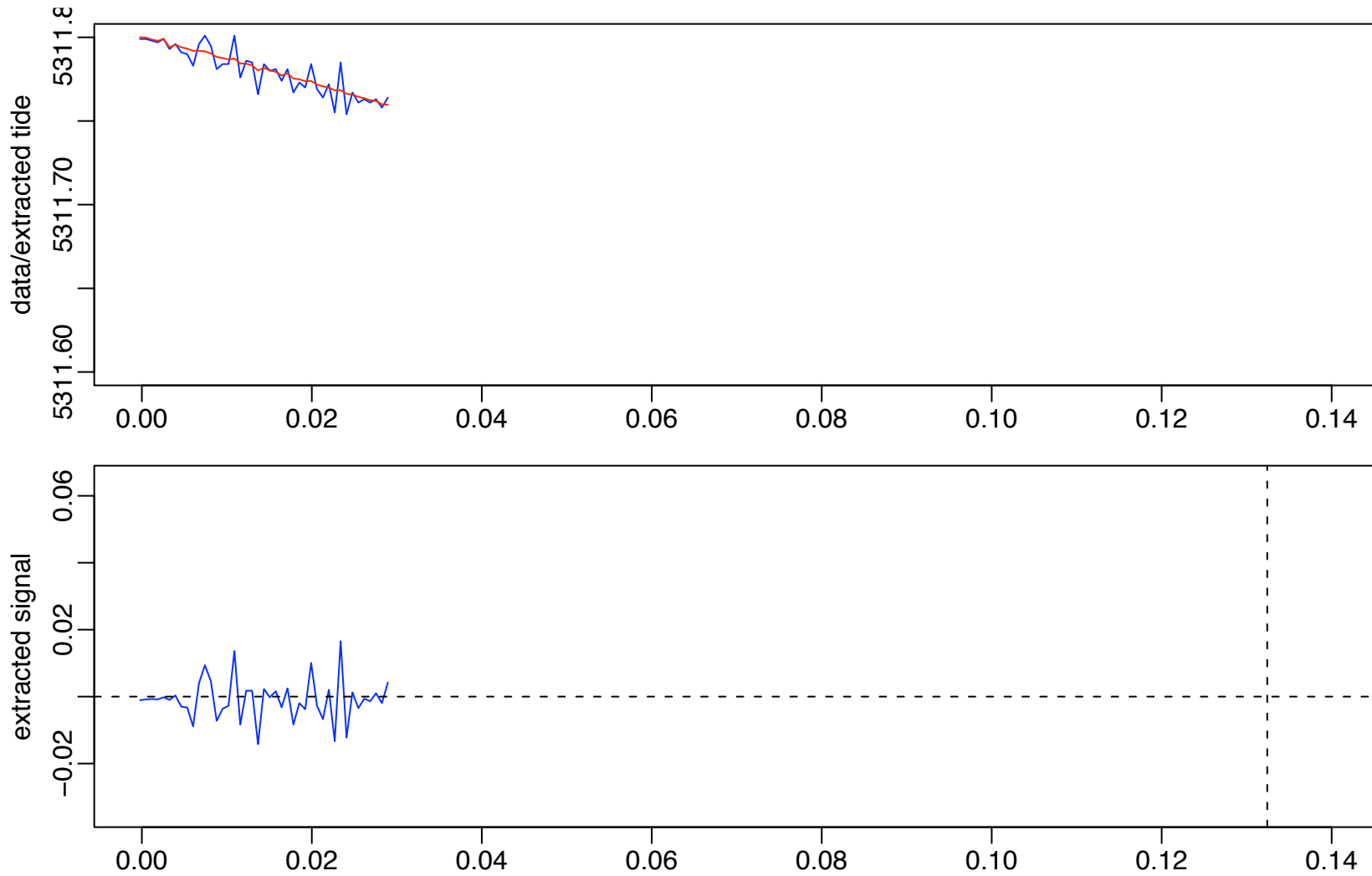
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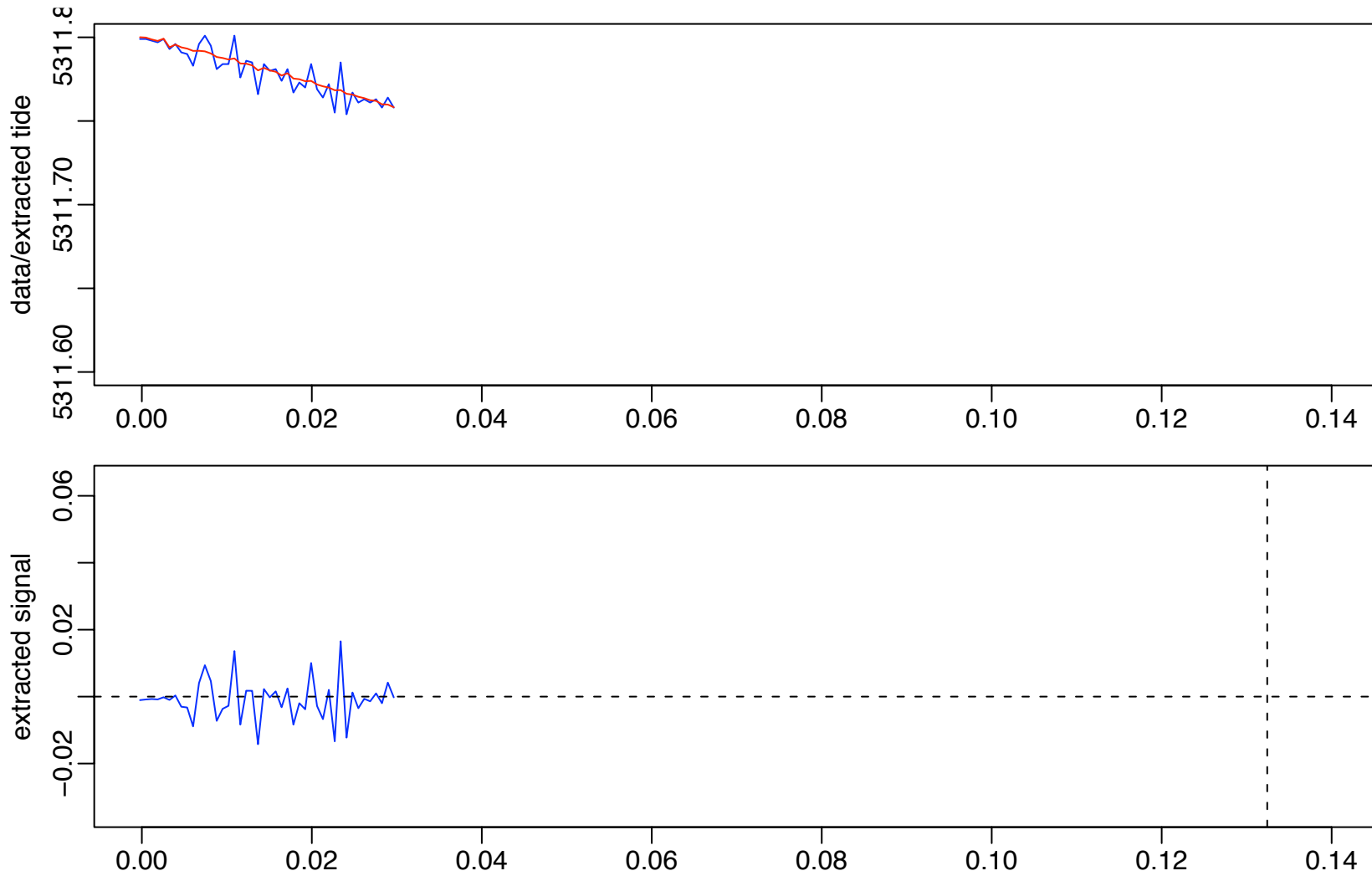
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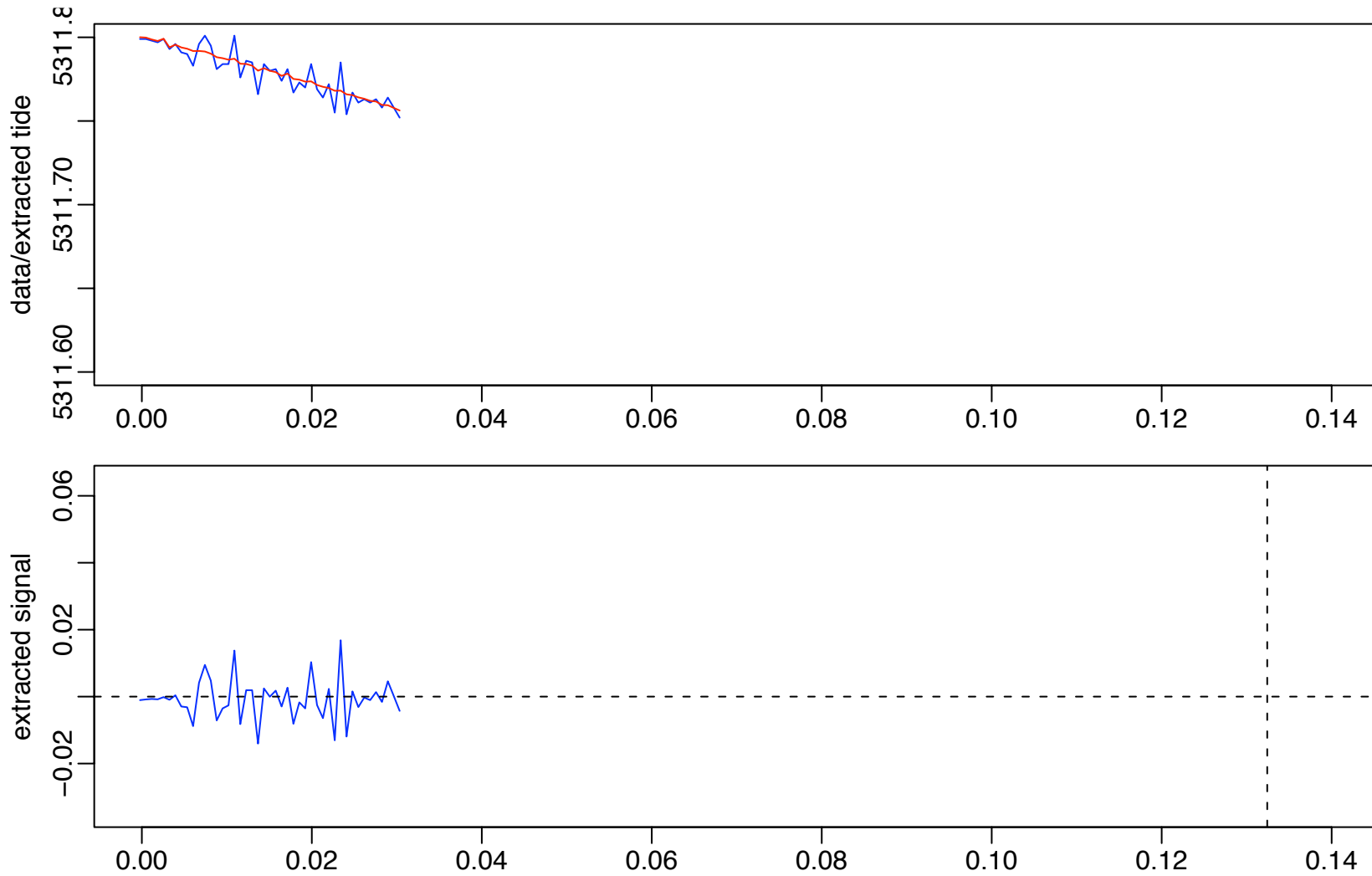
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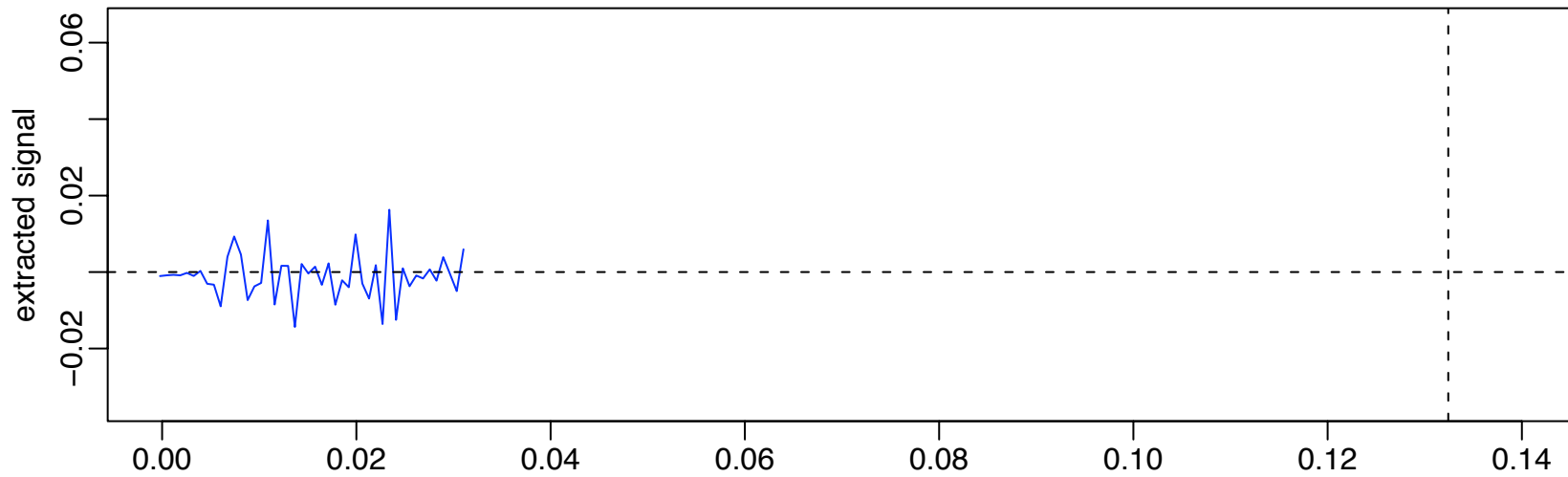
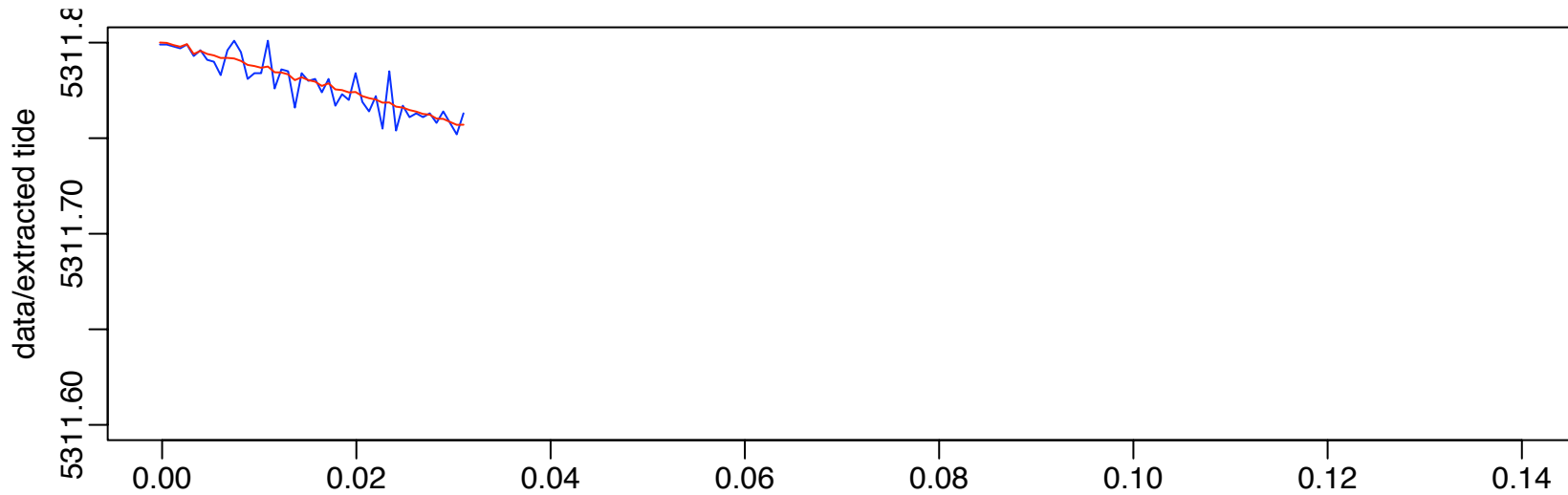
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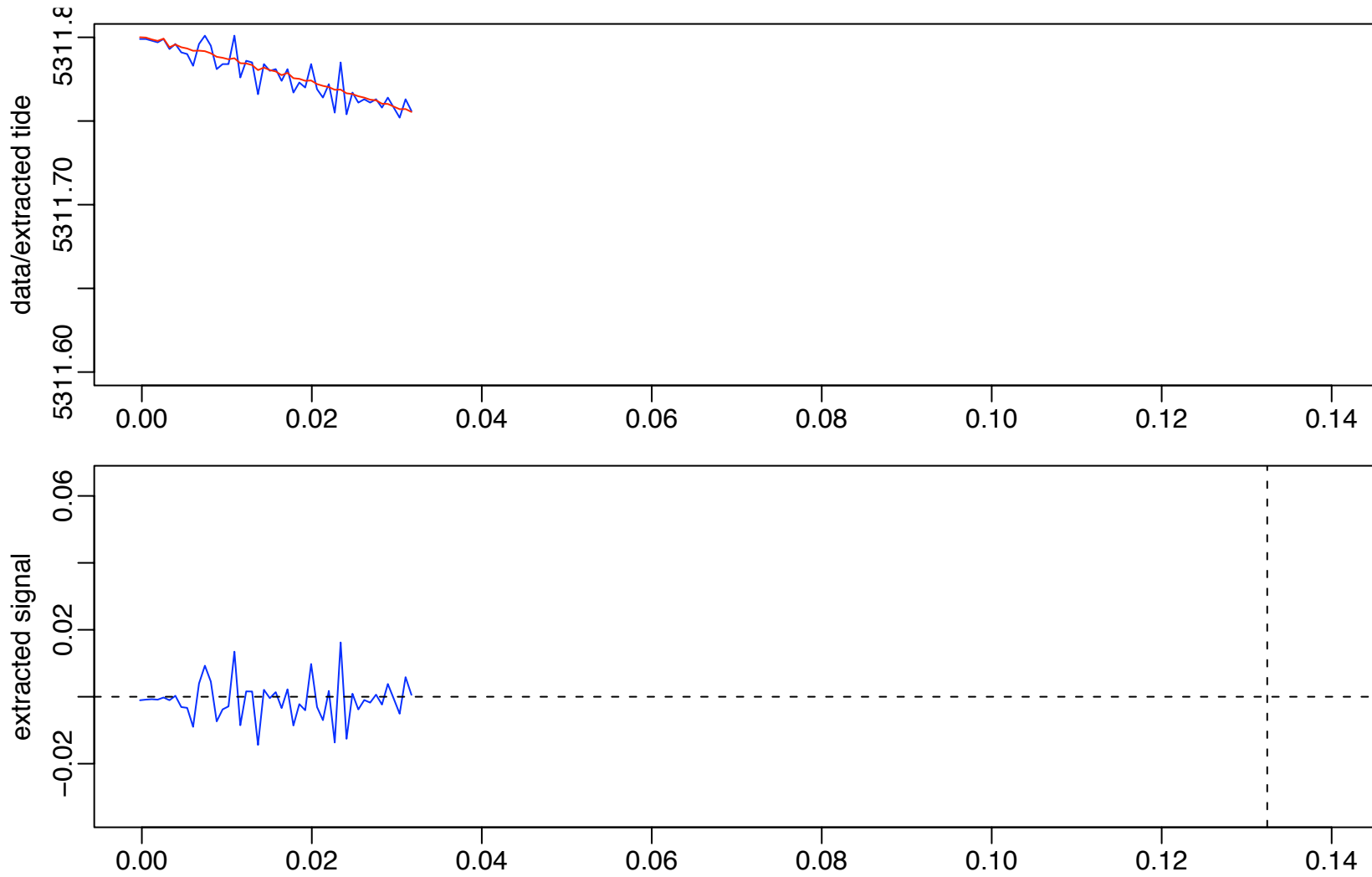
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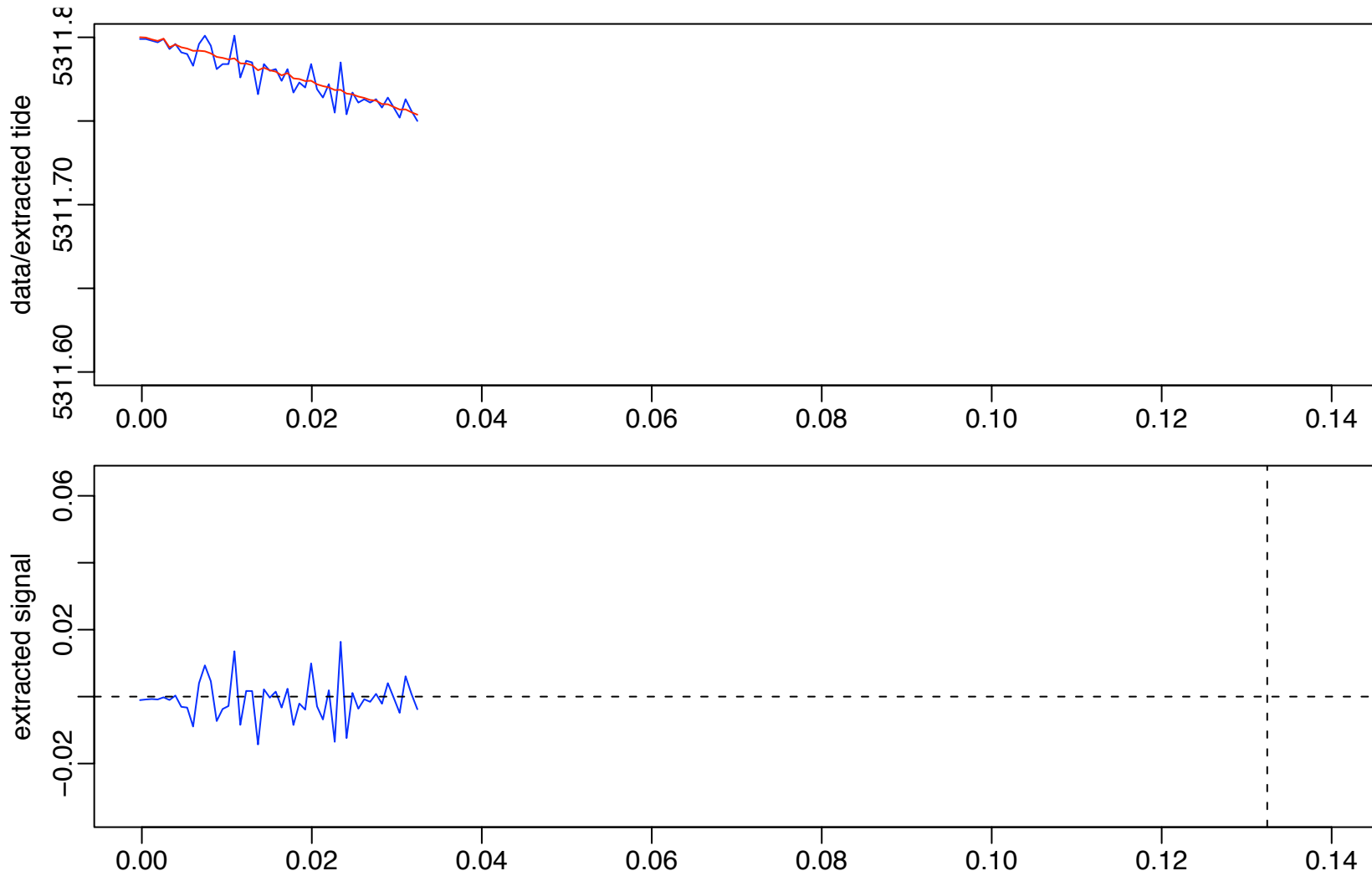


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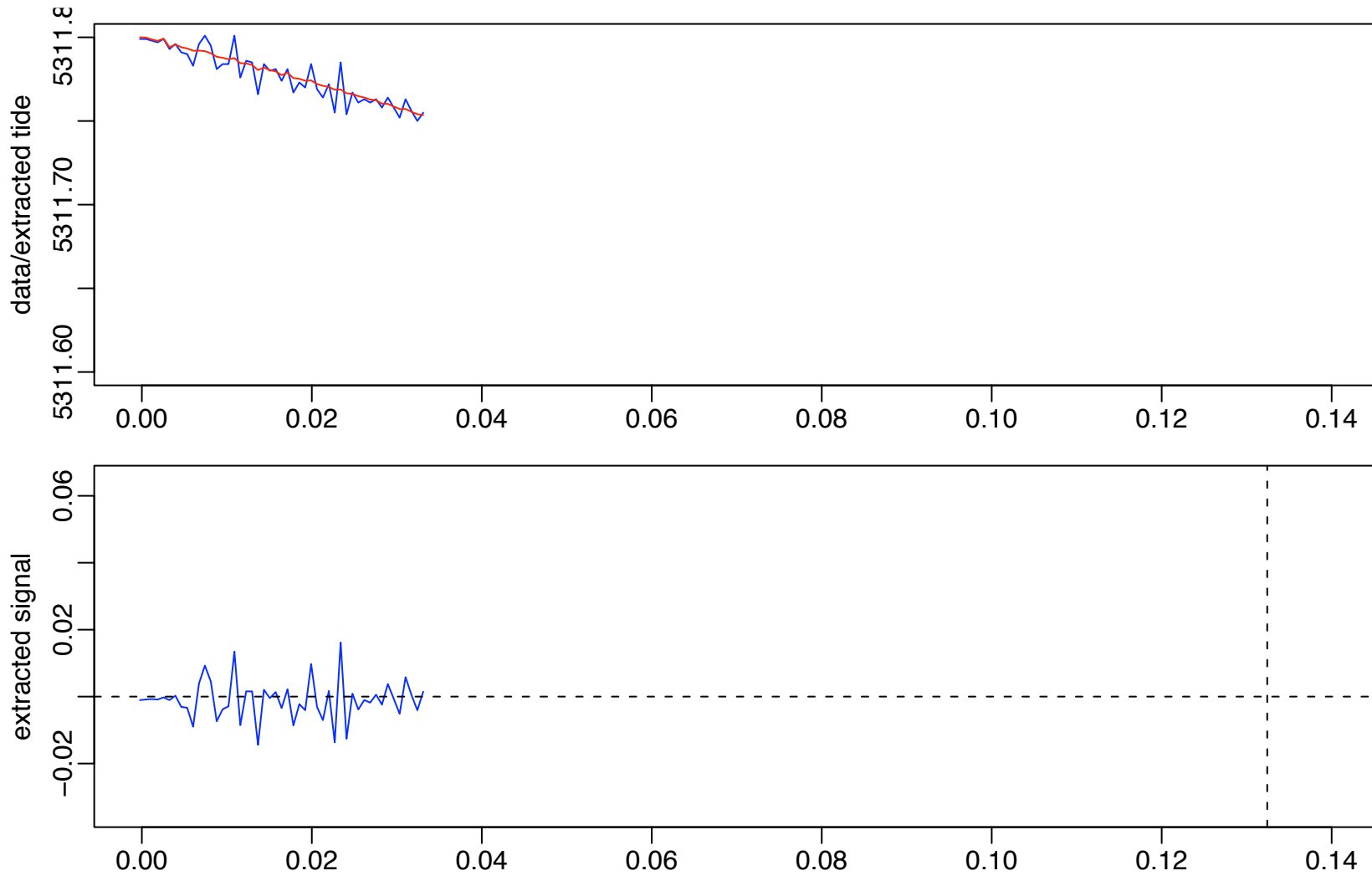




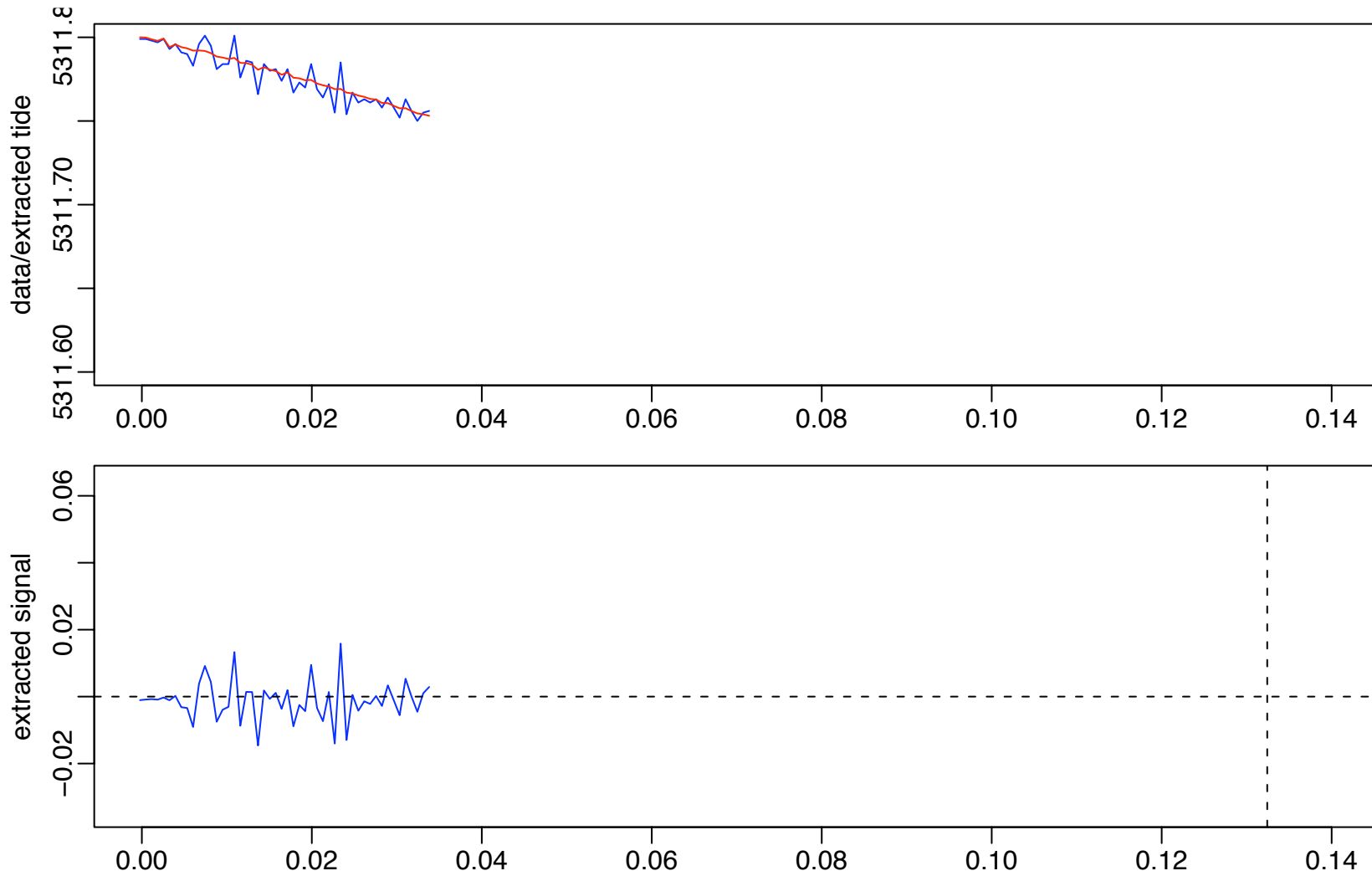
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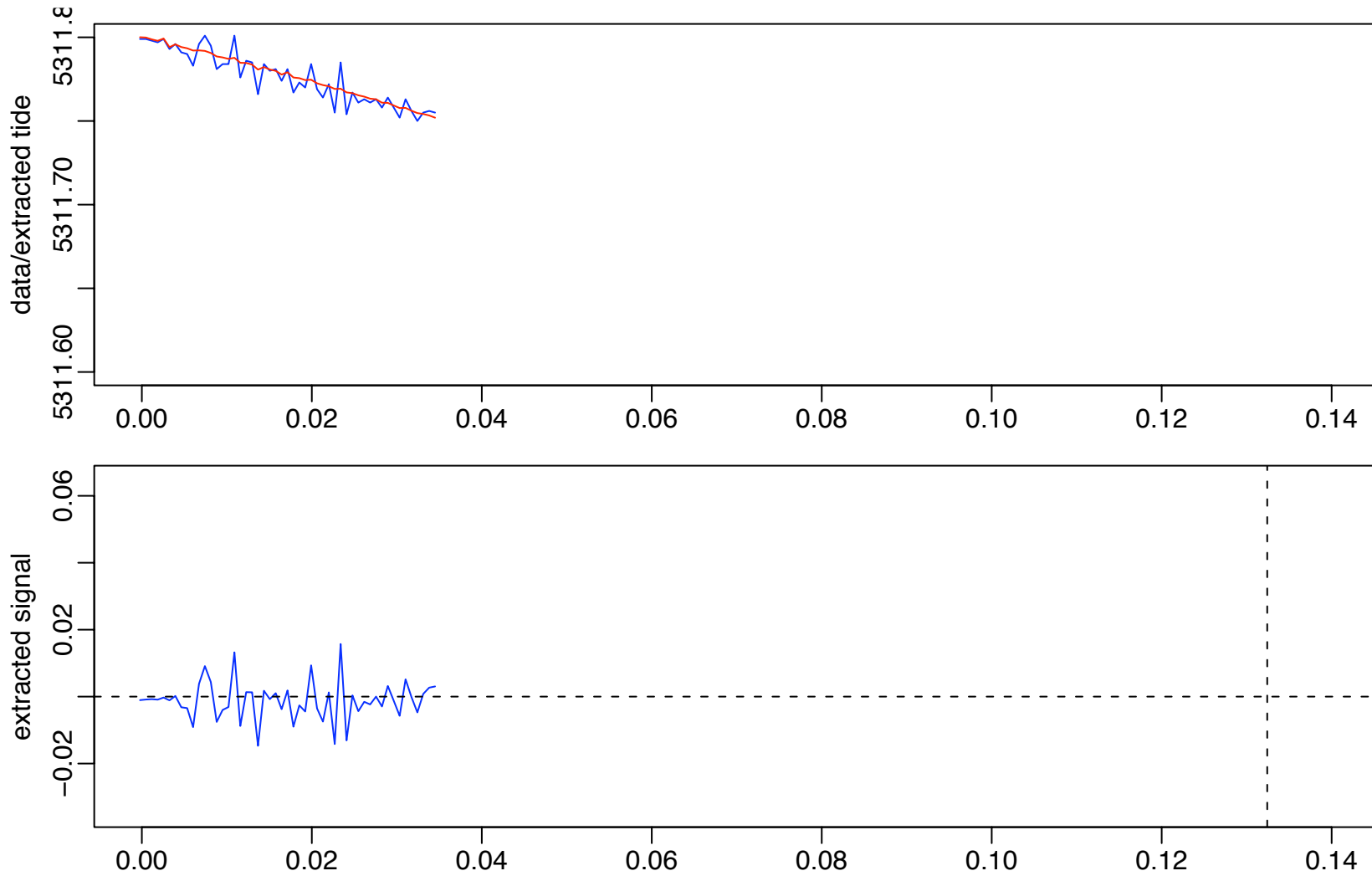
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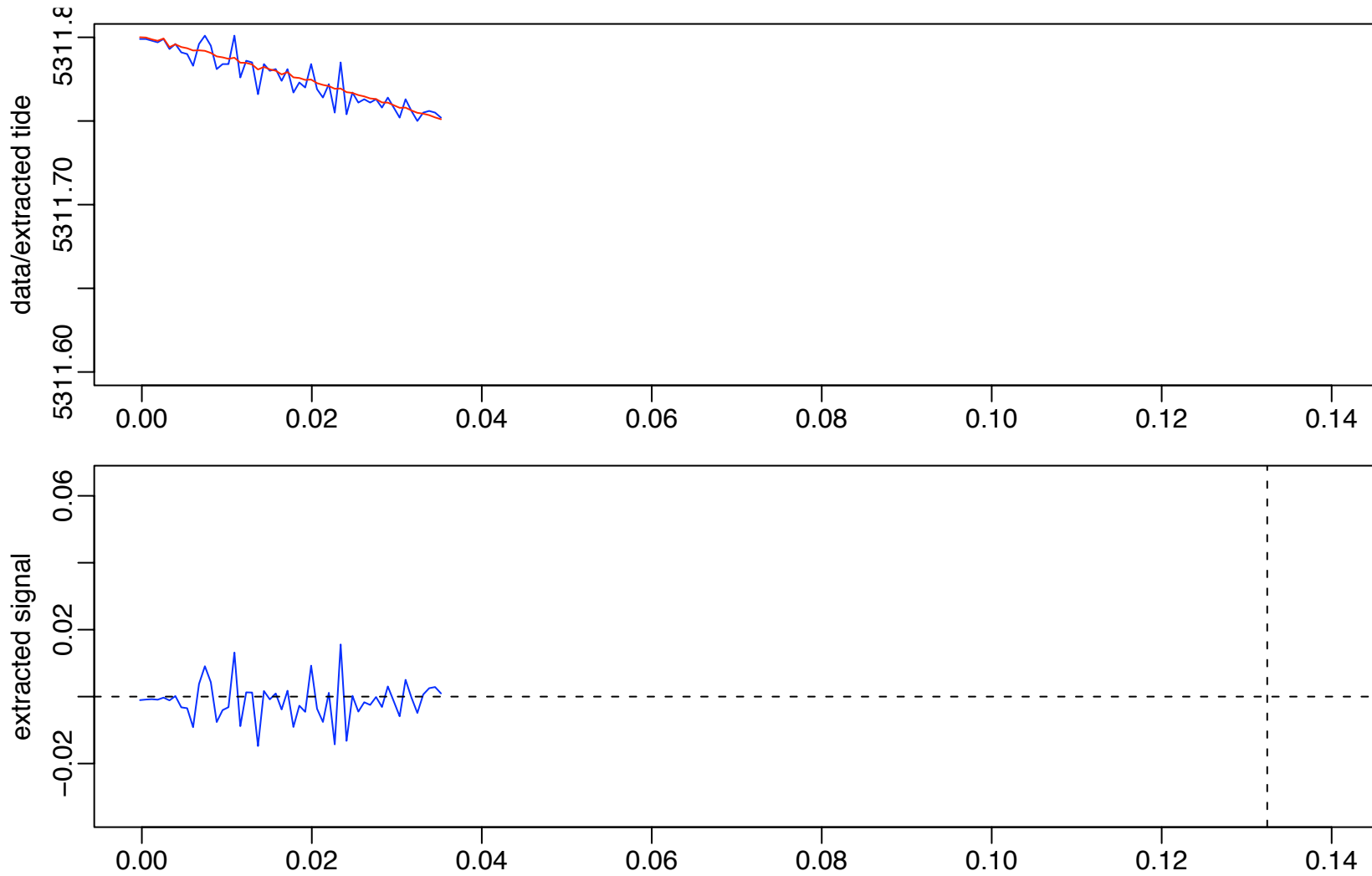
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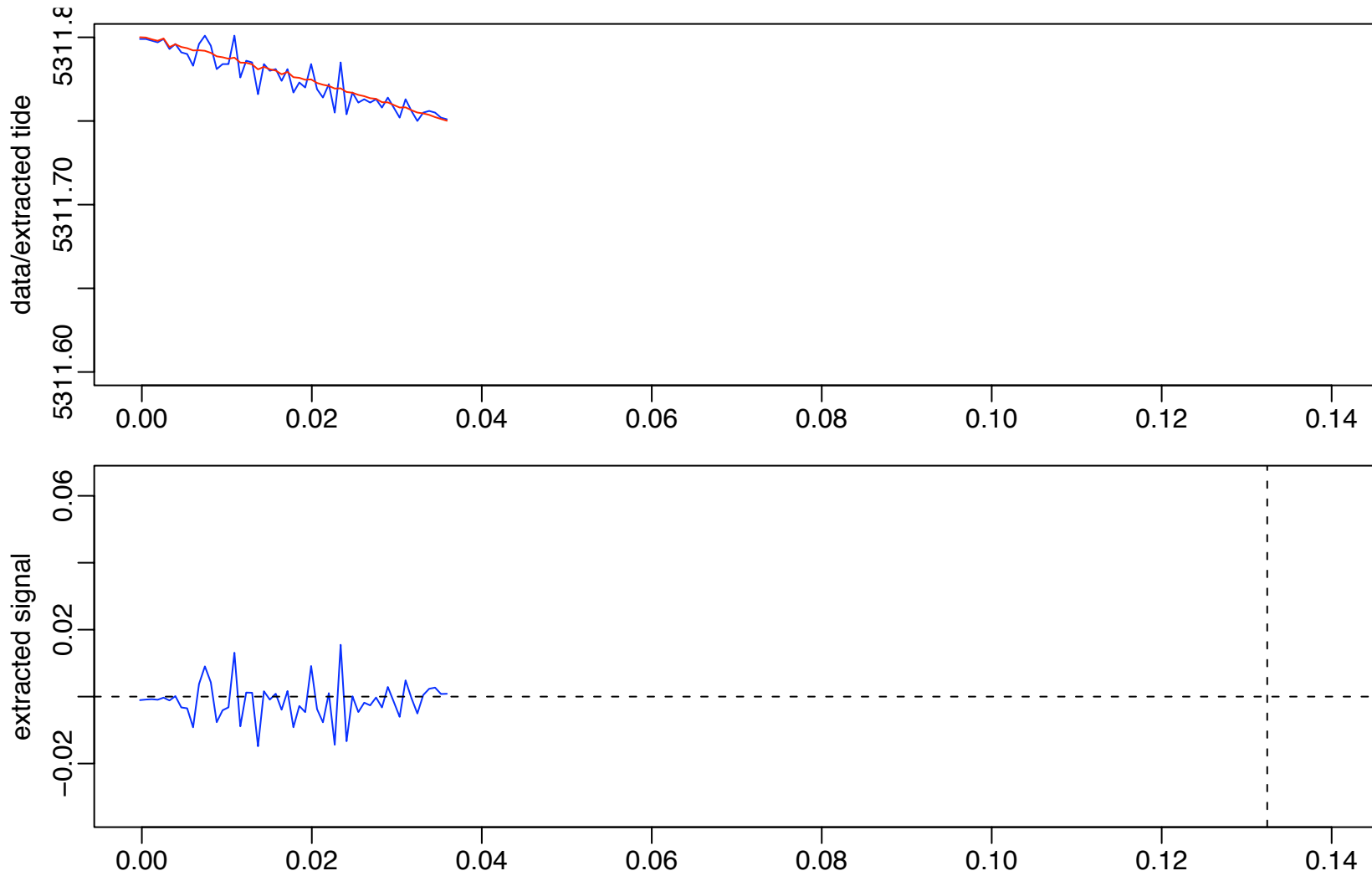
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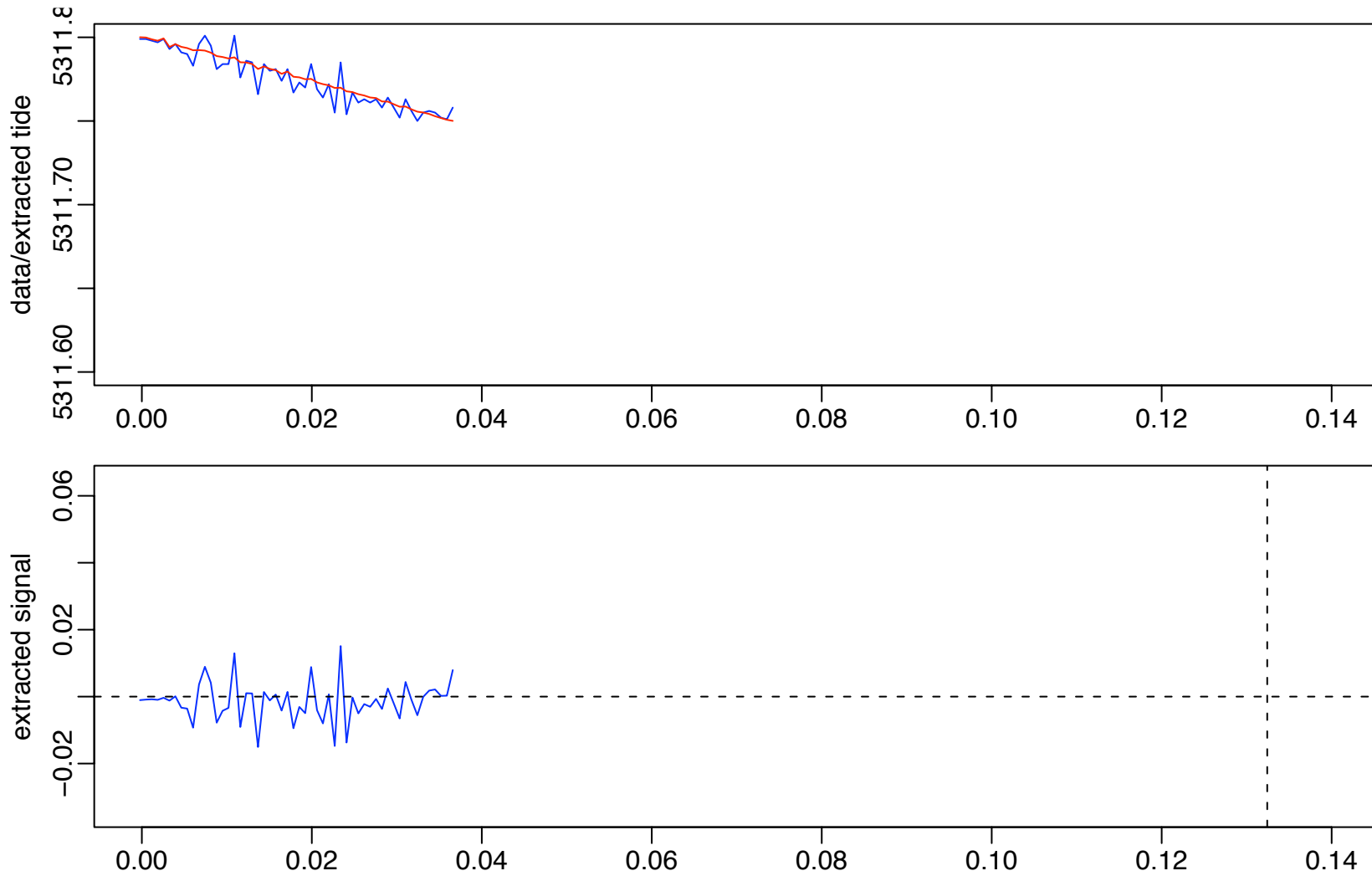
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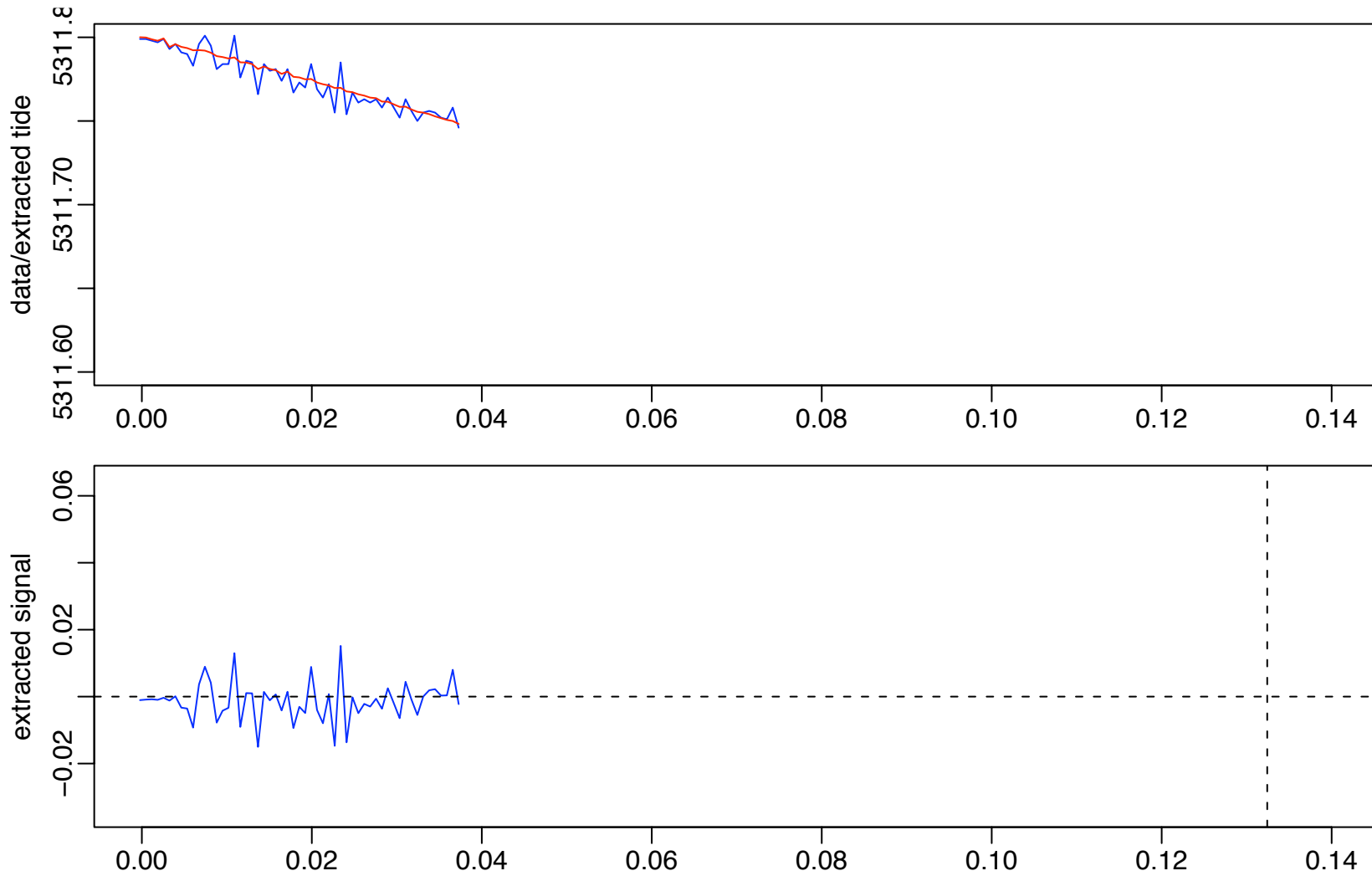
# Kalman Filtering/Smoothing – 14 Day Model



# Kalman Filtering/Smoothing – 14 Day Model

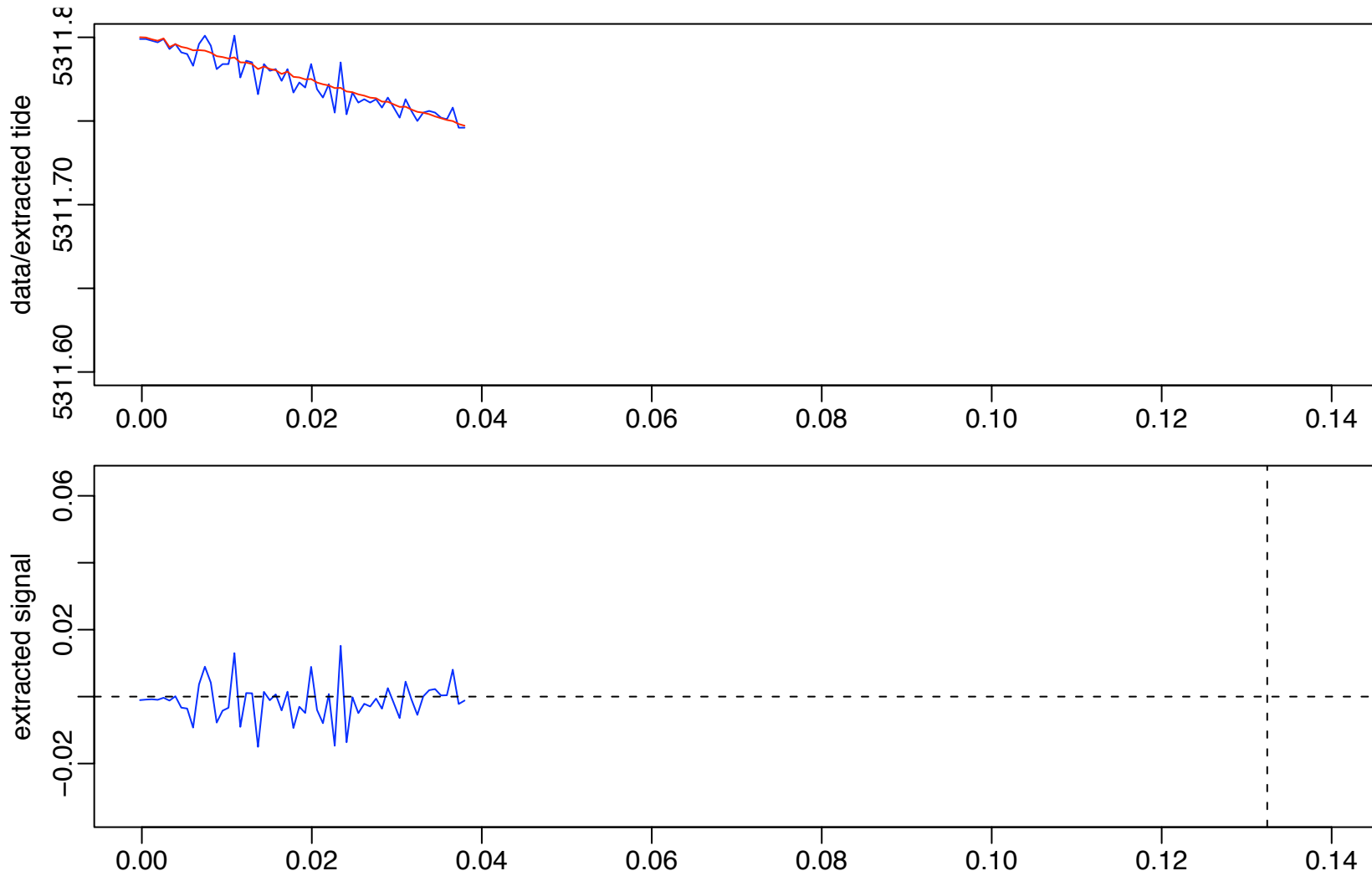


# Kalman Filtering/Smoothing – 14 Day Model

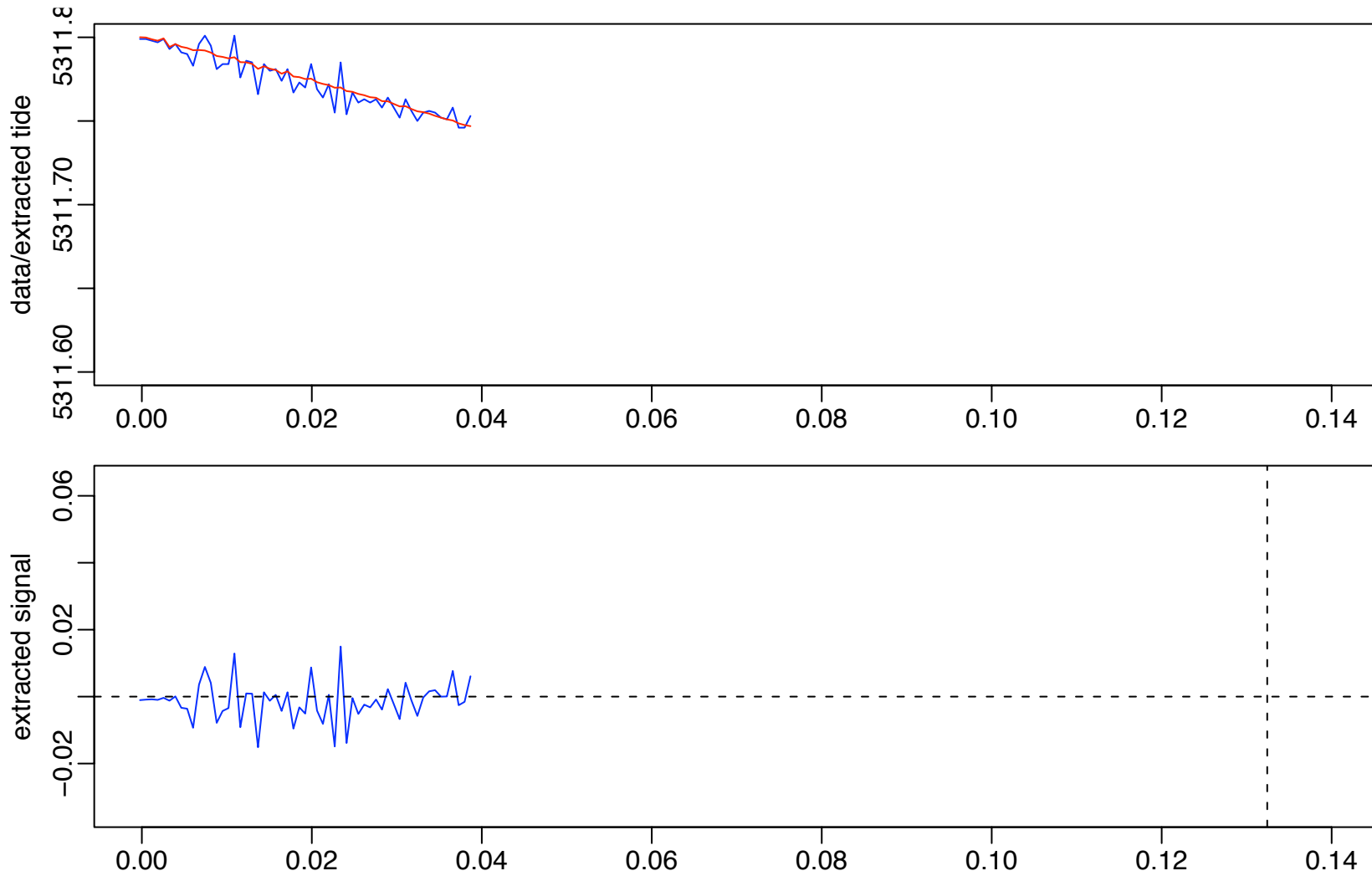




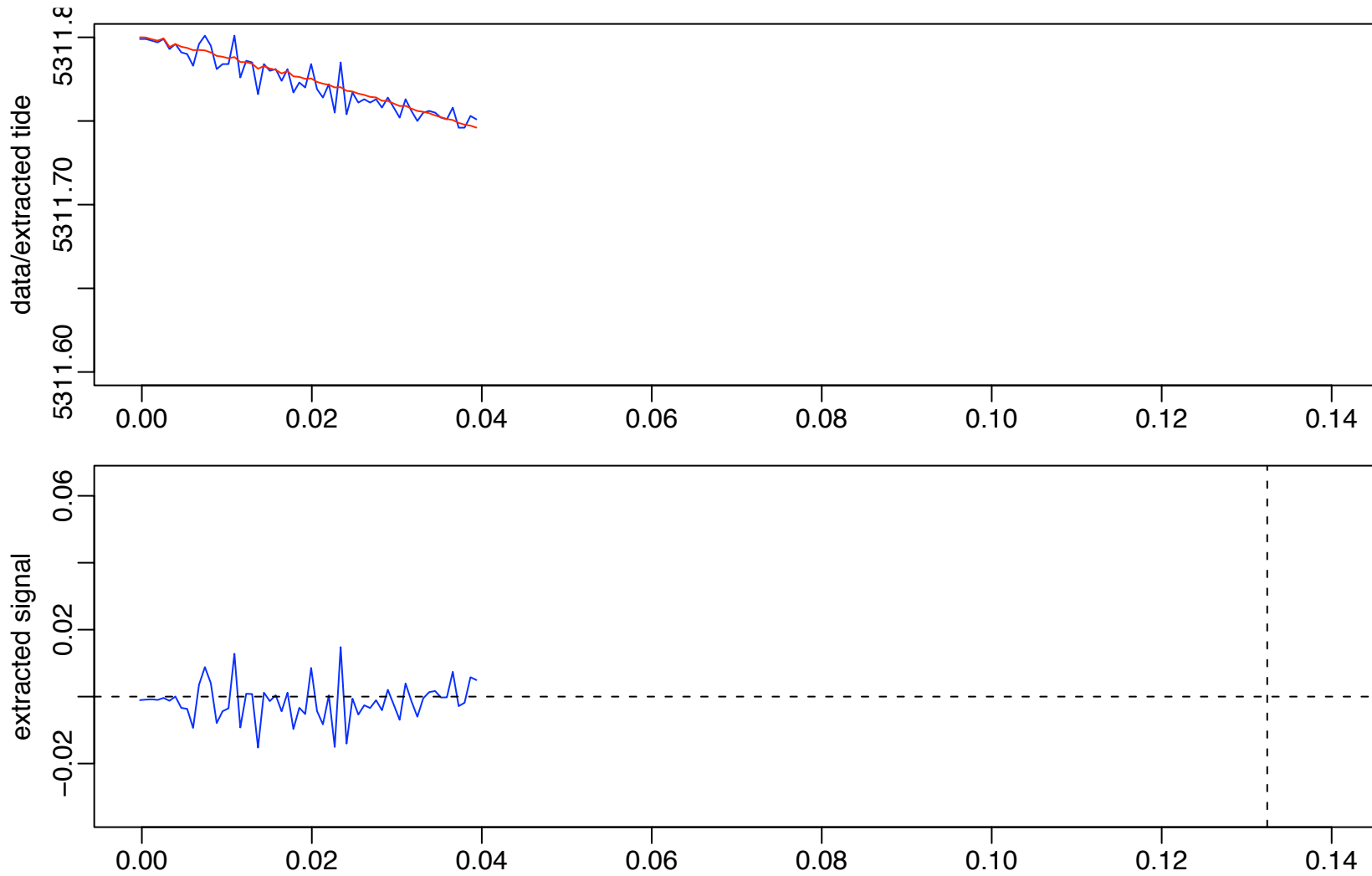
# Kalman Filtering/Smoothing – 14 Day Model



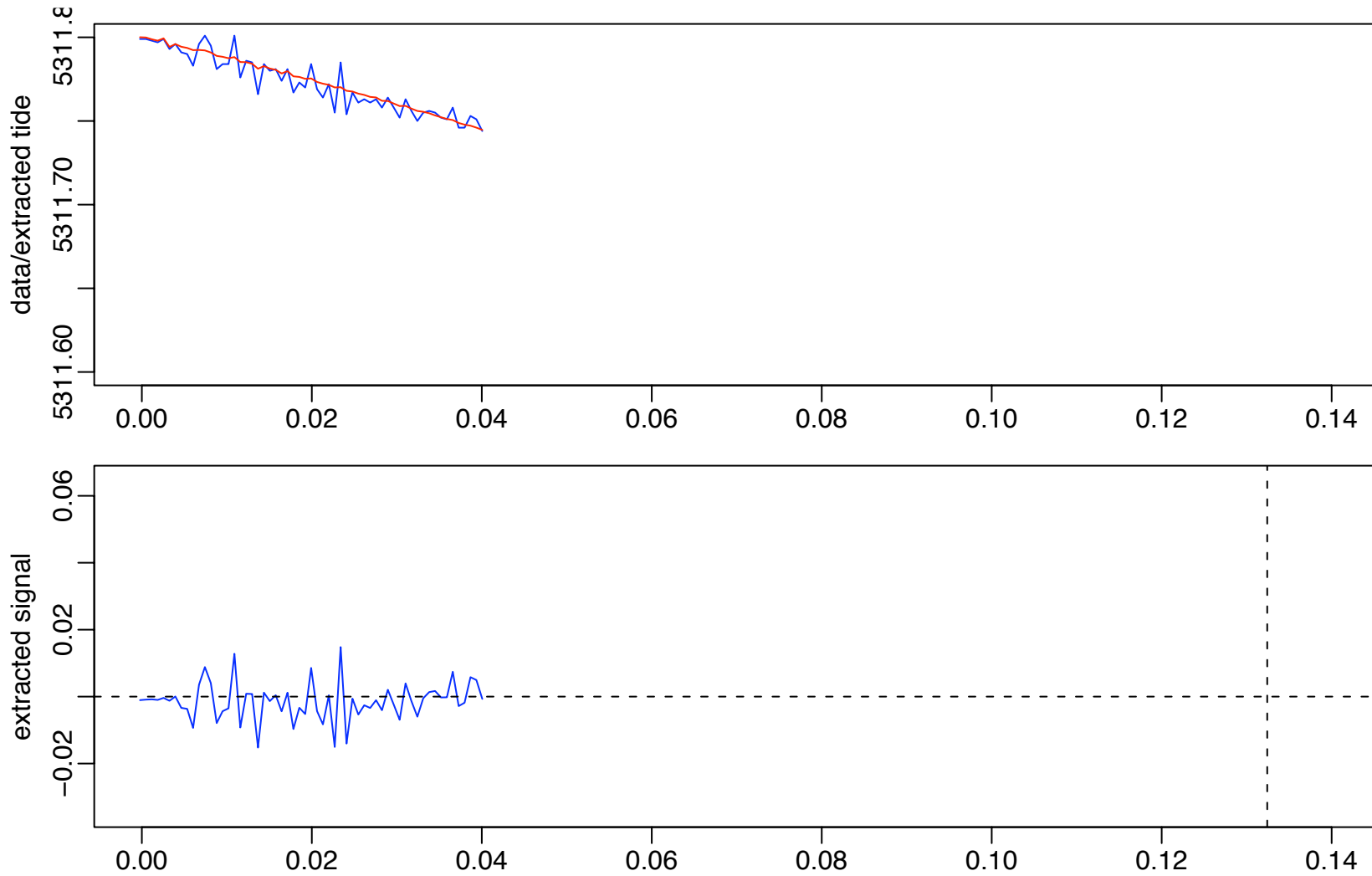
# Kalman Filtering/Smoothing – 14 Day Model



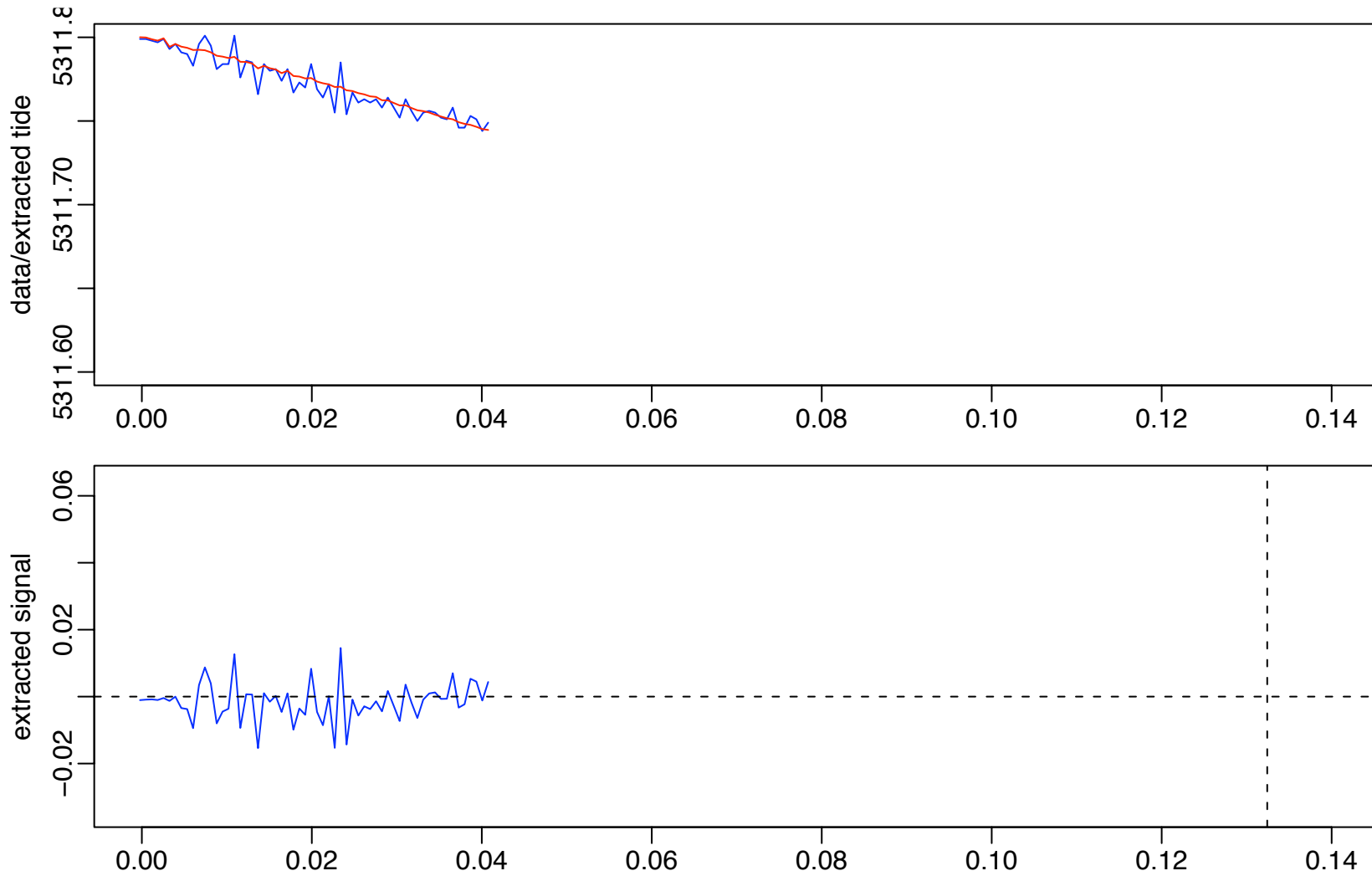
# Kalman Filtering/Smoothing – 14 Day Model



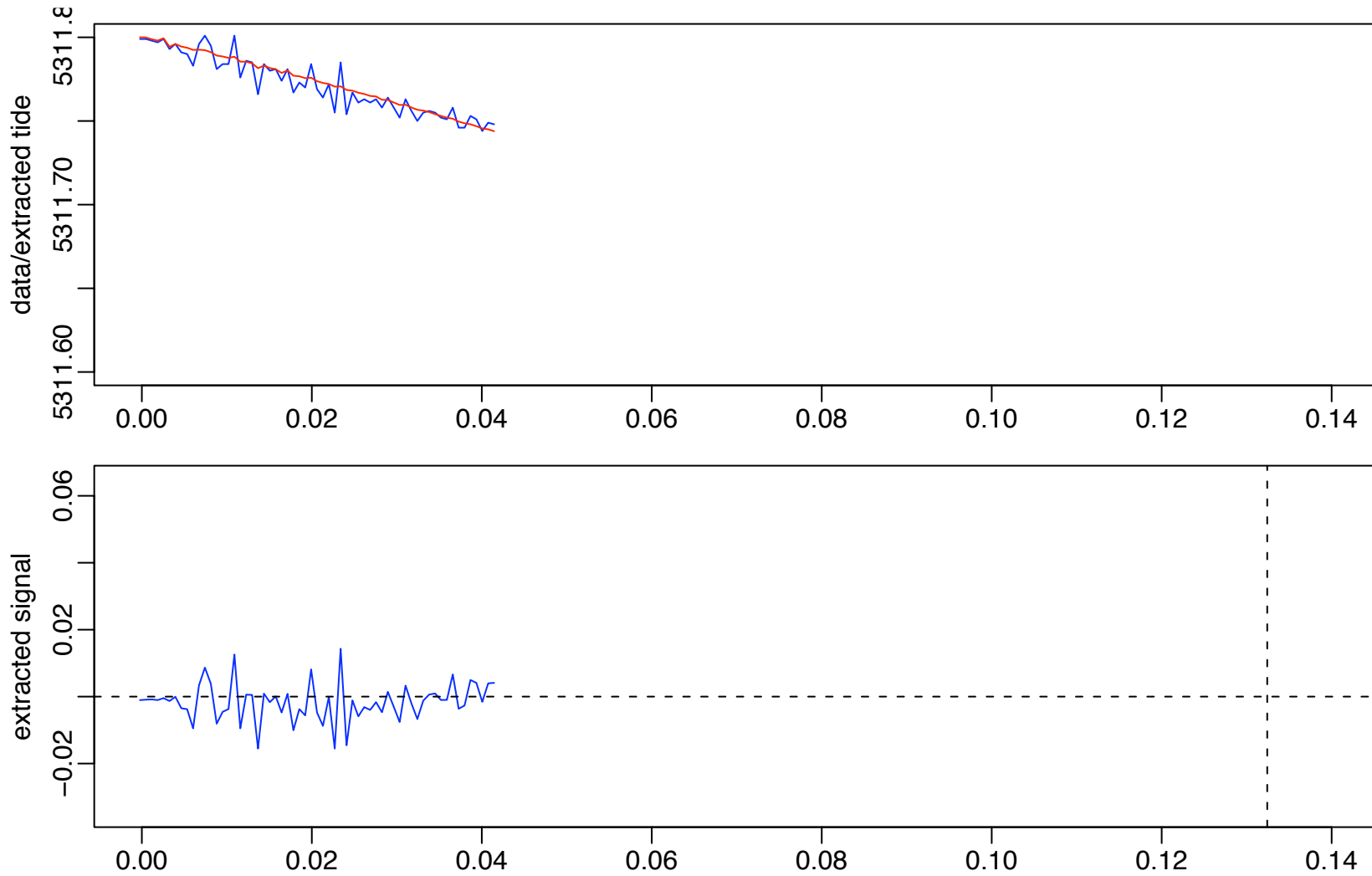
# Kalman Filtering/Smoothing – 14 Day Model



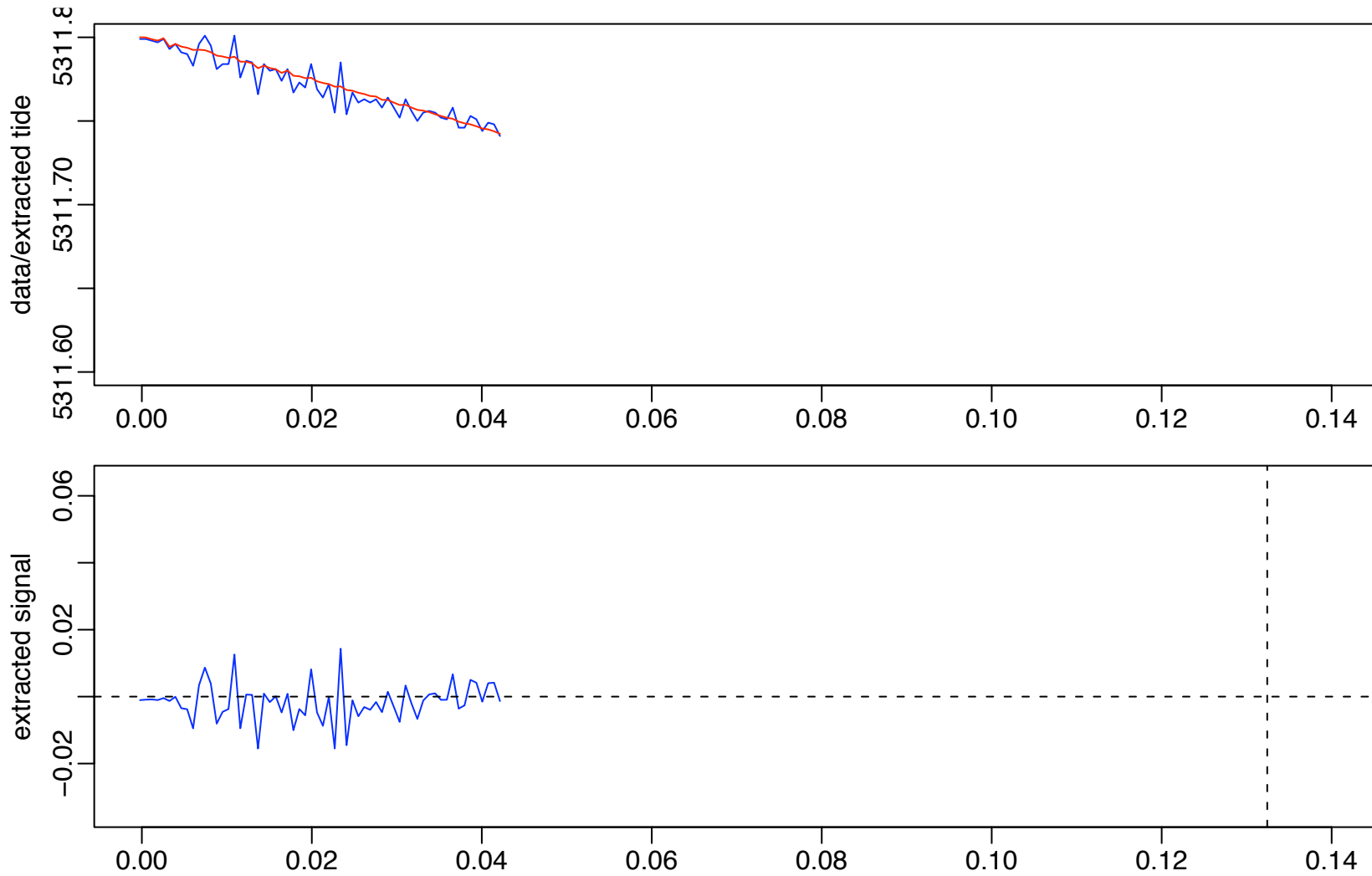
# Kalman Filtering/Smoothing – 14 Day Model



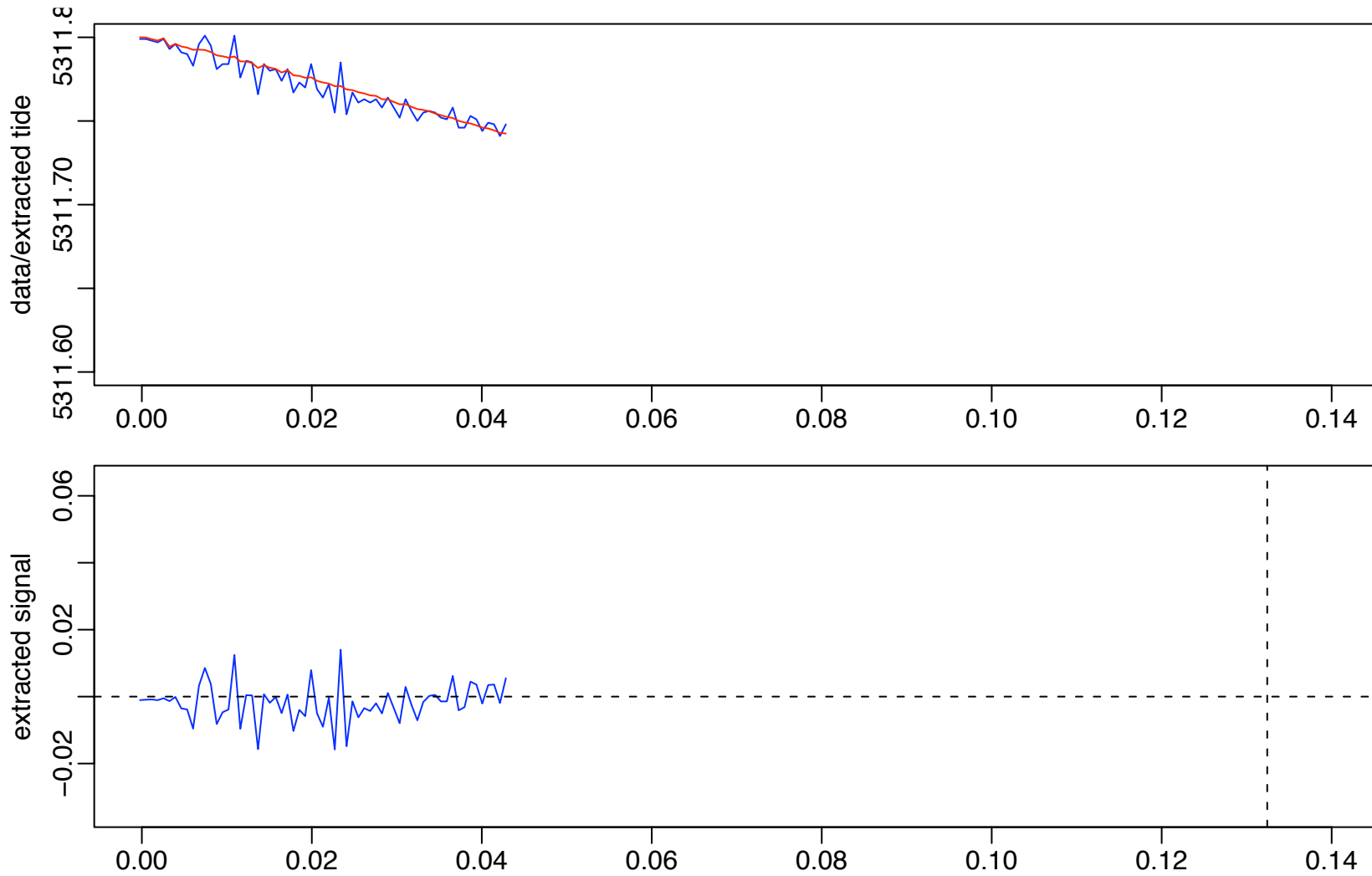
# Kalman Filtering/Smoothing – 14 Day Model



# Kalman Filtering/Smoothing – 14 Day Model

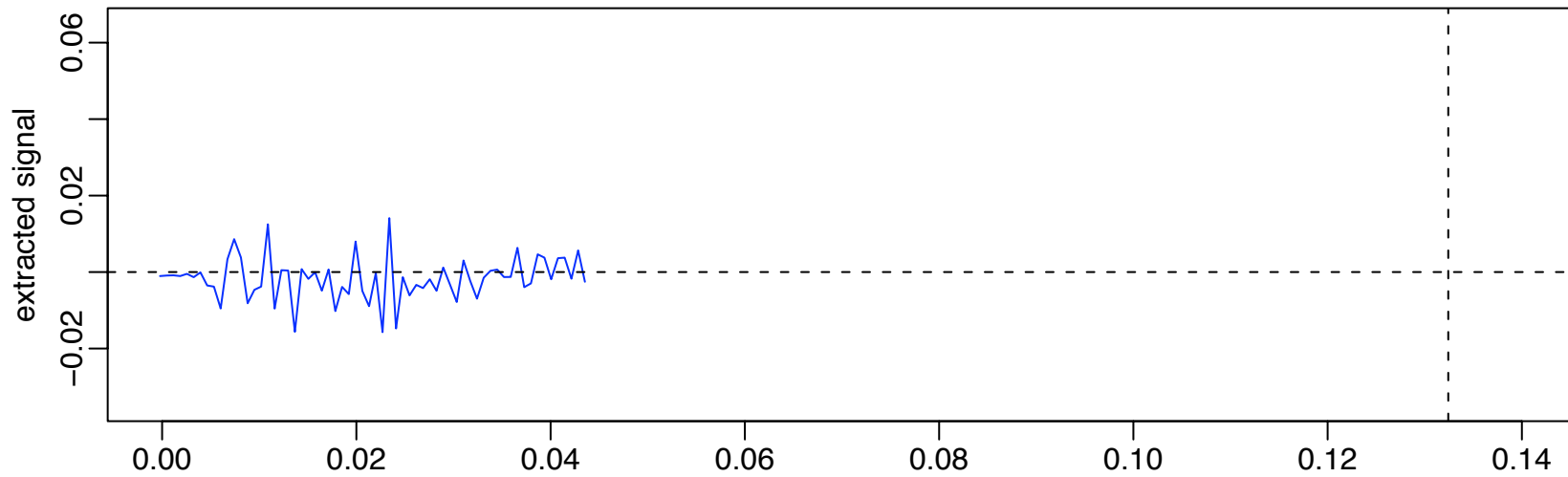
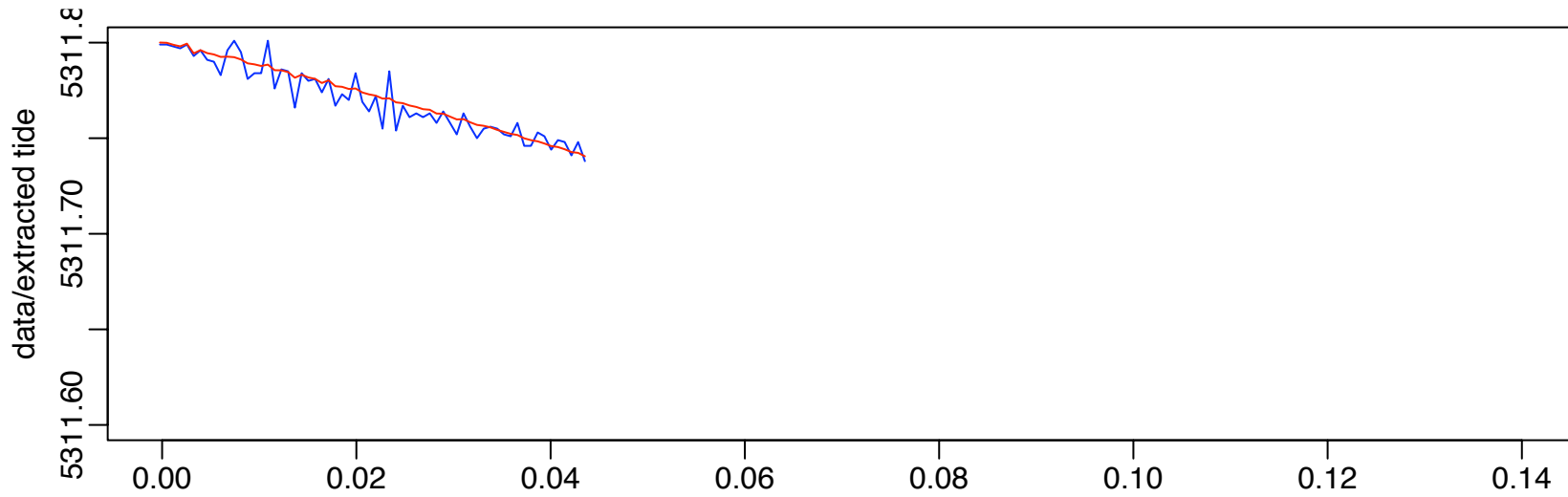


# Kalman Filtering/Smoothing – 14 Day Model

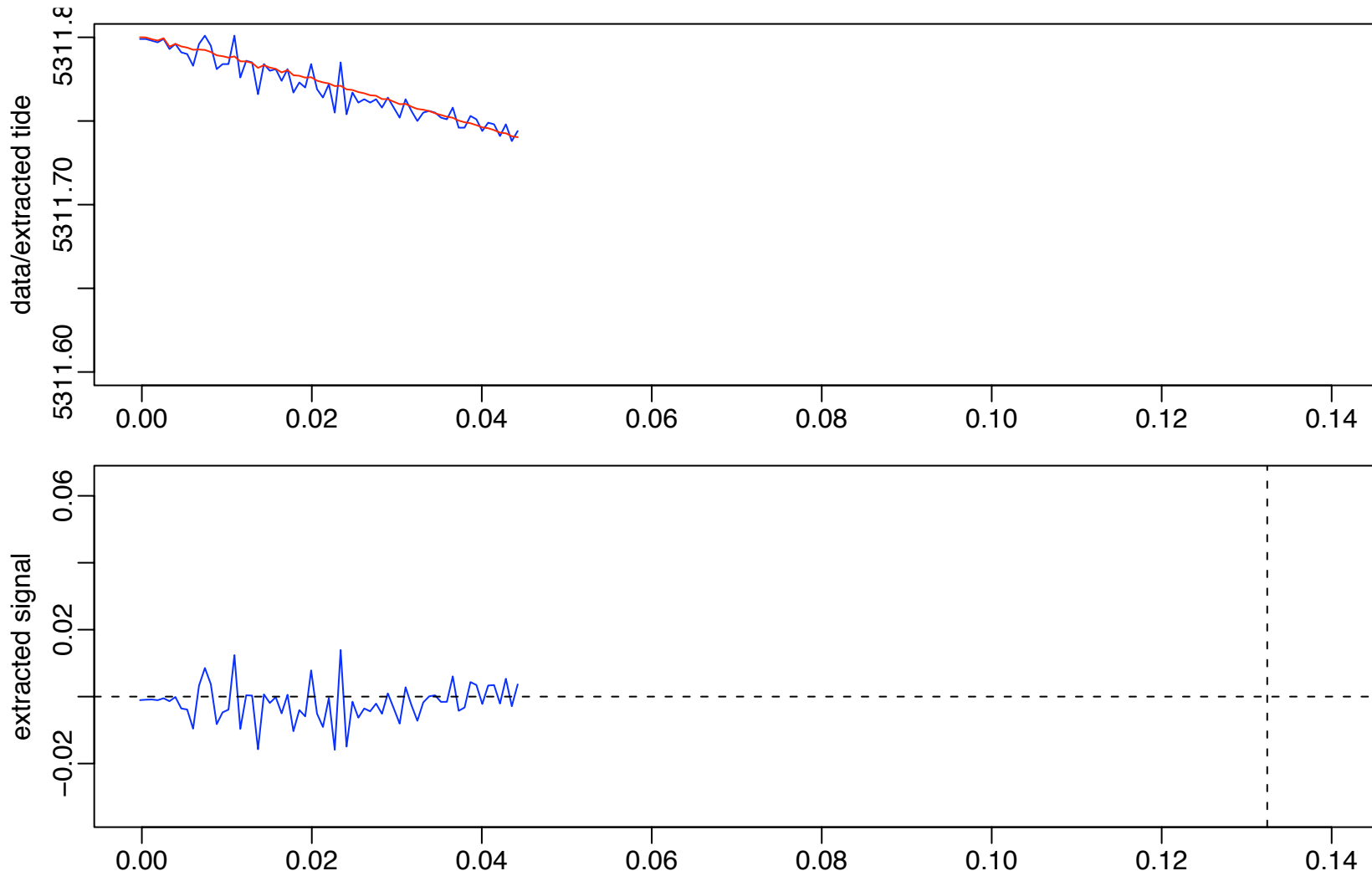




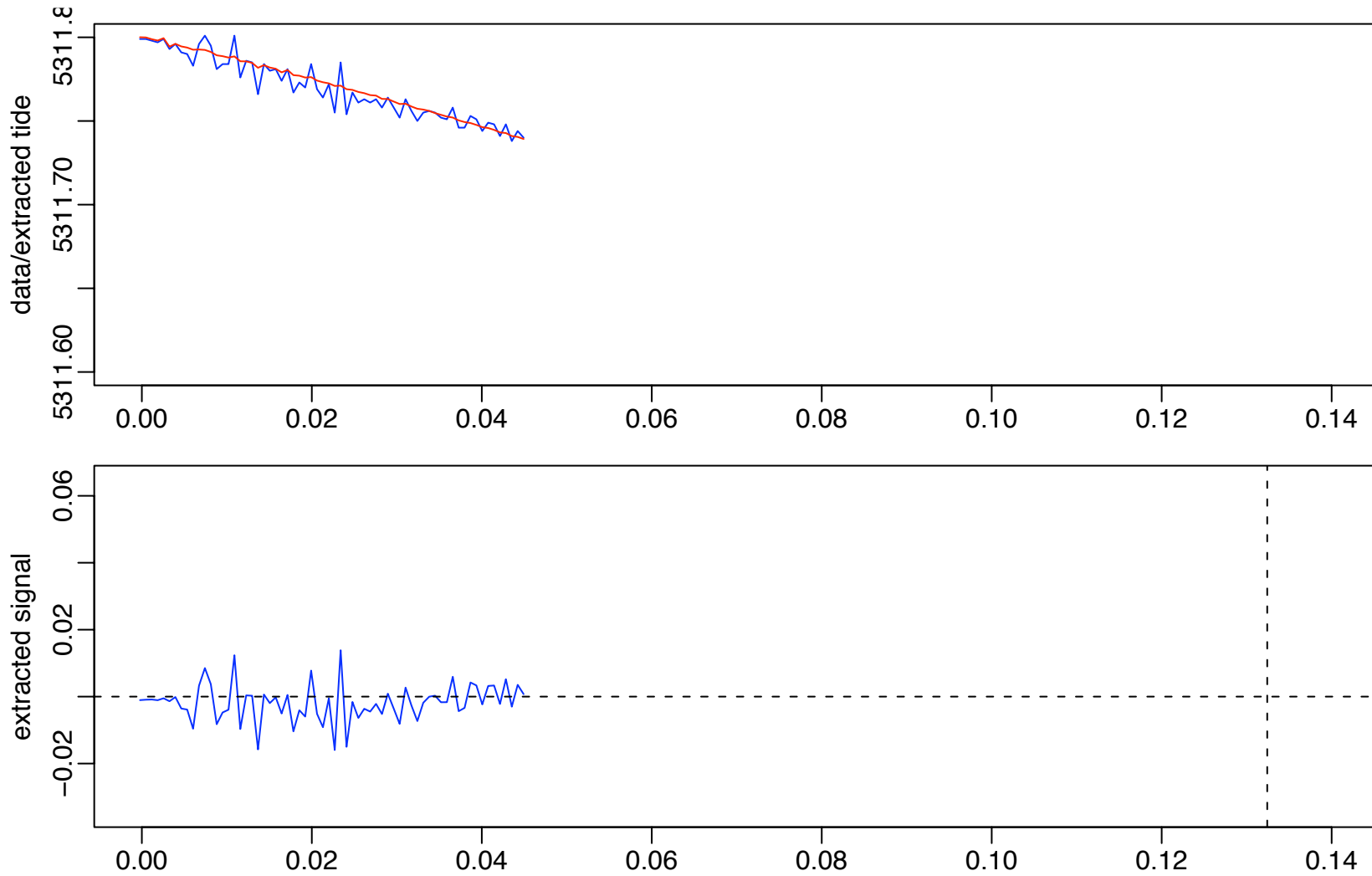
# Kalman Filtering/Smoothing – 14 Day Model



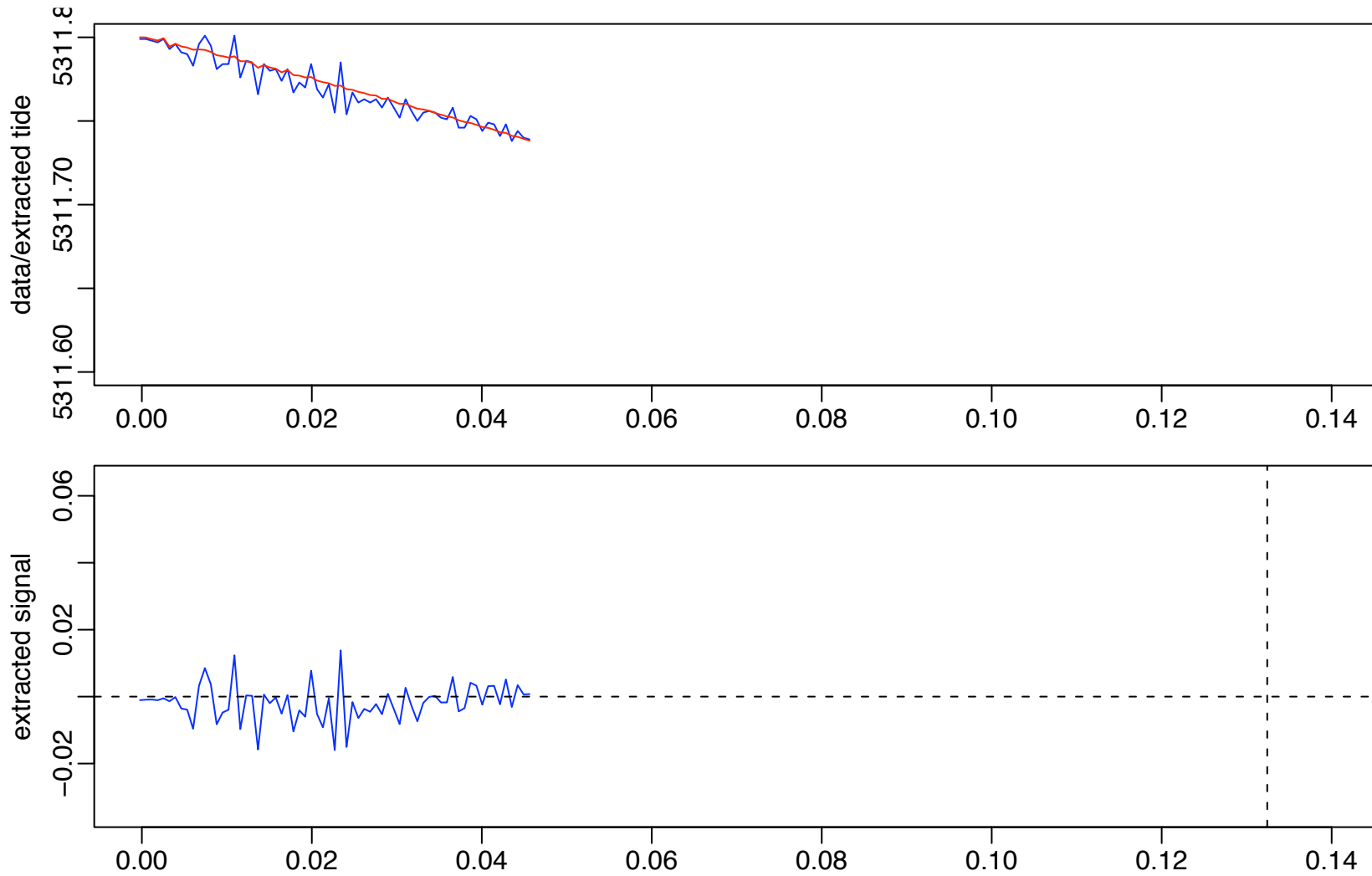
# Kalman Filtering/Smoothing – 14 Day Model



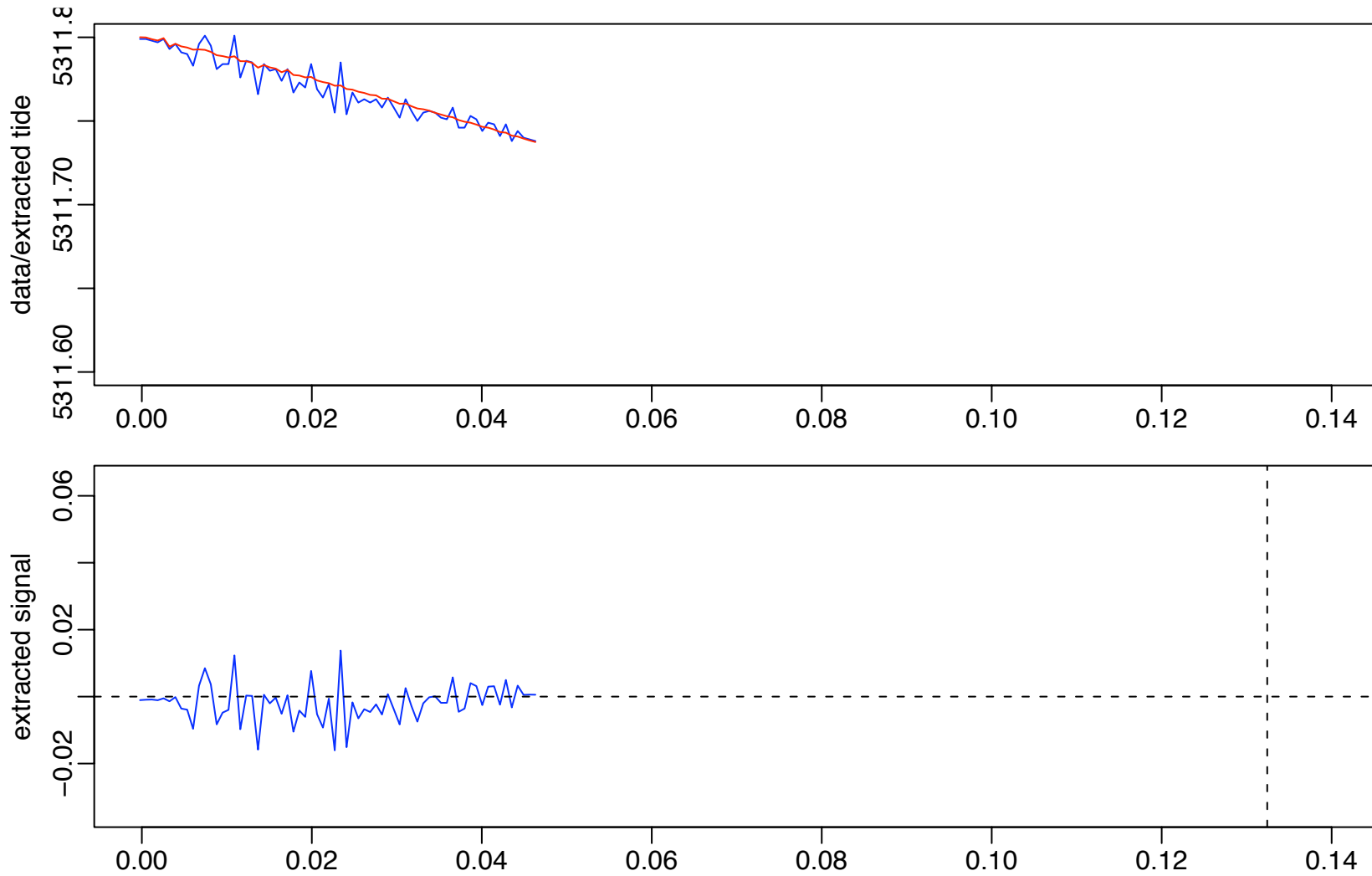
# Kalman Filtering/Smoothing – 14 Day Model



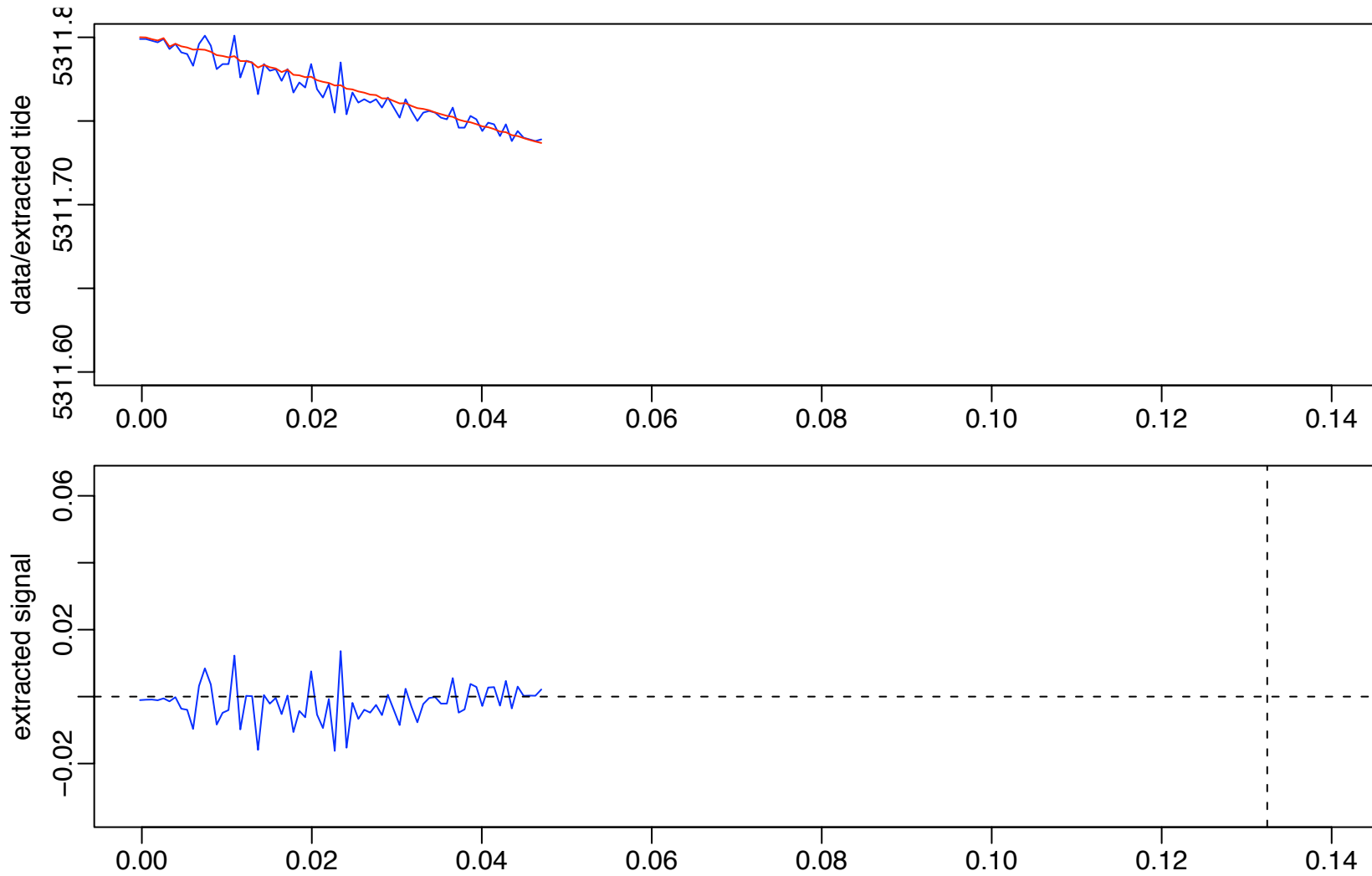
# Kalman Filtering/Smoothing – 14 Day Model



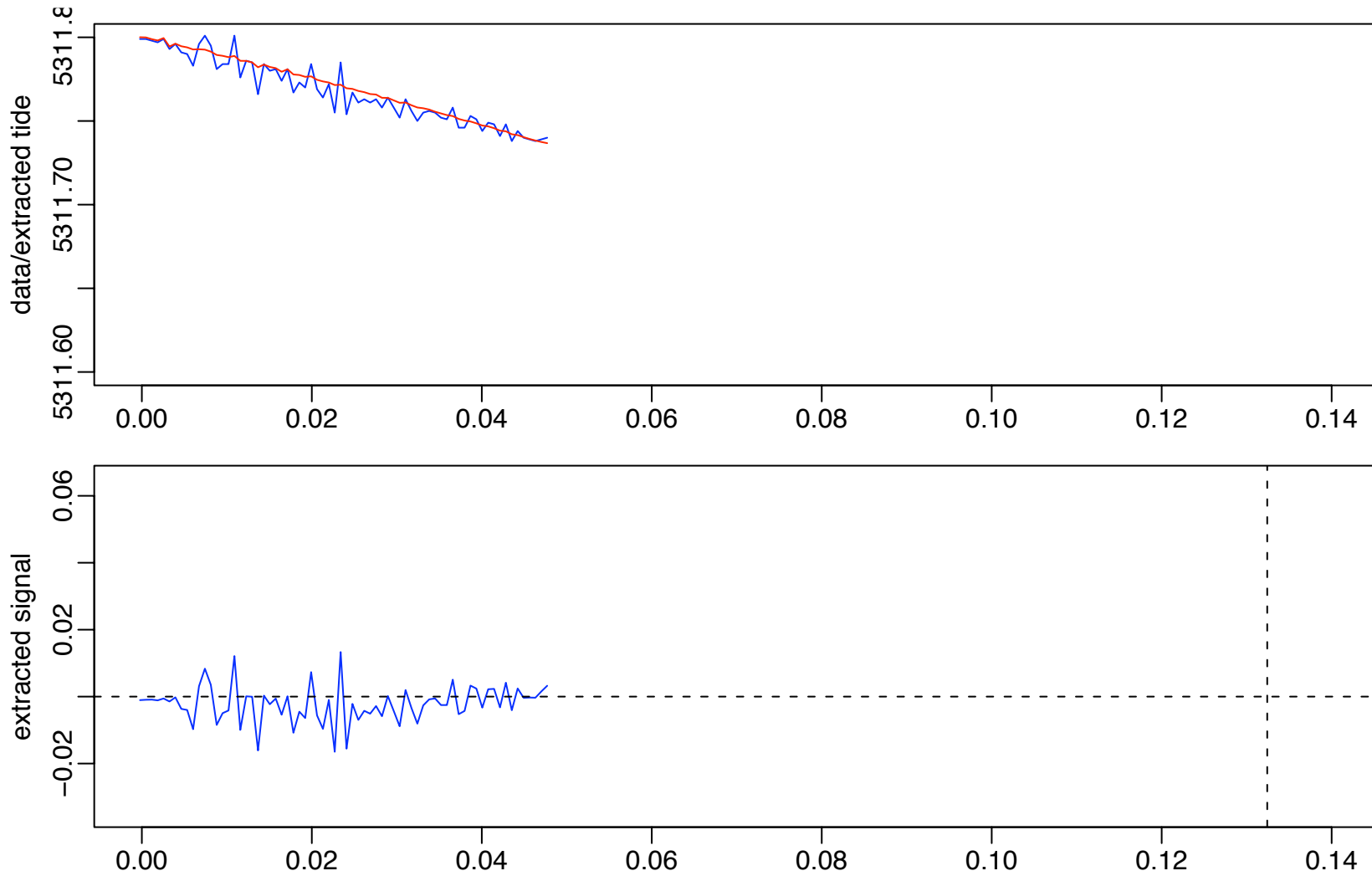
# Kalman Filtering/Smoothing – 14 Day Model



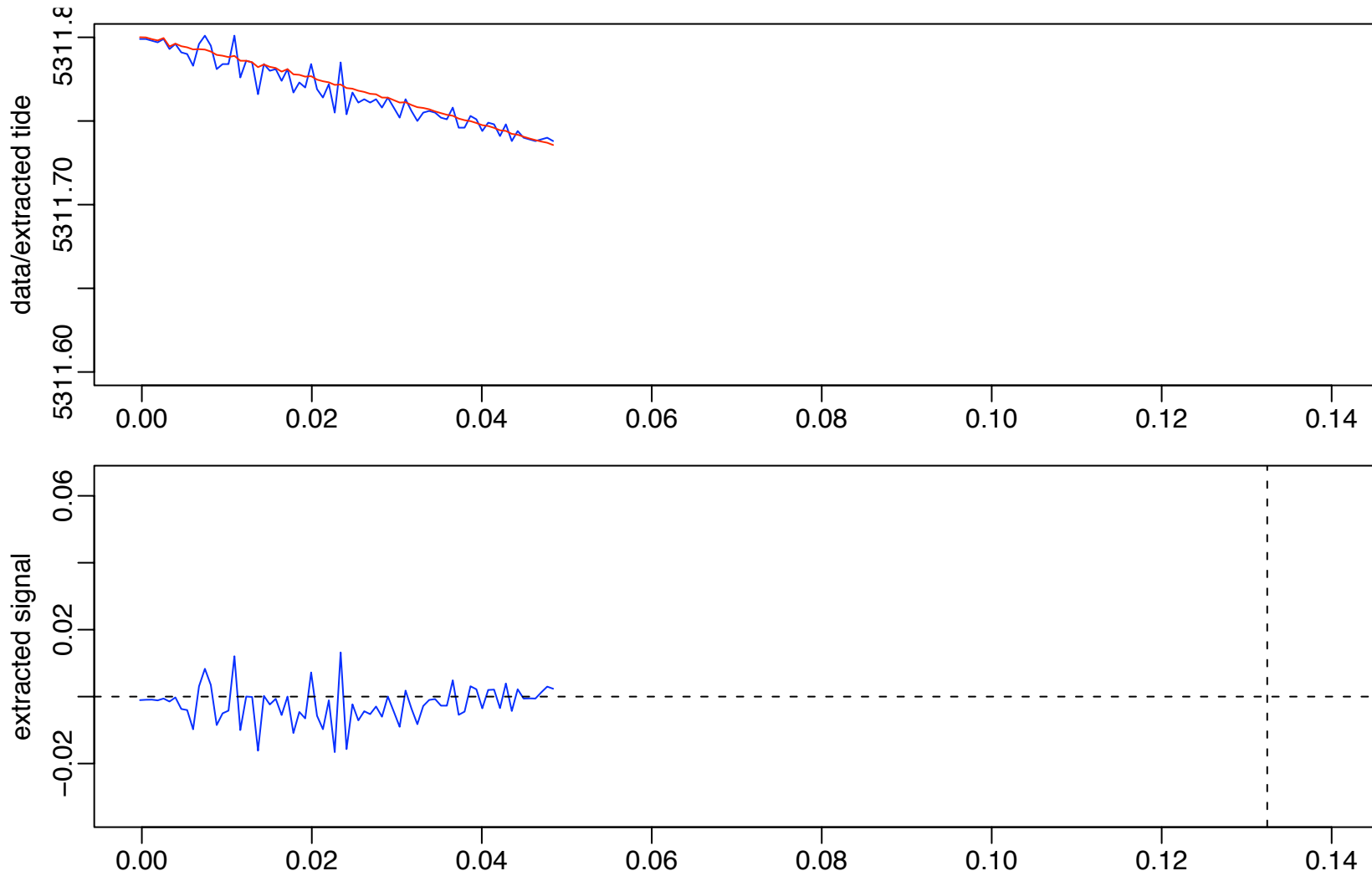
# Kalman Filtering/Smoothing – 14 Day Model



# Kalman Filtering/Smoothing – 14 Day Model

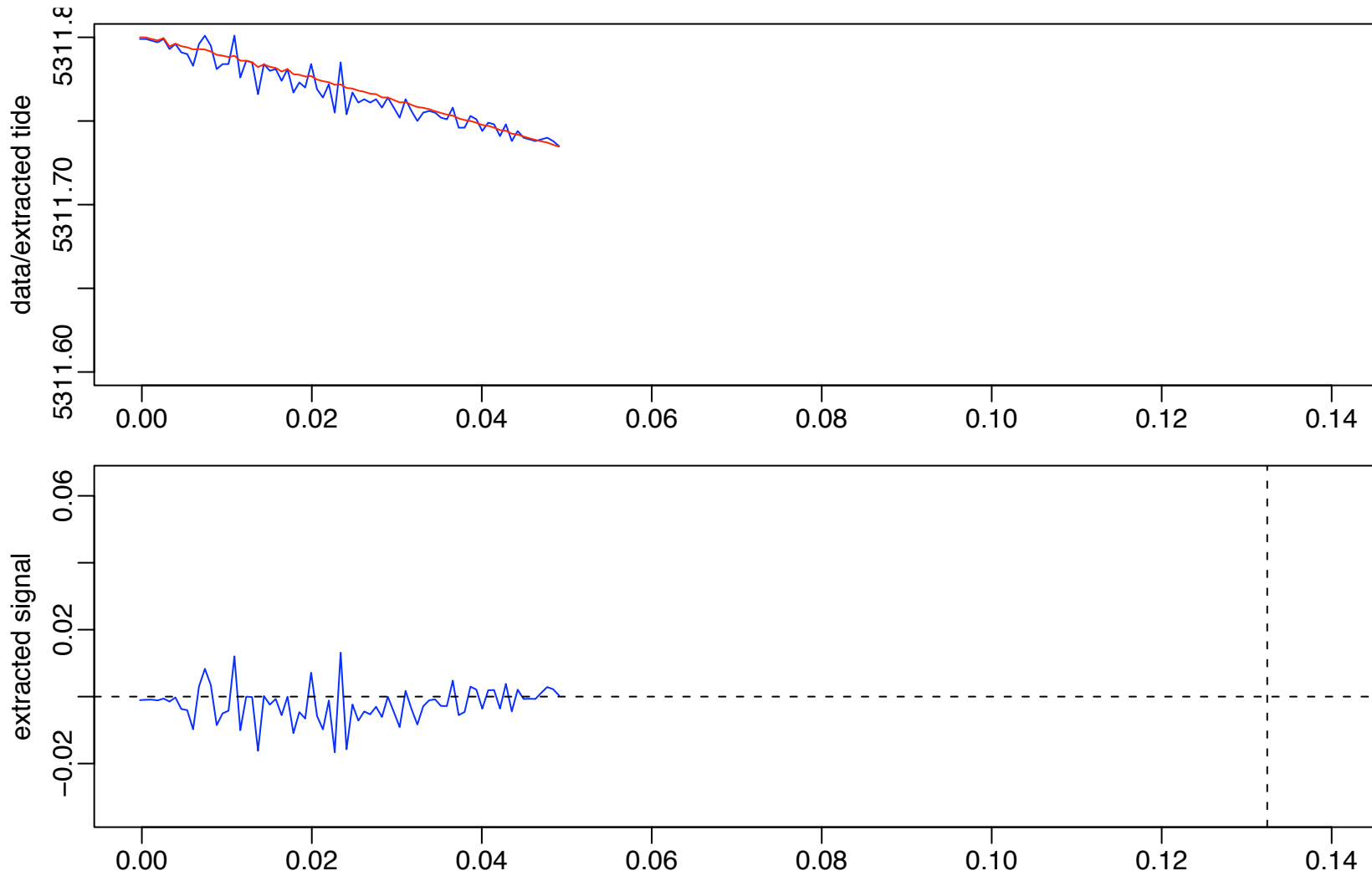


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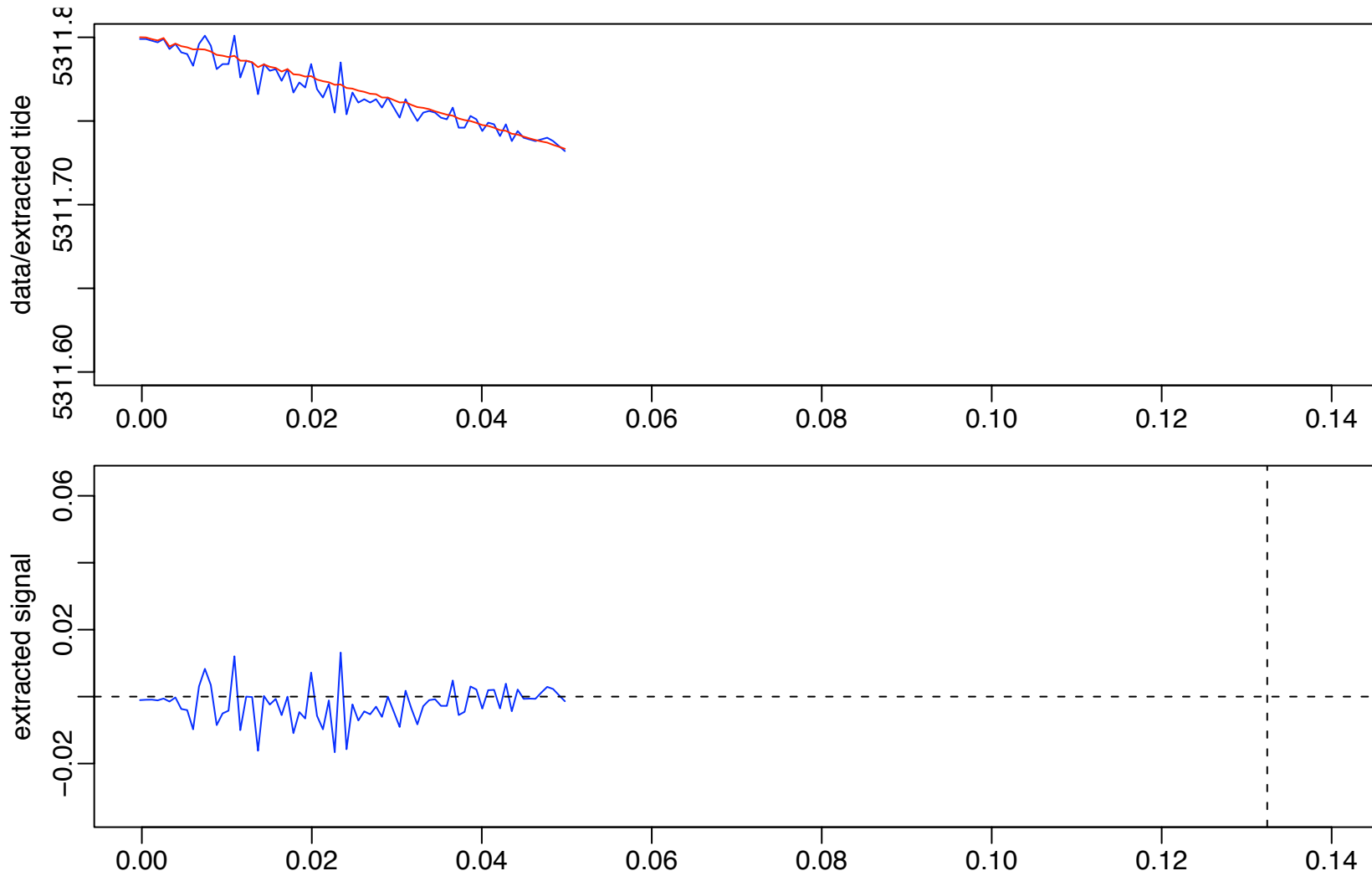




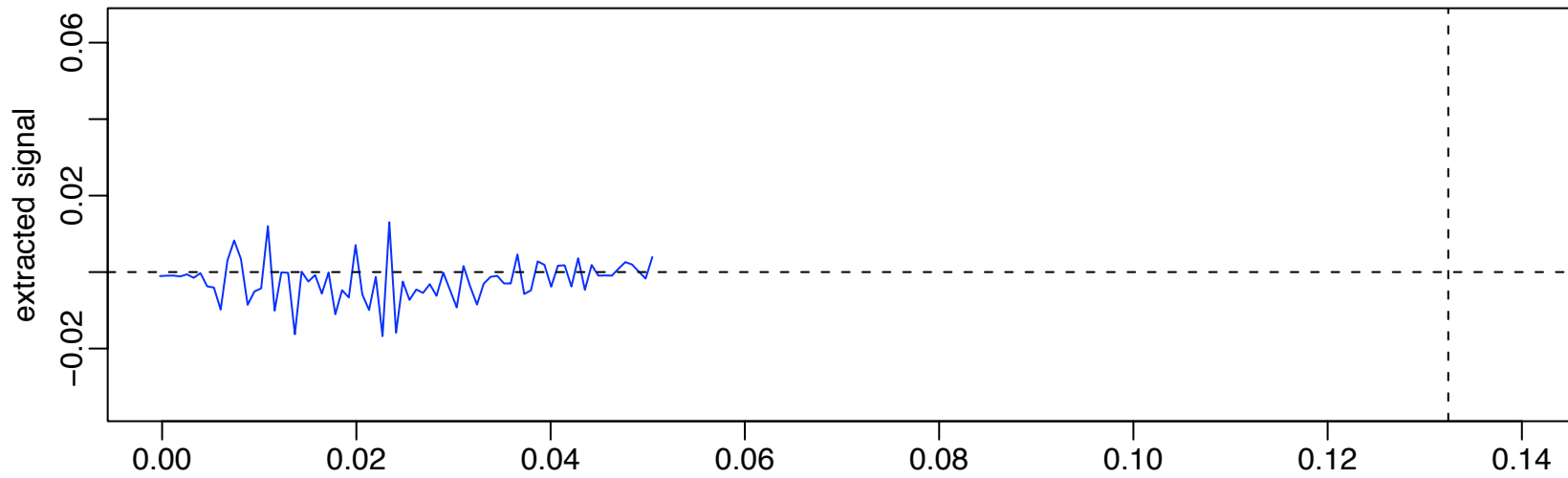
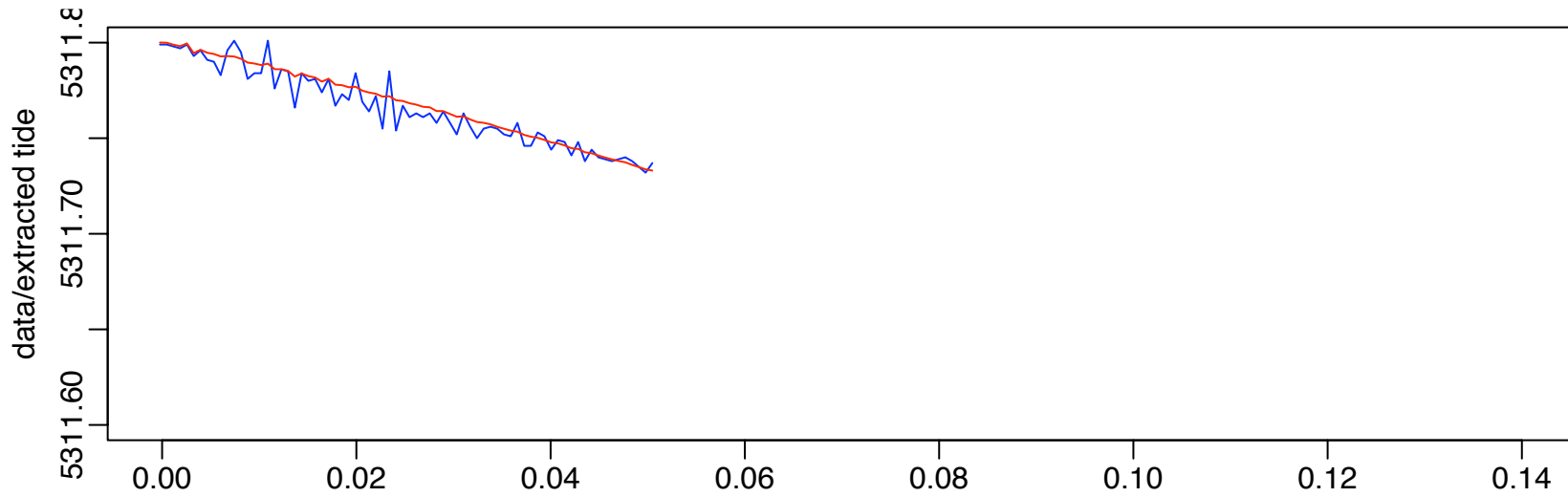
# Kalman Filtering/Smoothing – 14 Day Model



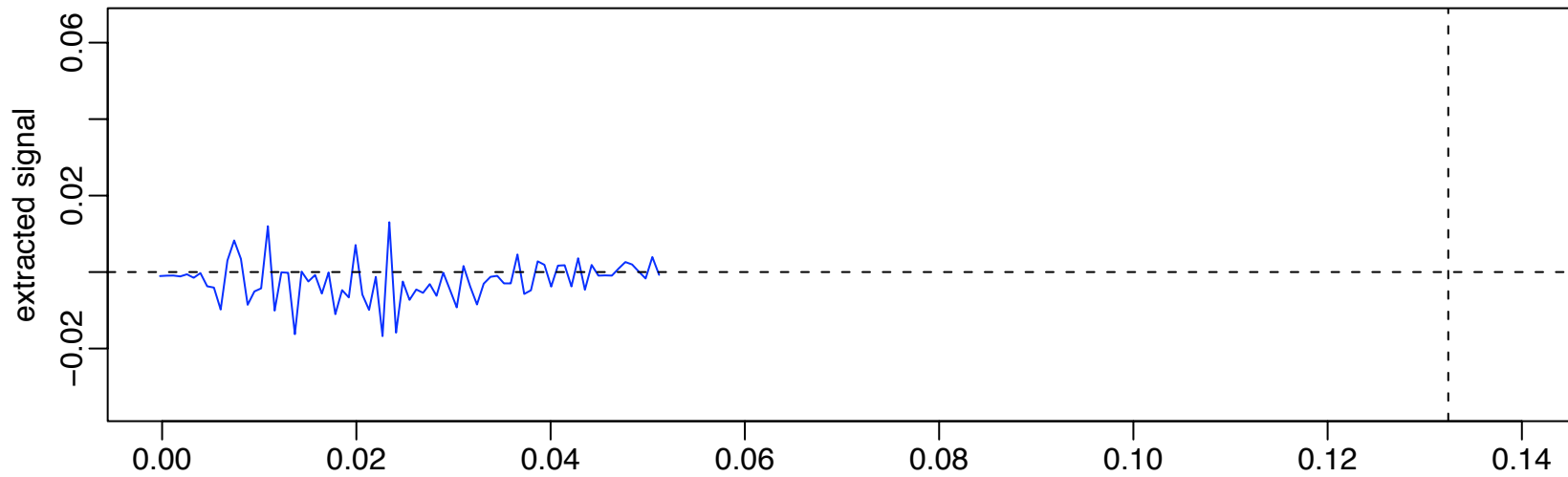
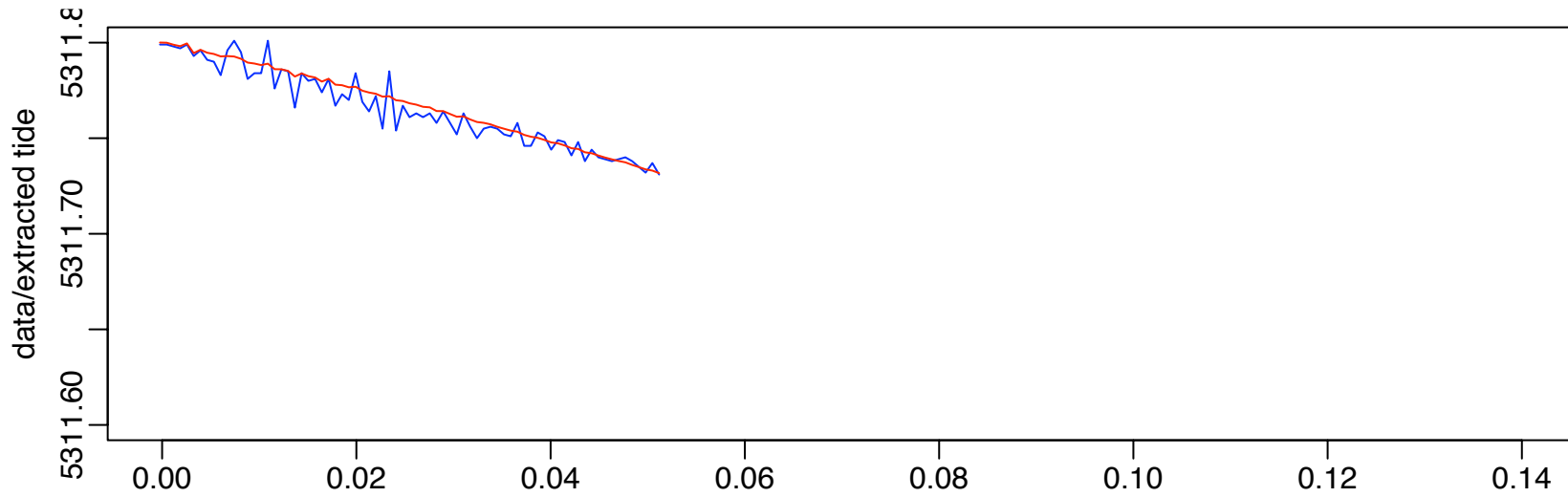
# Kalman Filtering/Smoothing – 14 Day Model



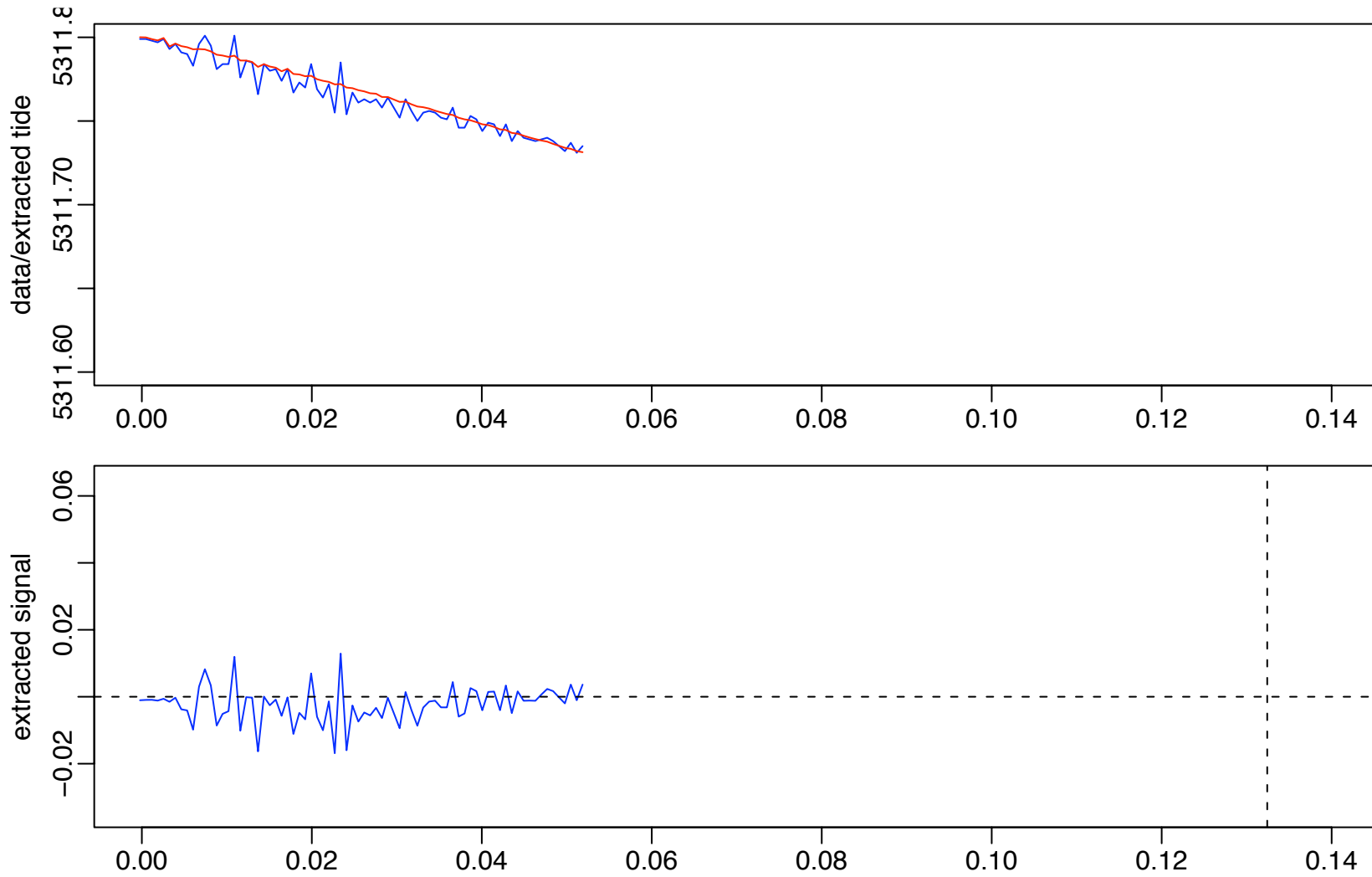
# Kalman Filtering/Smoothing – 14 Day Model



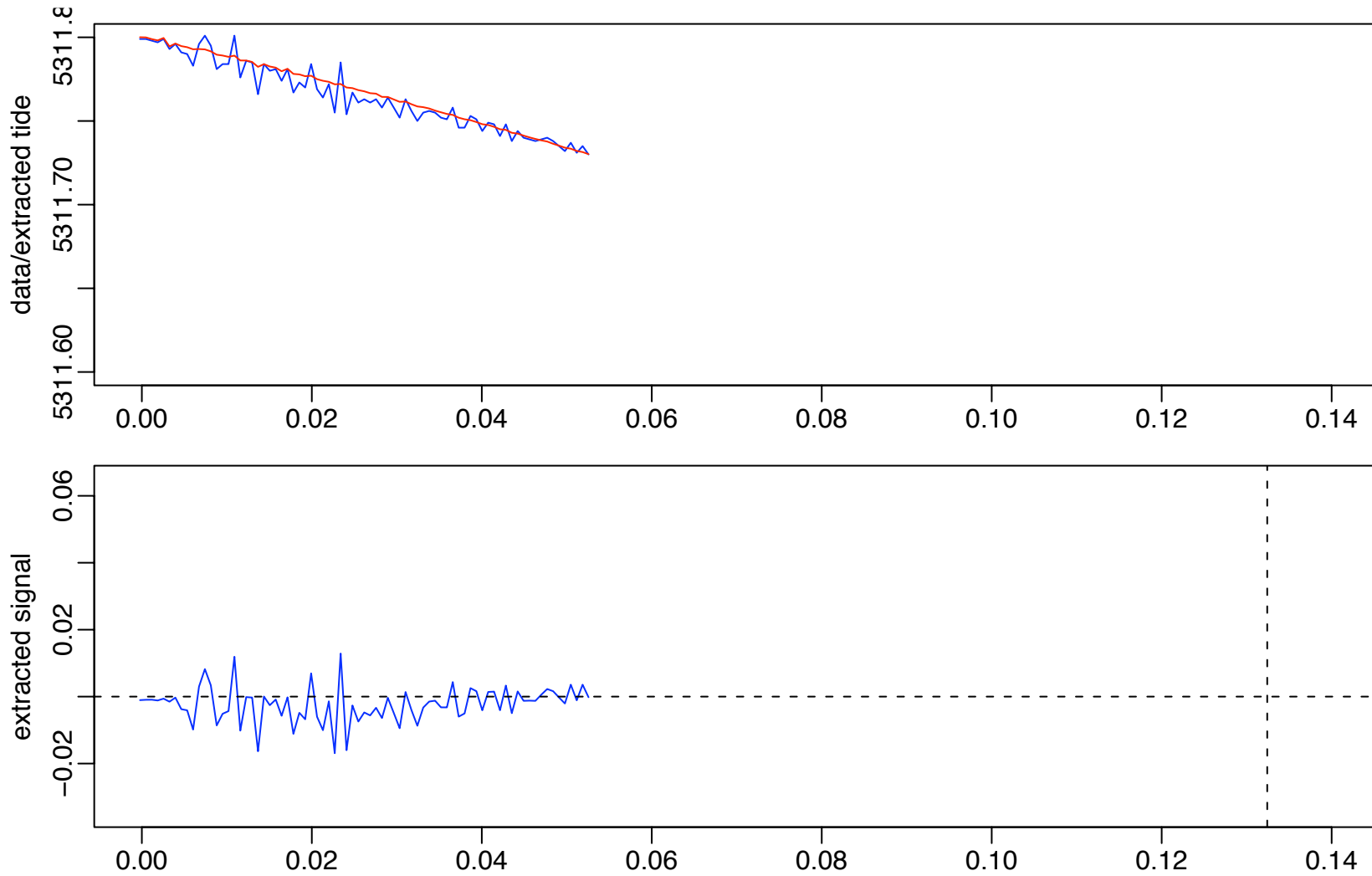
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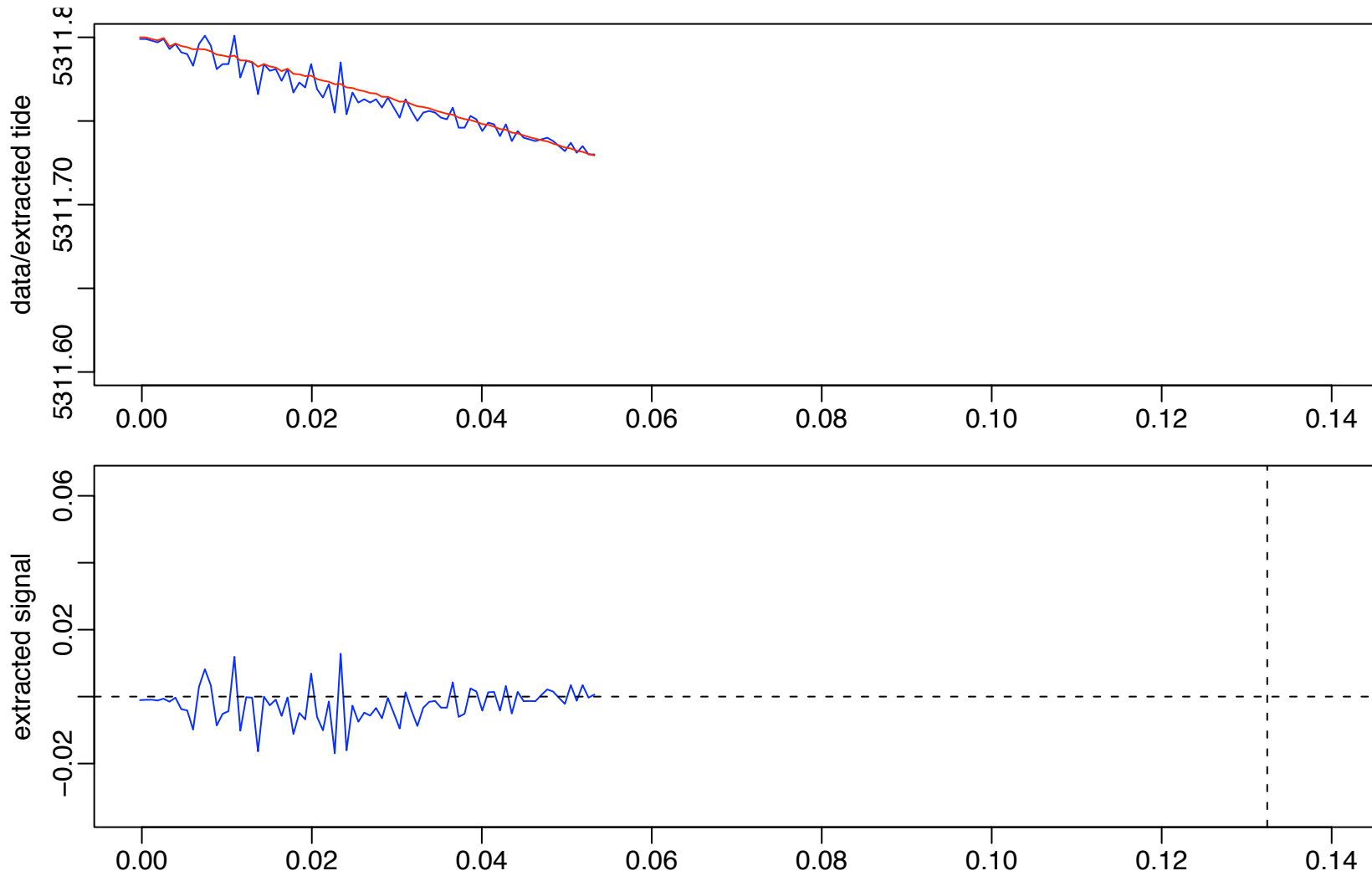
# Kalman Filtering/Smoothing – 14 Day Model



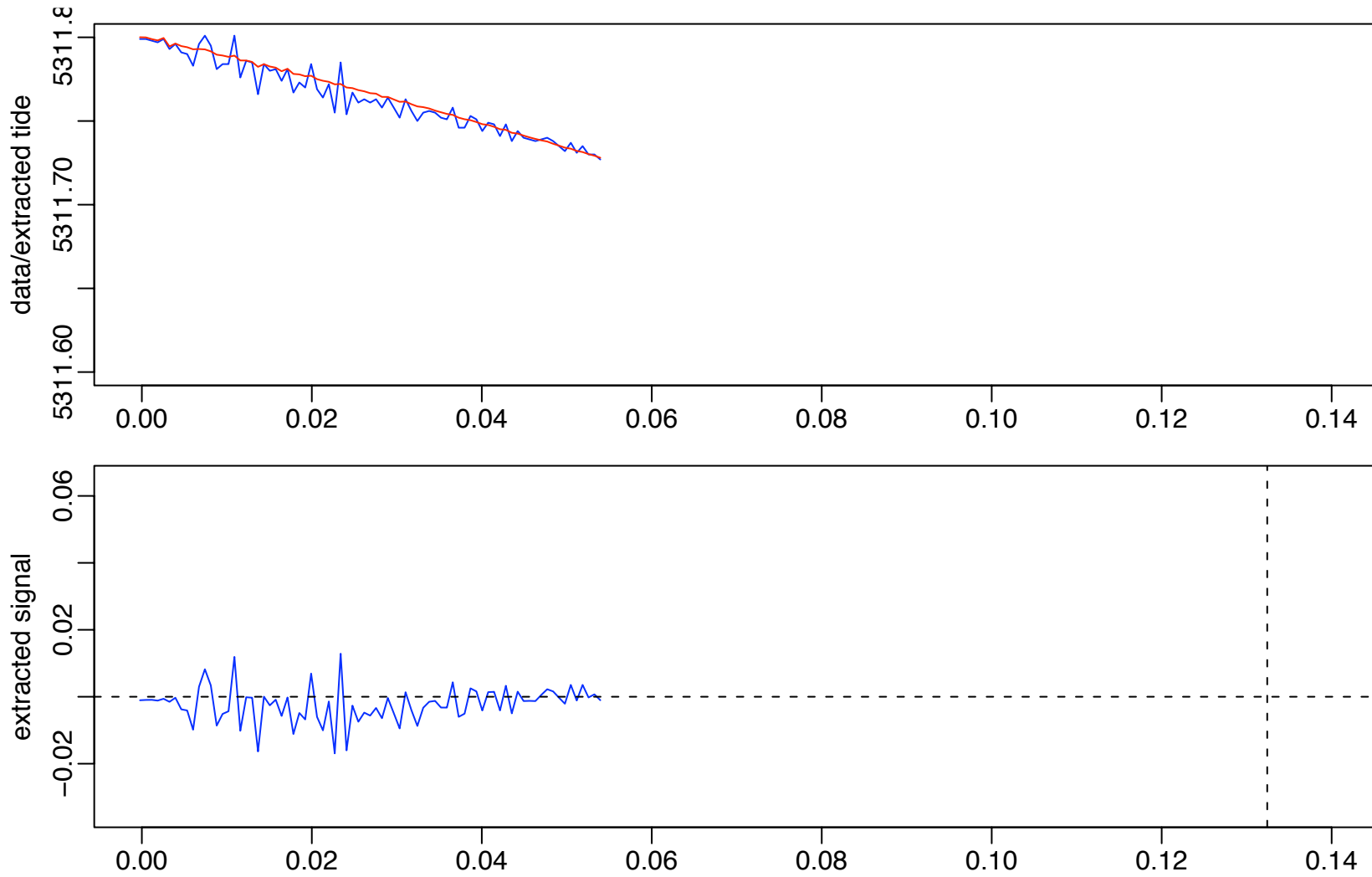
# Kalman Filtering/Smoothing – 14 Day Model



# Kalman Filtering/Smoothing – 14 Day Model

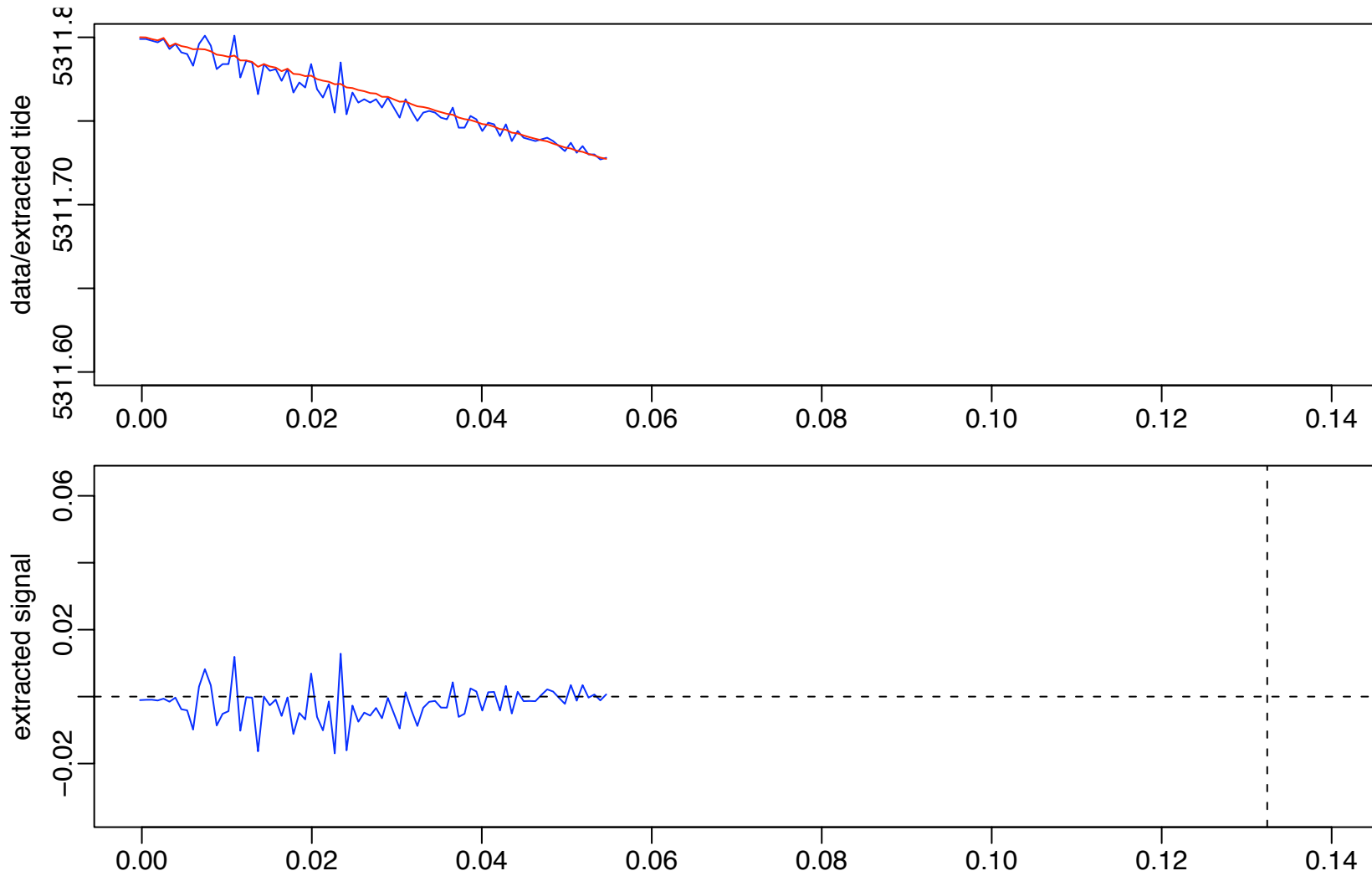


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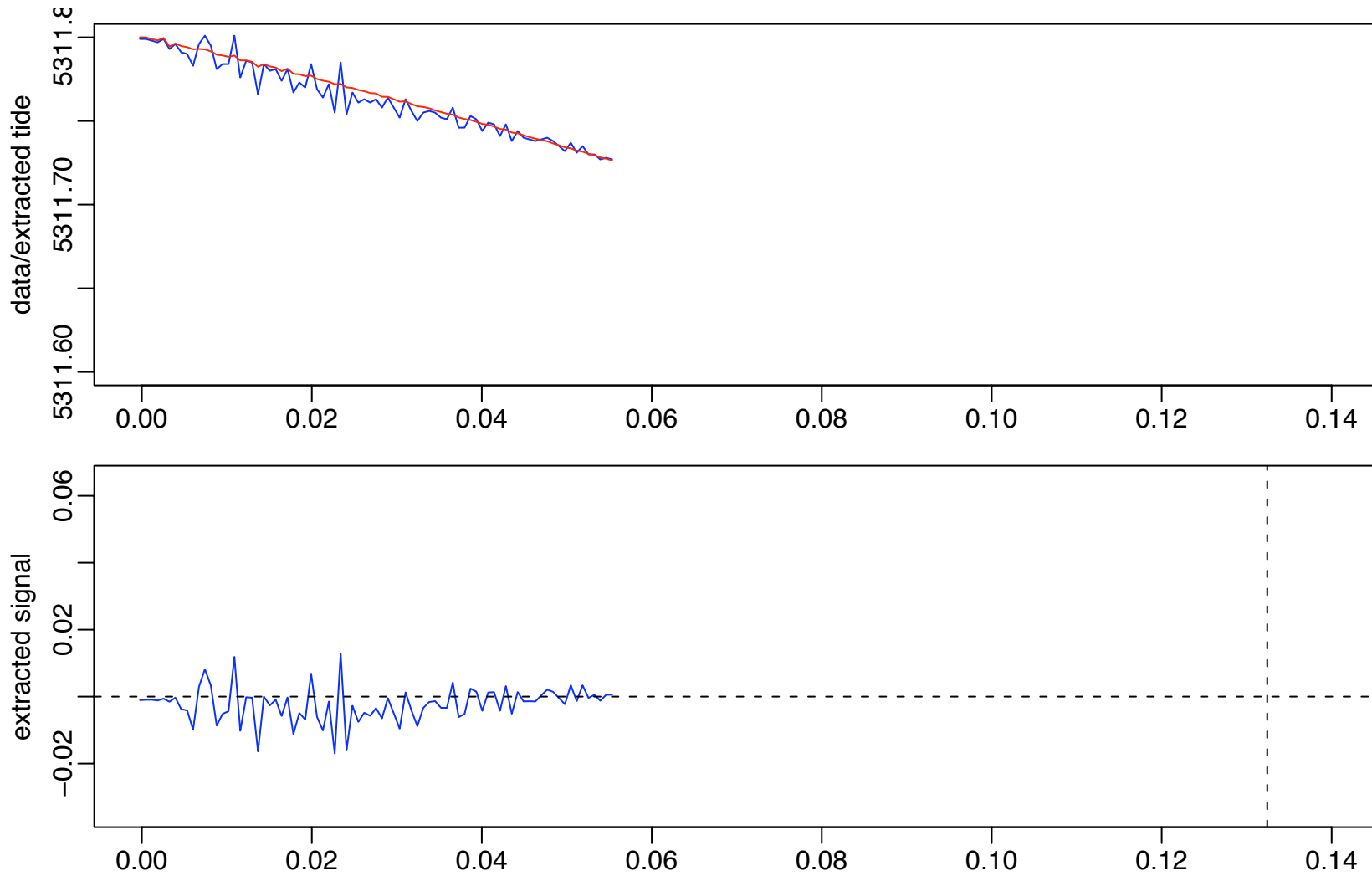




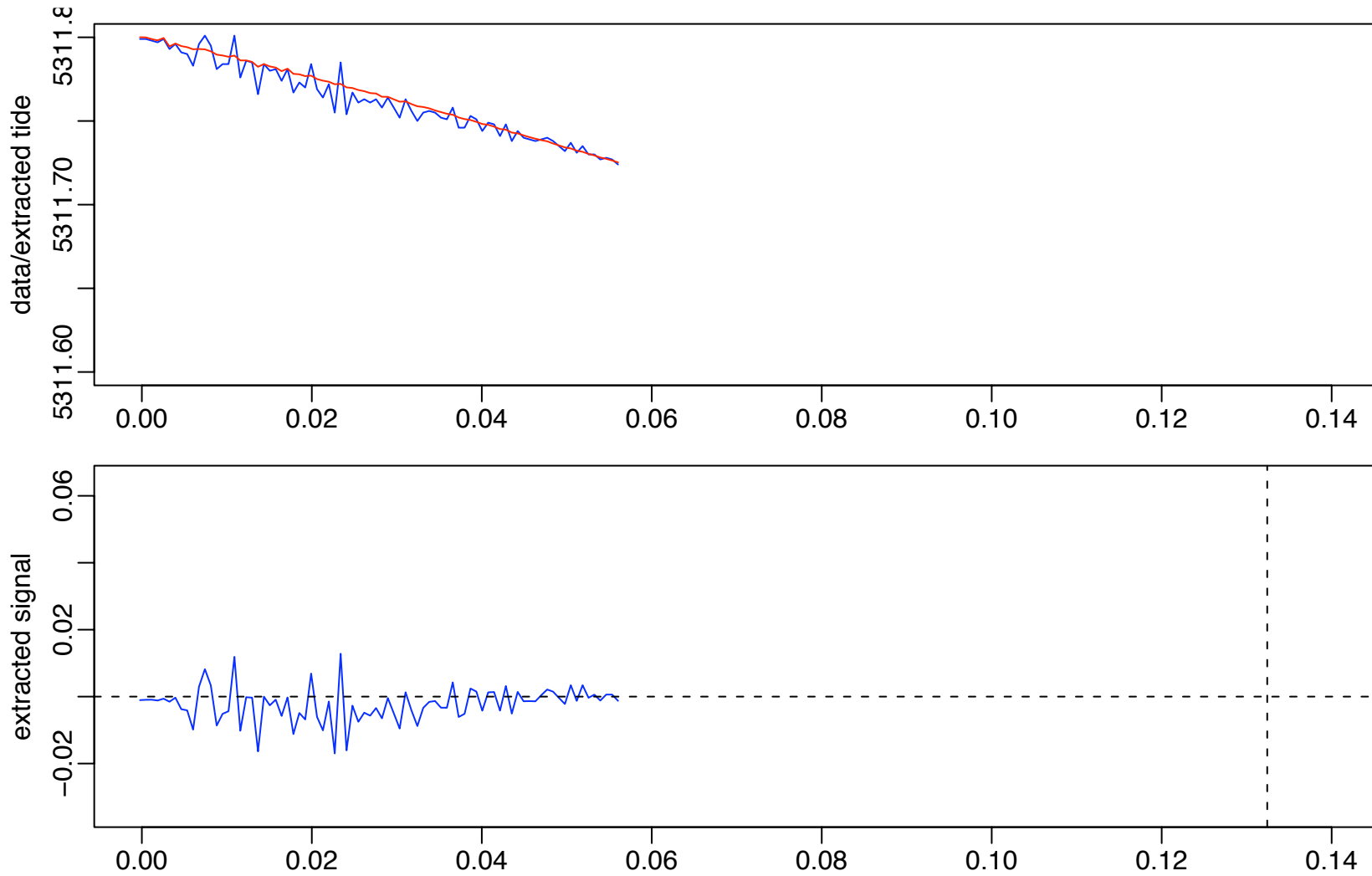
# Kalman Filtering/Smoothing – 14 Day Model



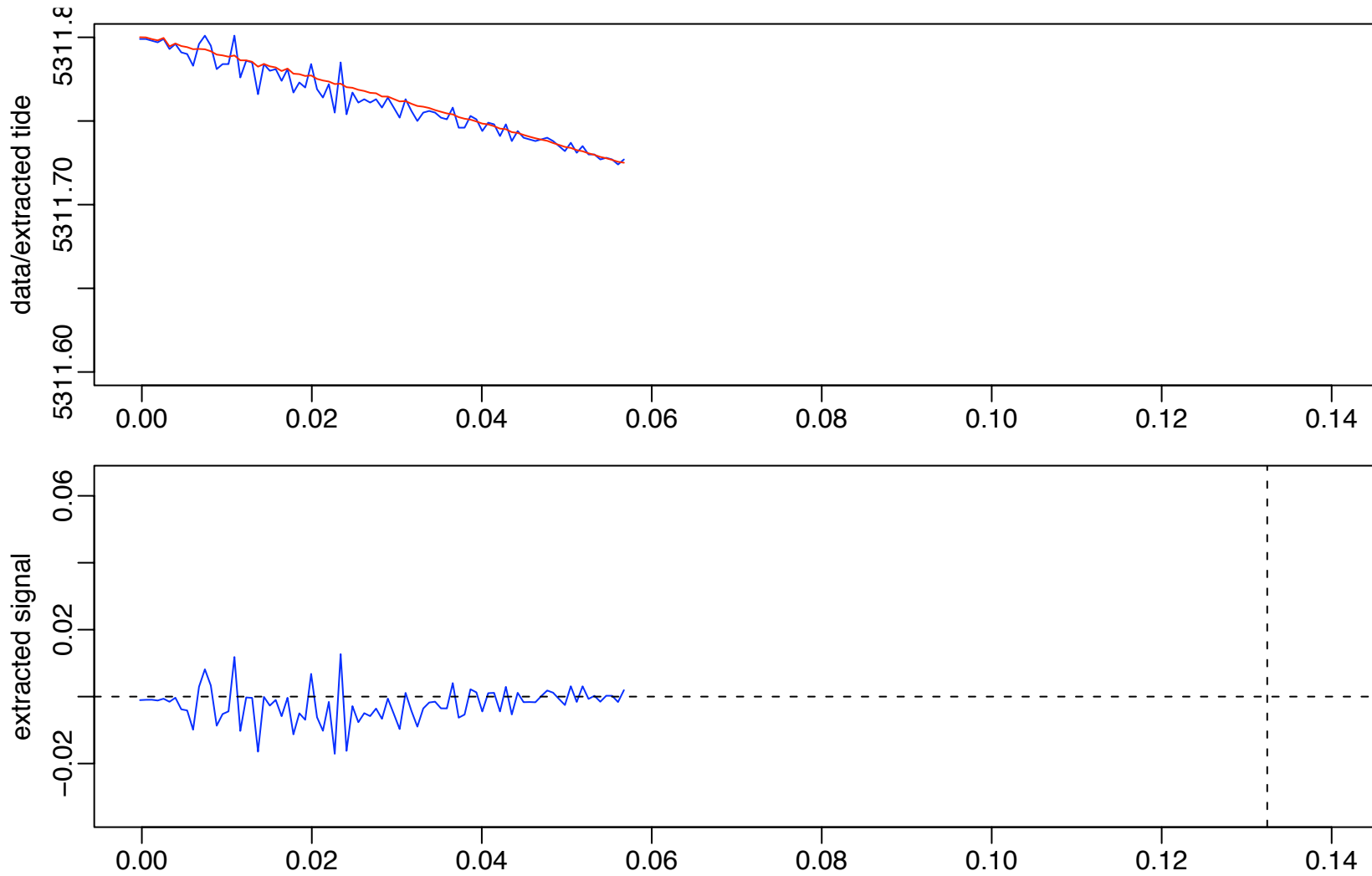
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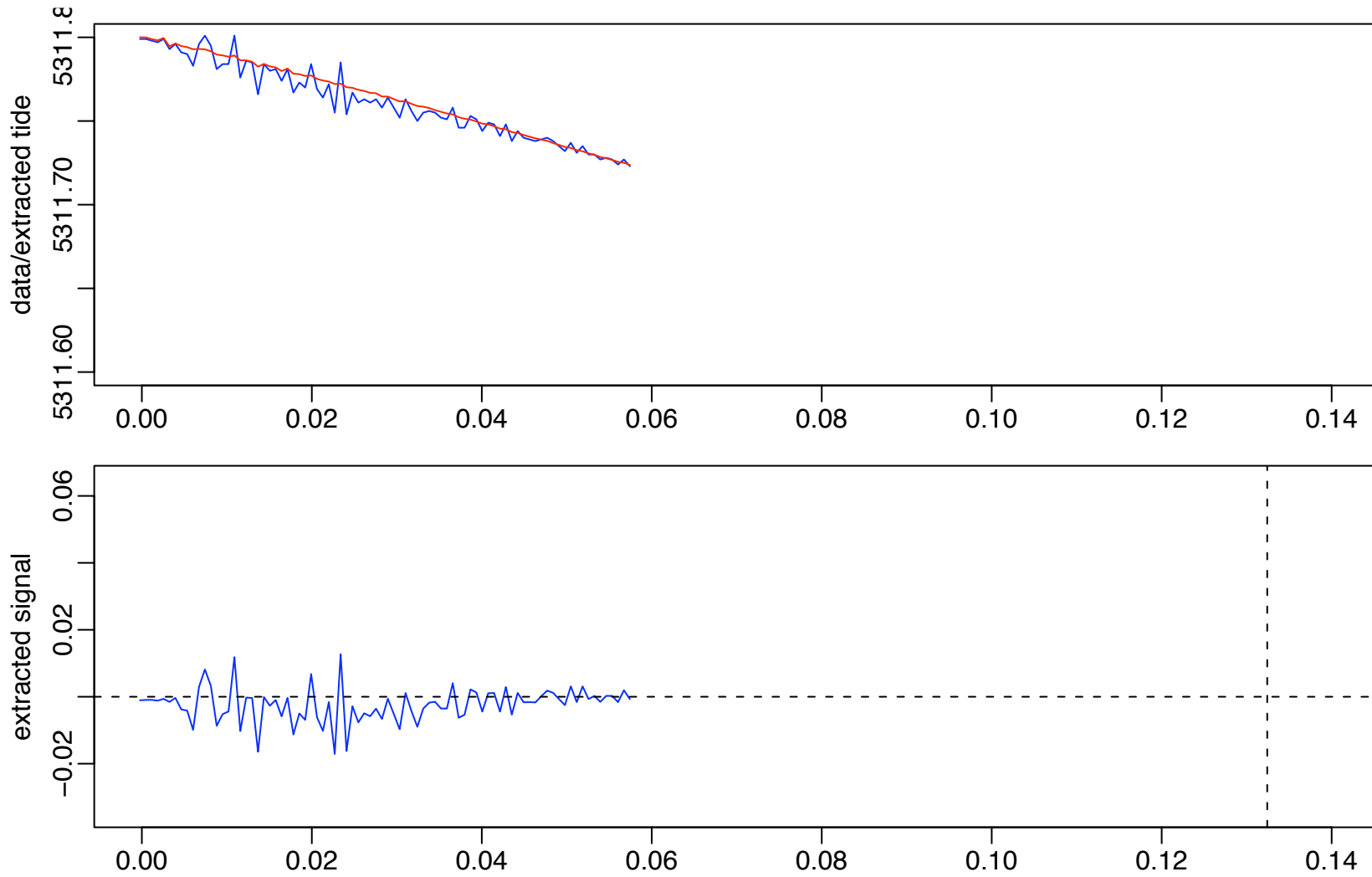
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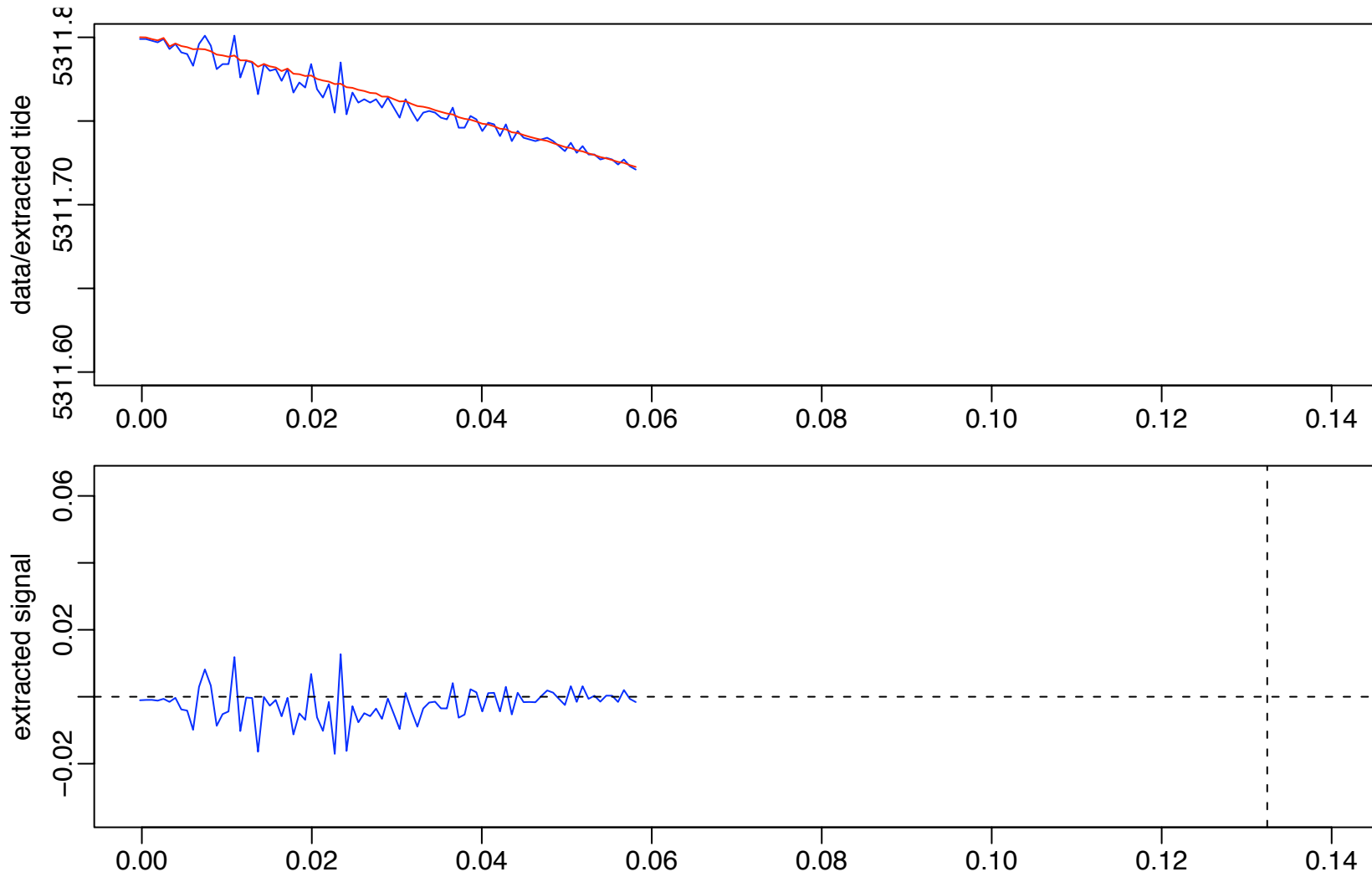
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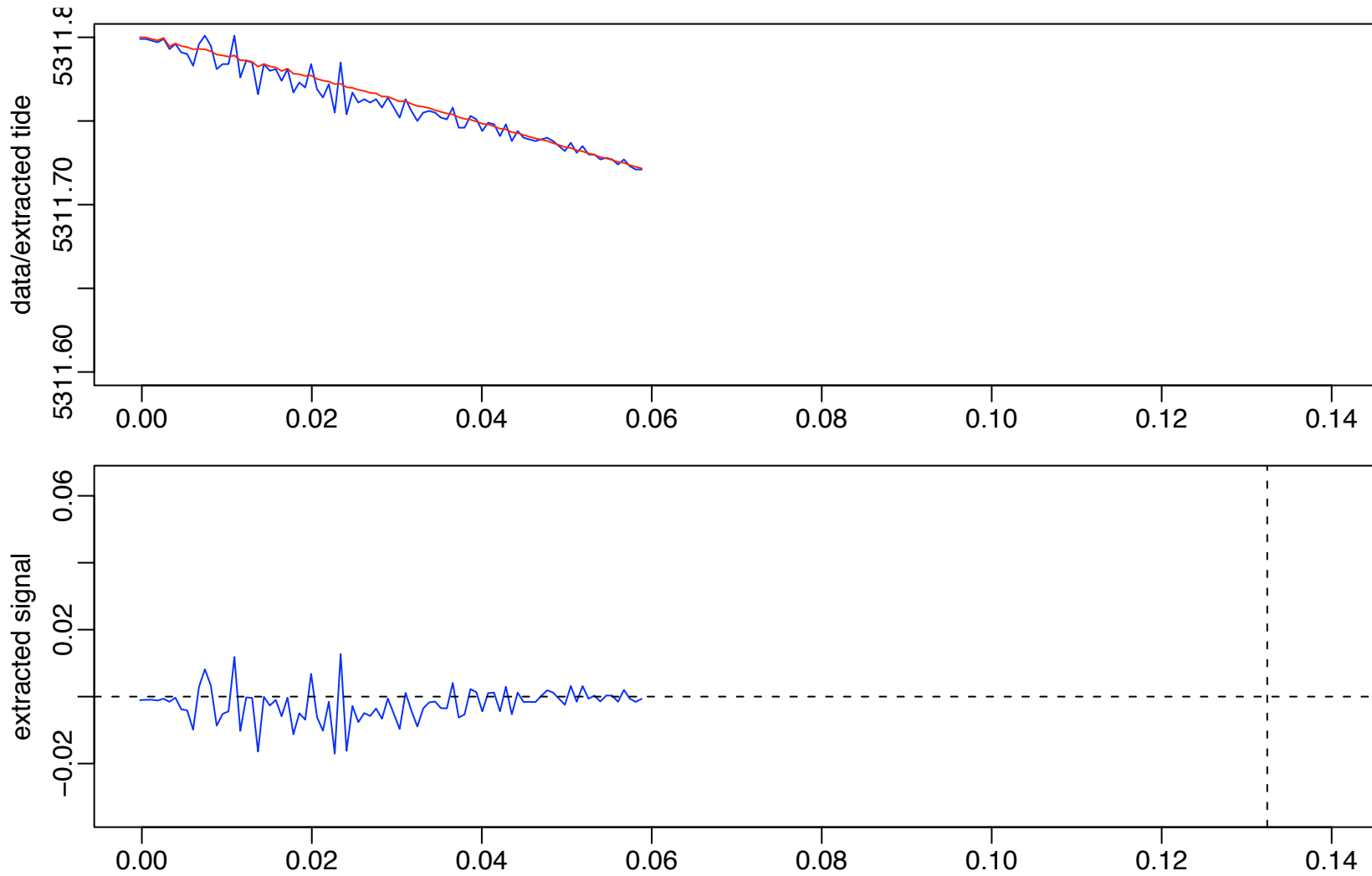
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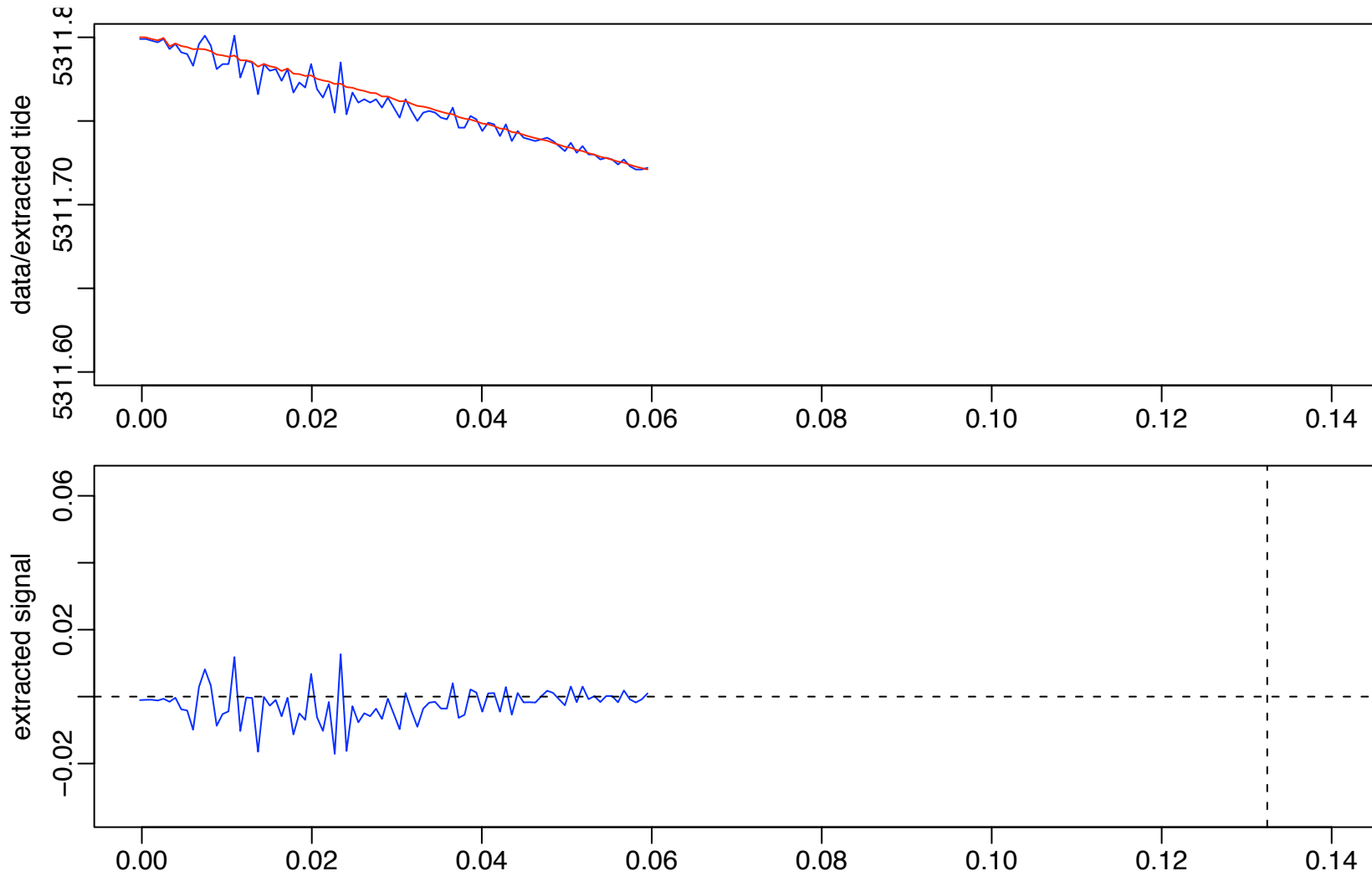
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# Kalman Filtering/Smoothing – 14 Day Model

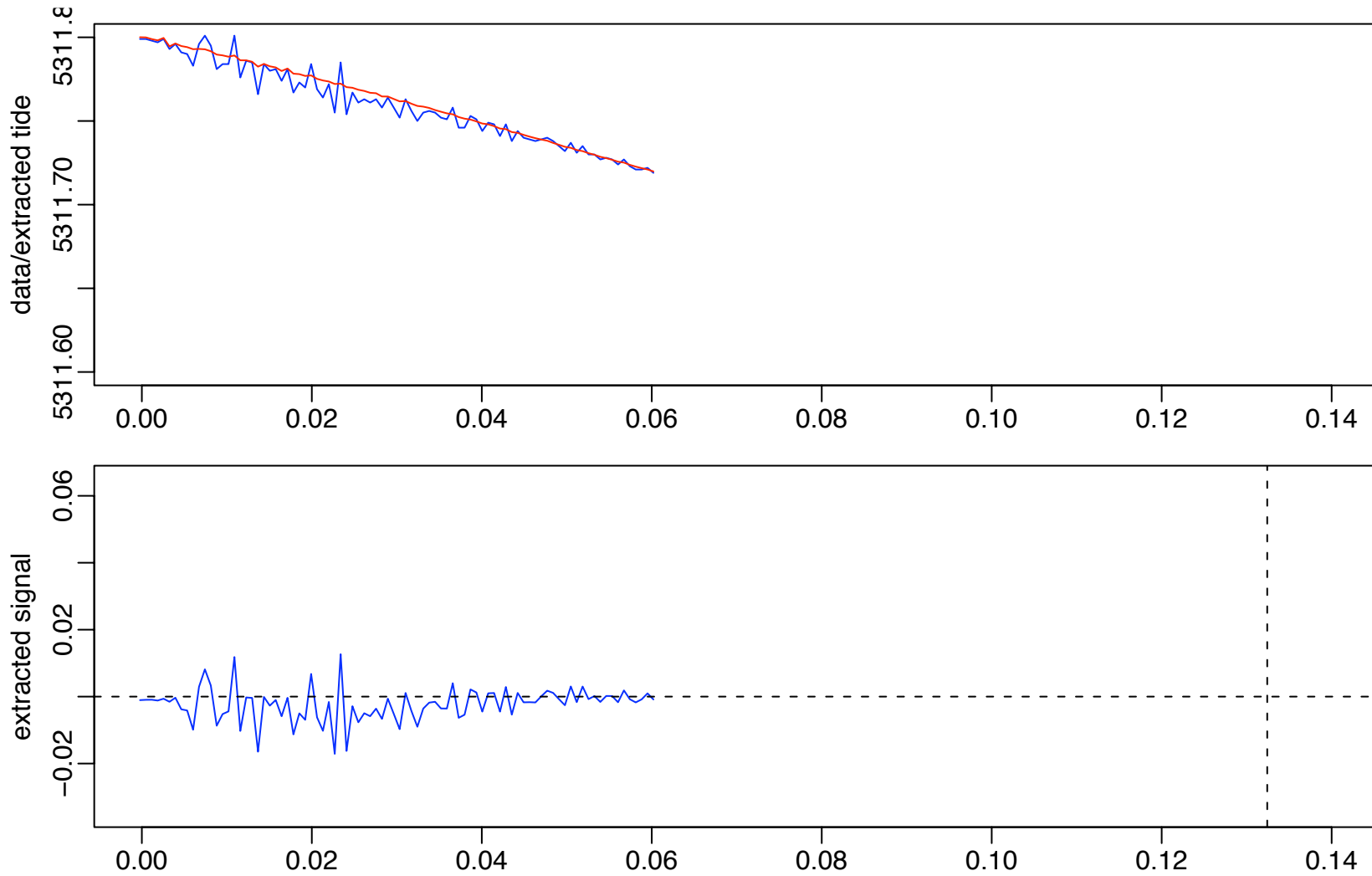


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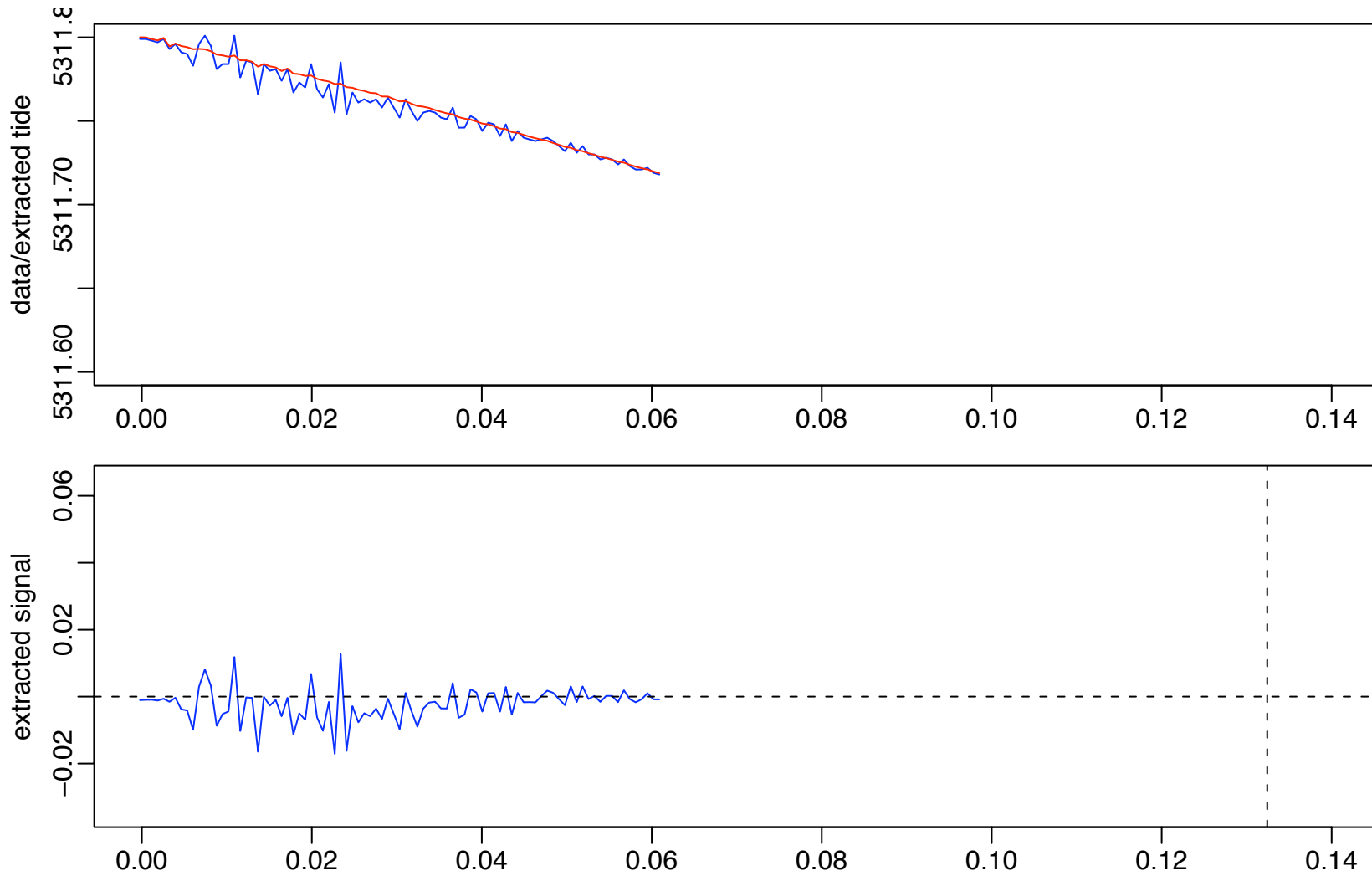




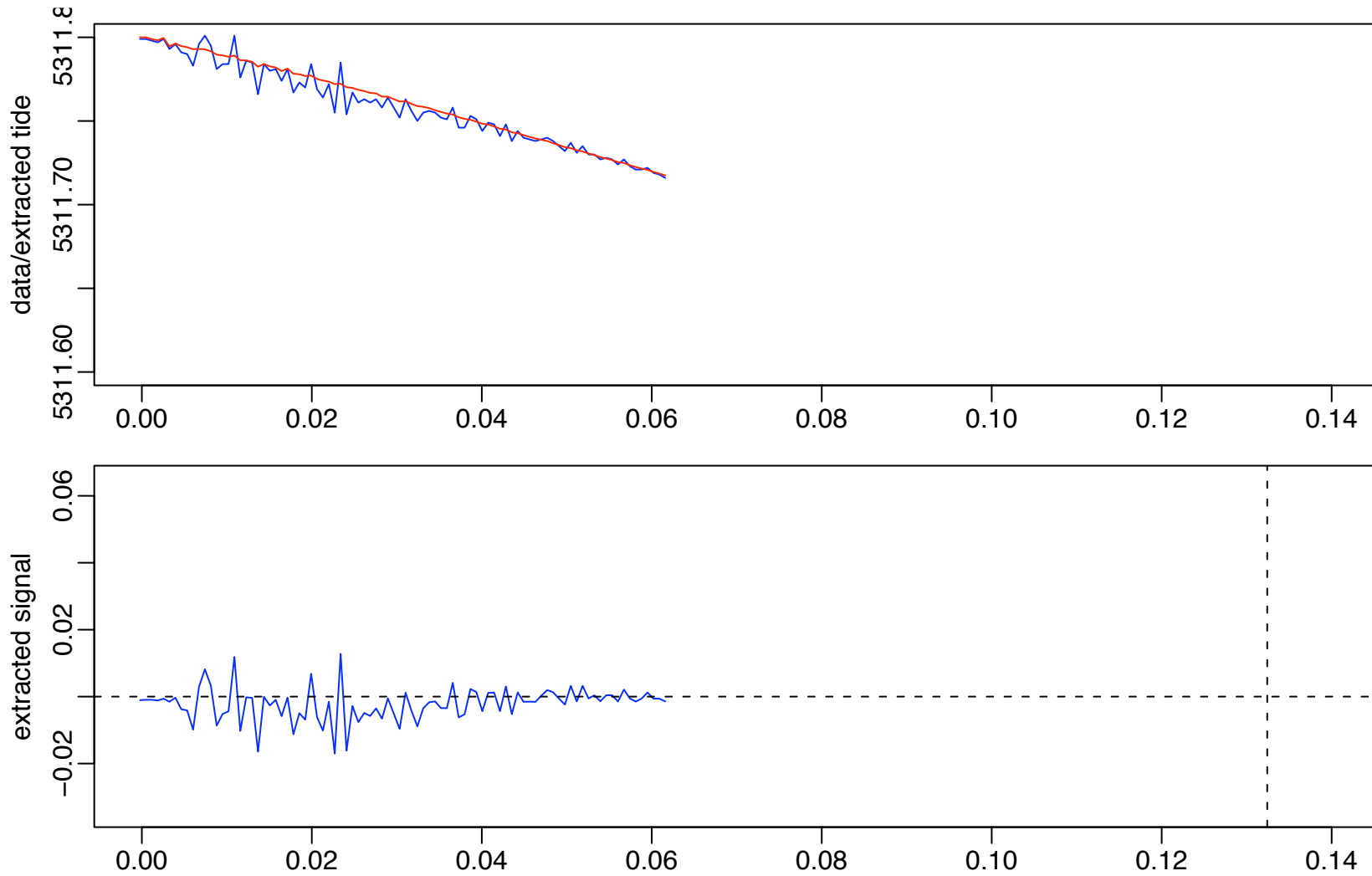
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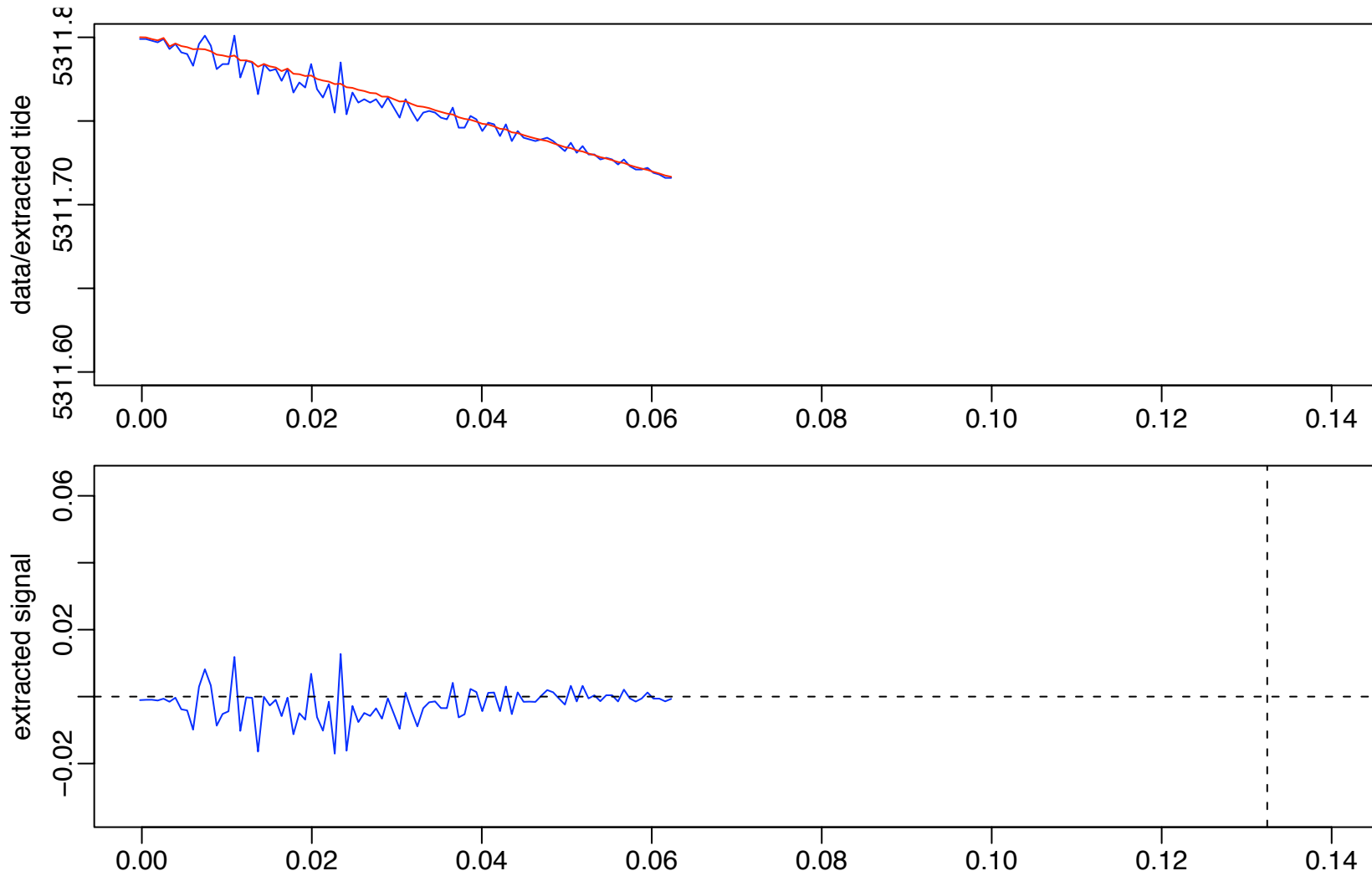
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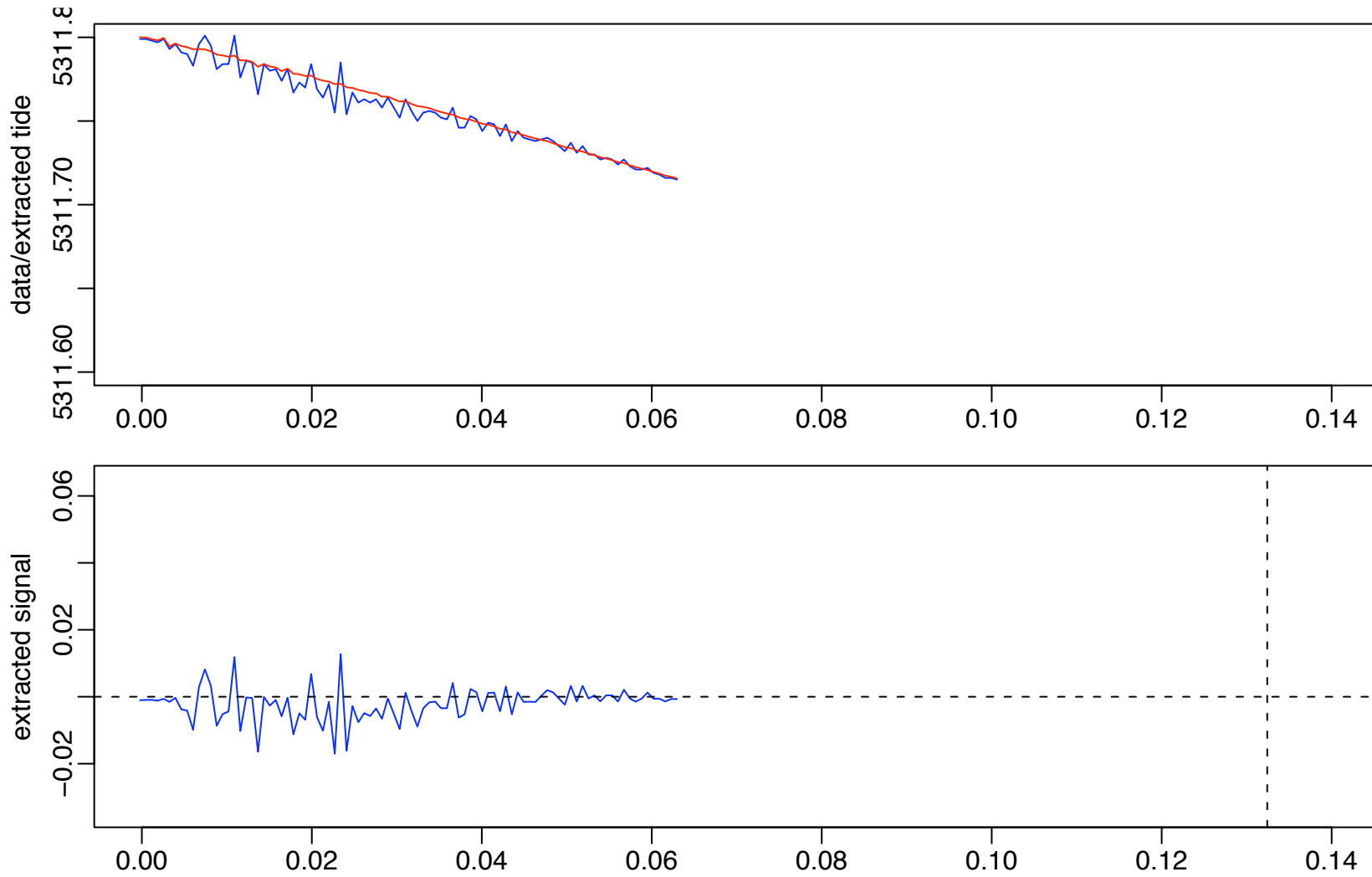
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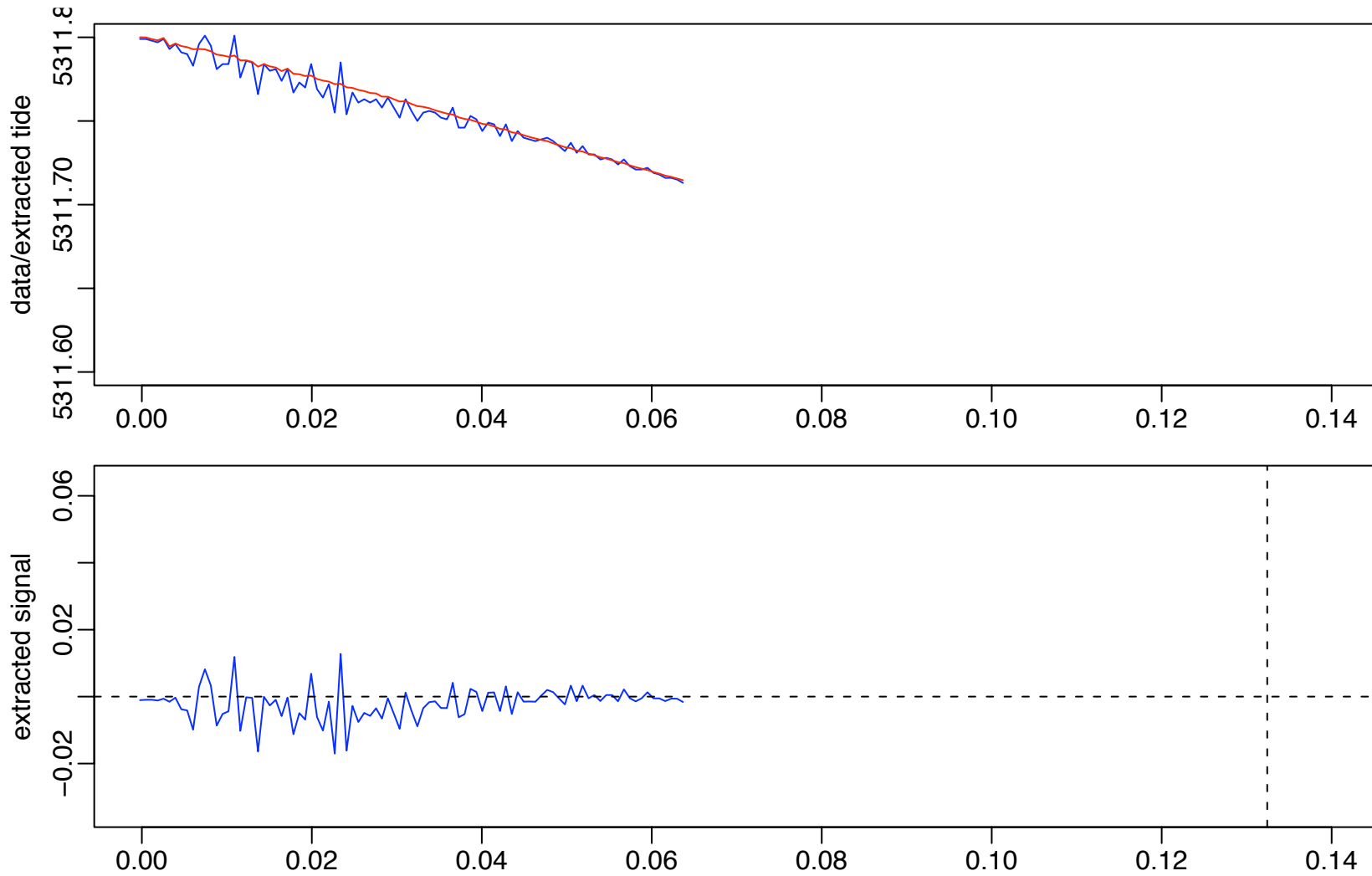
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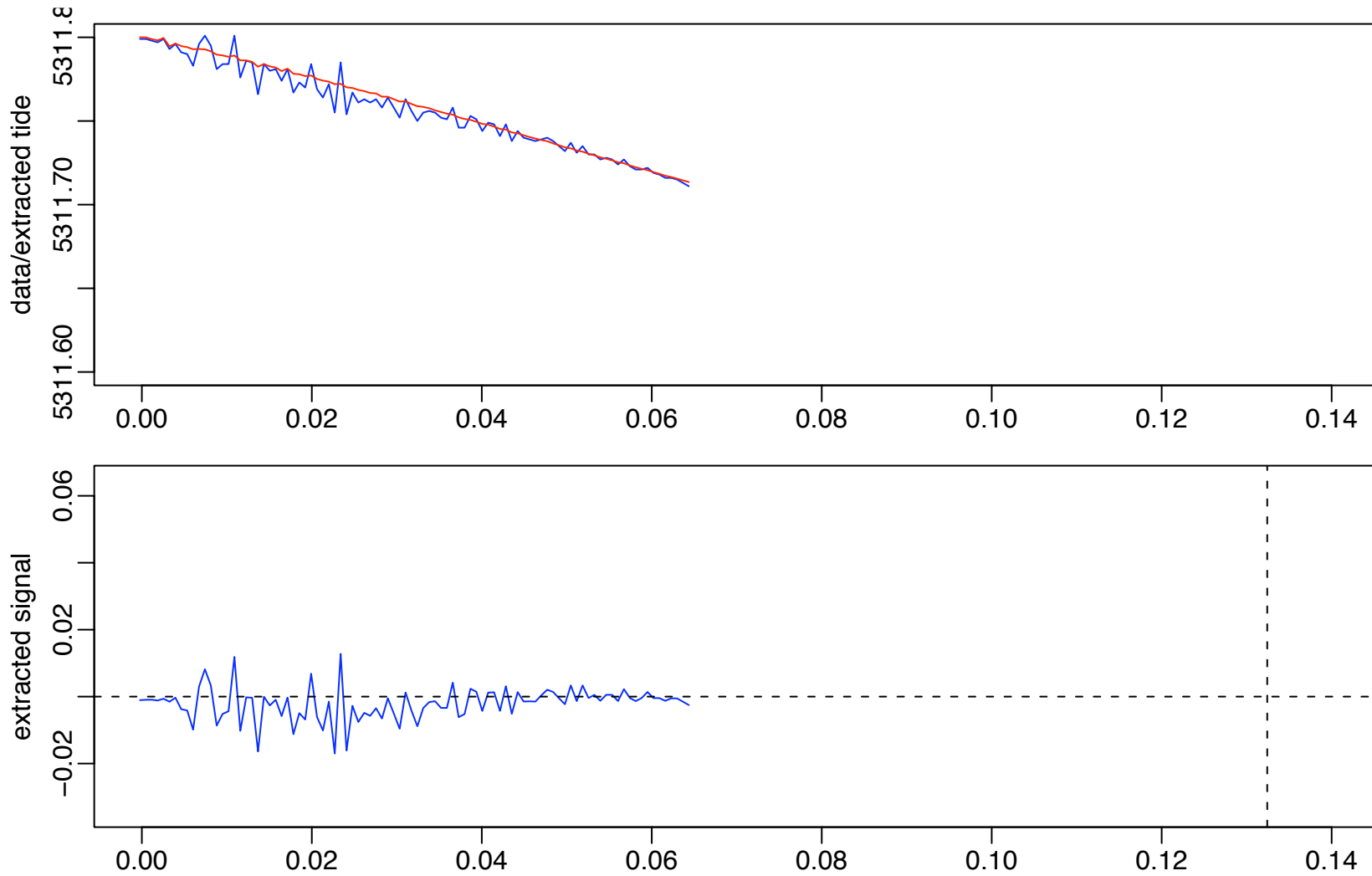
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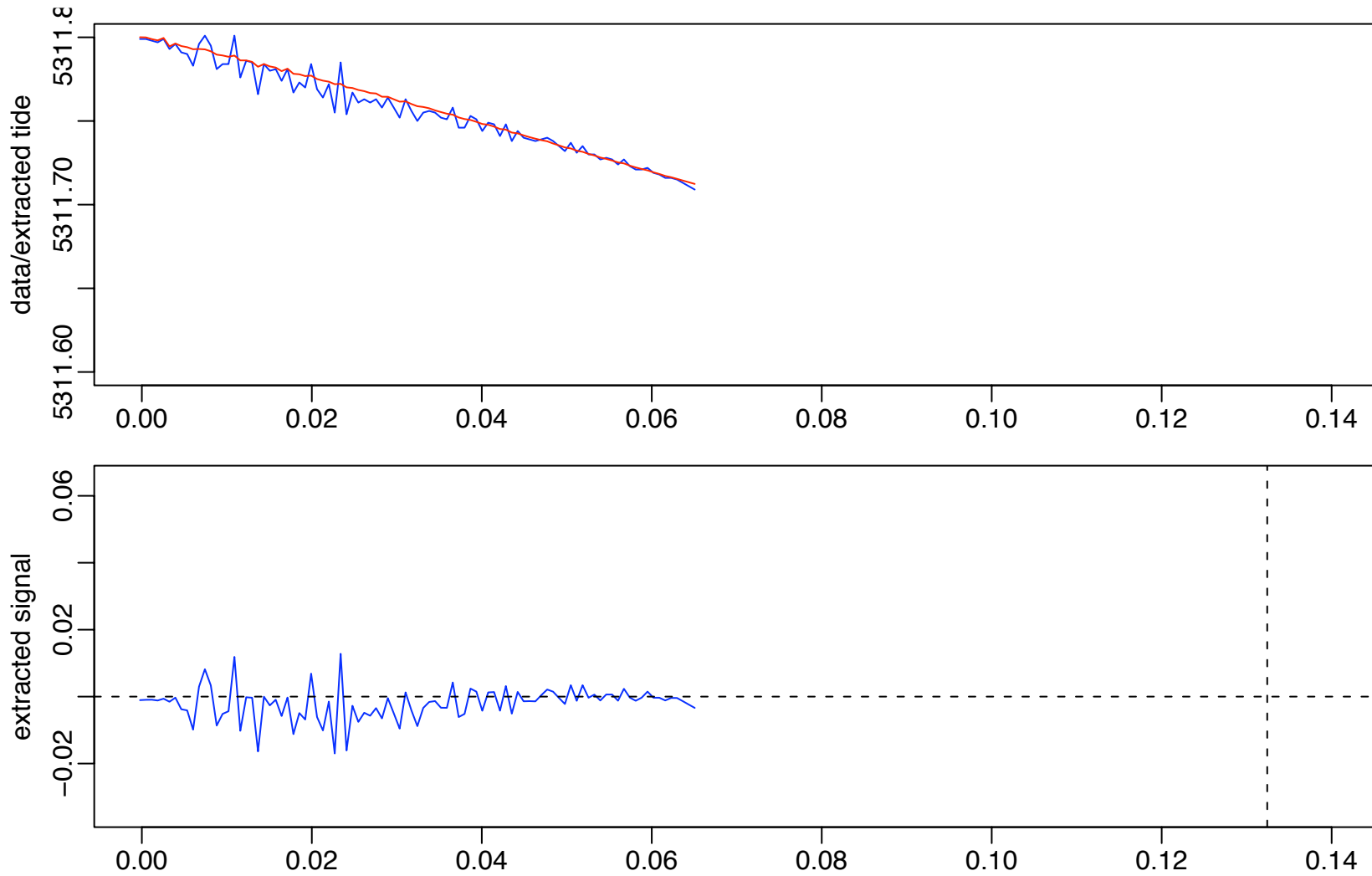
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# Kalman Filtering/Smoothing – 14 Day Model

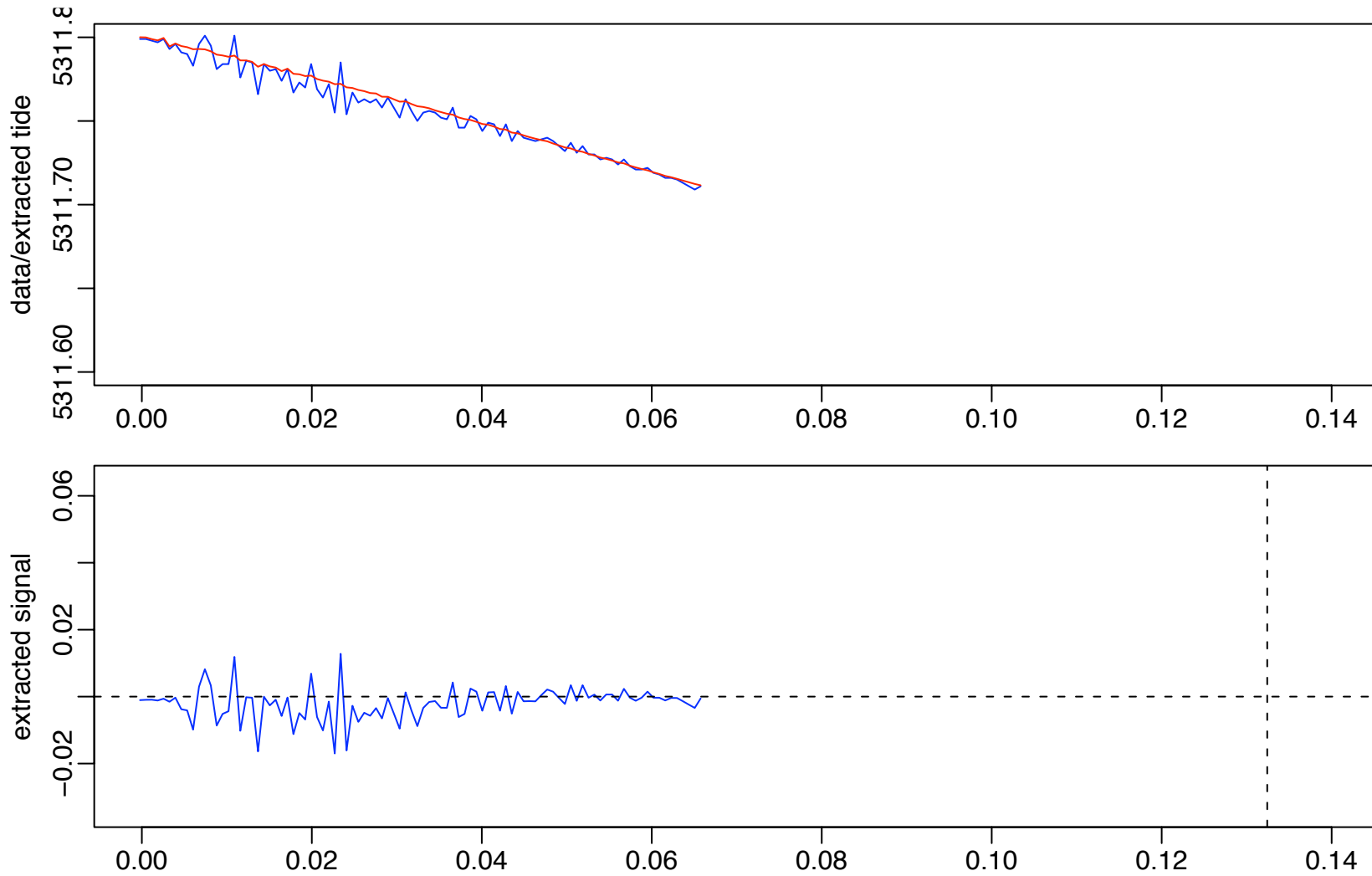


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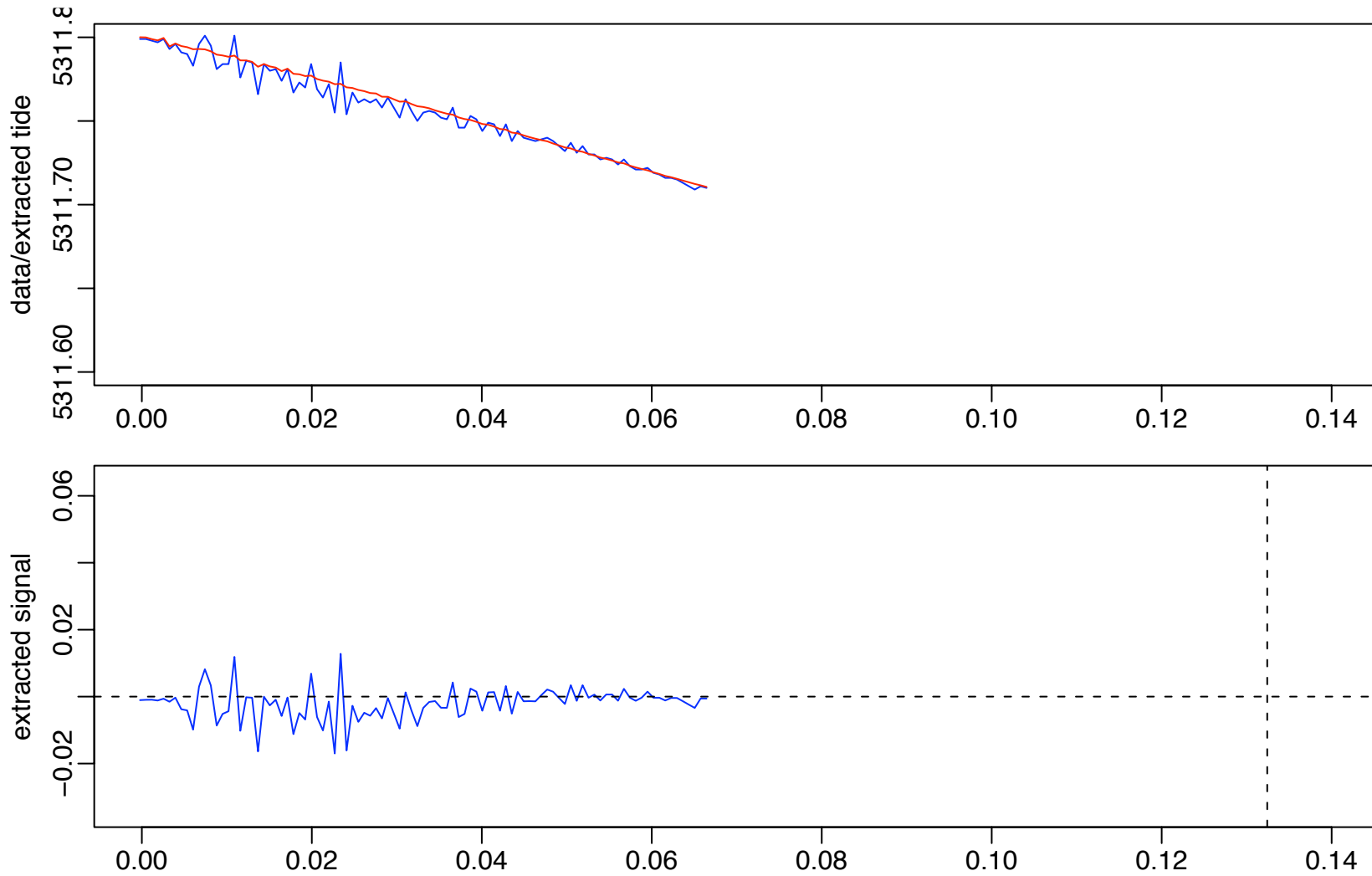




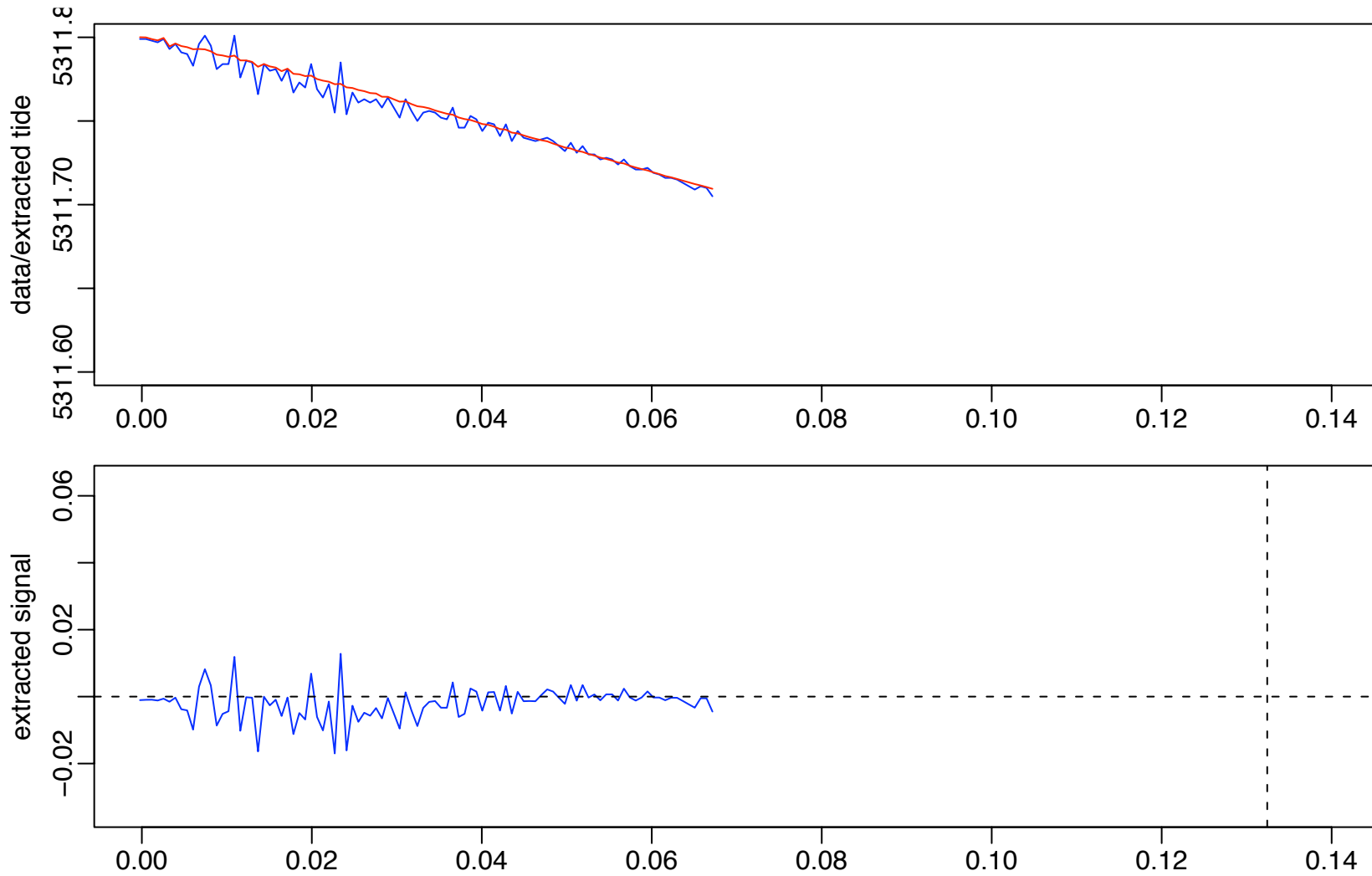
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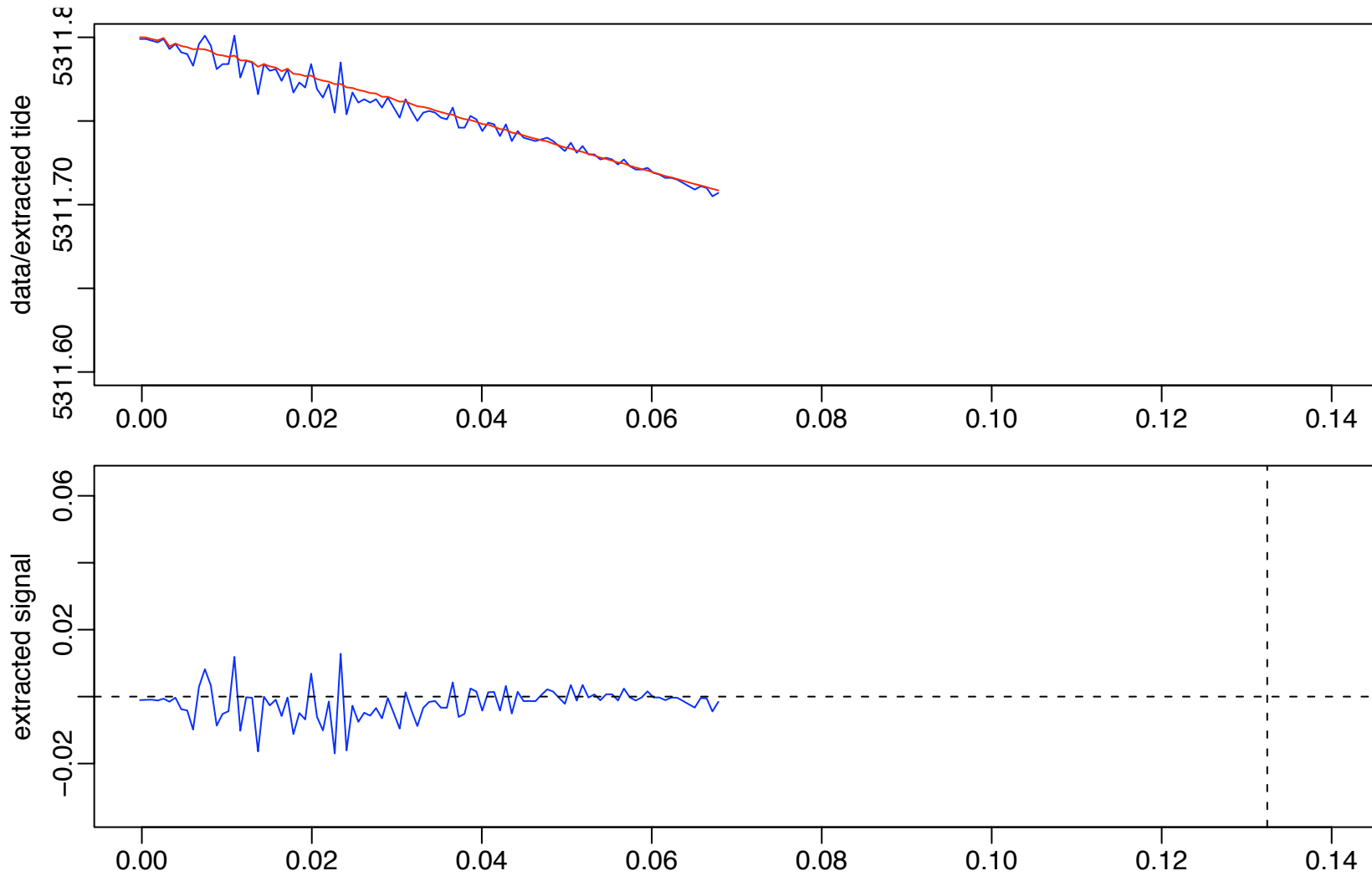
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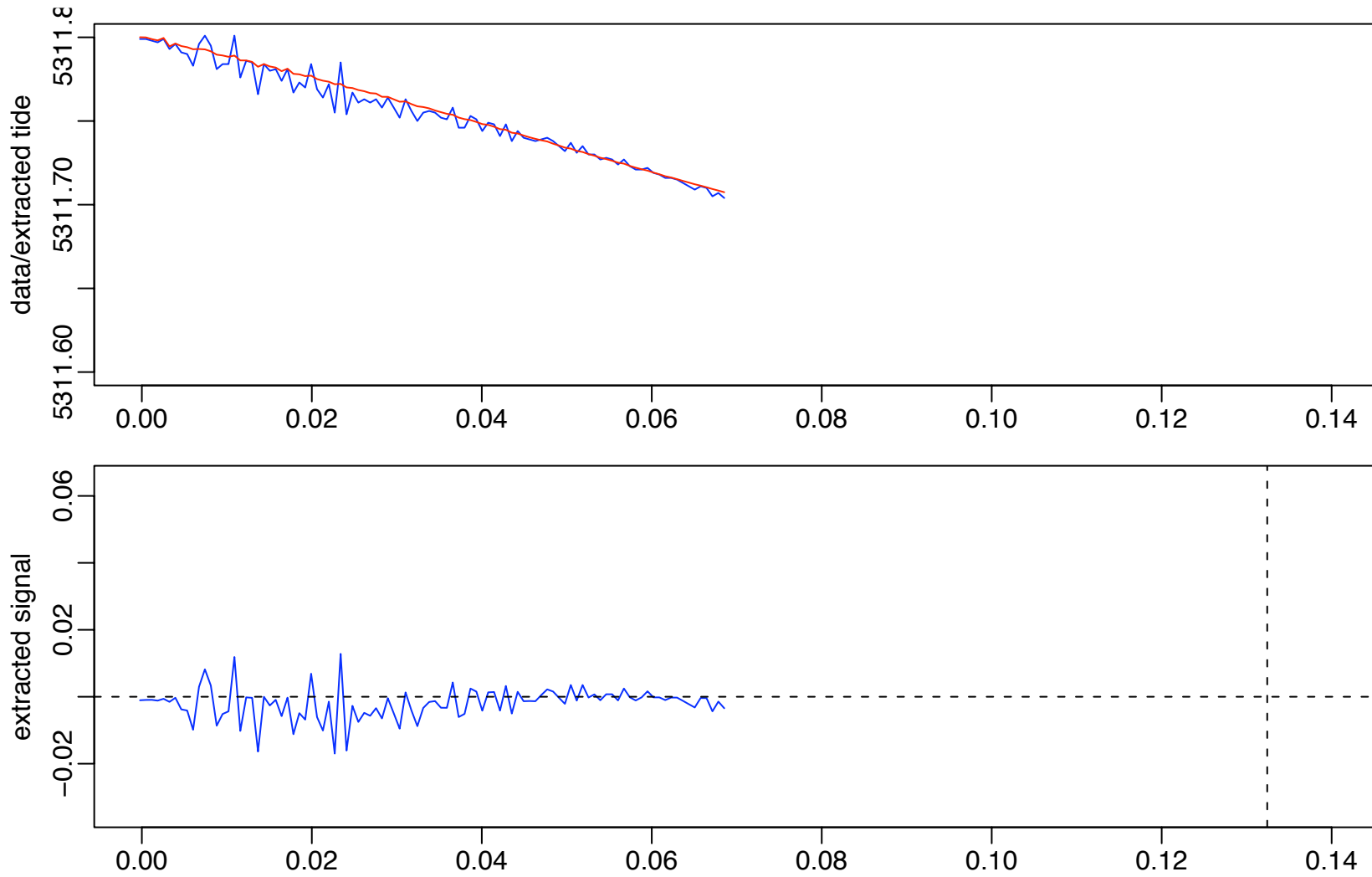
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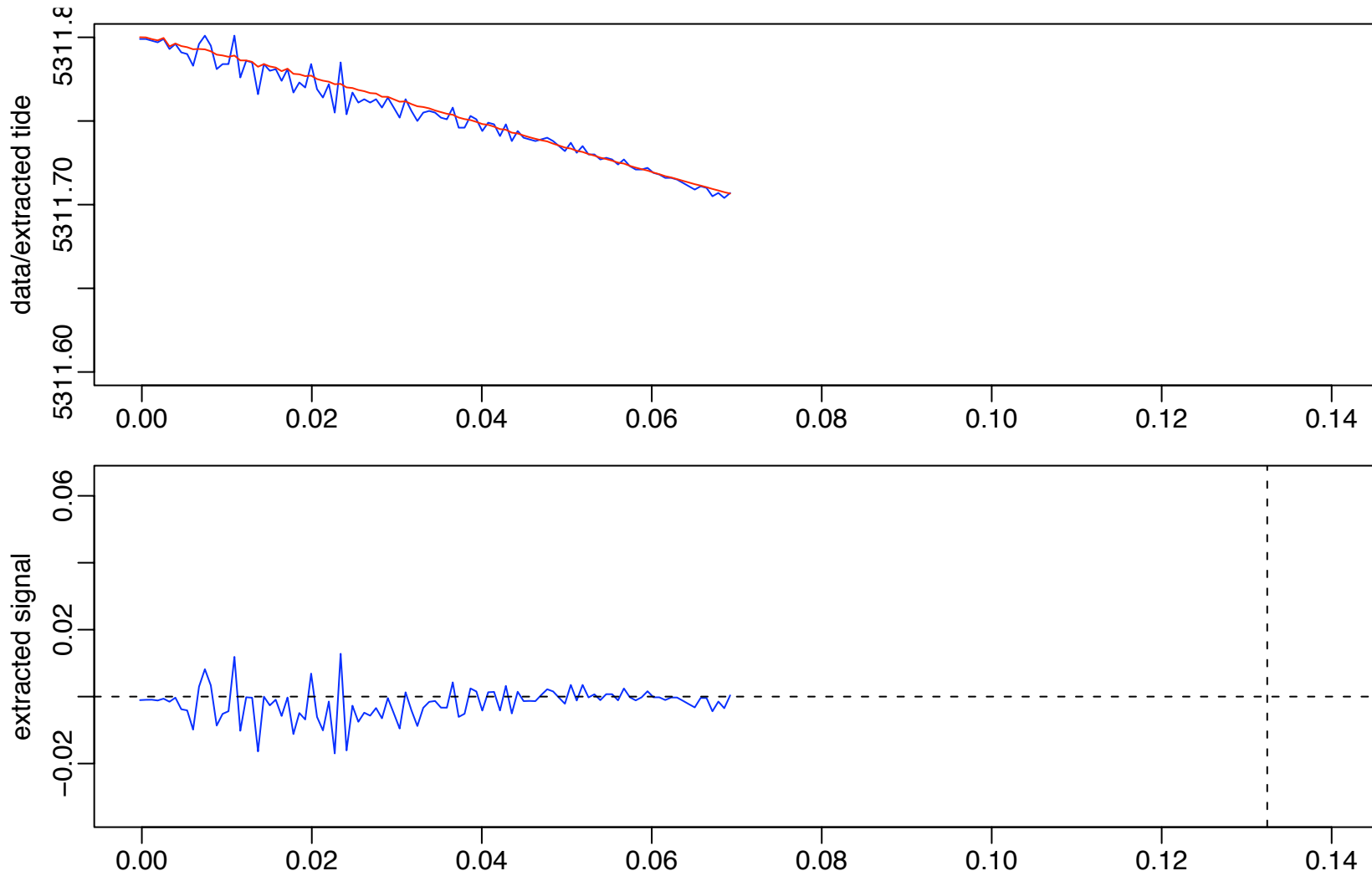
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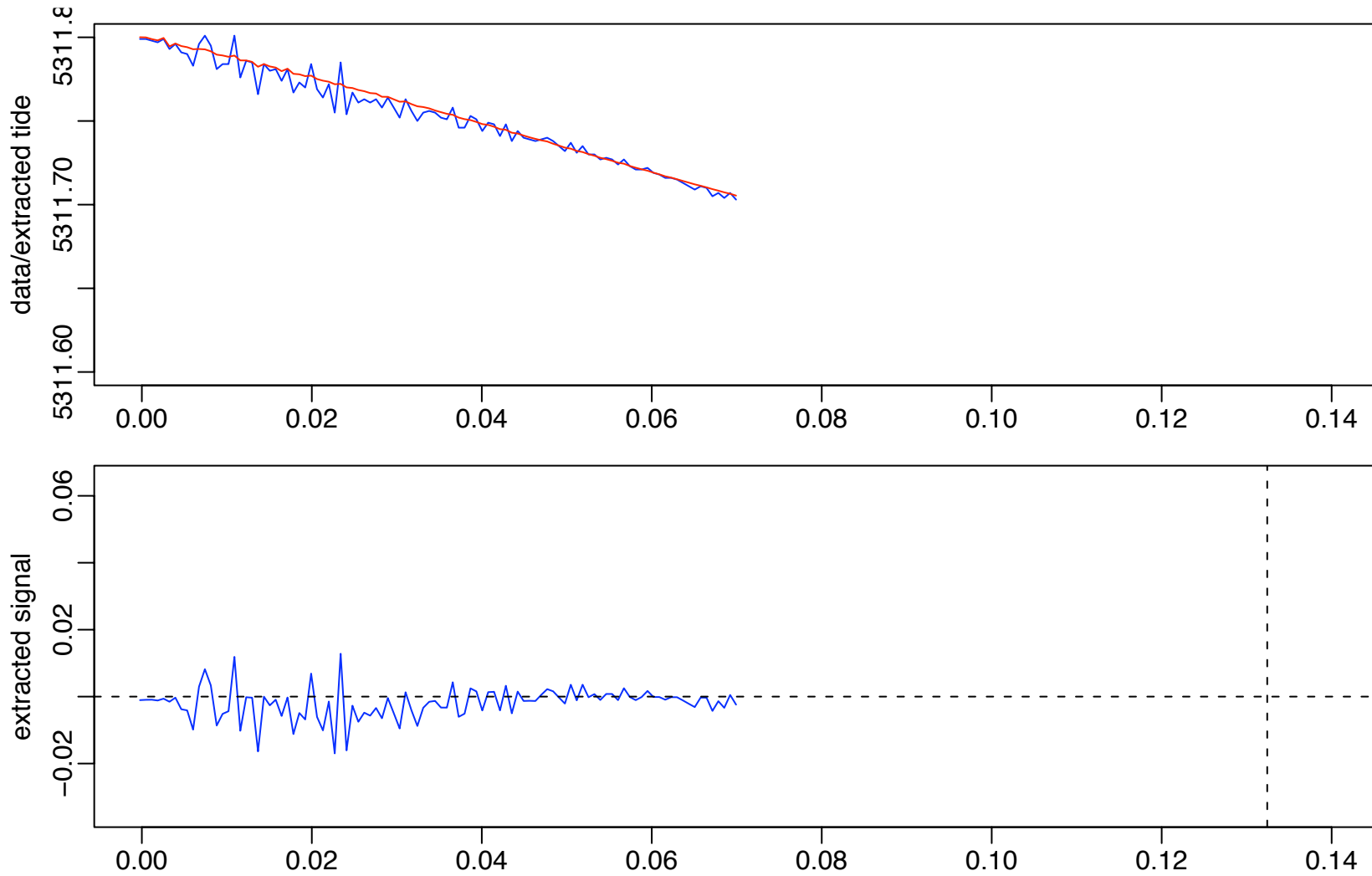
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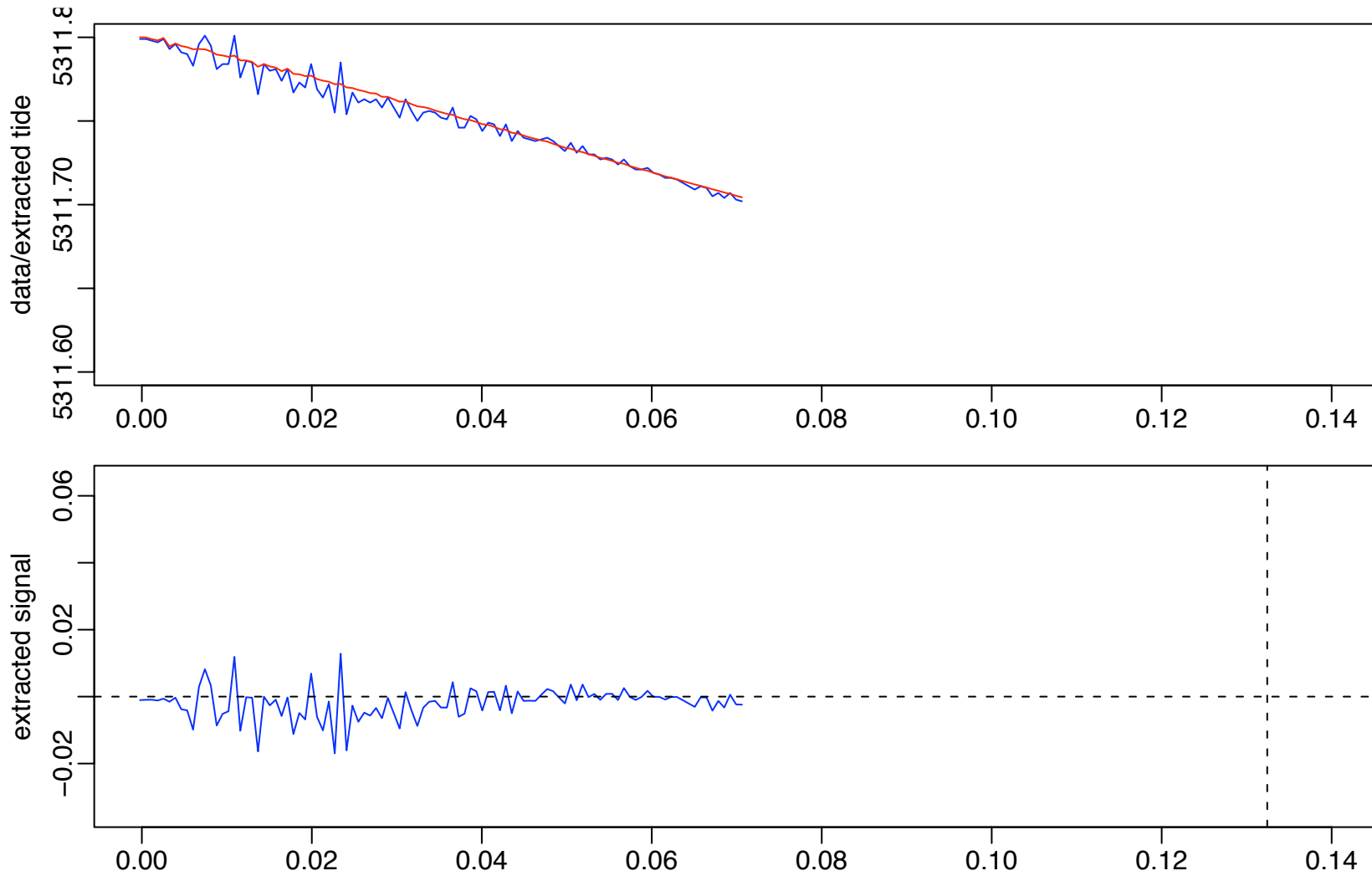
# Kalman Filtering/Smoothing – 14 Day Model



# Kalman Filtering/Smoothing – 14 Day Model

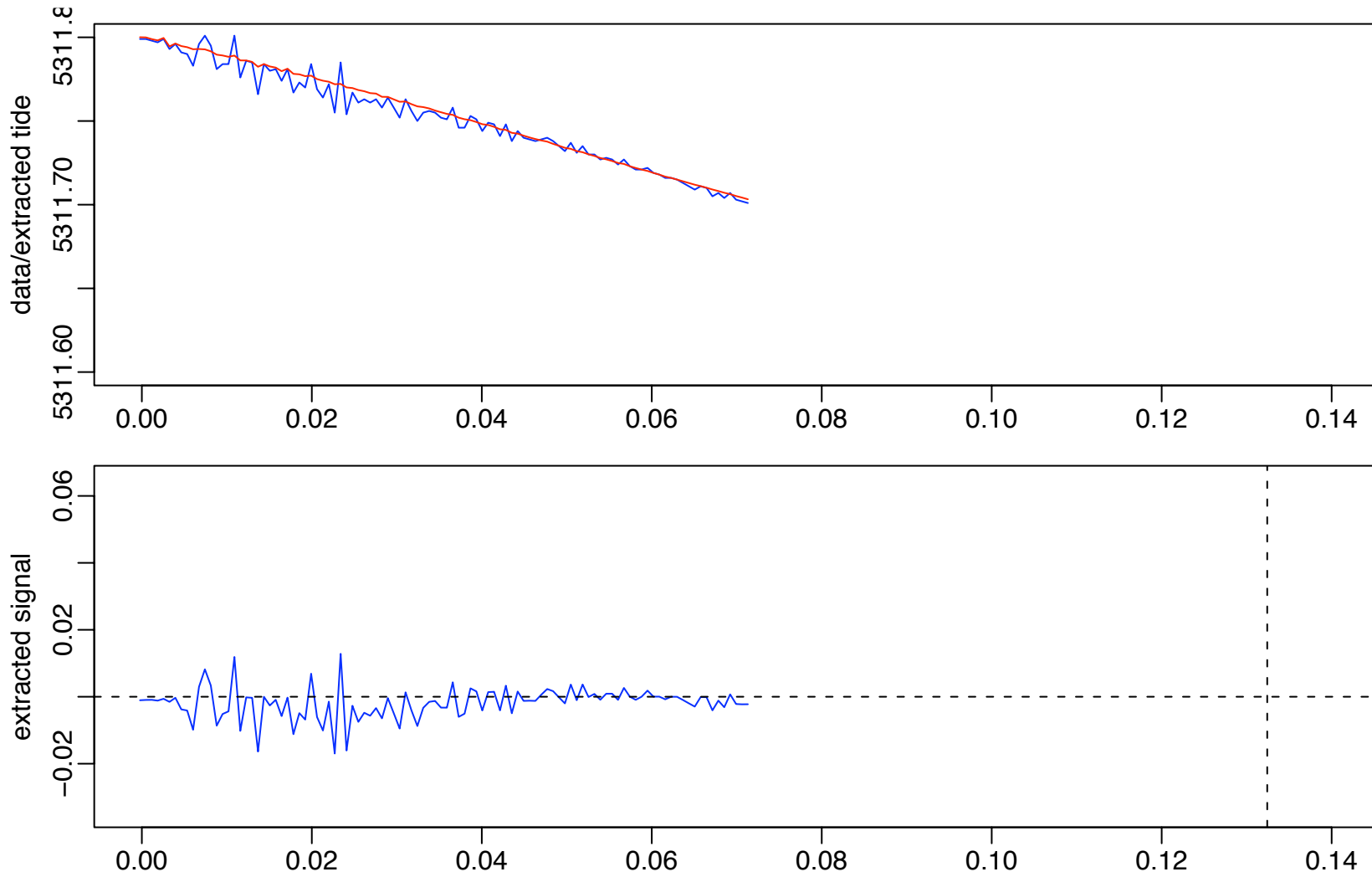


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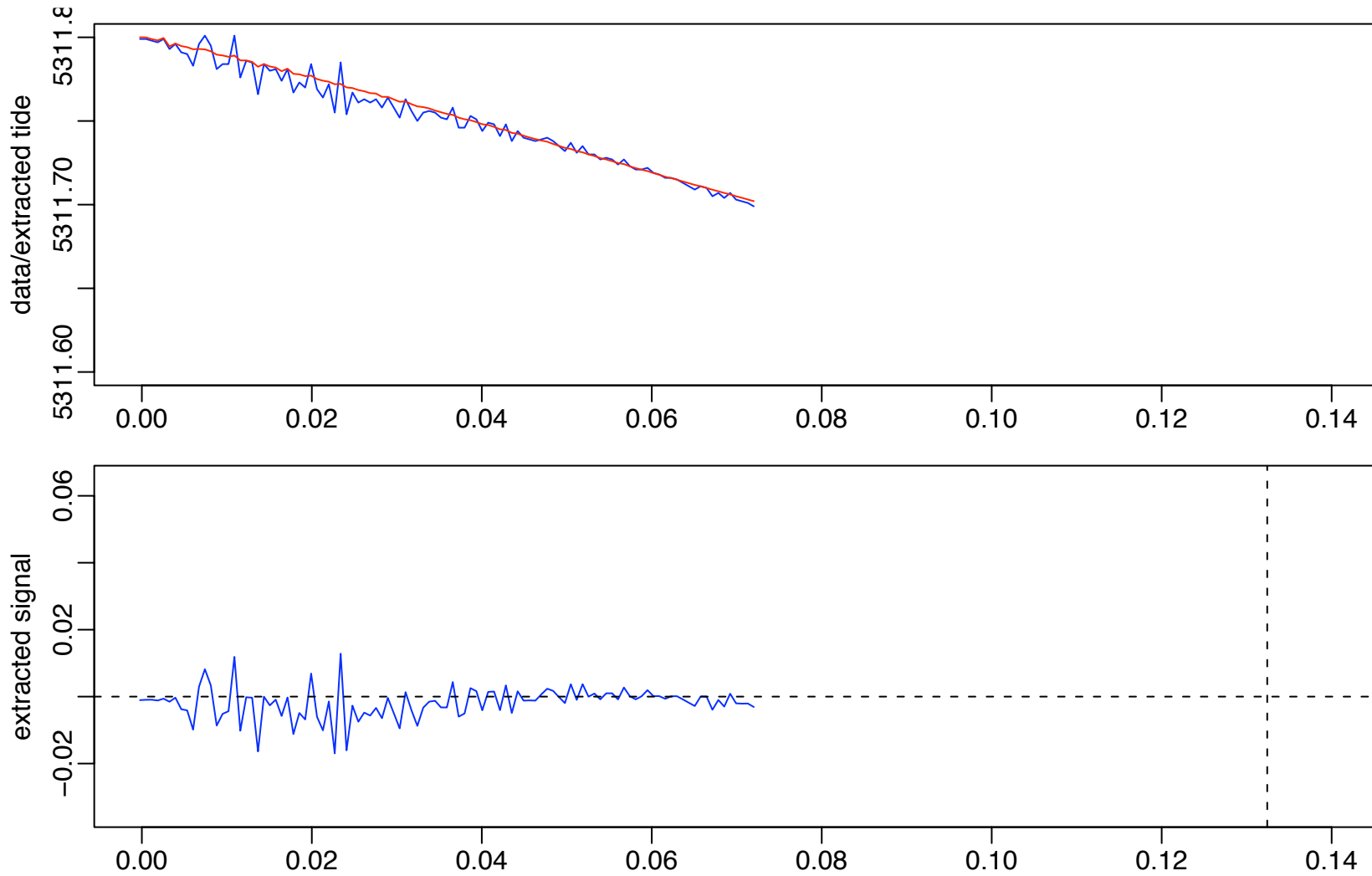




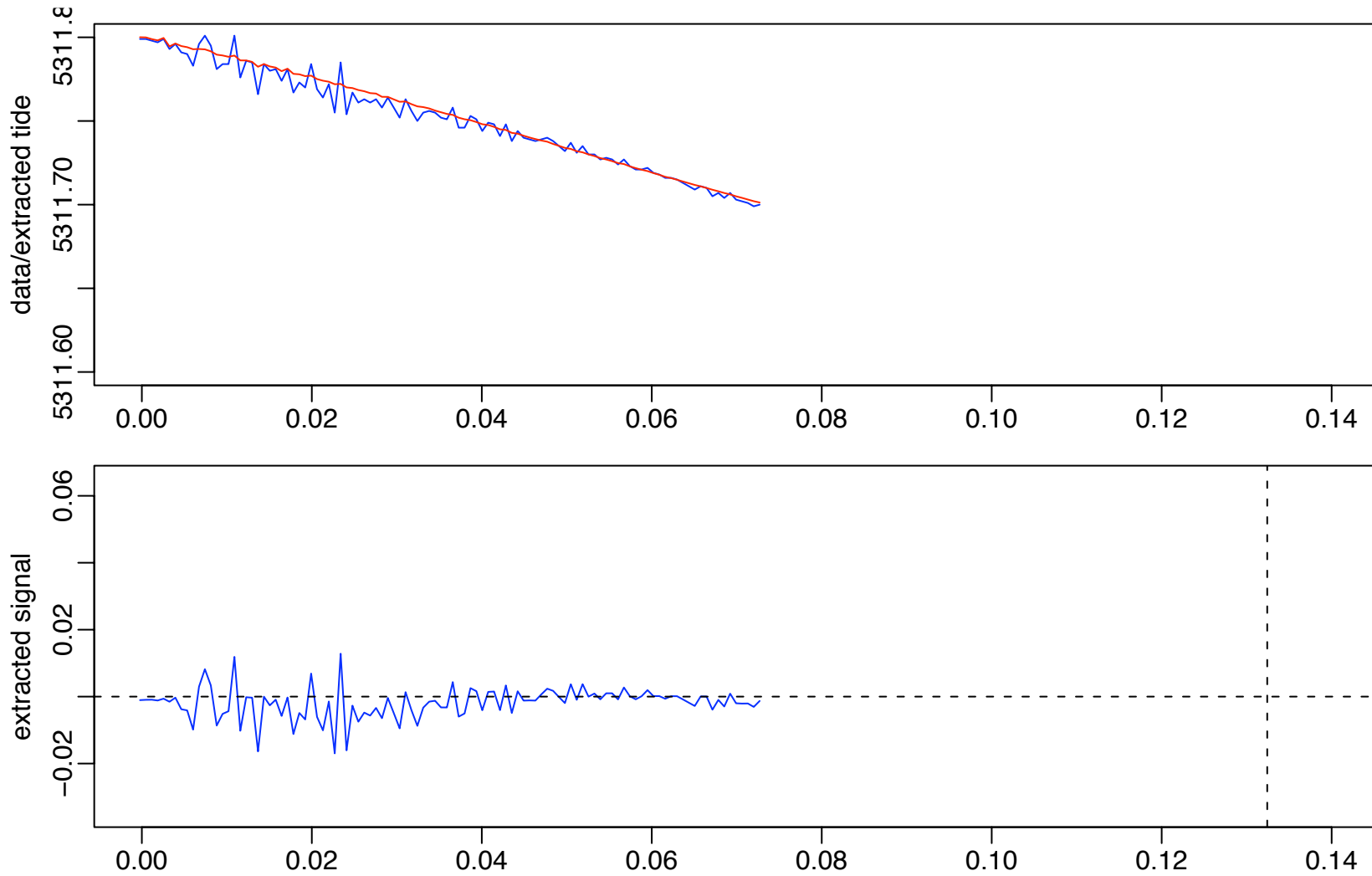
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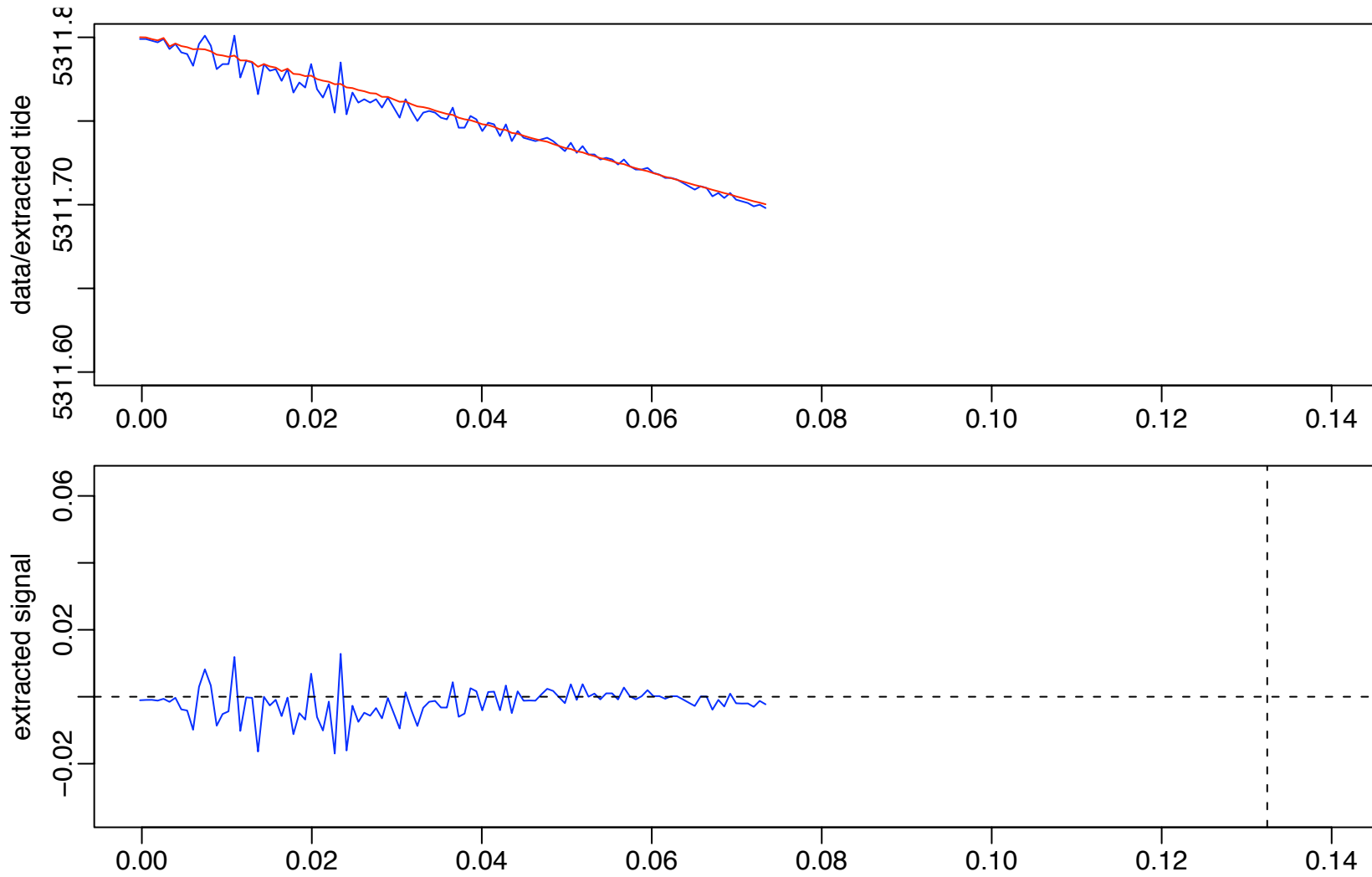
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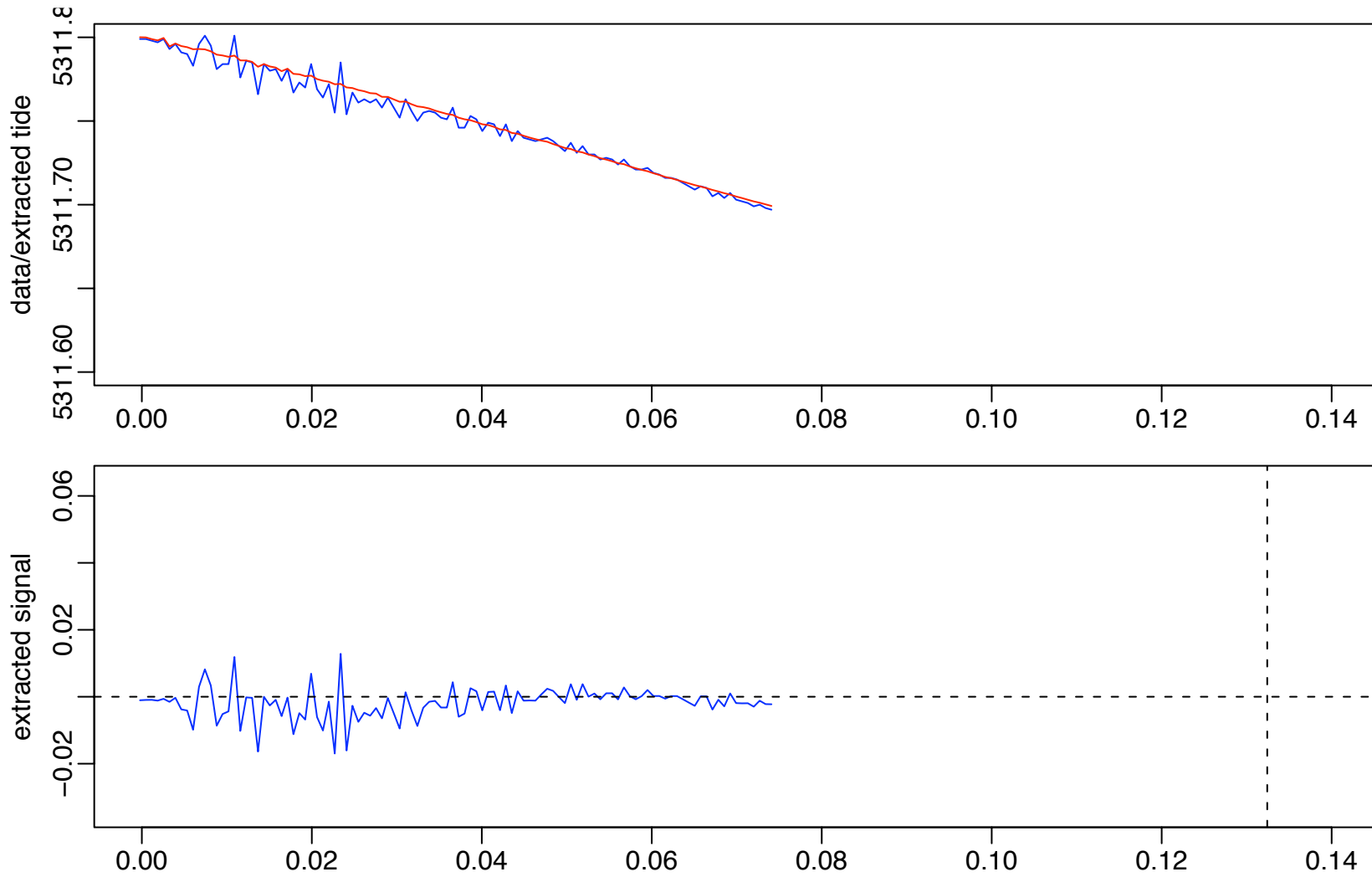
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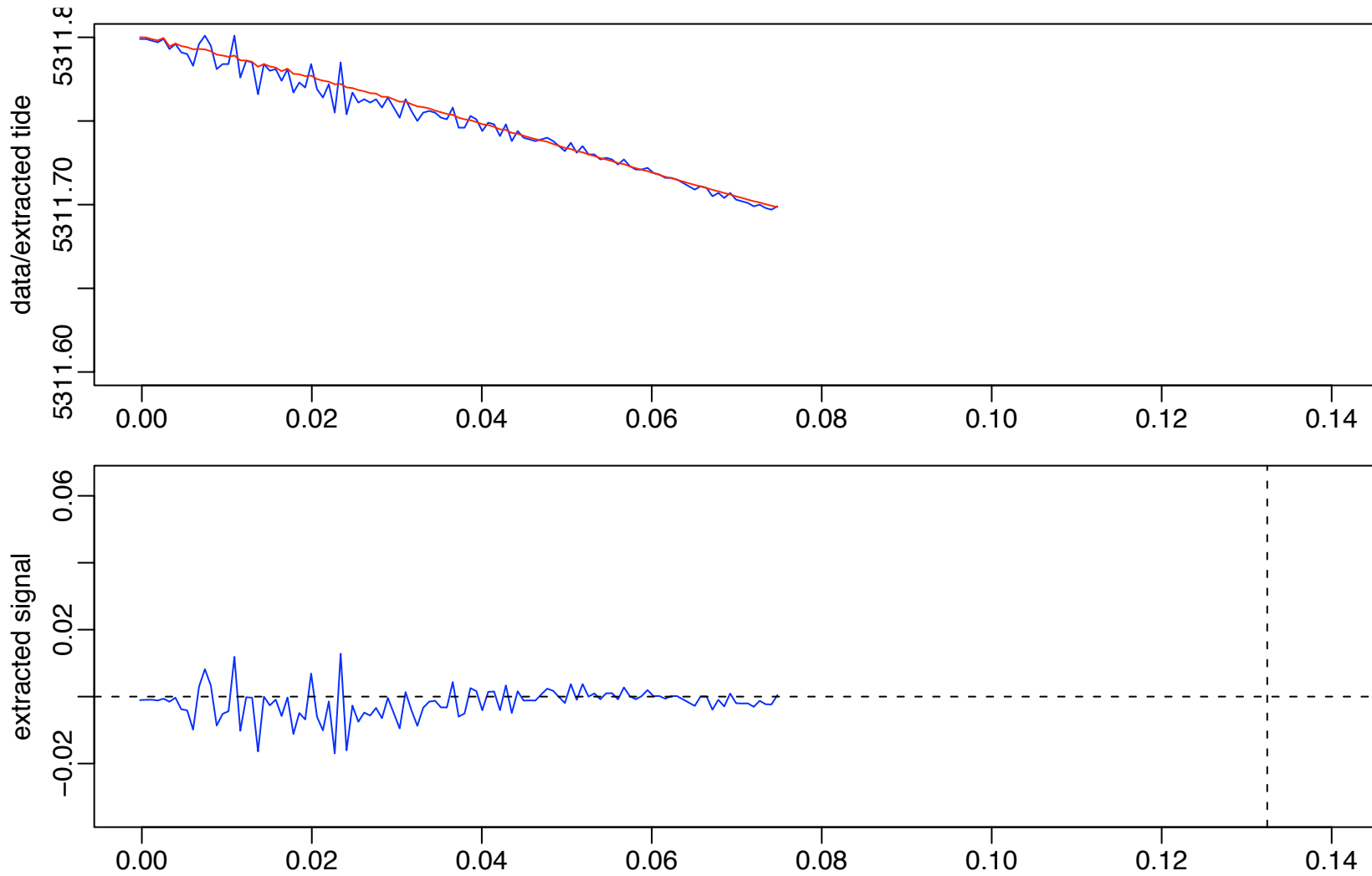
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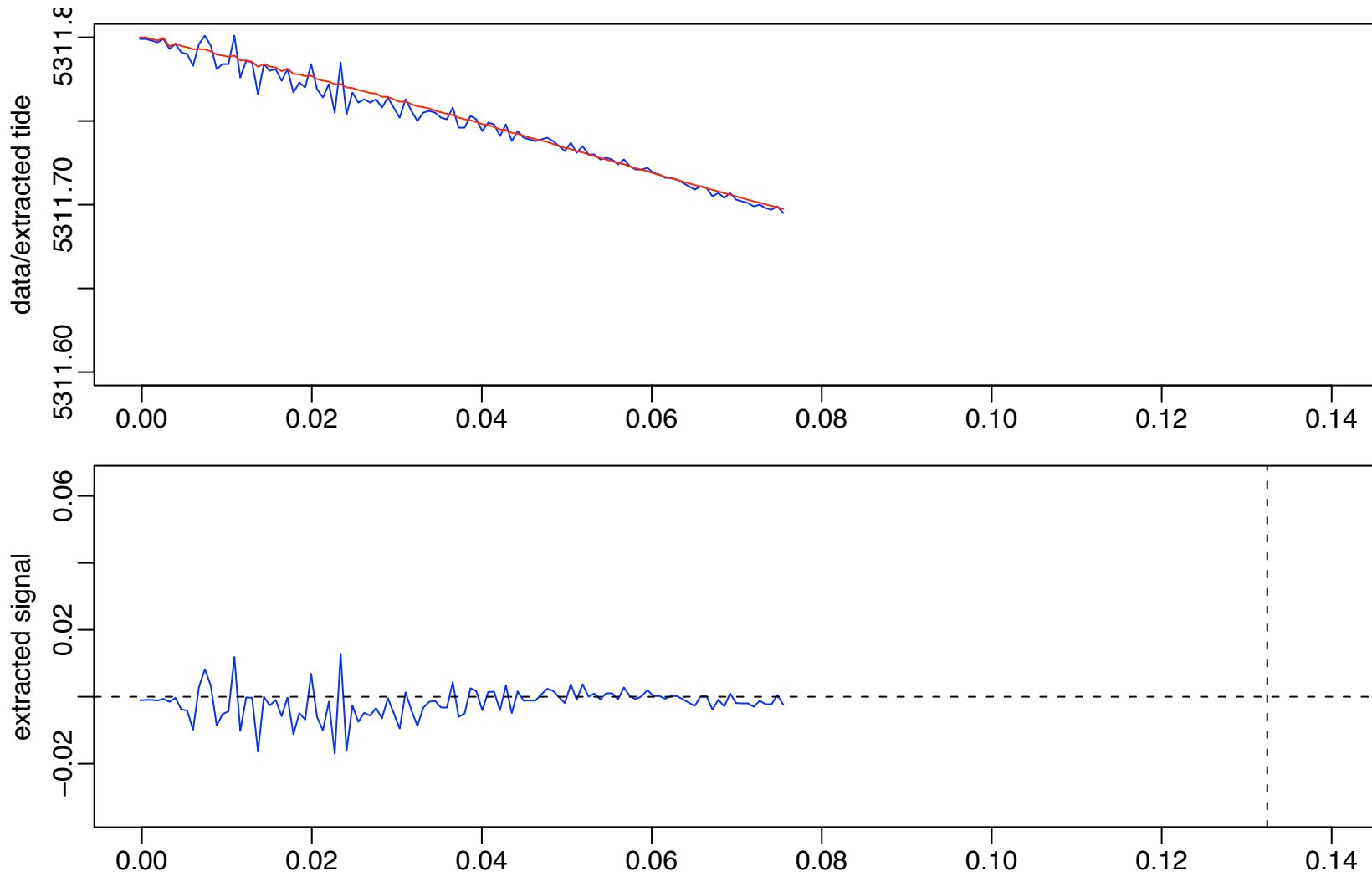
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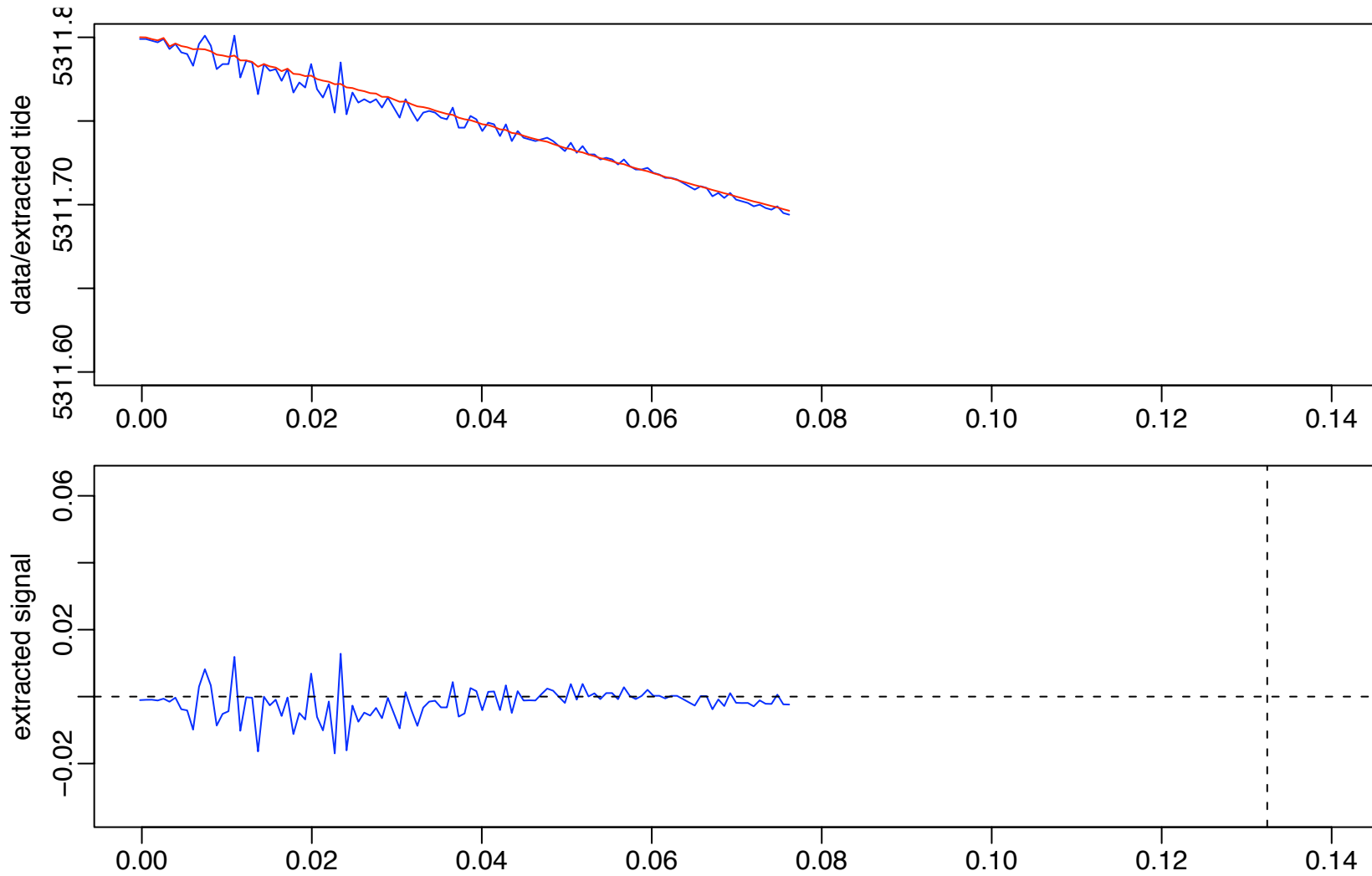
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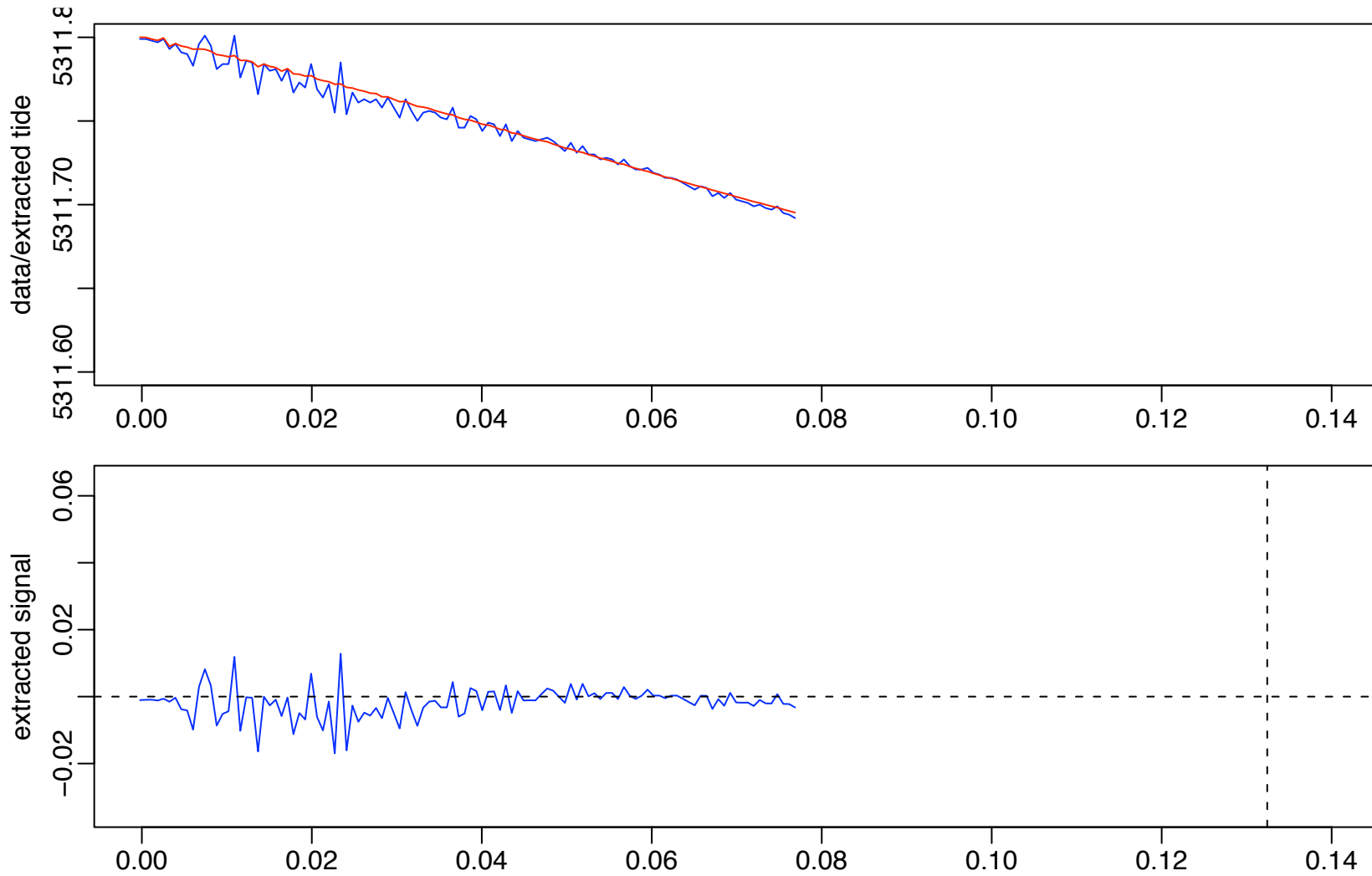


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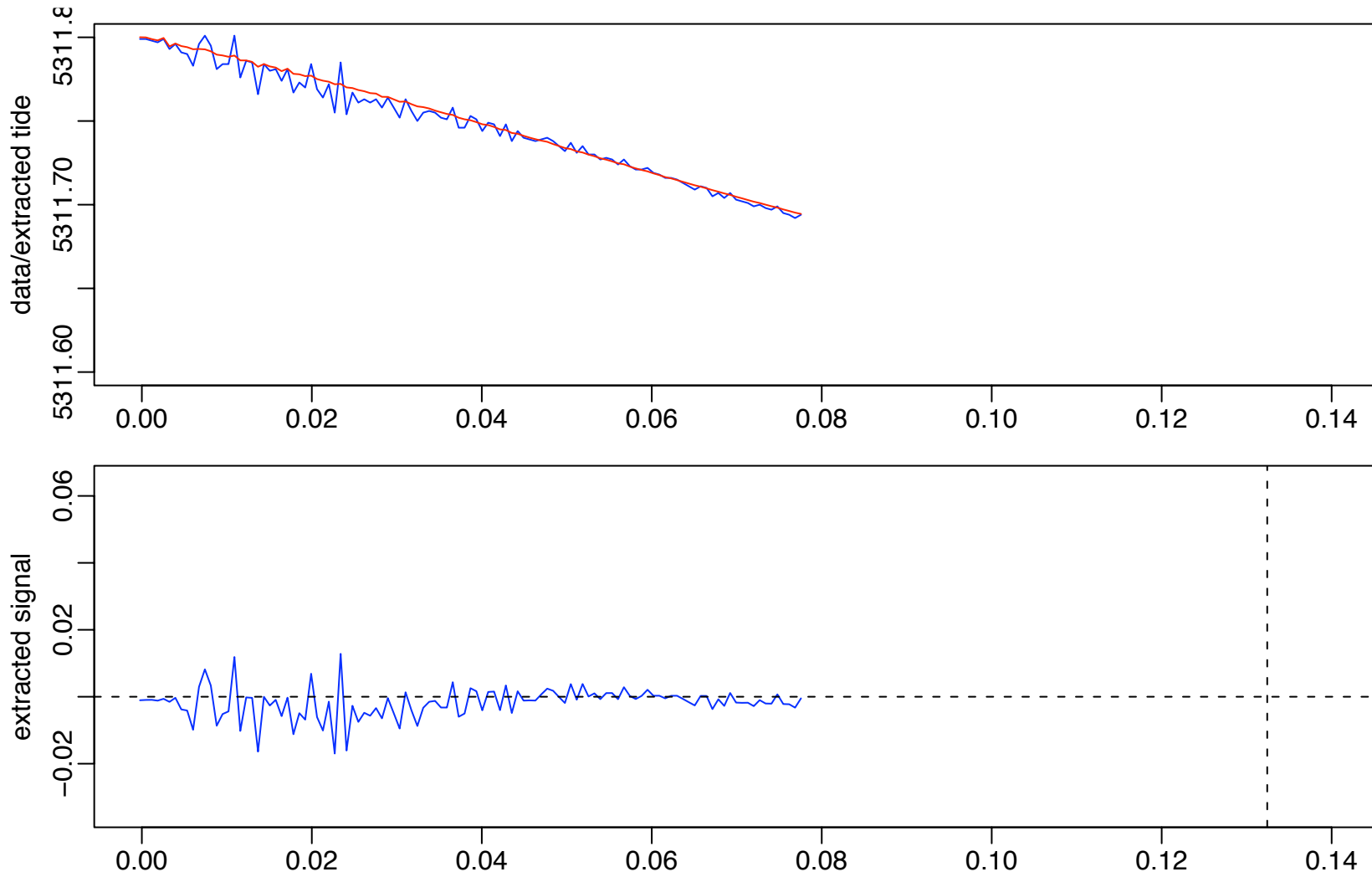




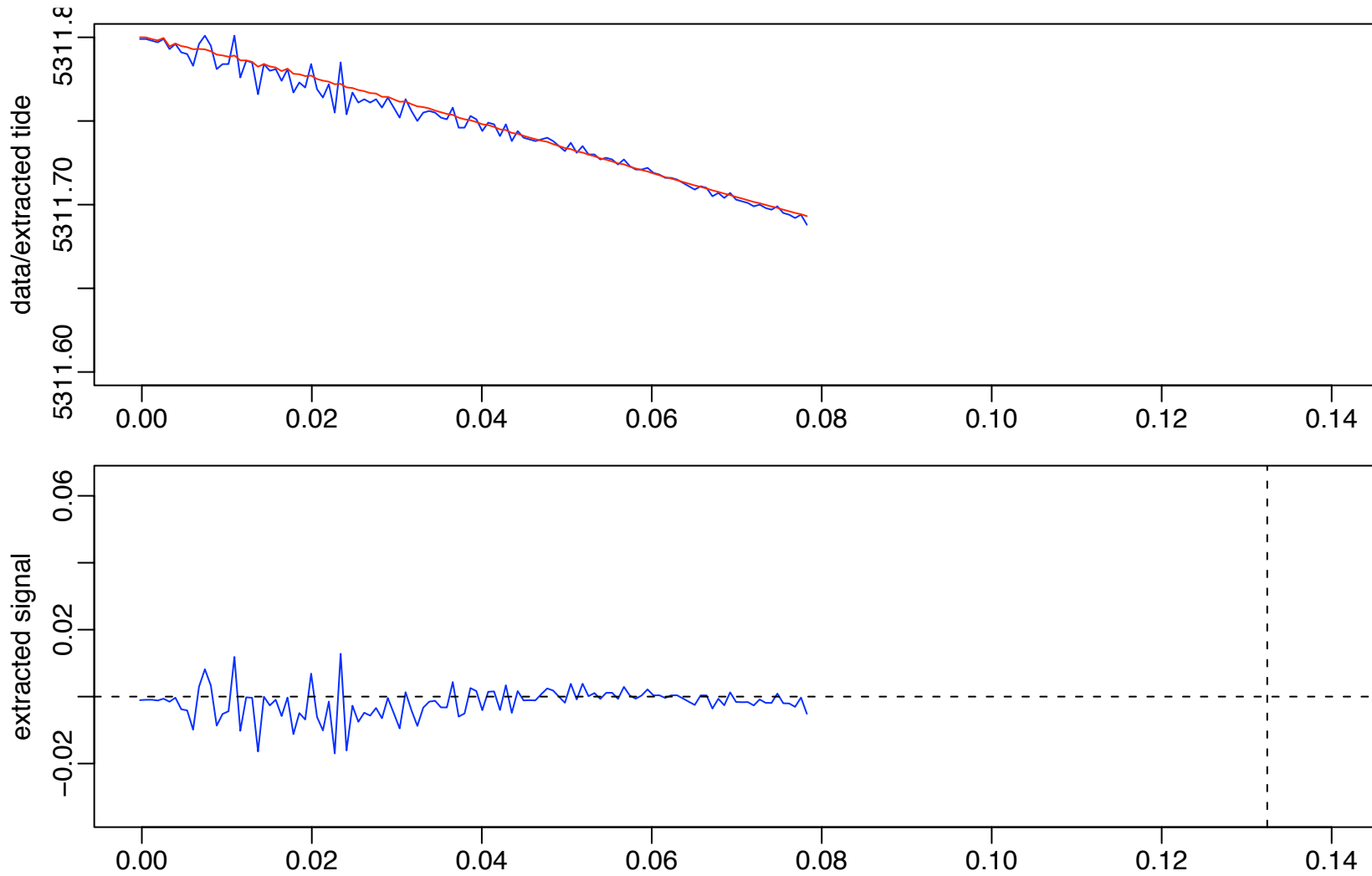
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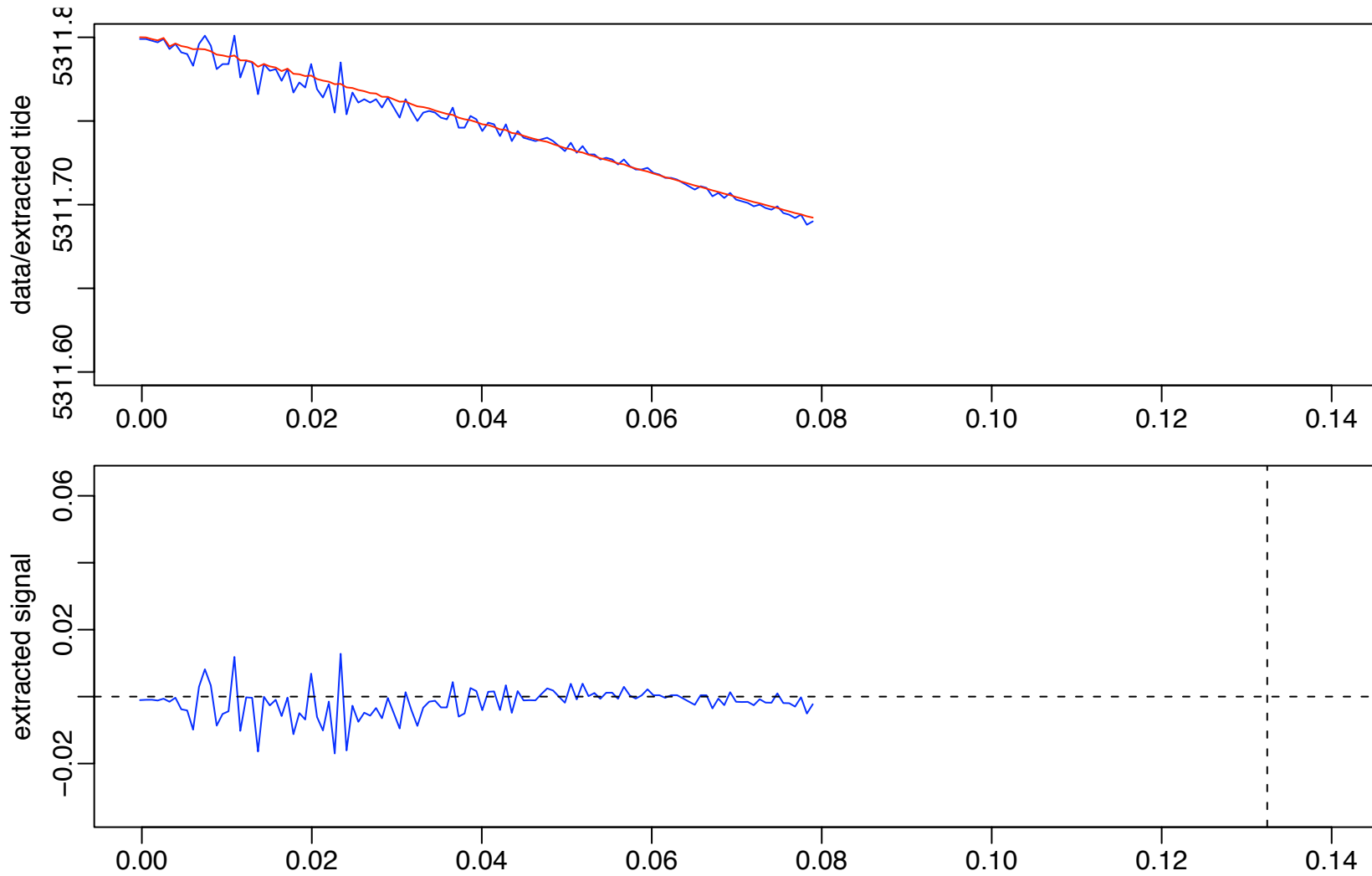
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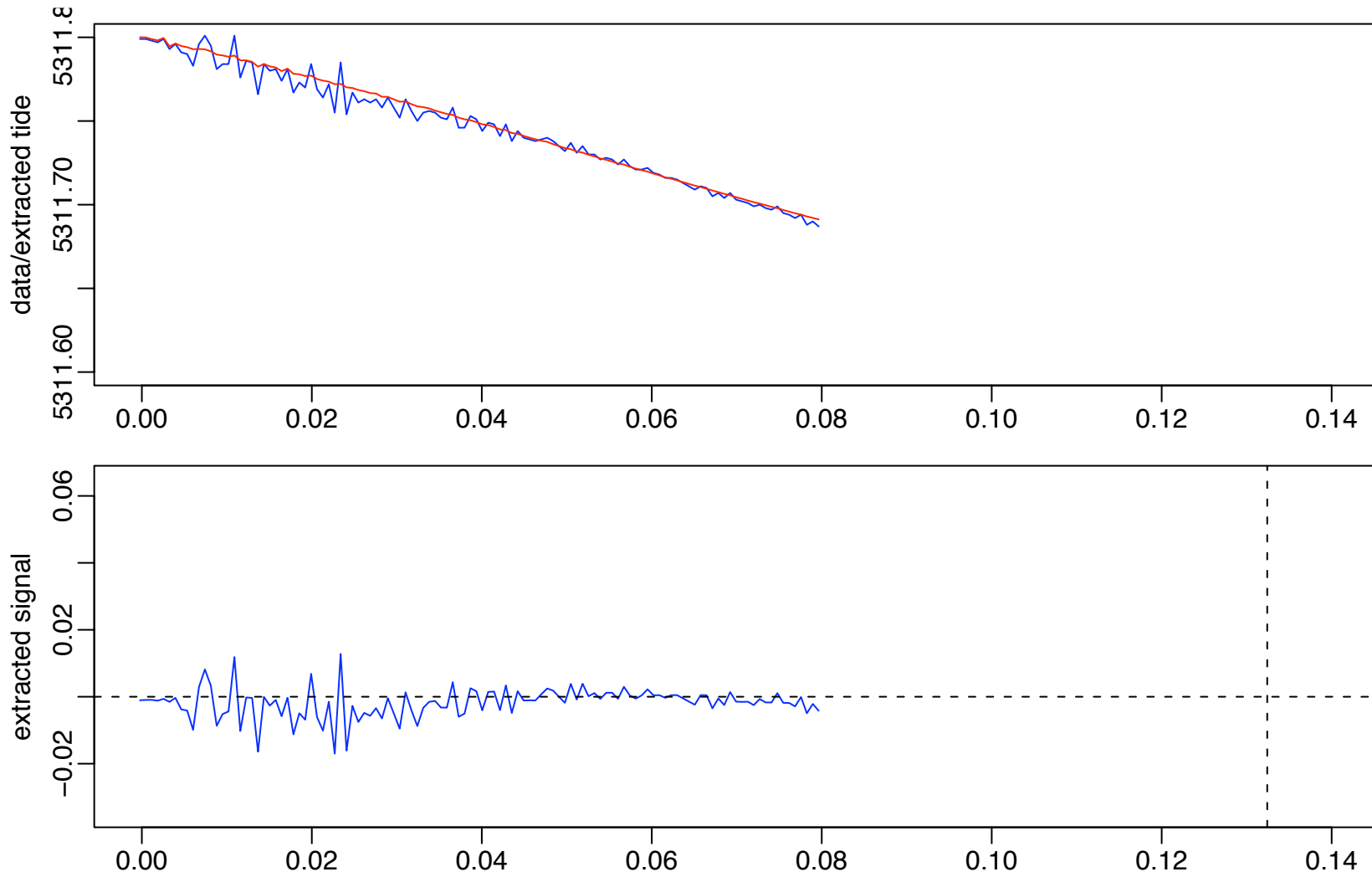
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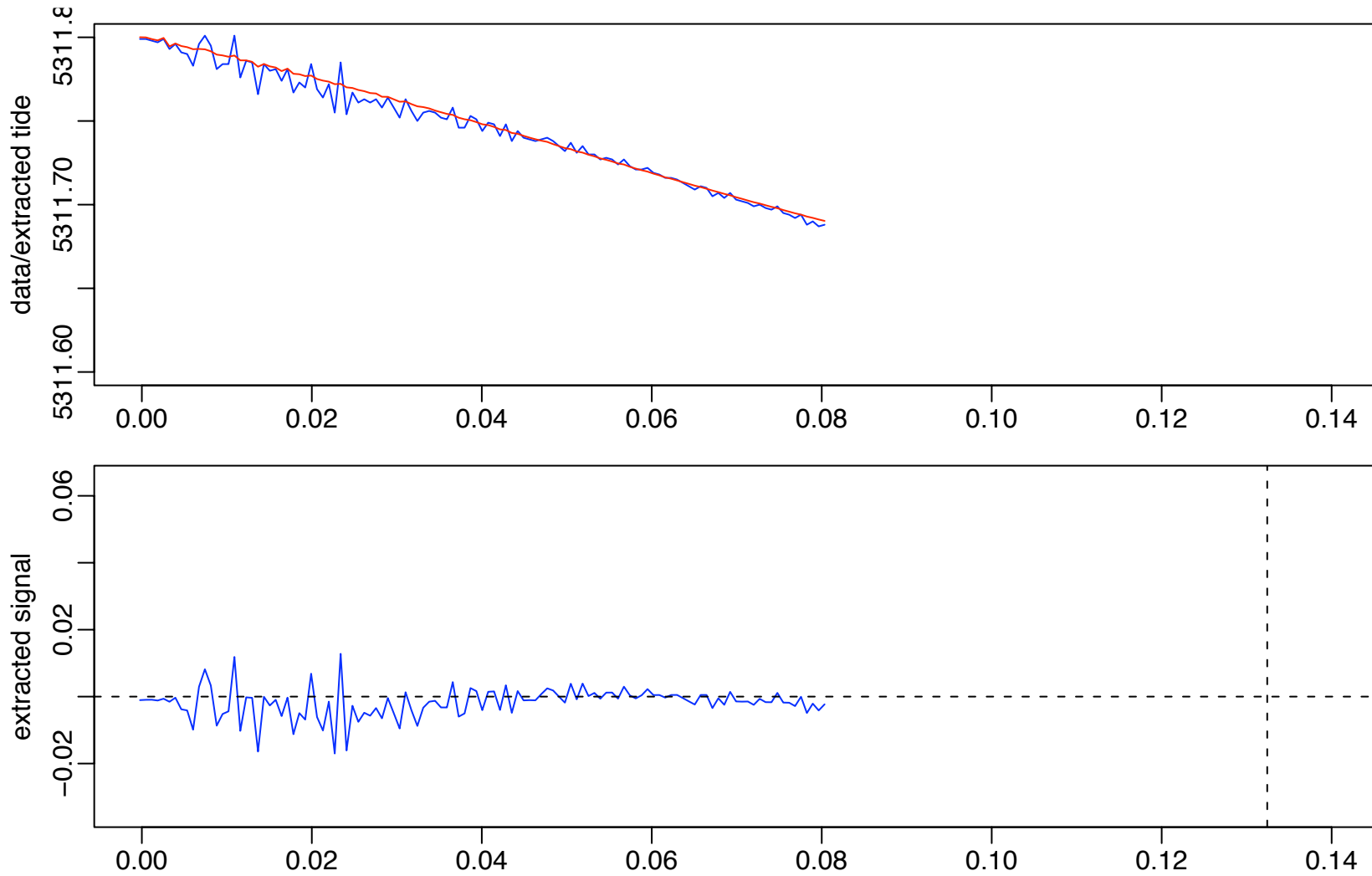
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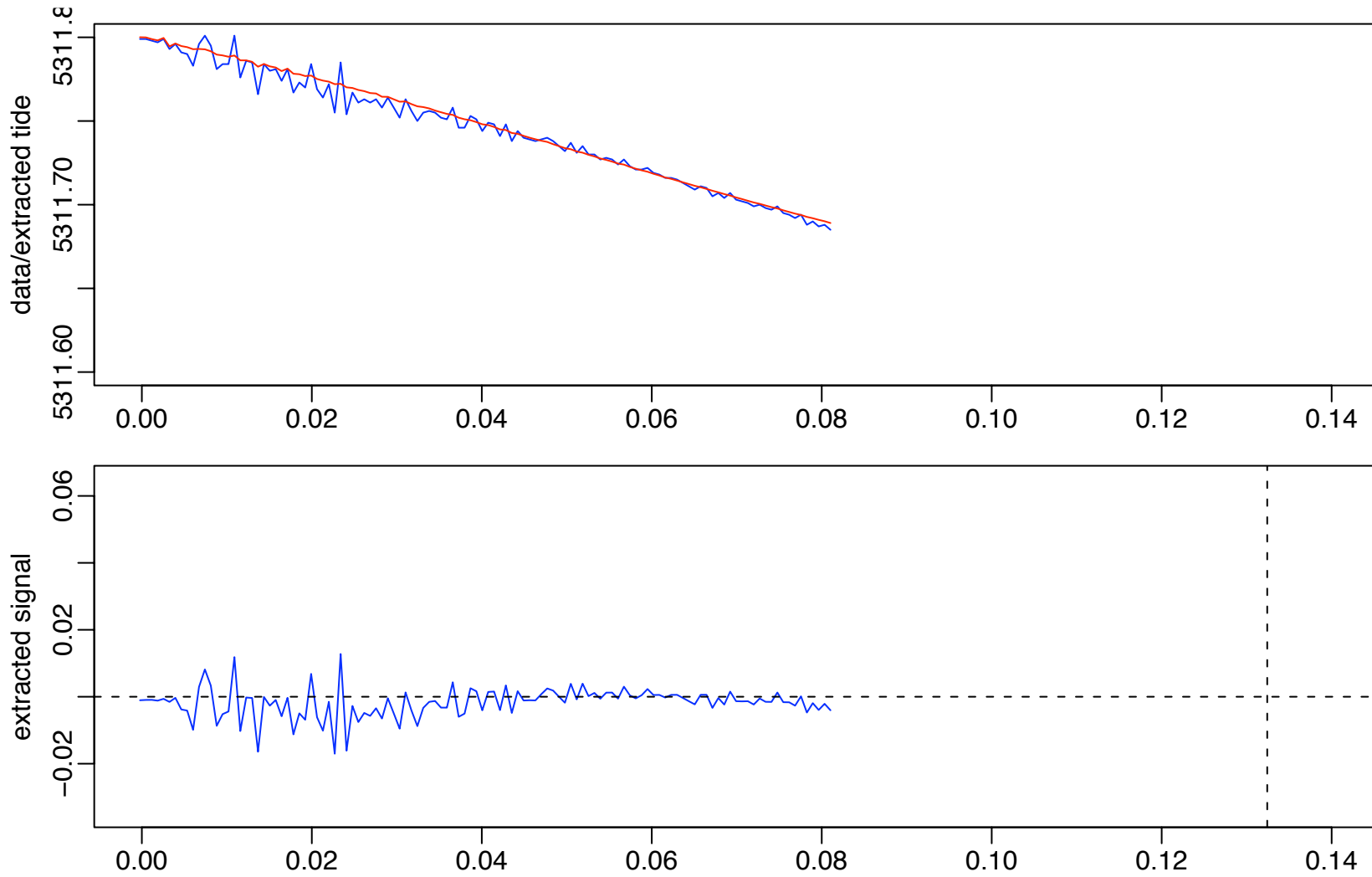
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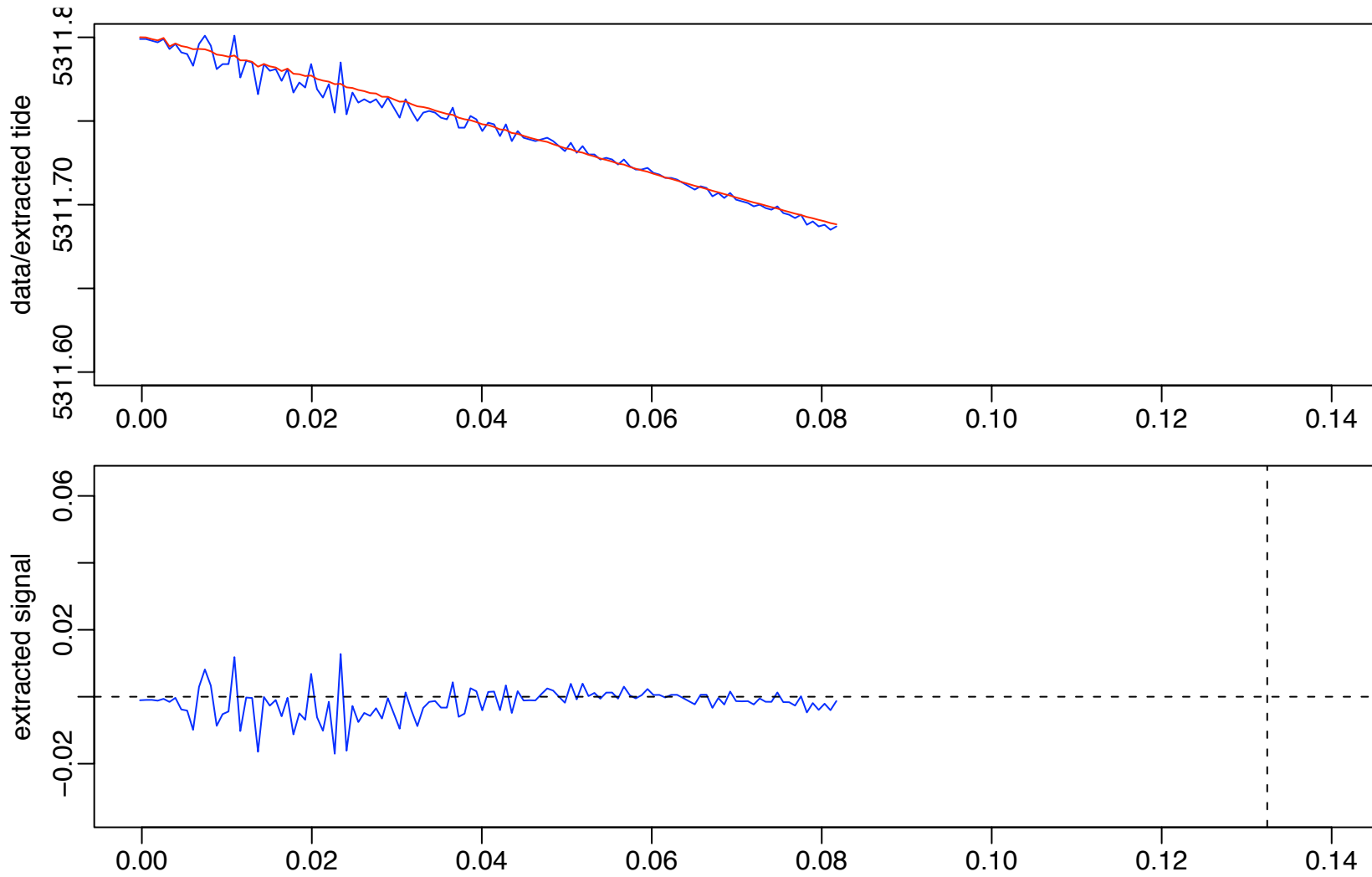
# Kalman Filtering/Smoothing – 14 Day Model



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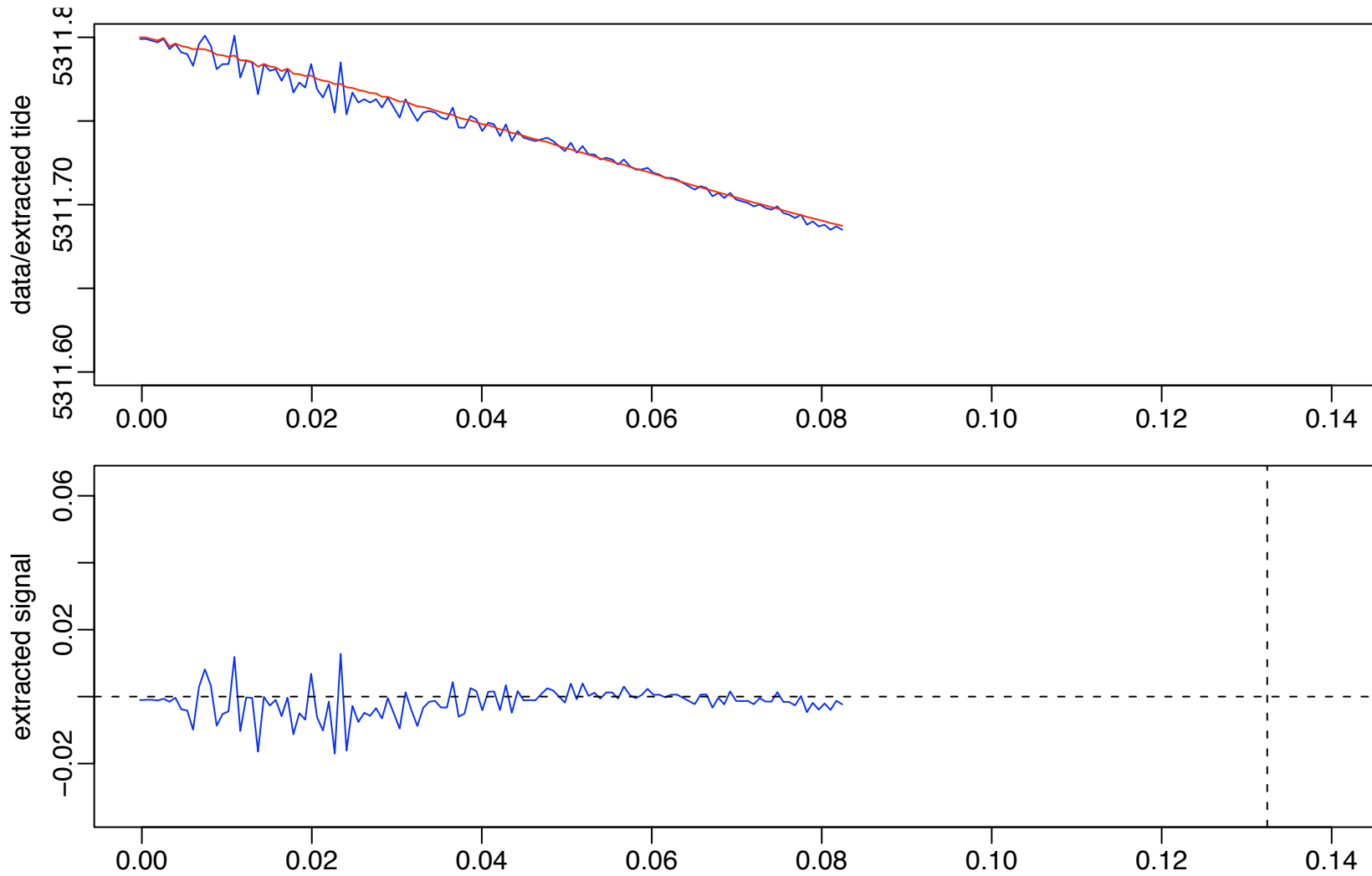


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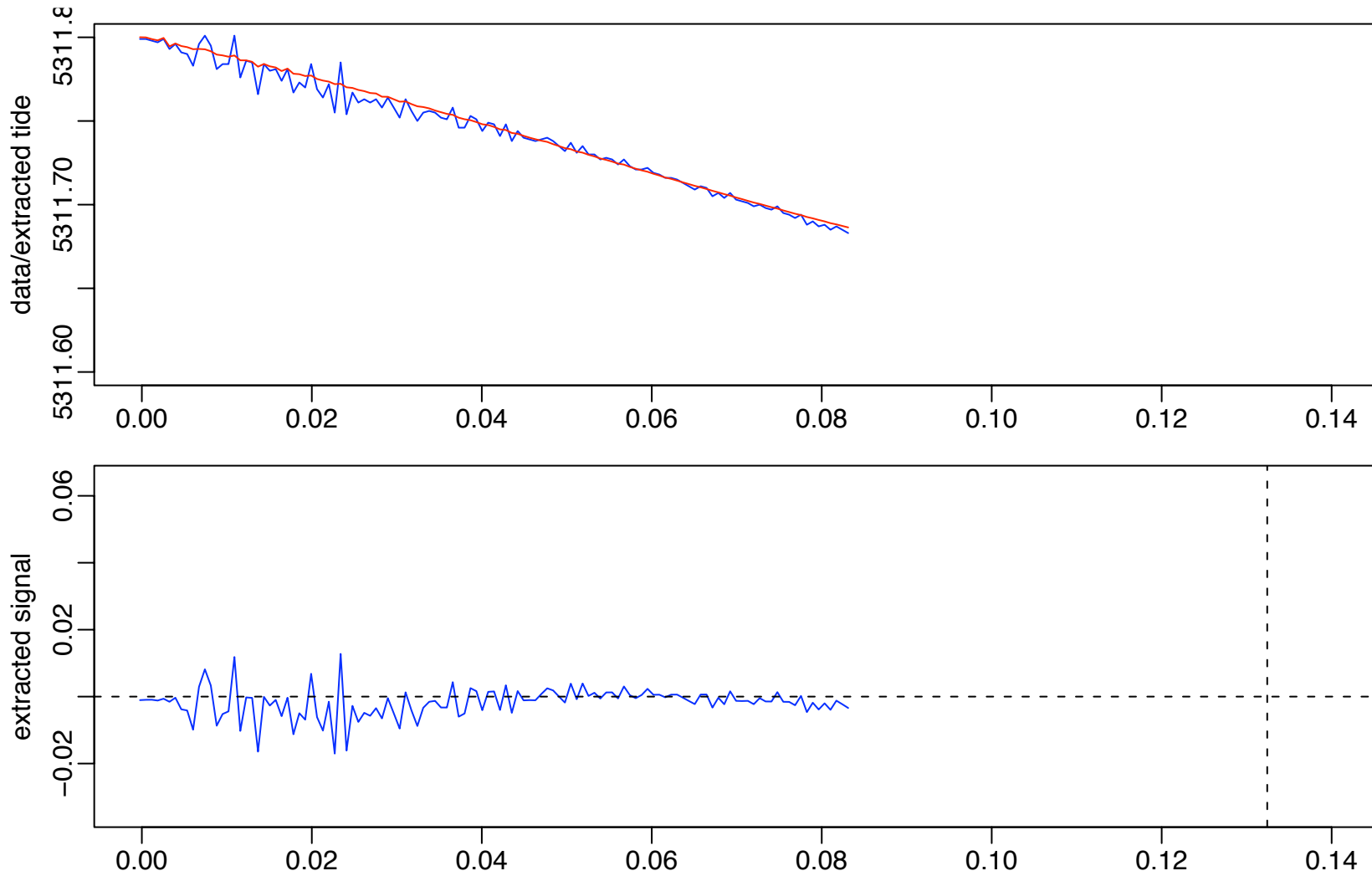




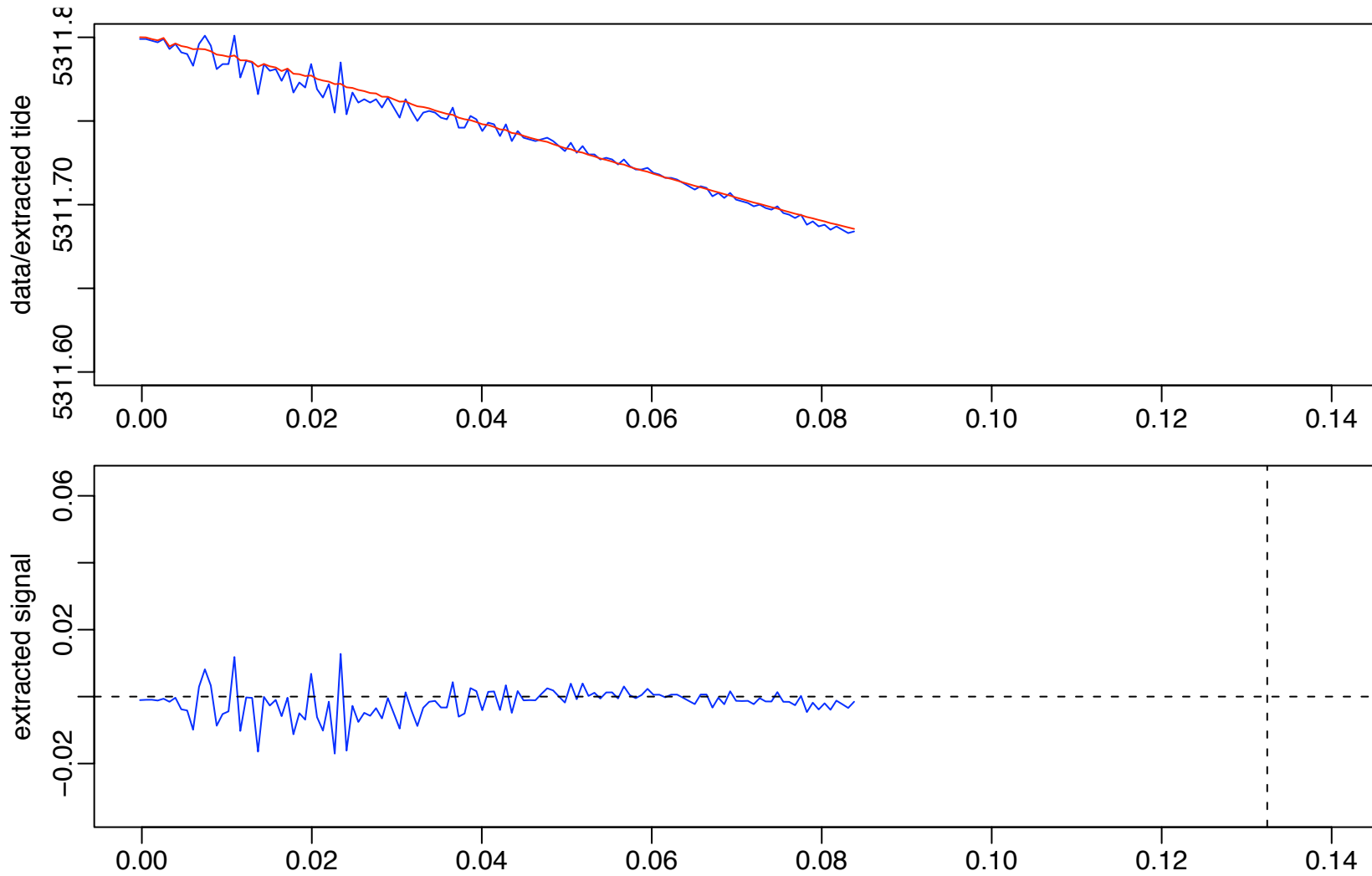
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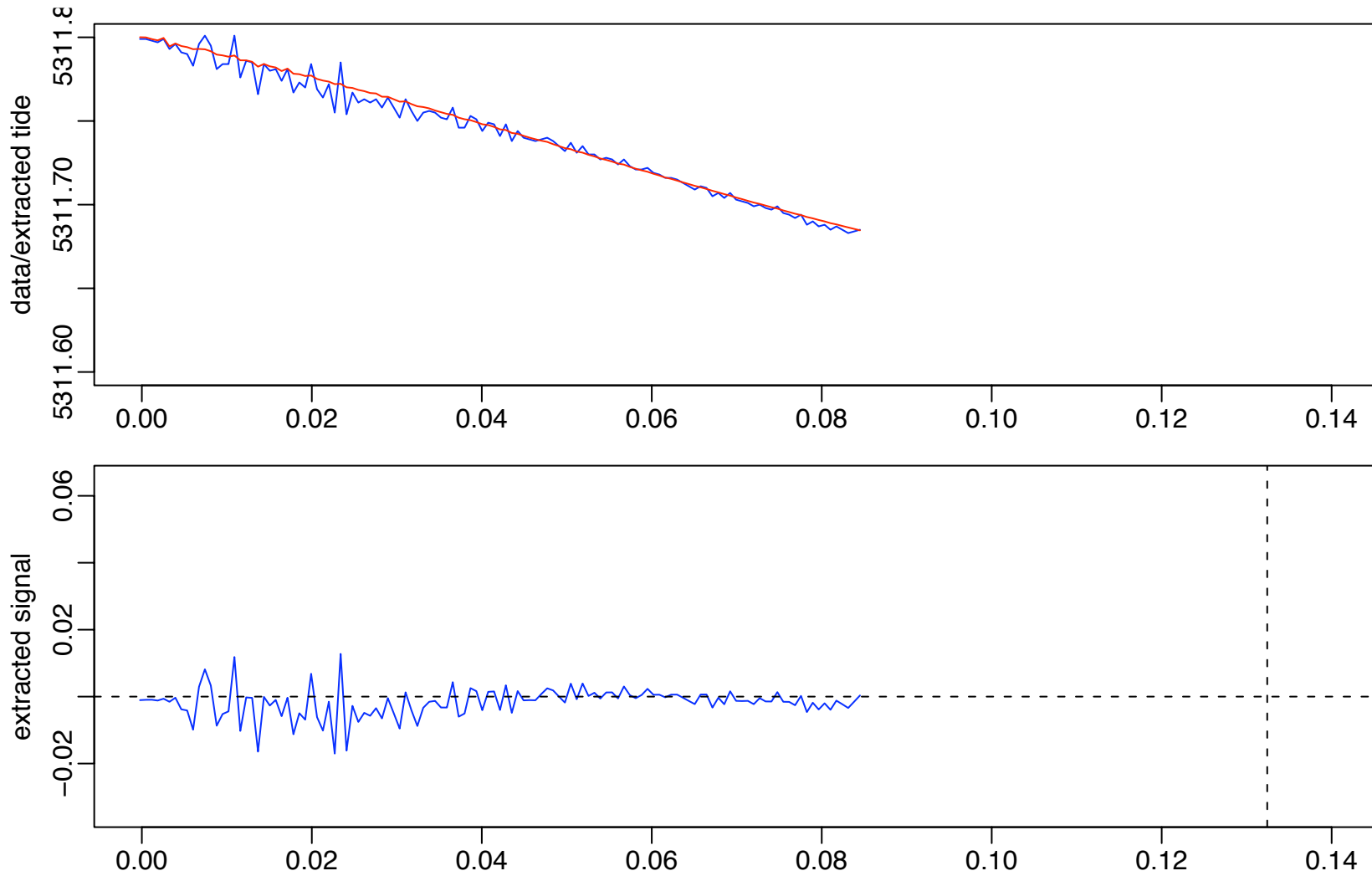
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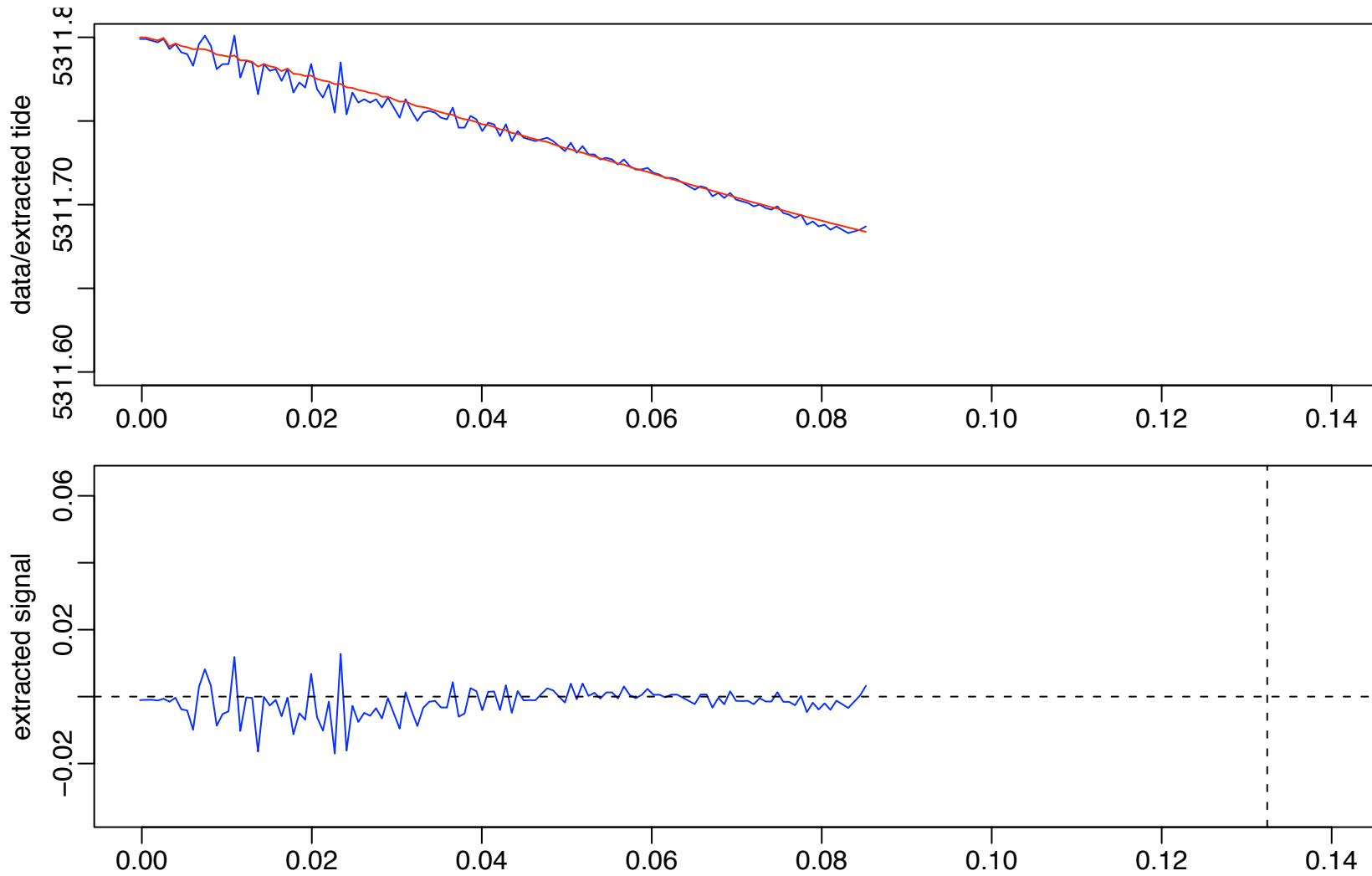
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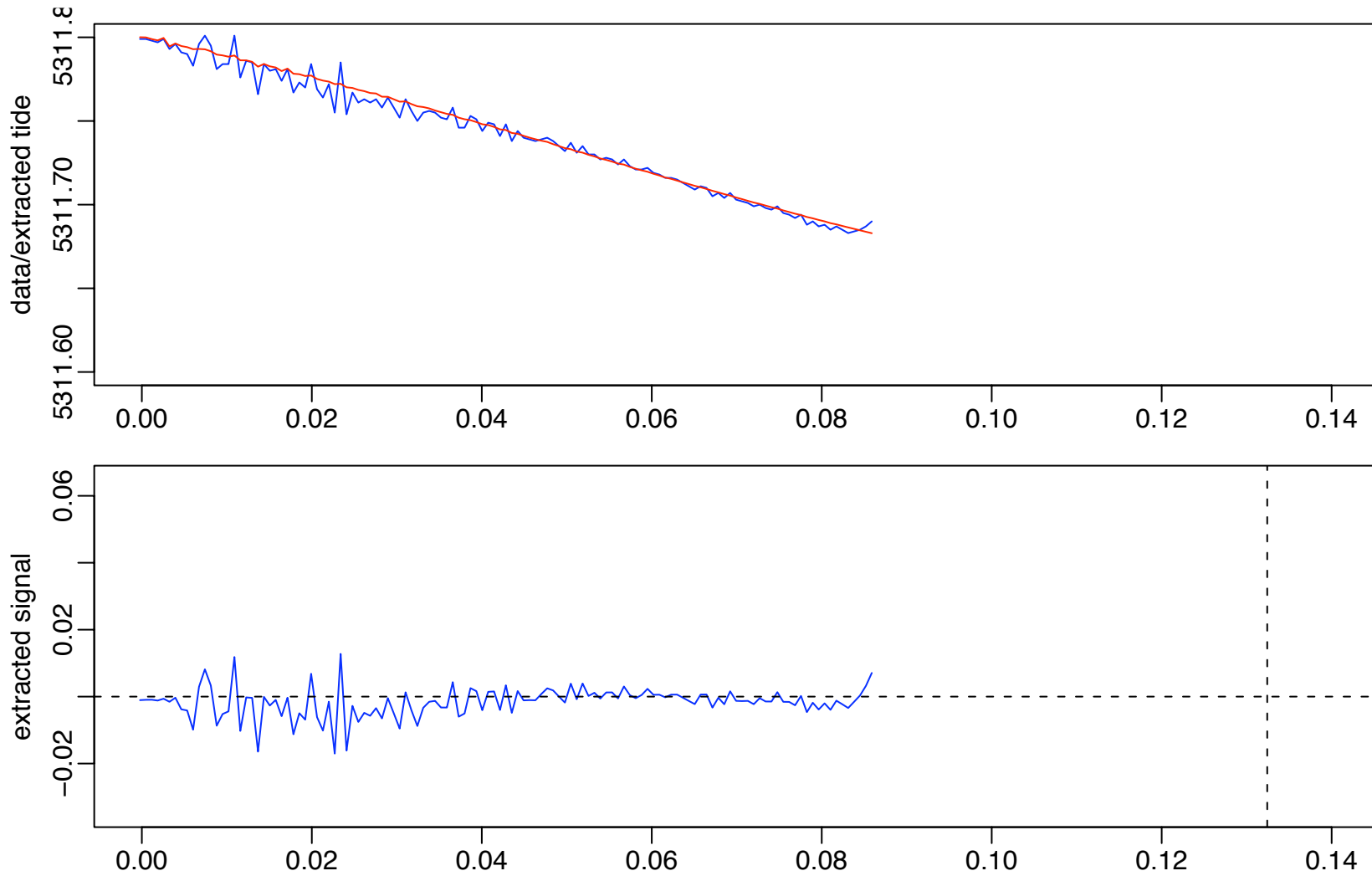
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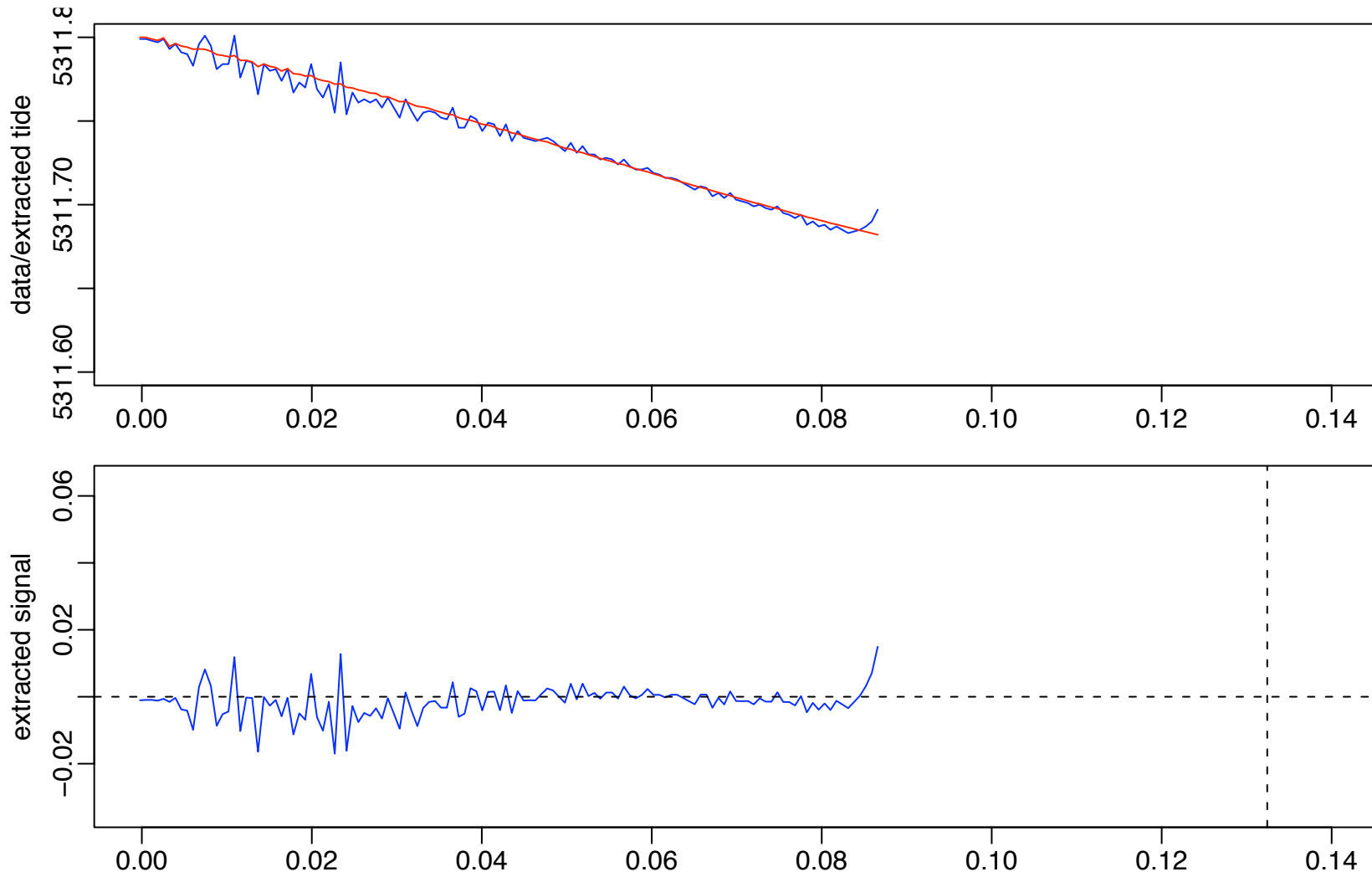
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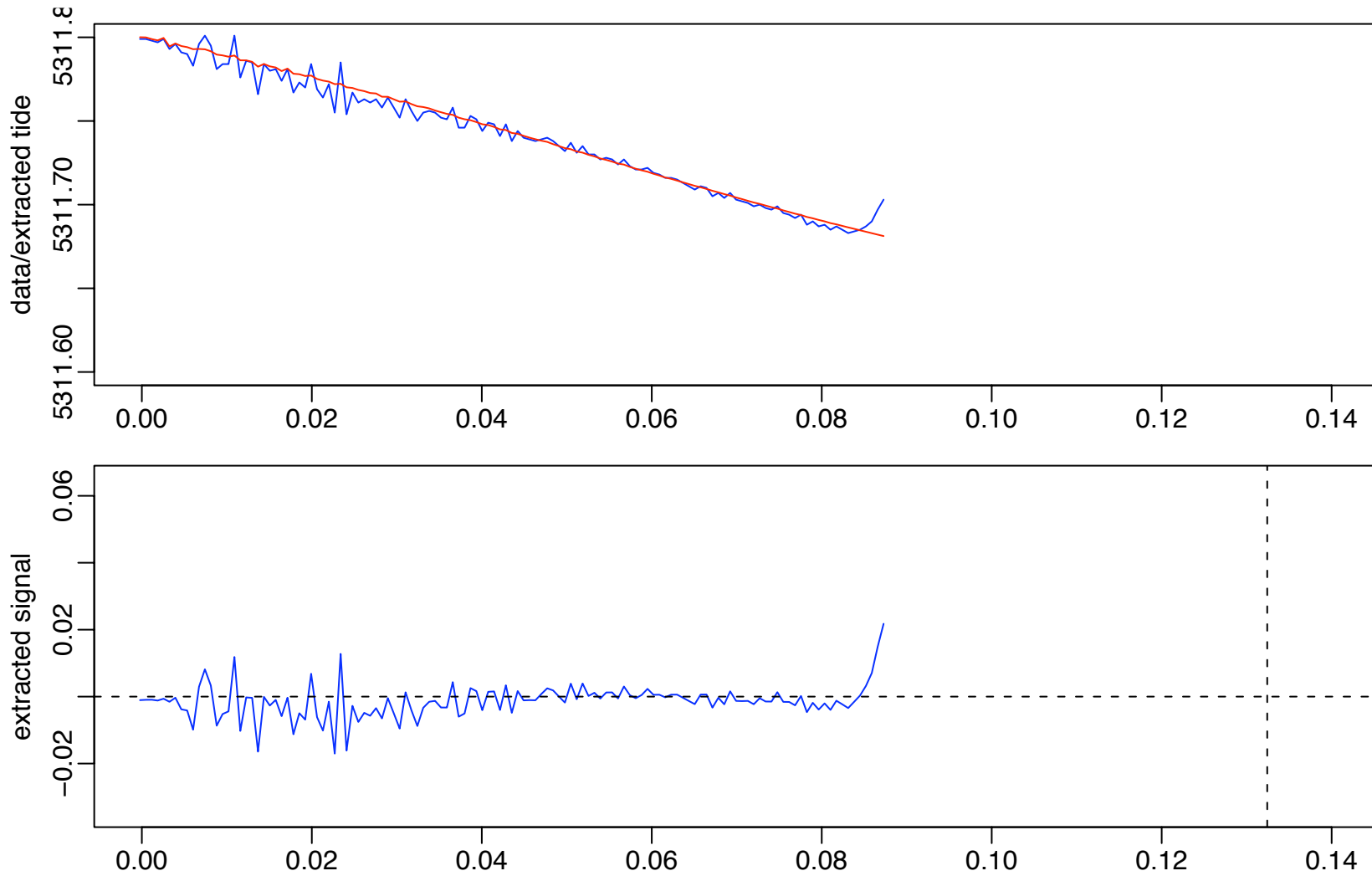
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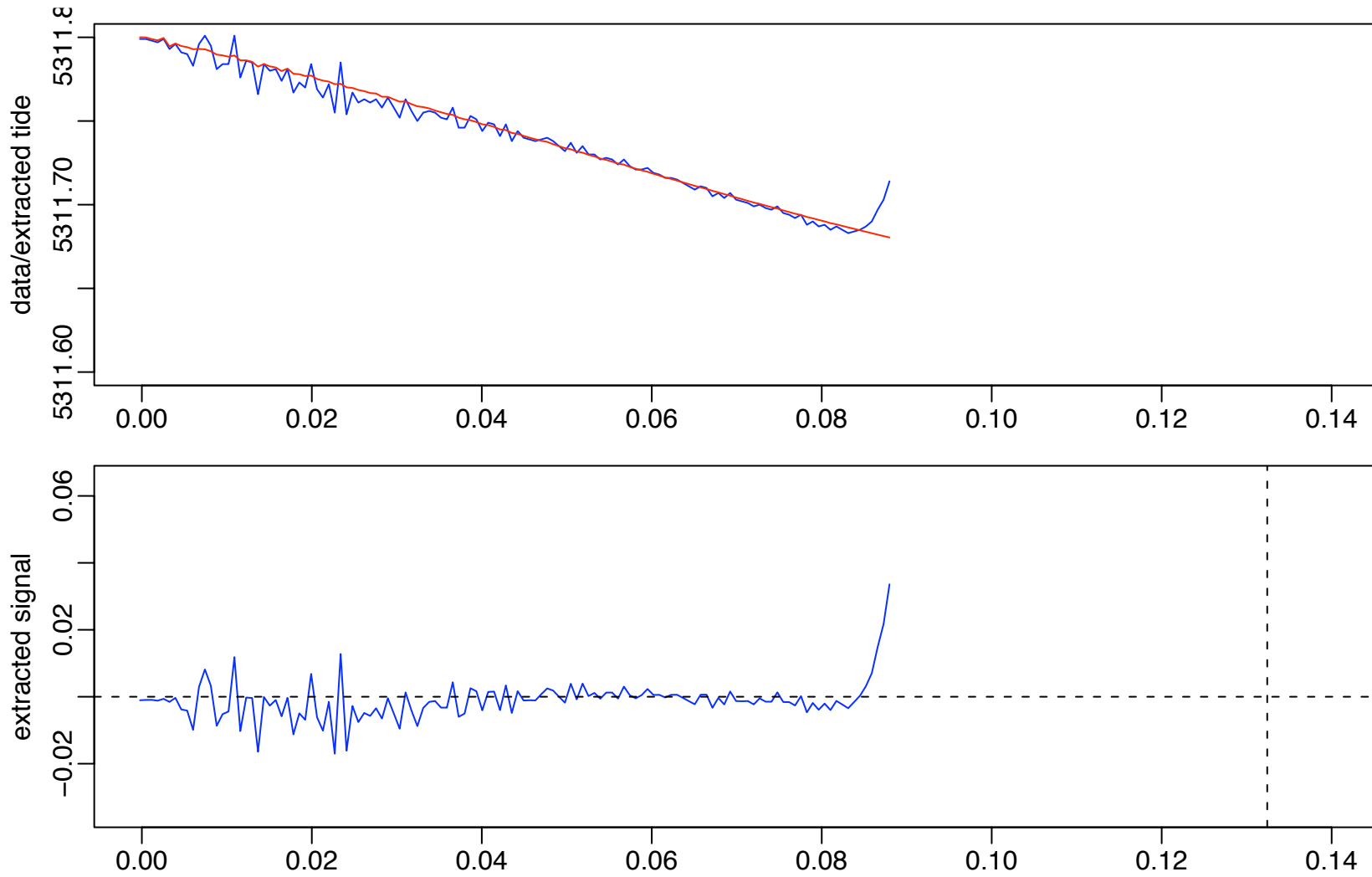


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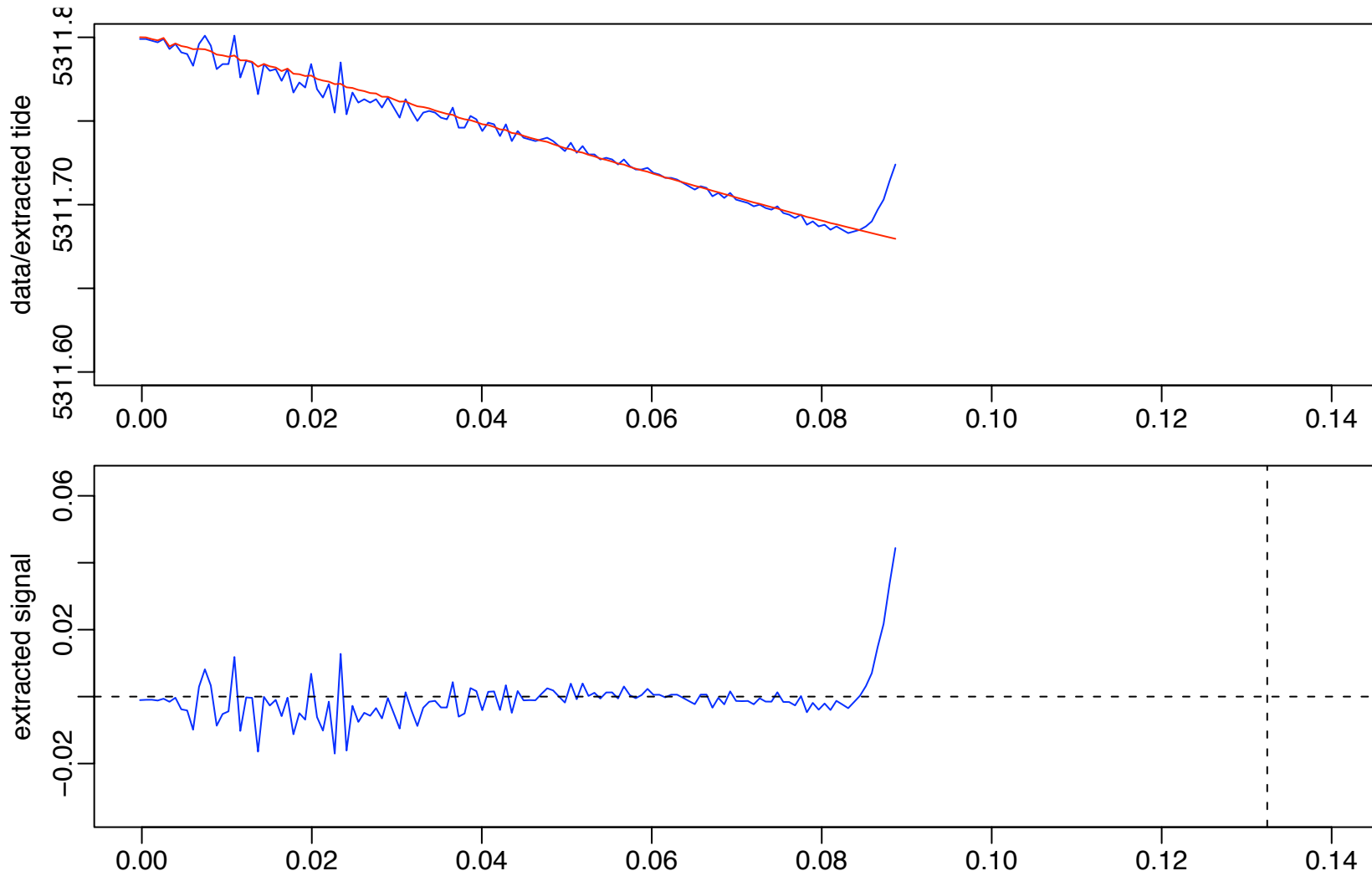




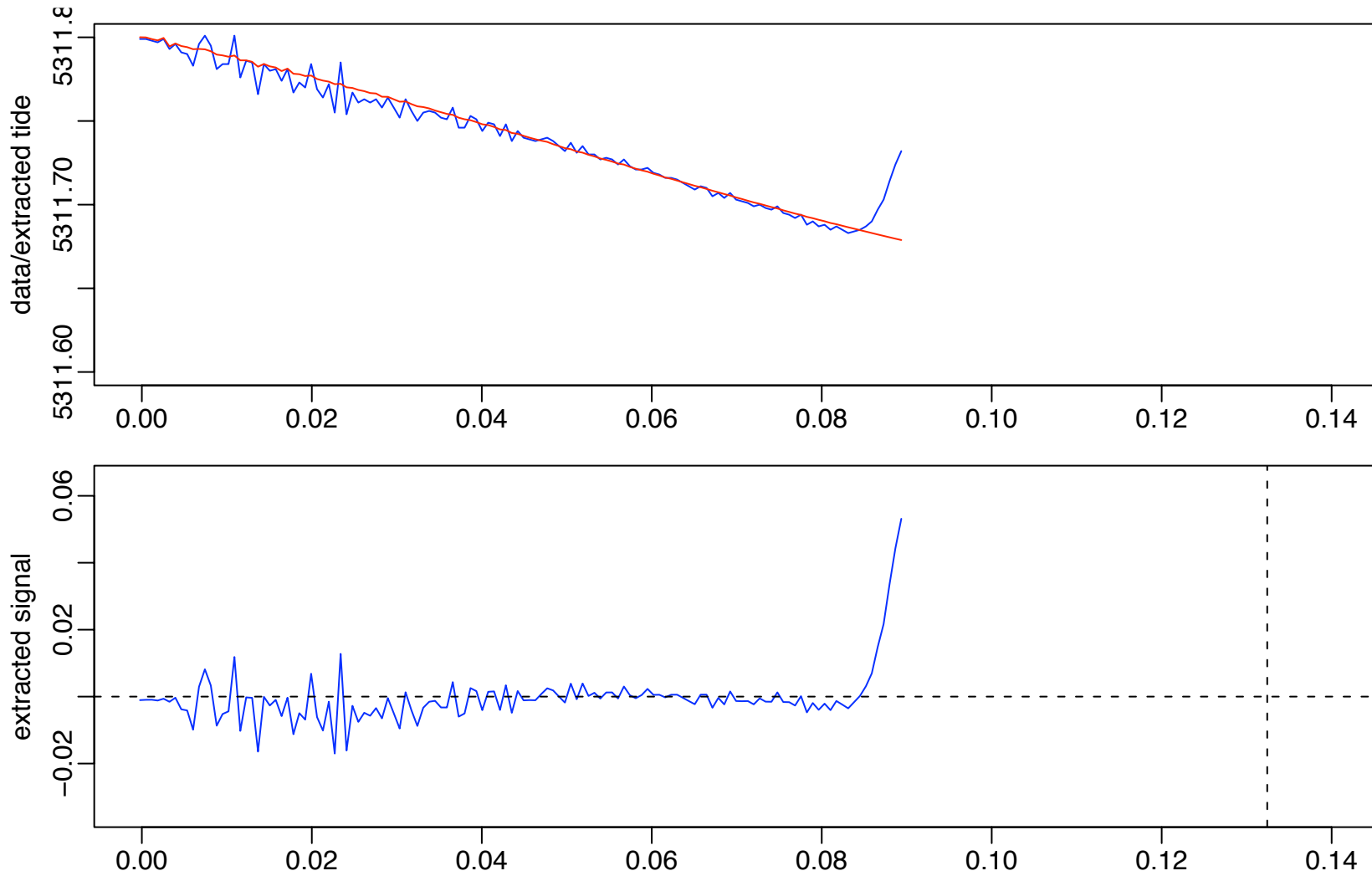
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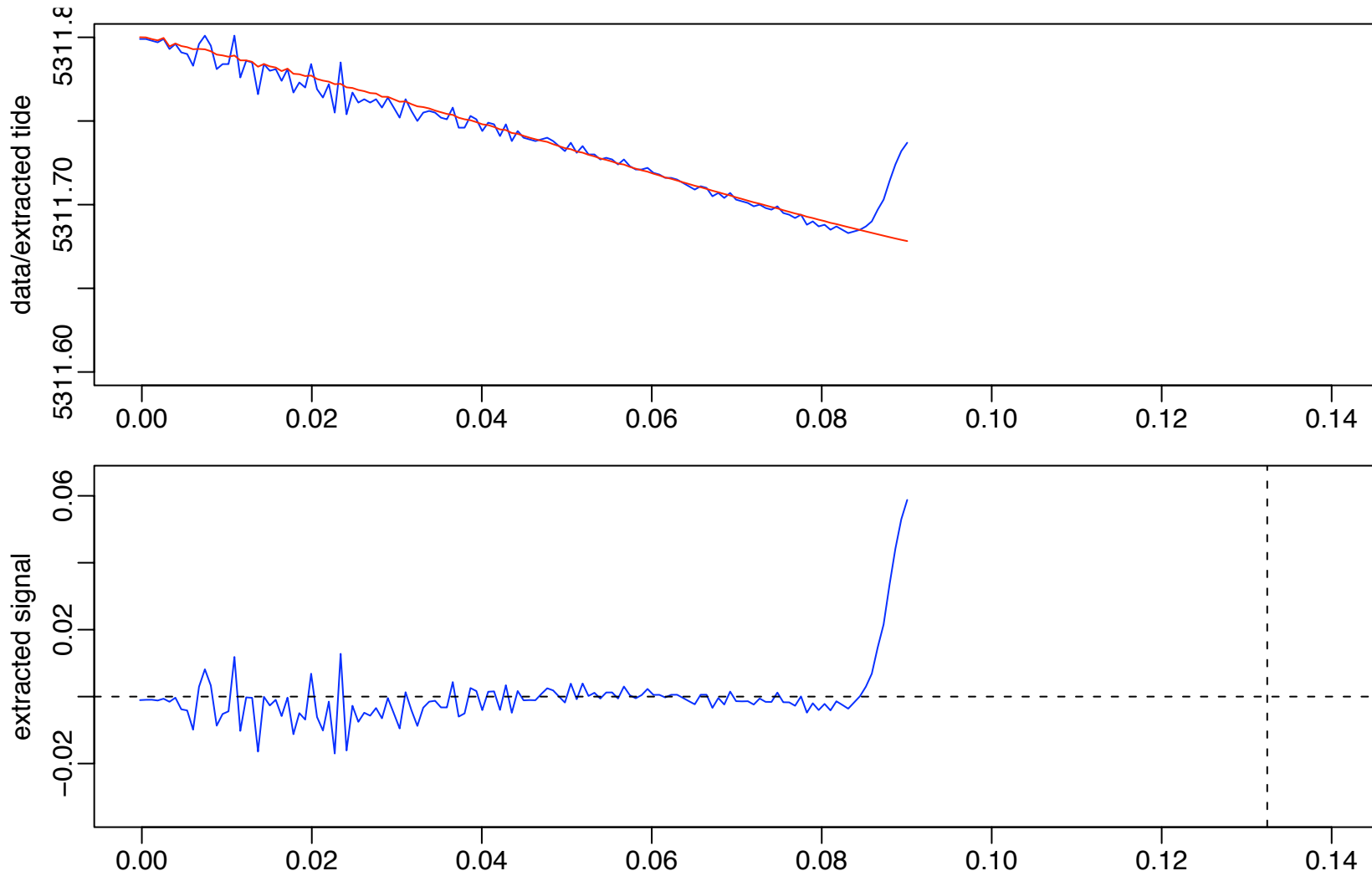
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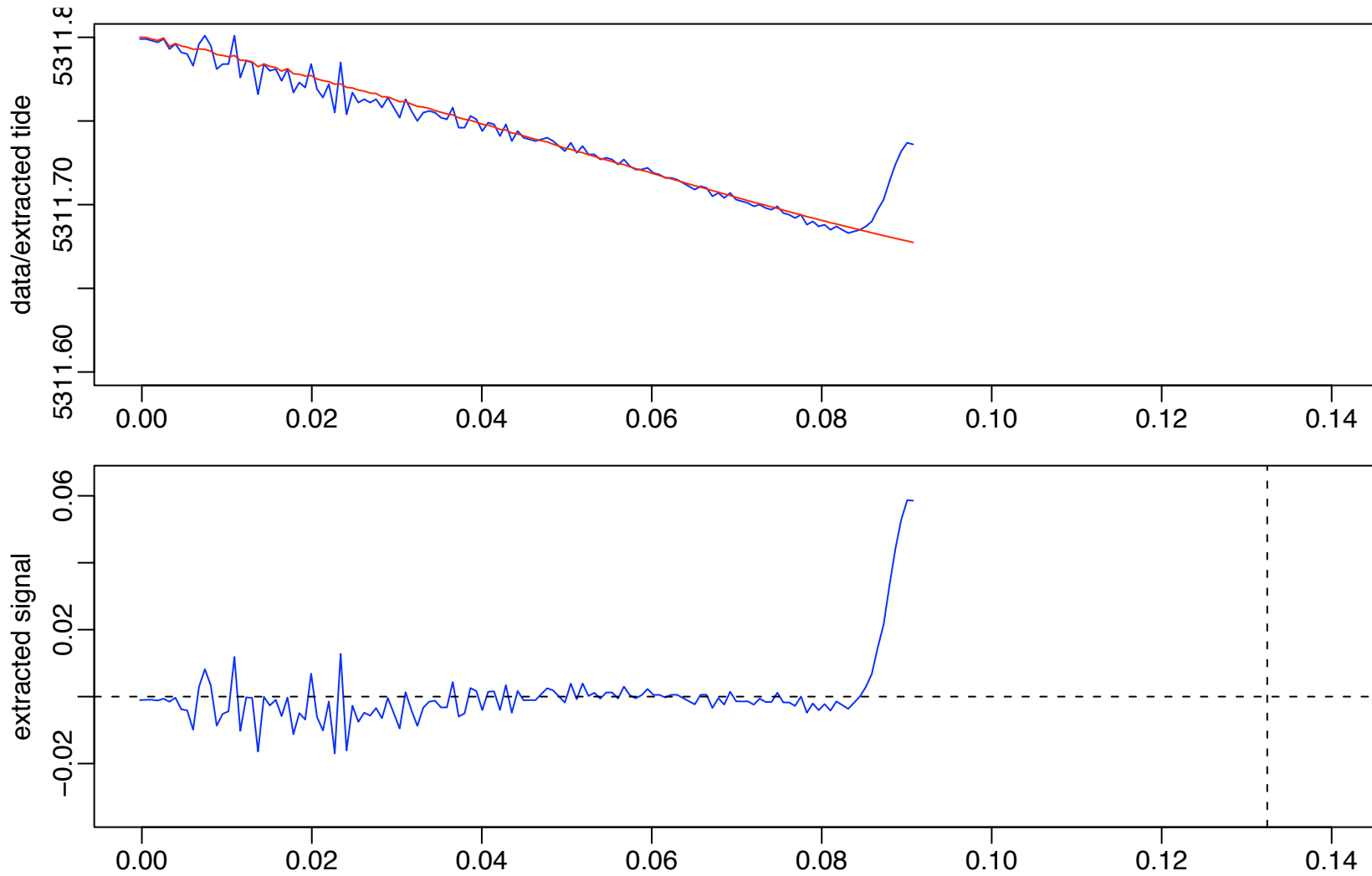
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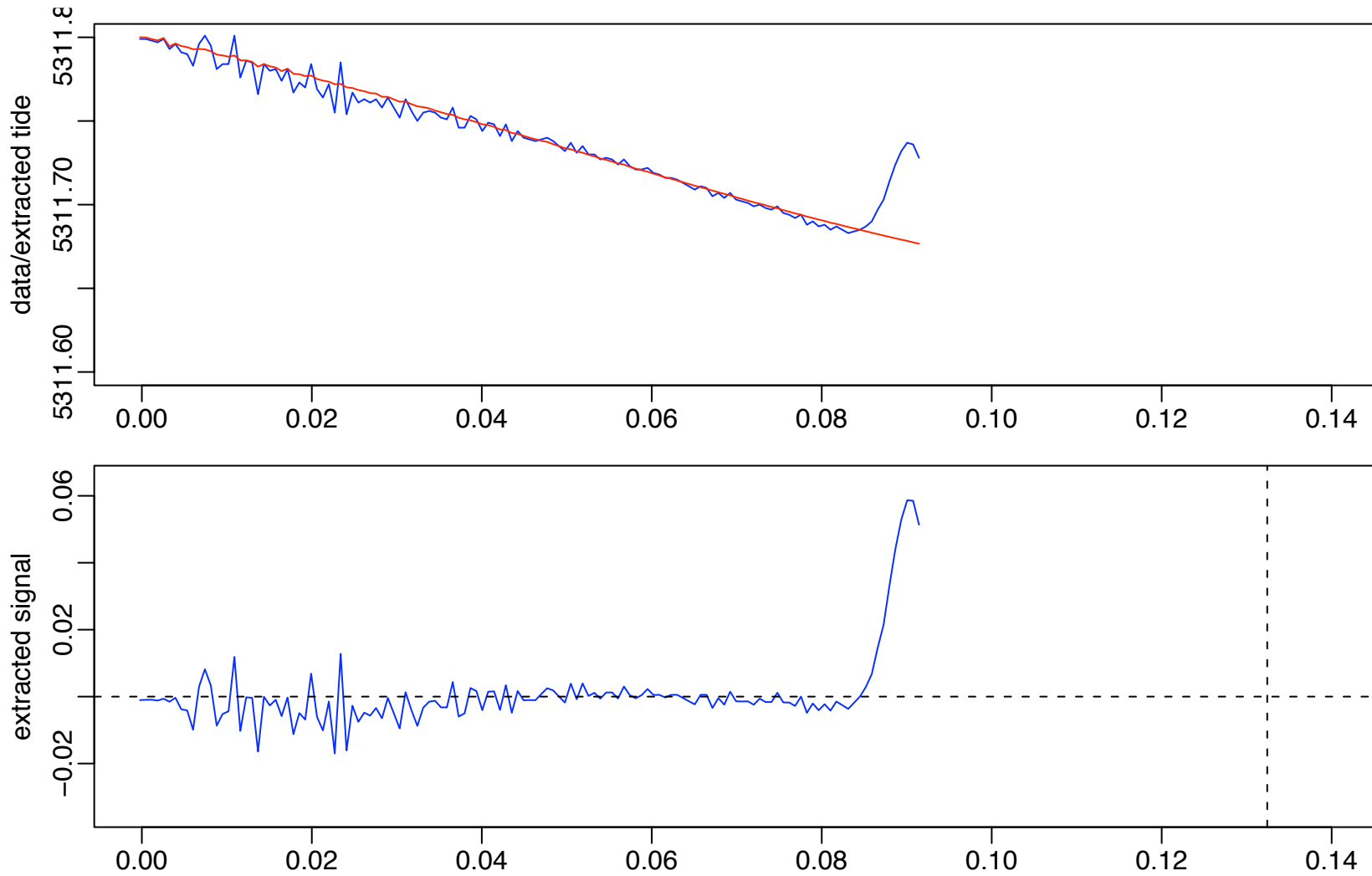
# Kalman Filtering/Smoothing – 14 Day Model



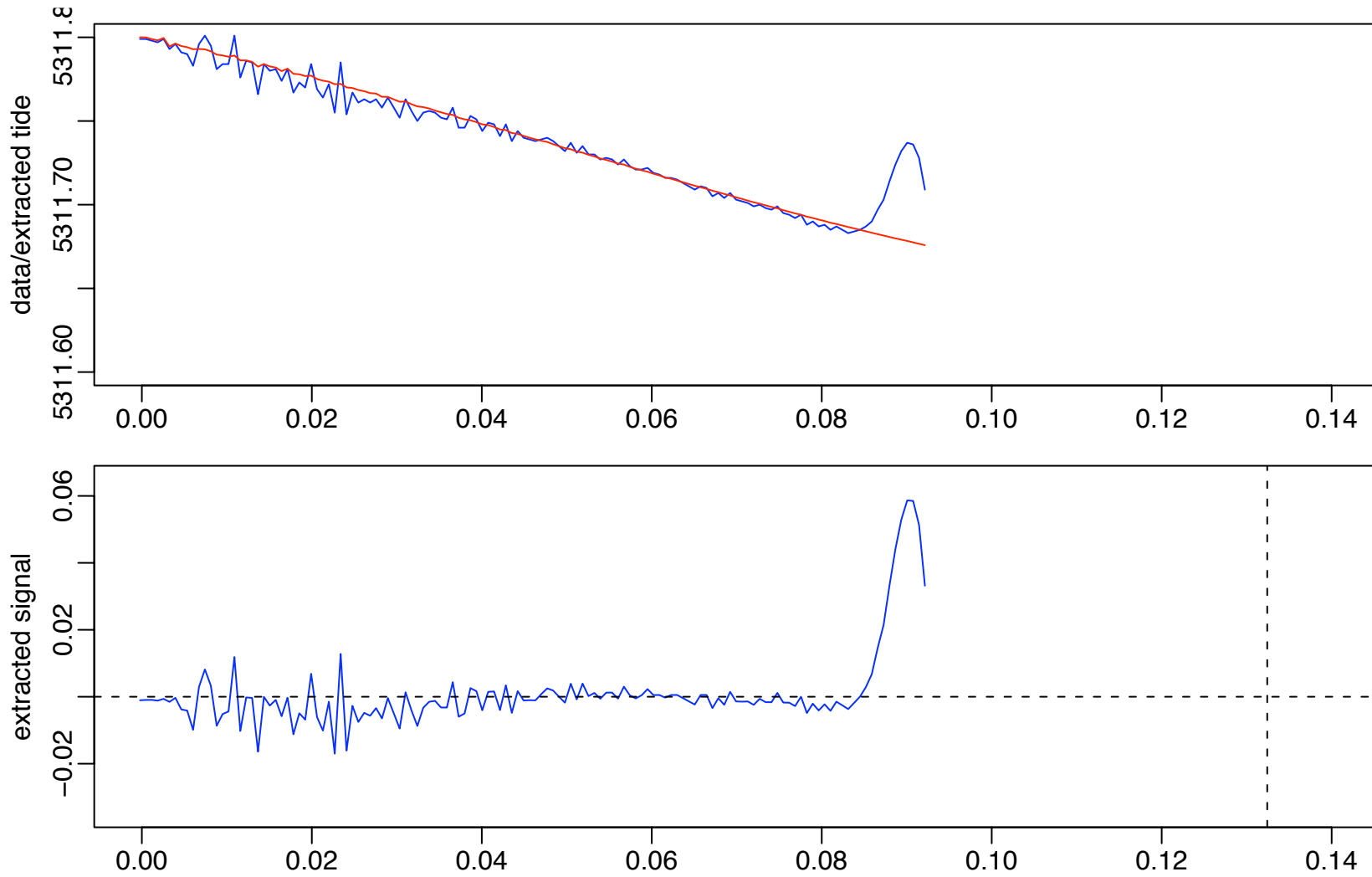
# Kalman Filtering/Smoothing – 14 Day Model



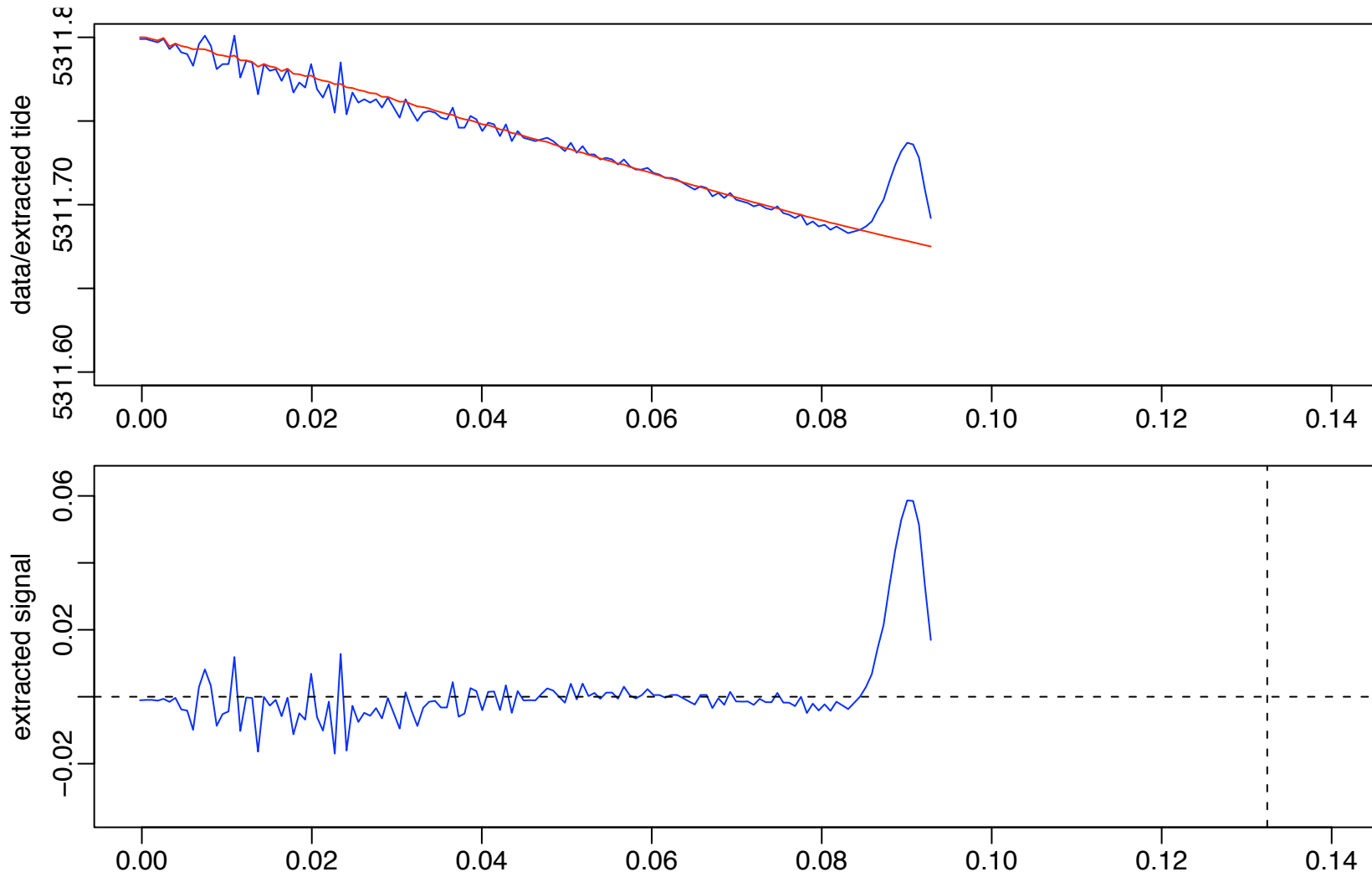
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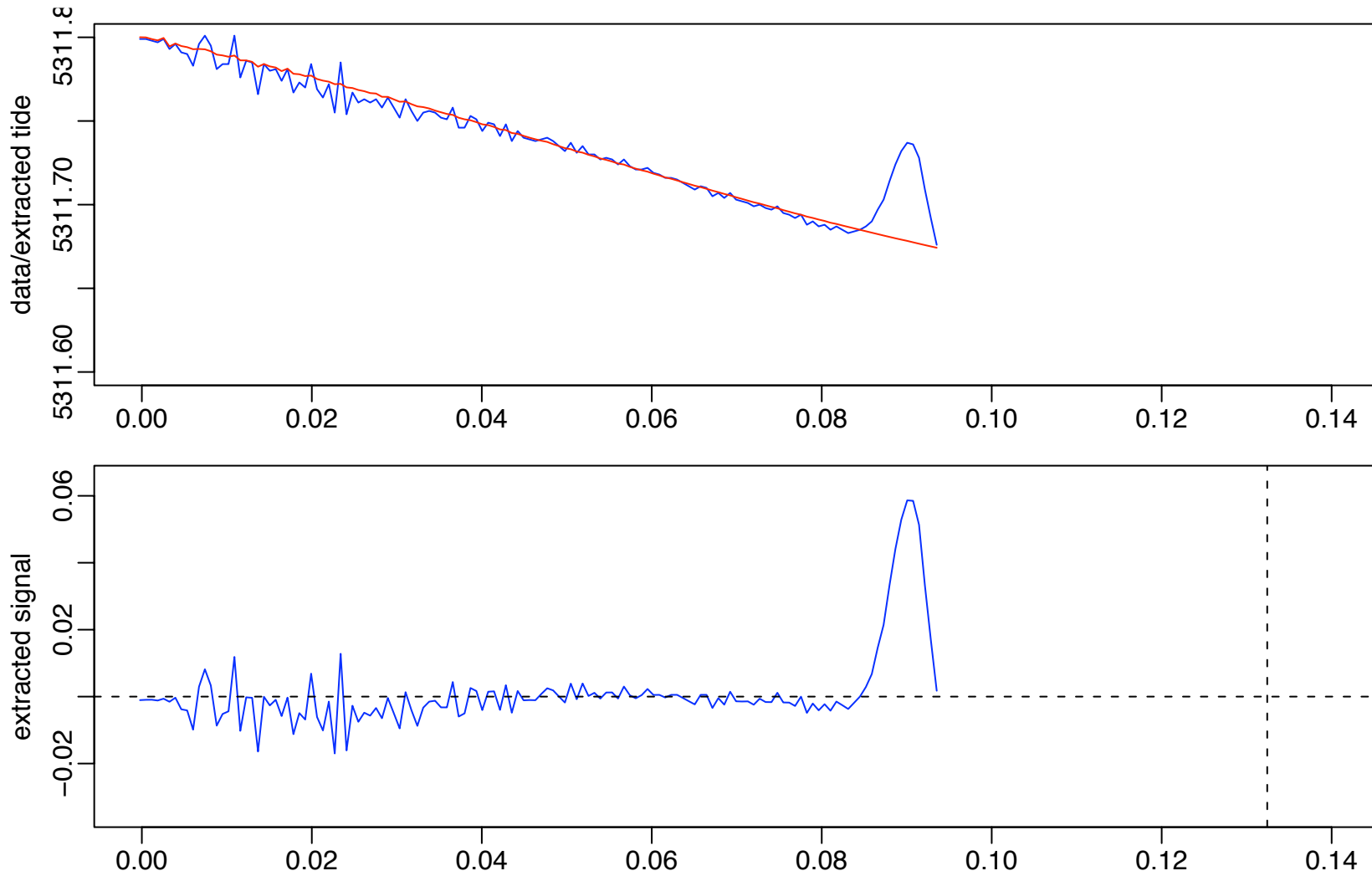


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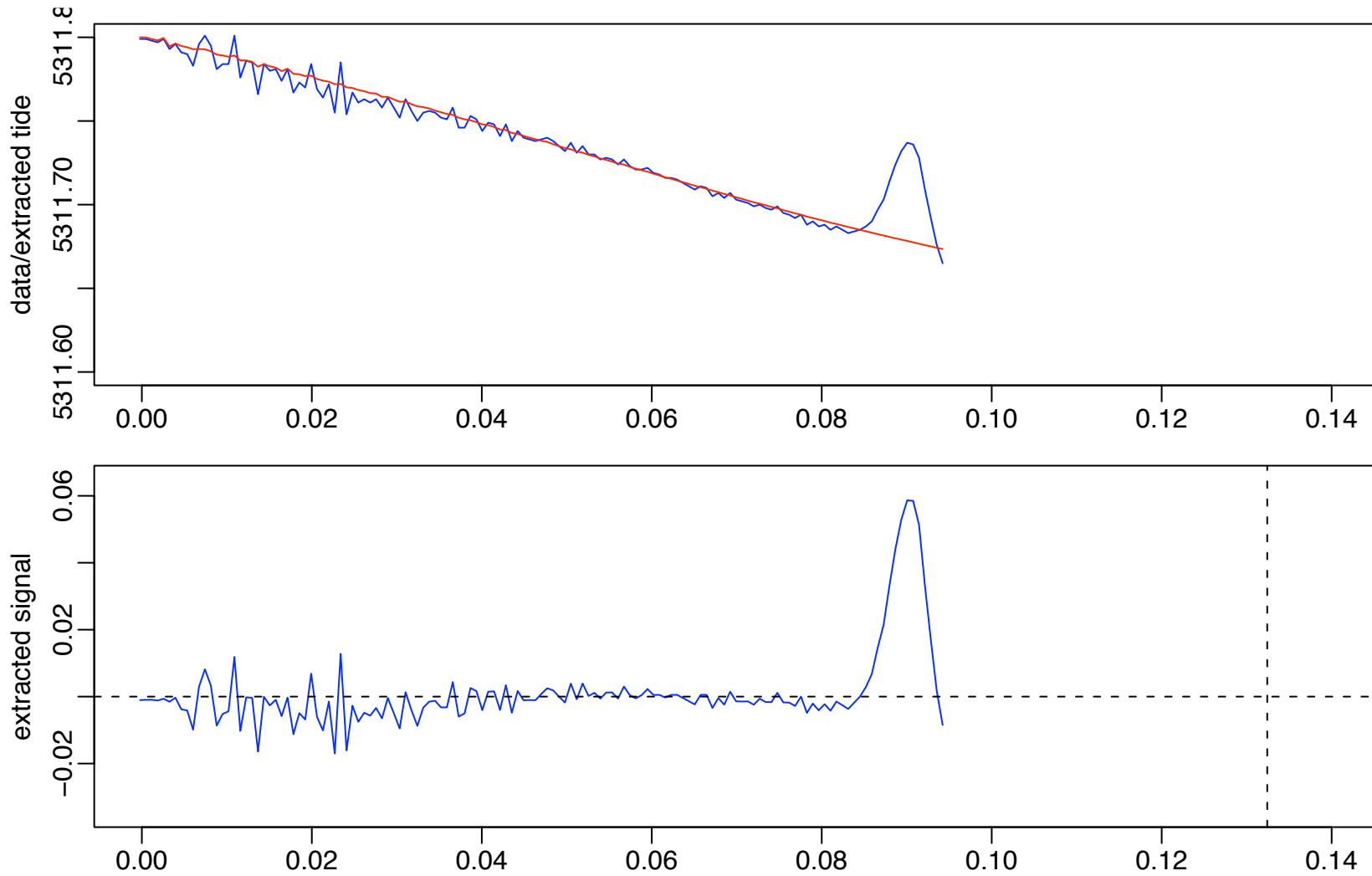




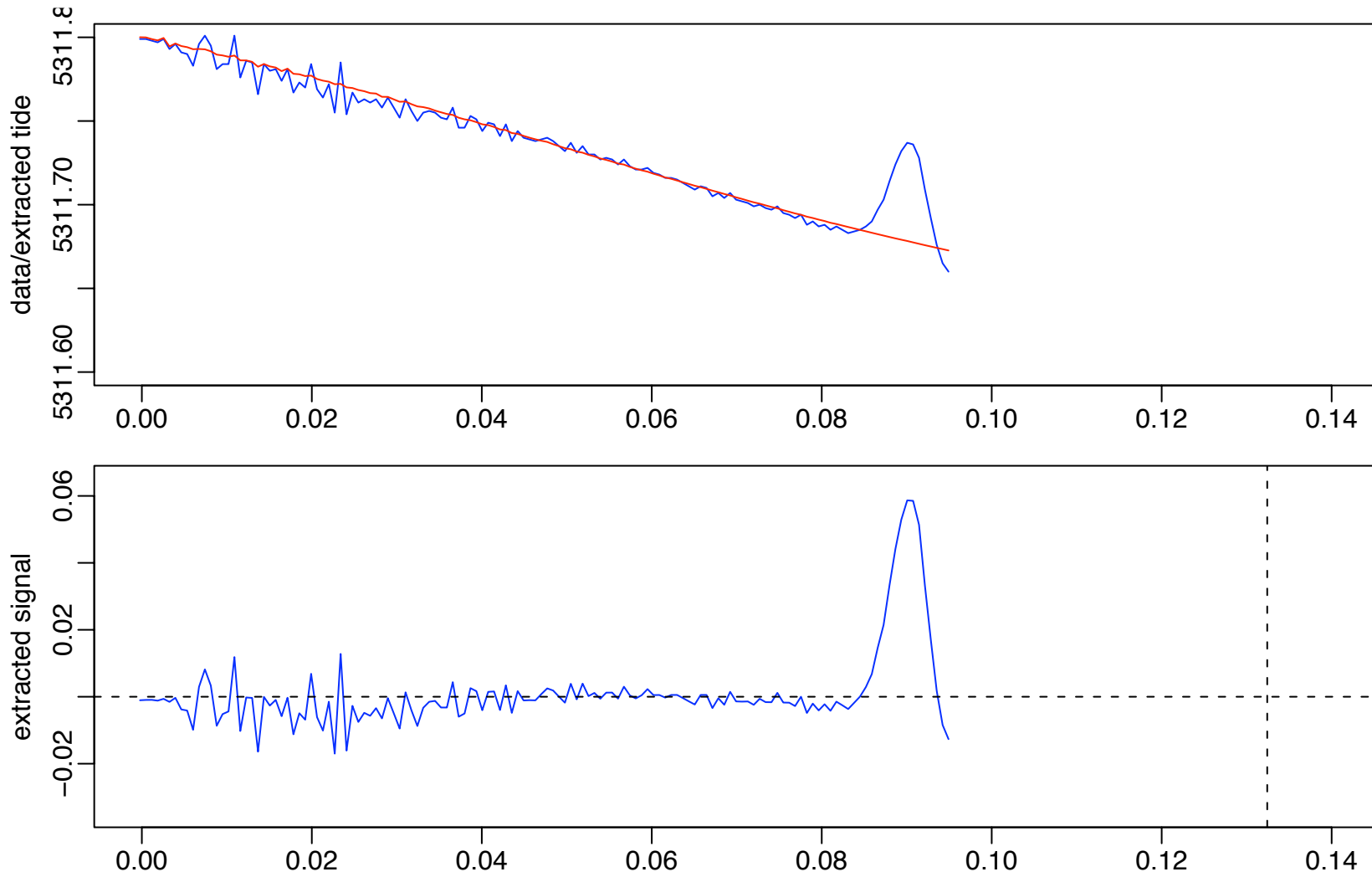
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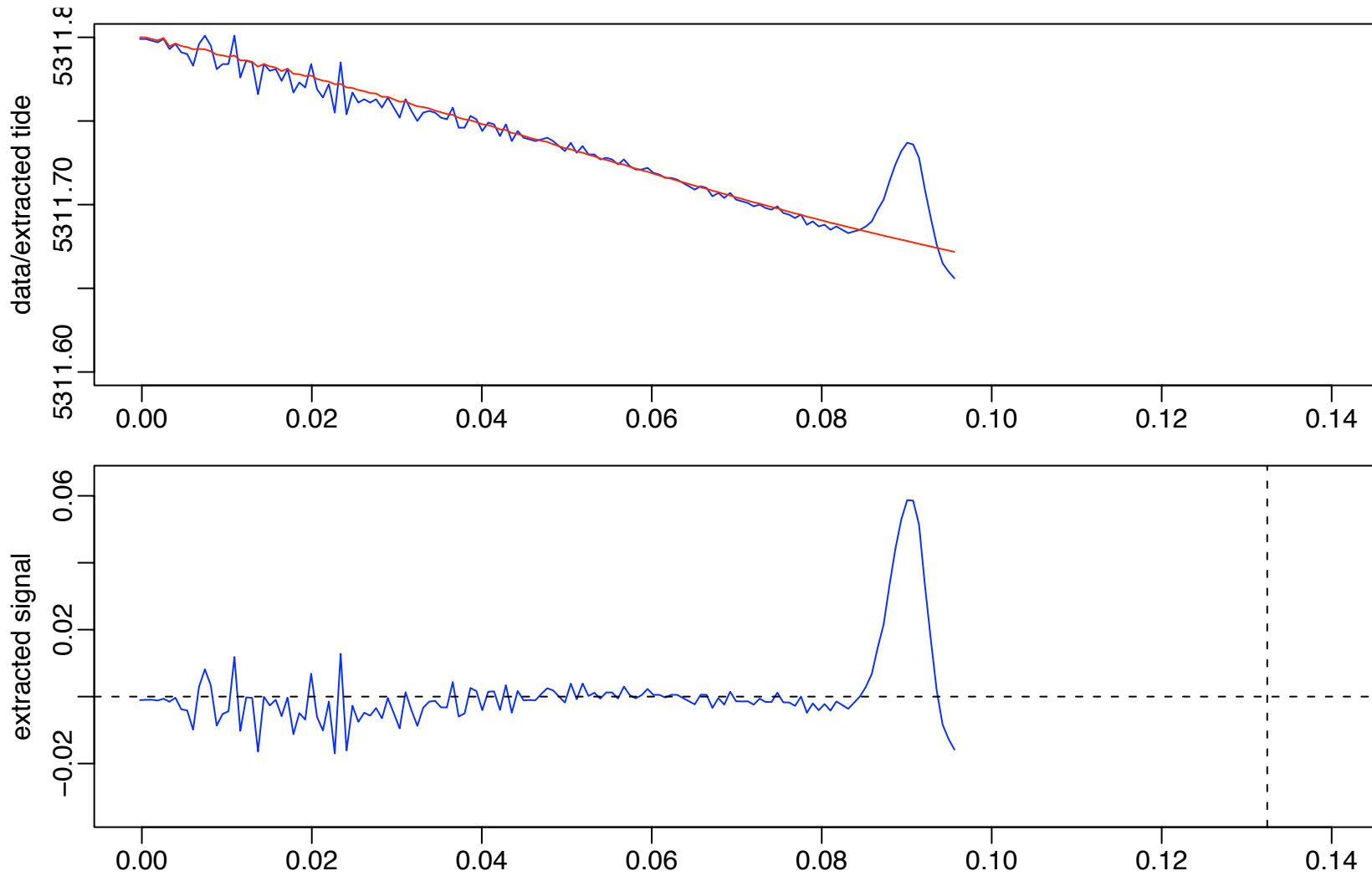
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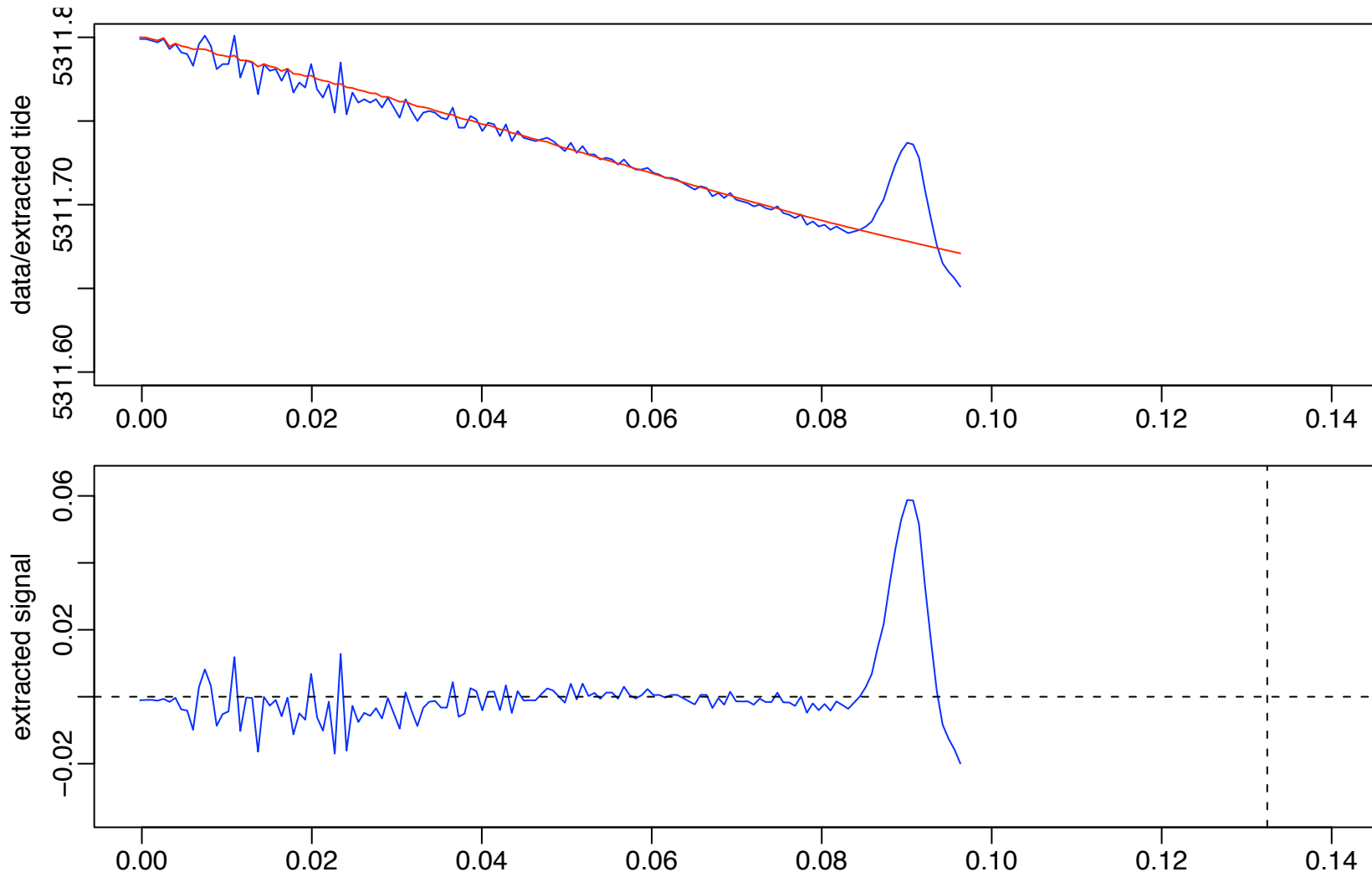
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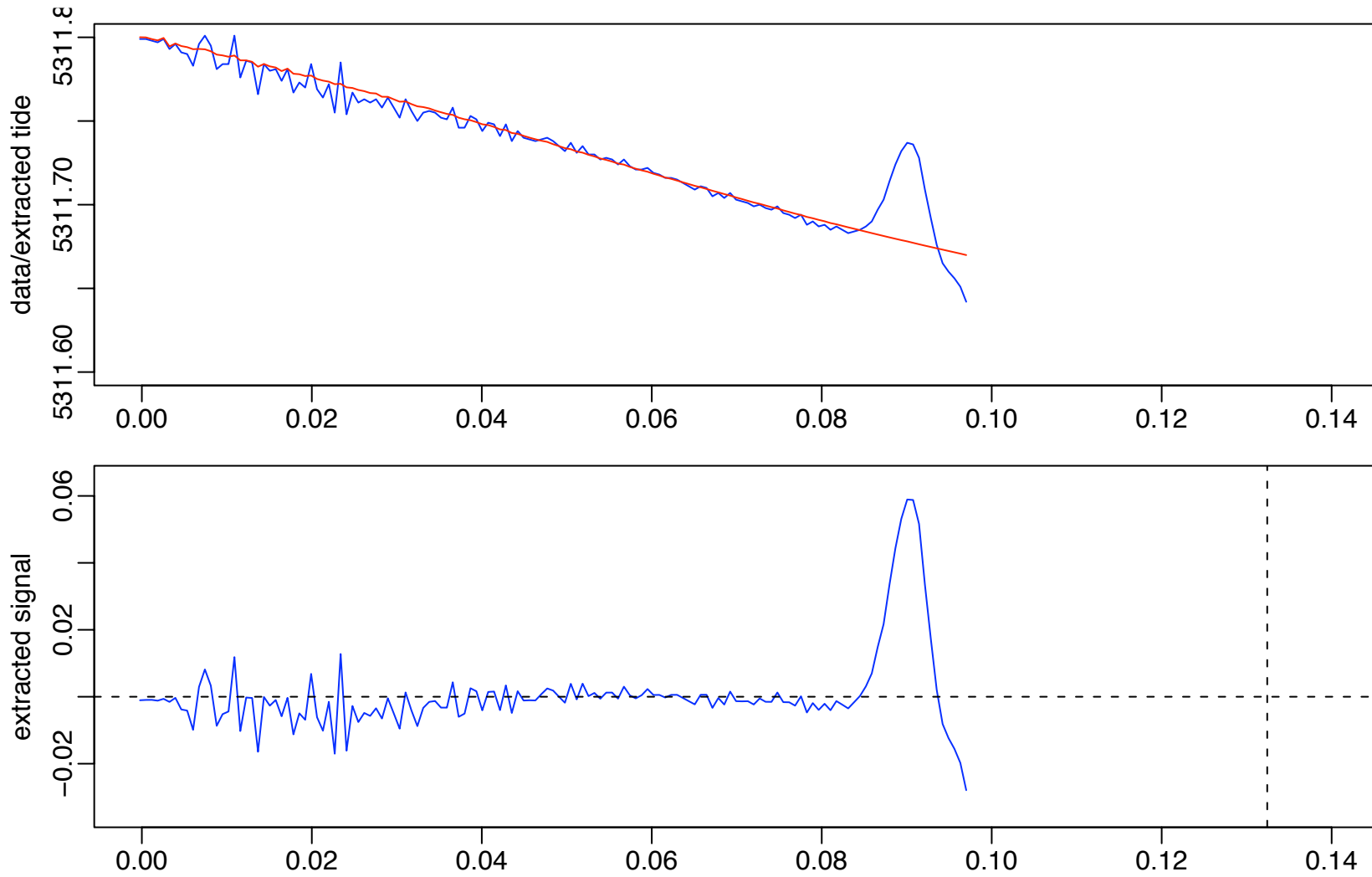
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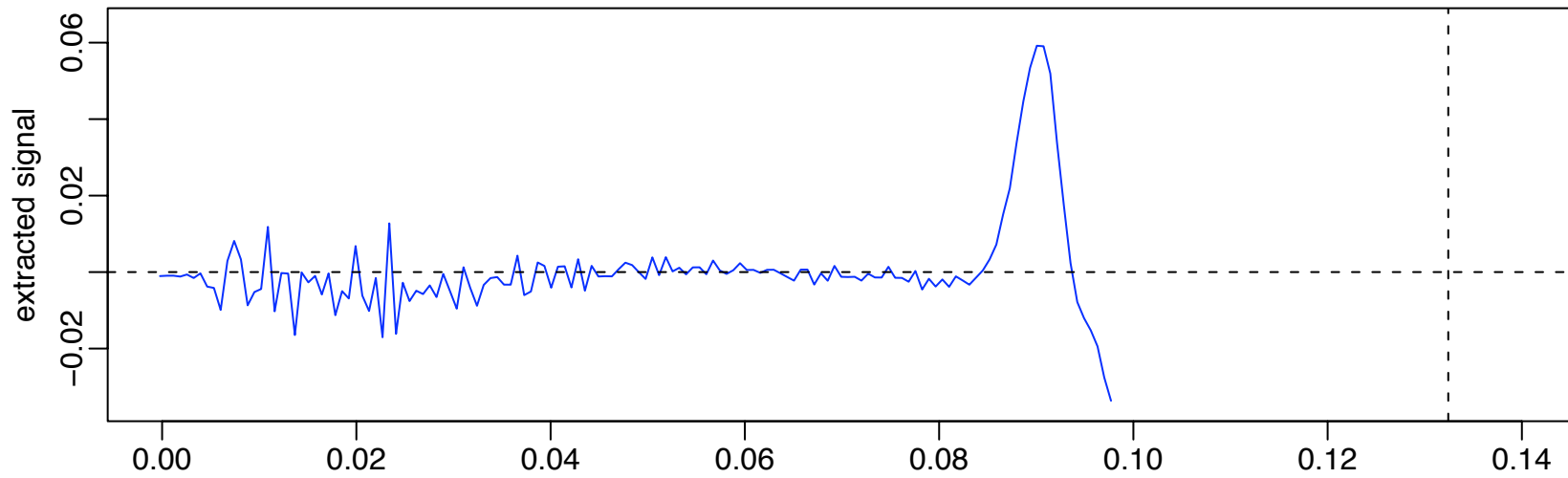
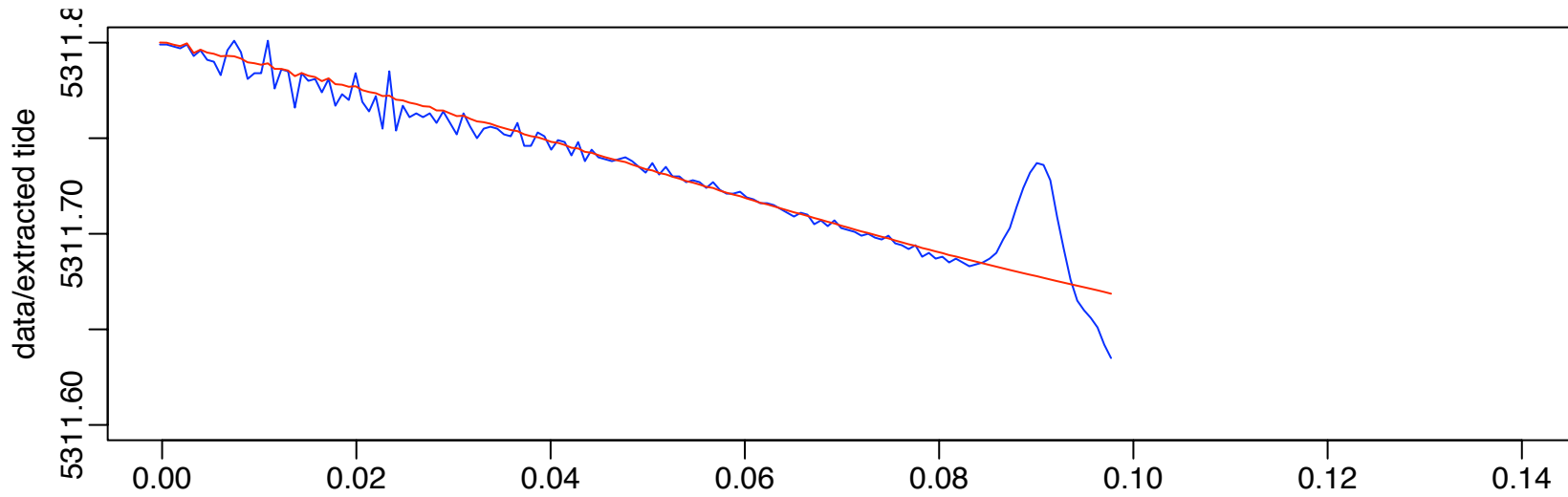
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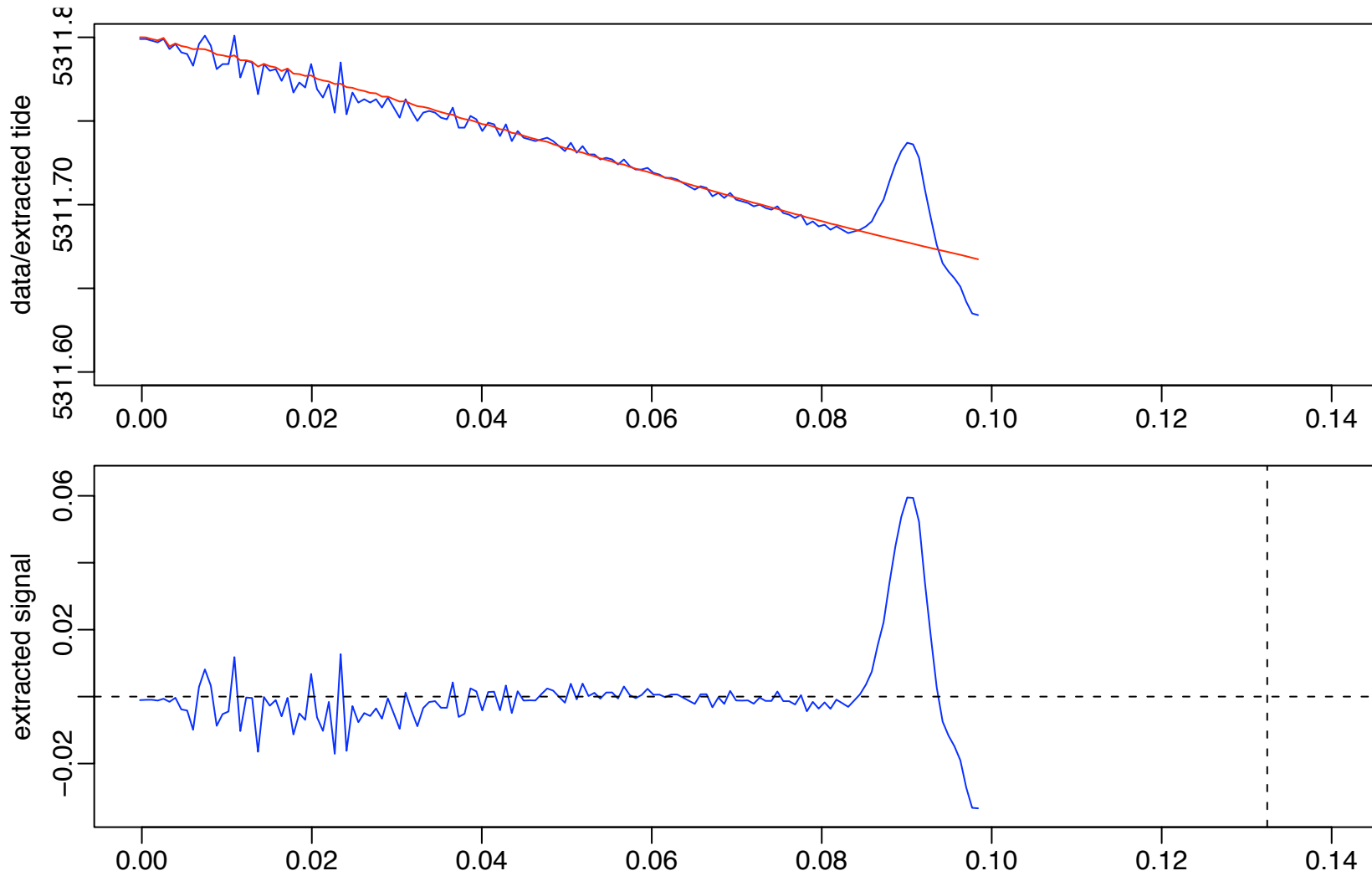
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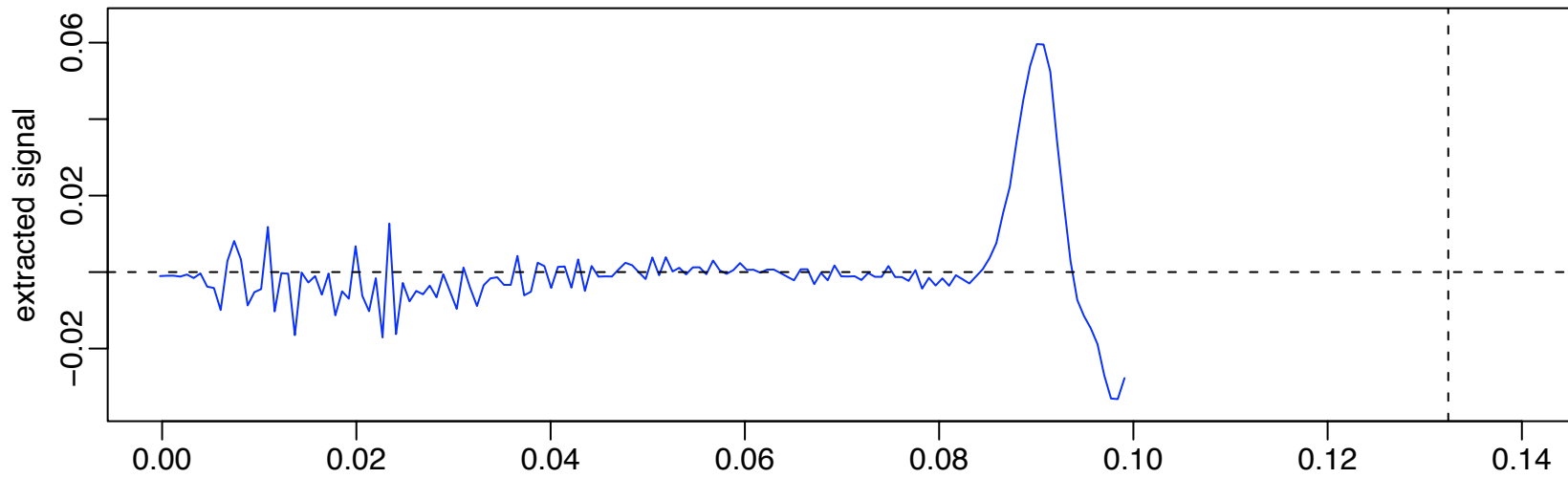
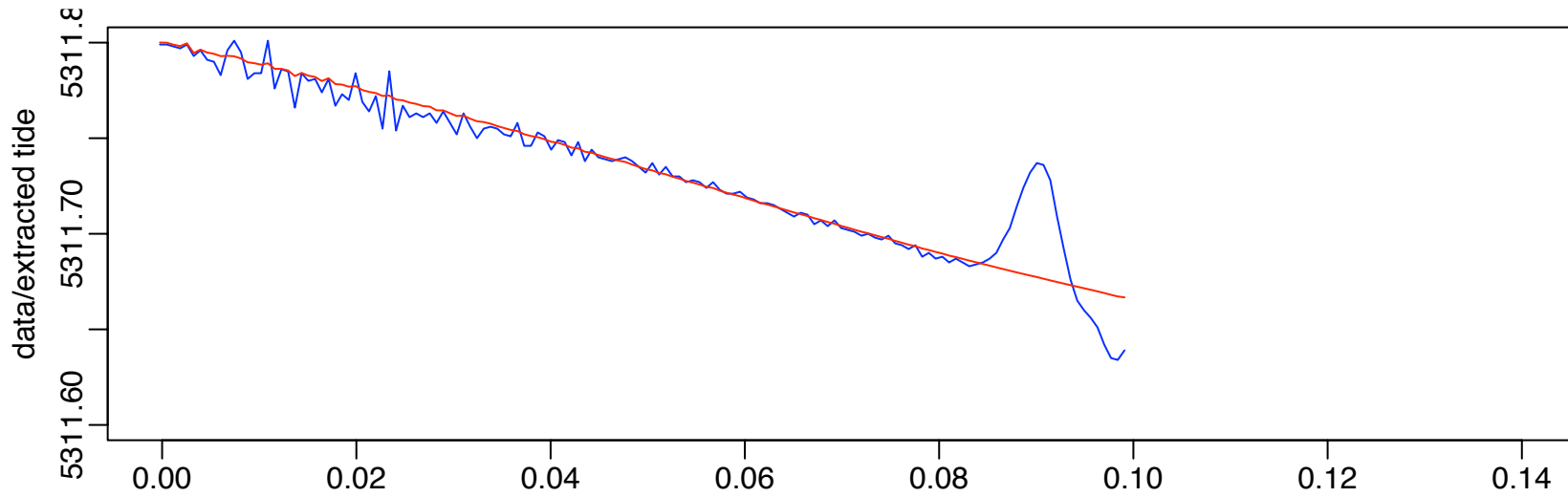


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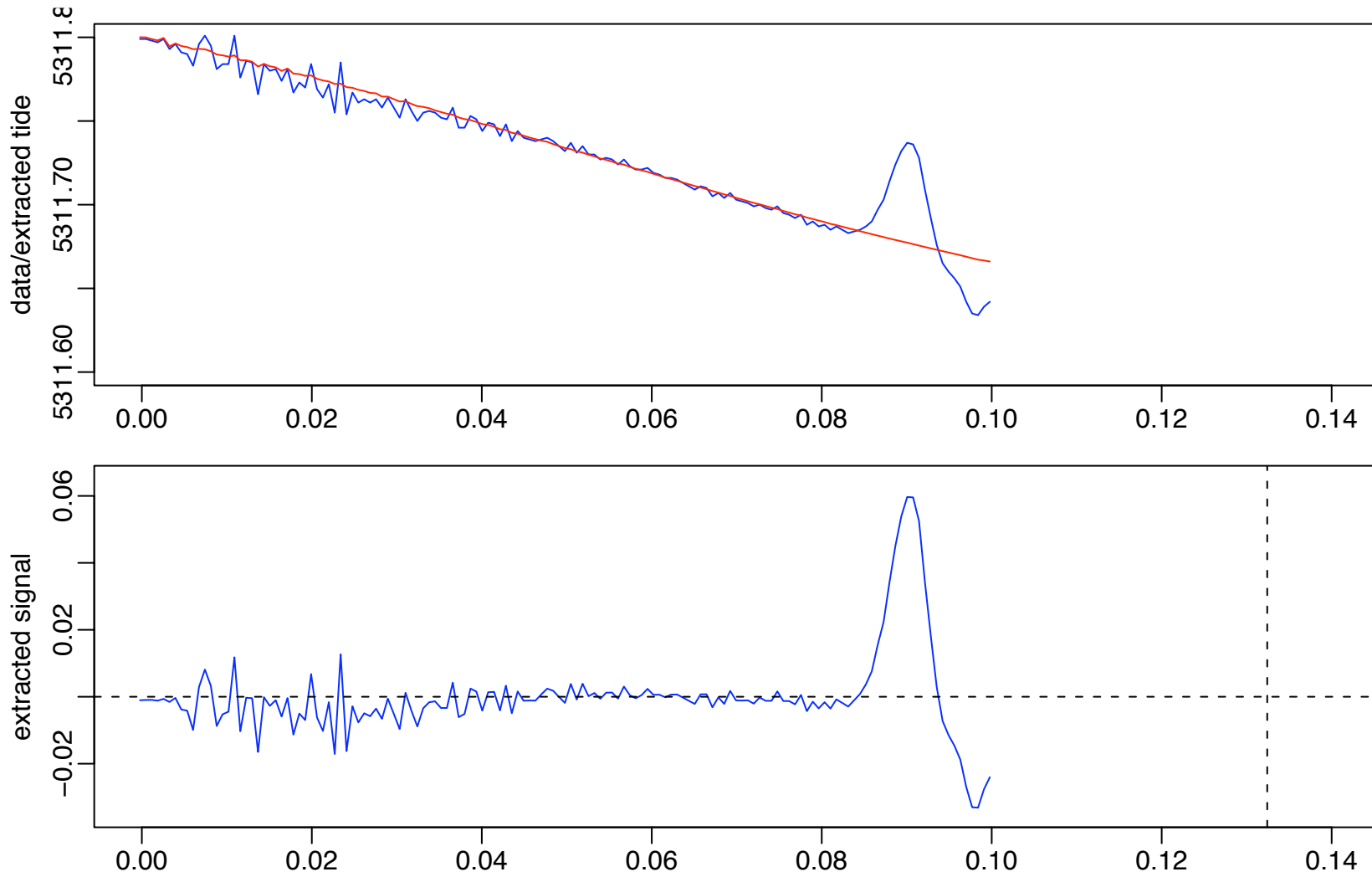




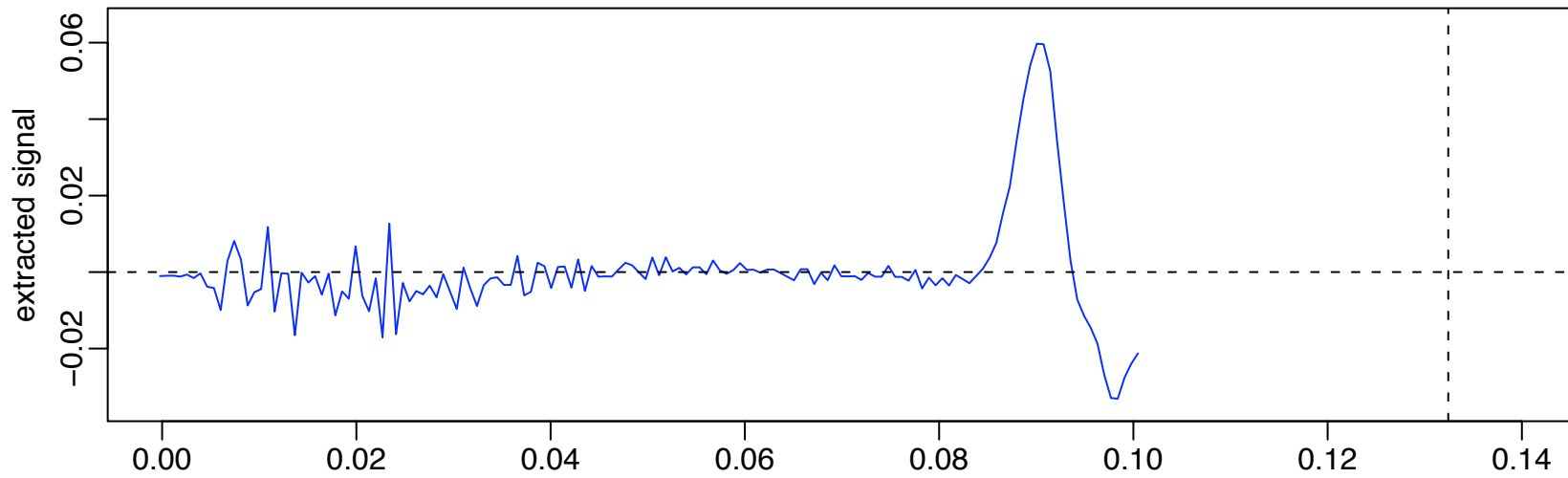
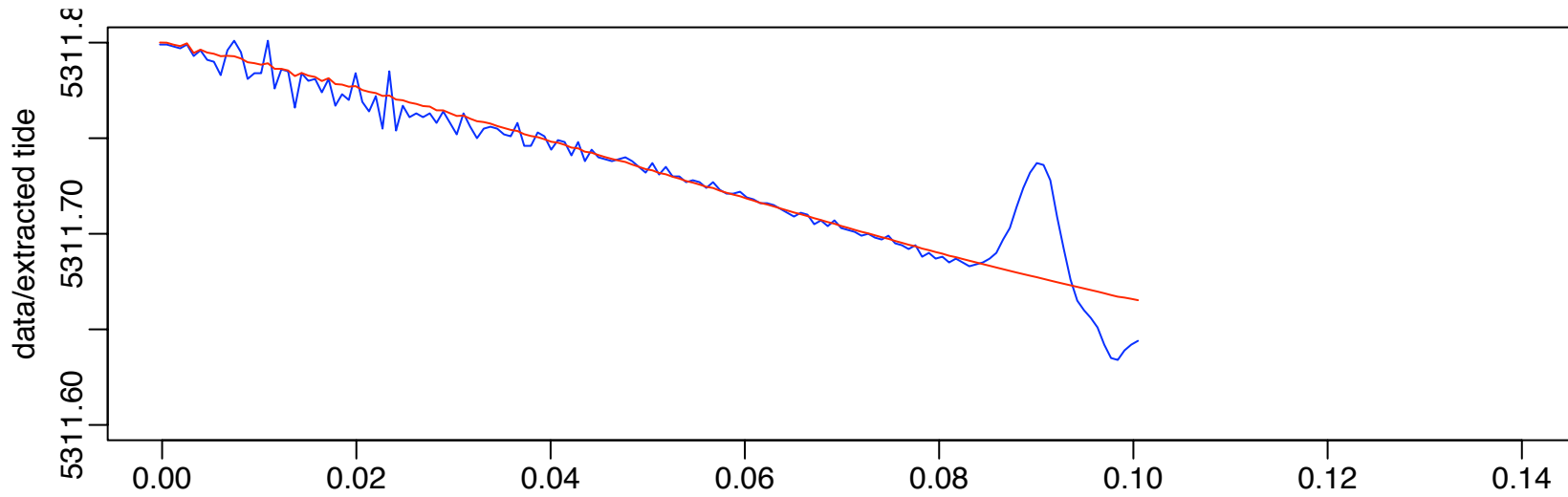
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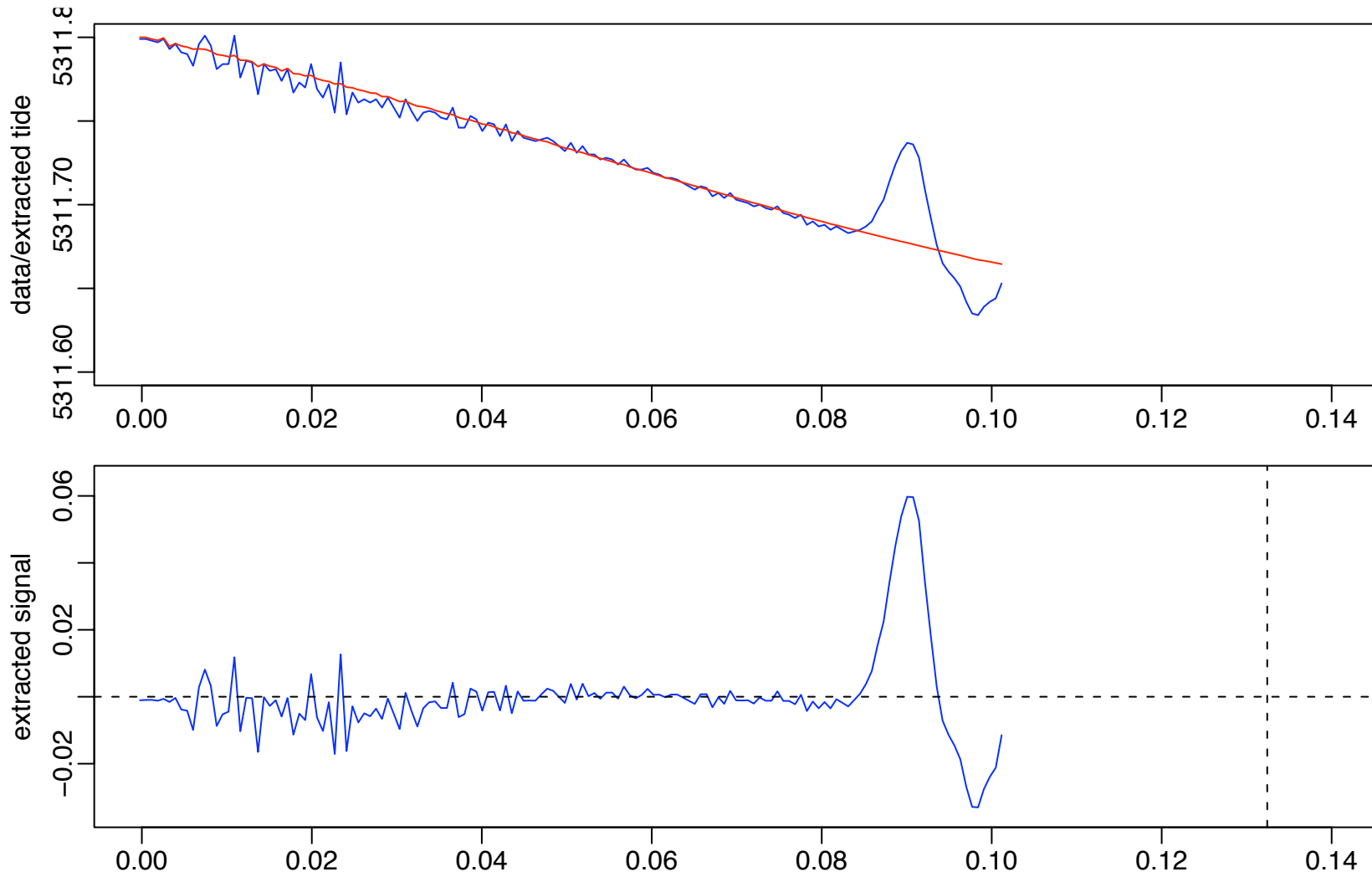
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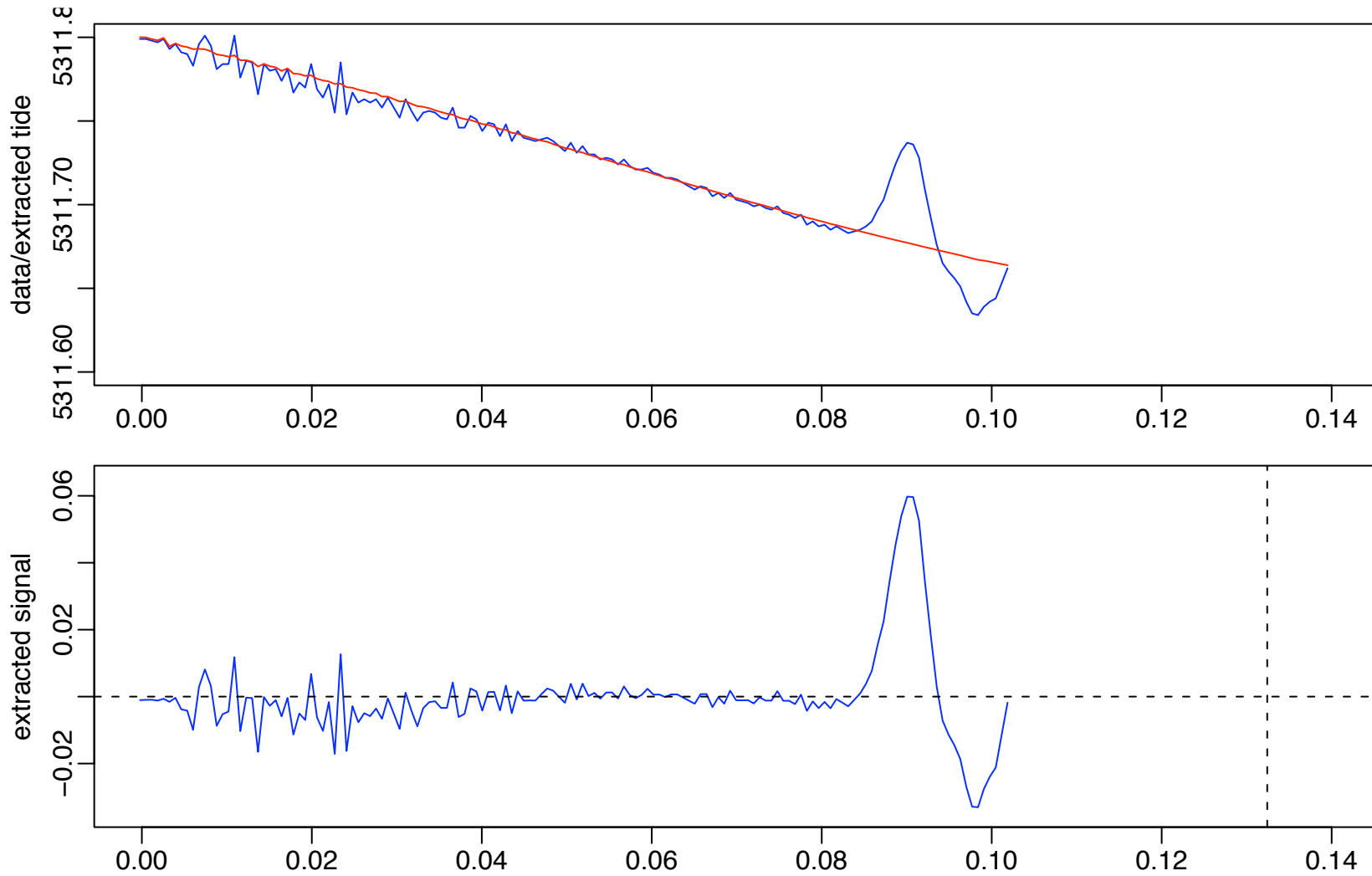
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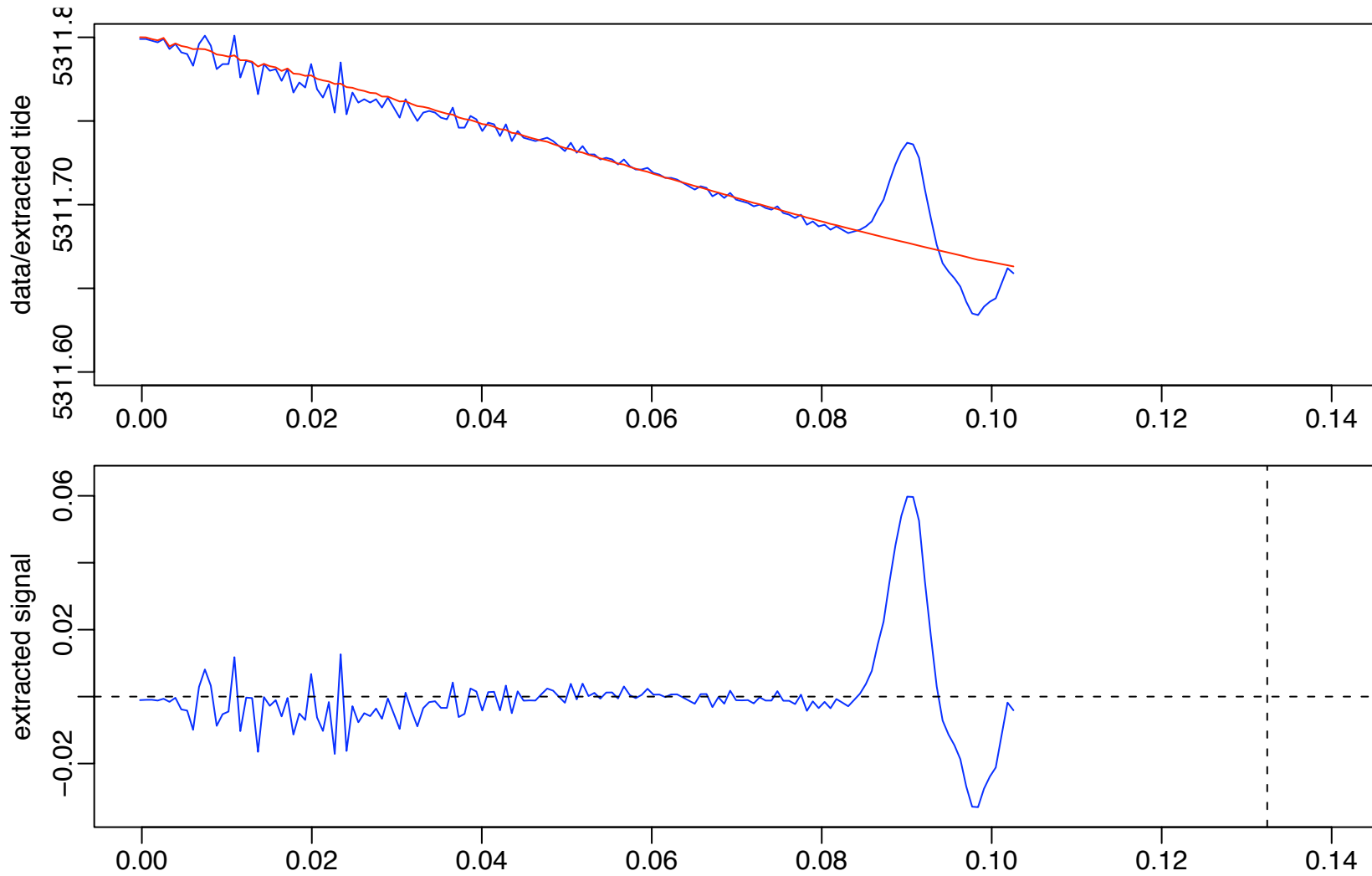
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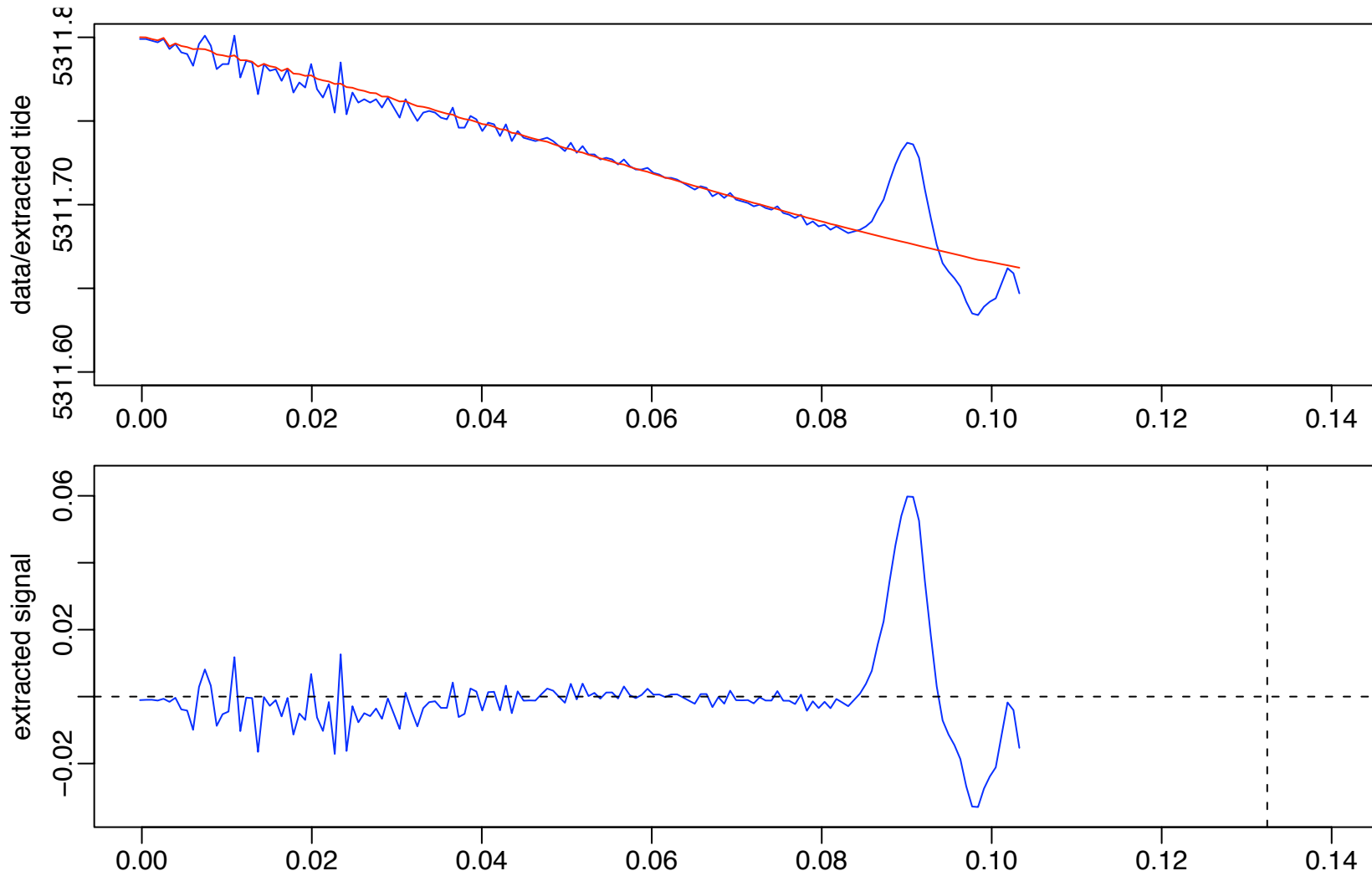
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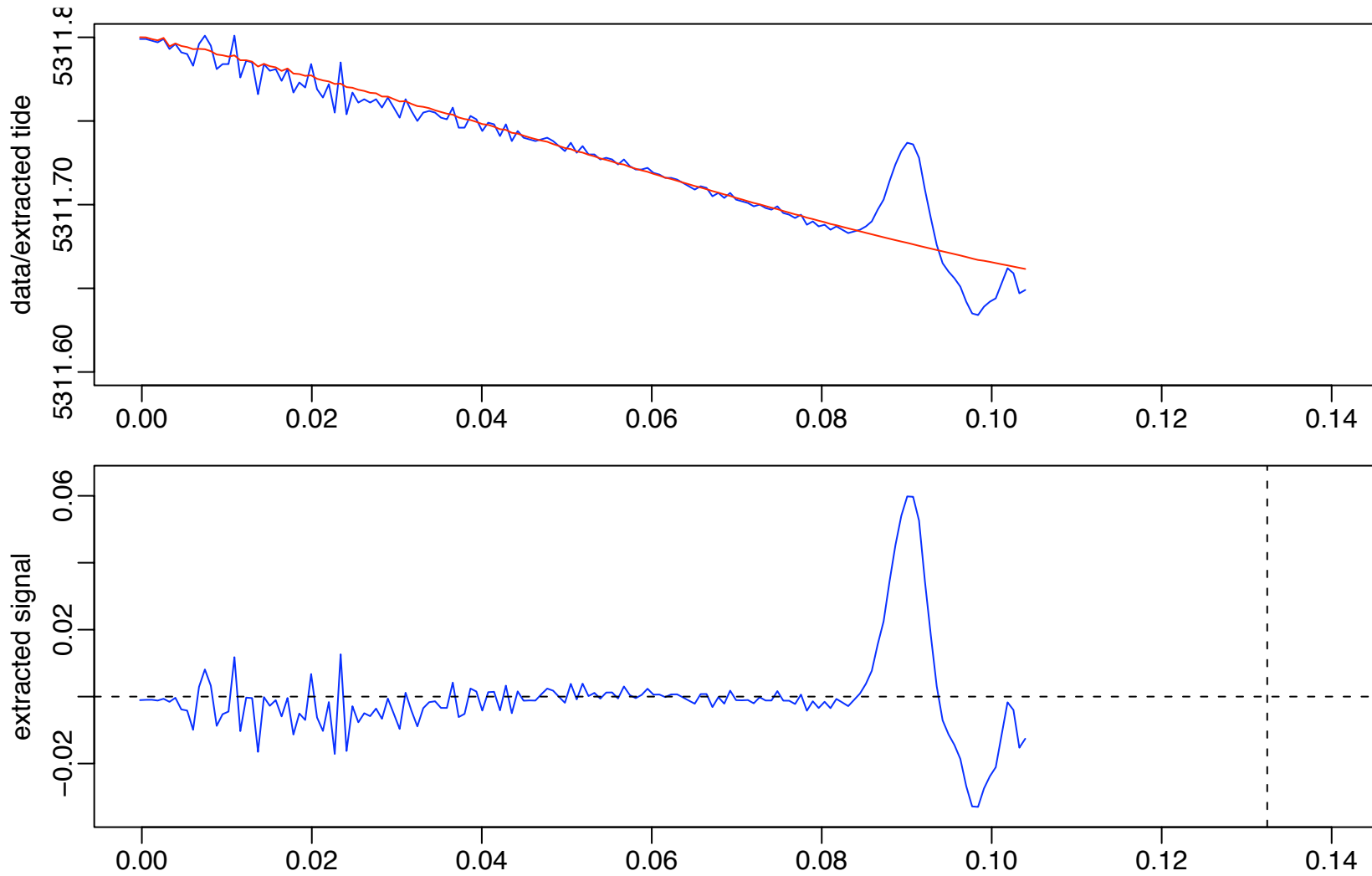
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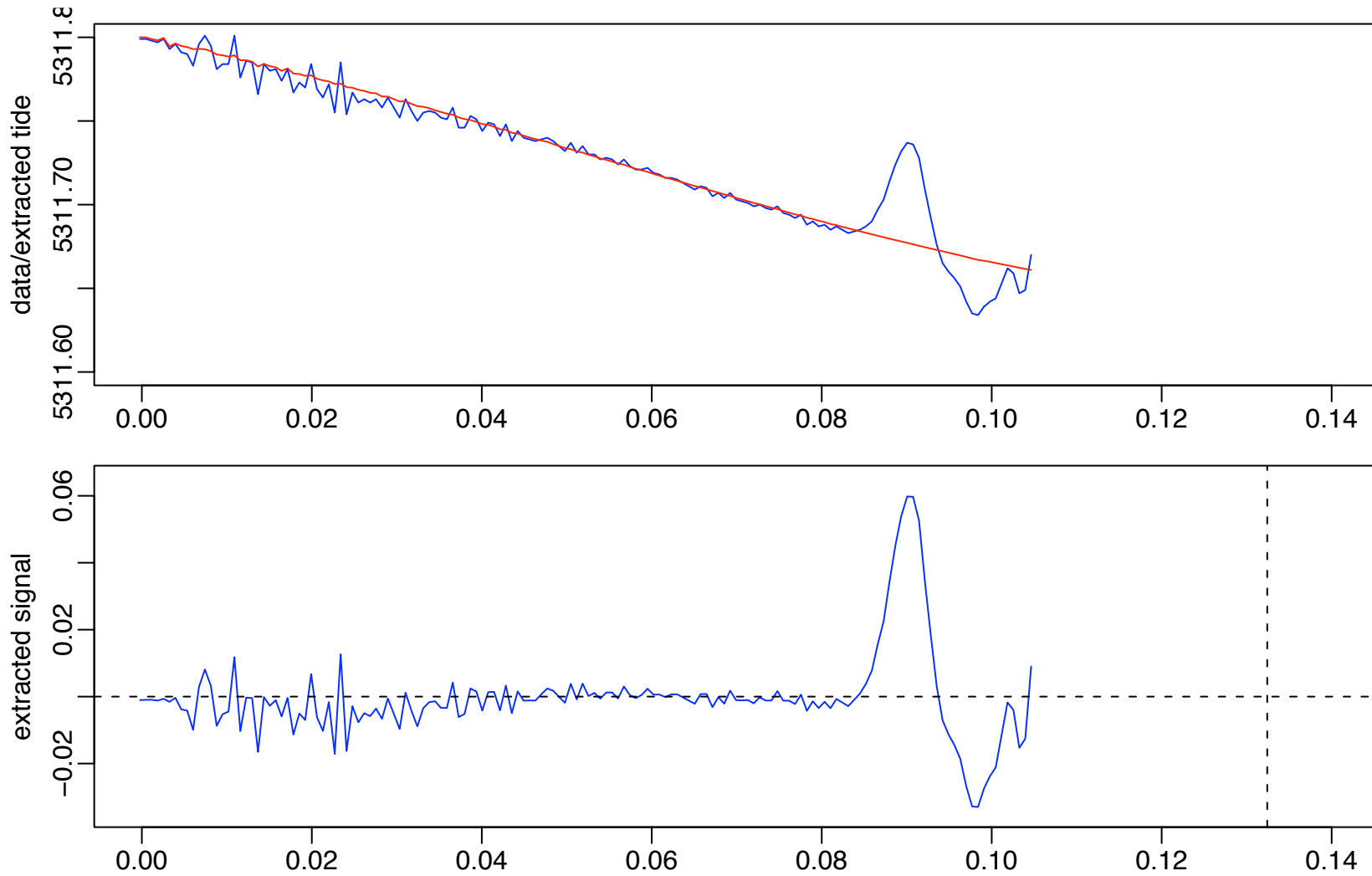


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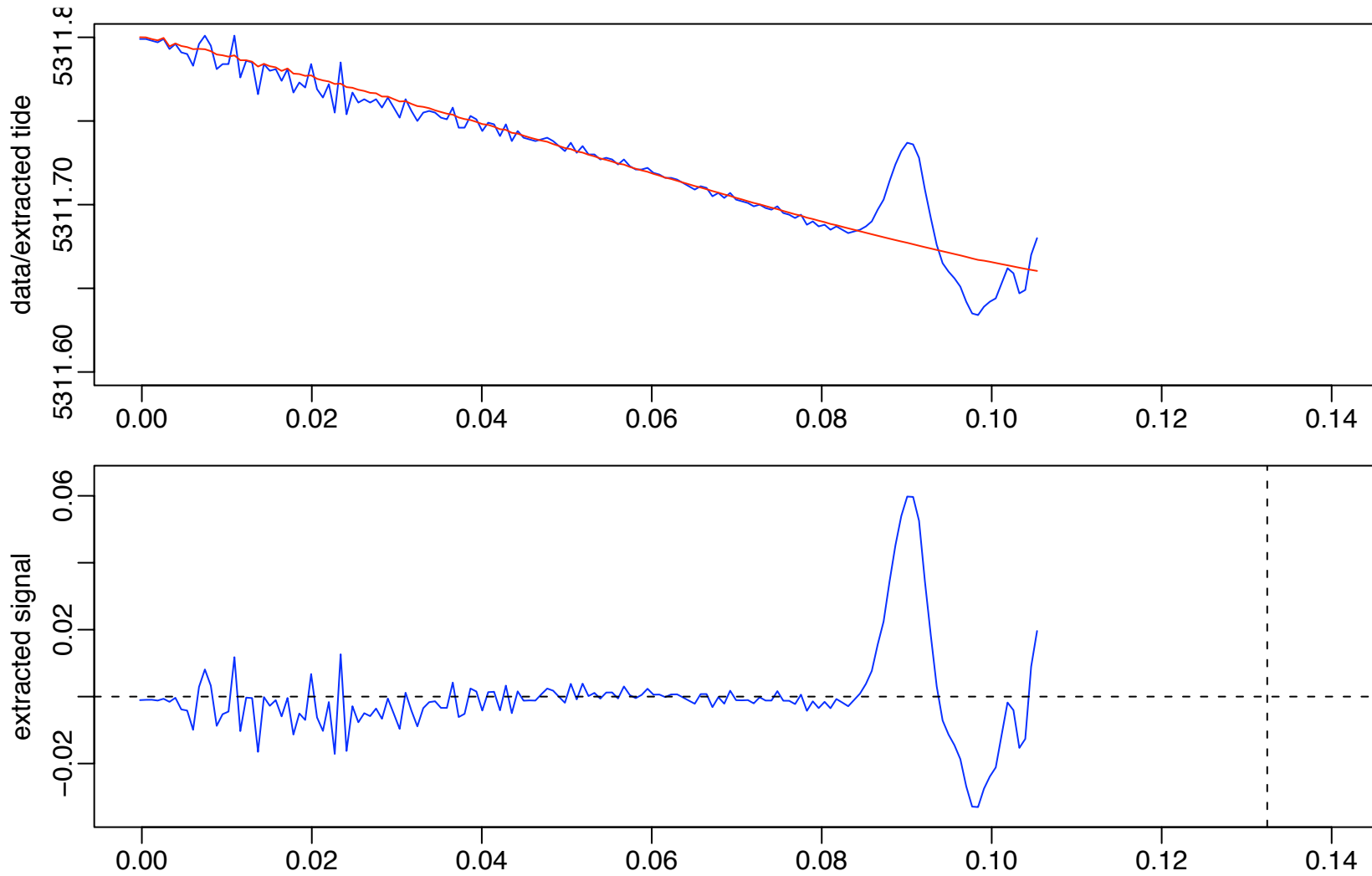




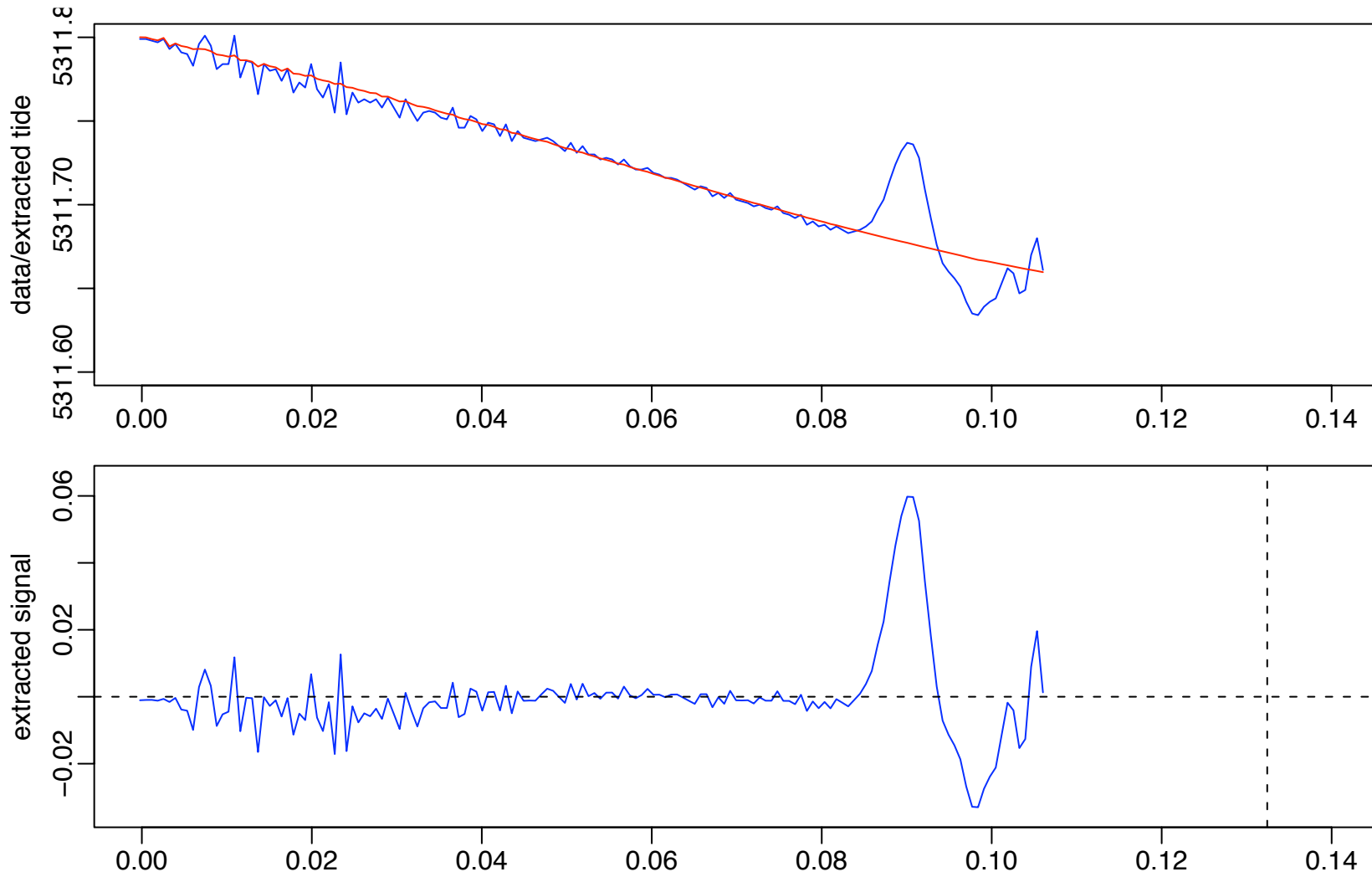
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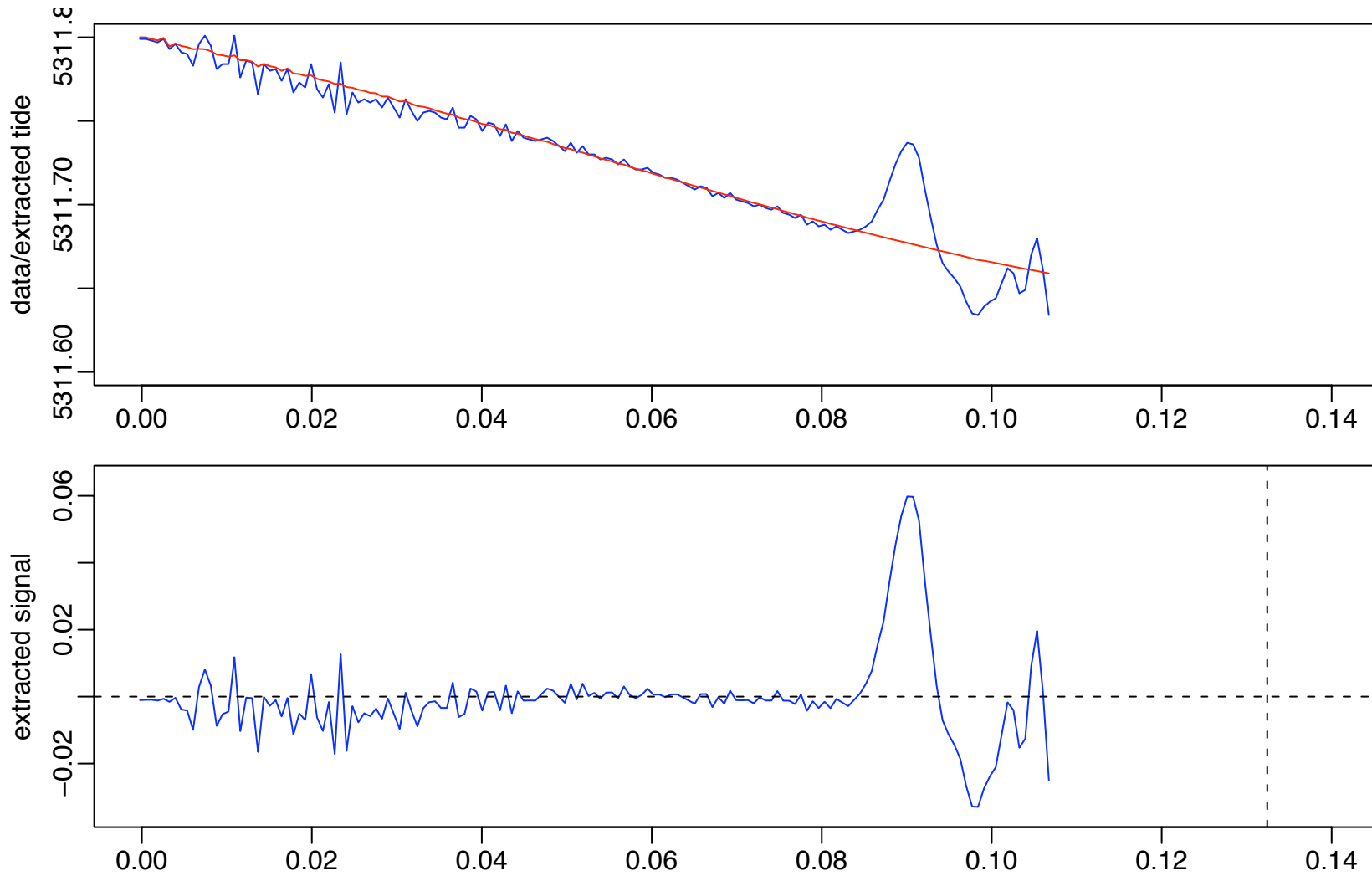
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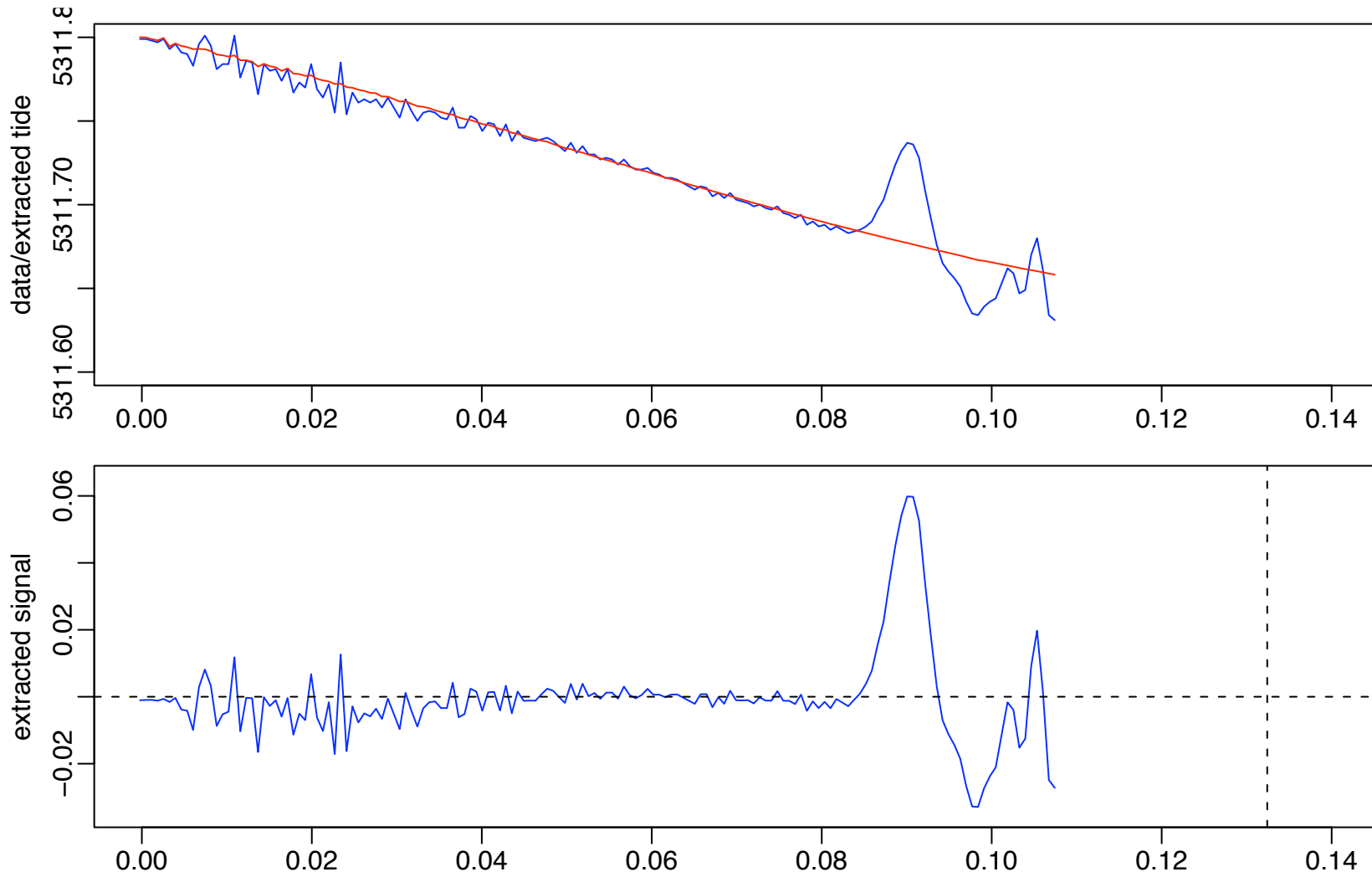
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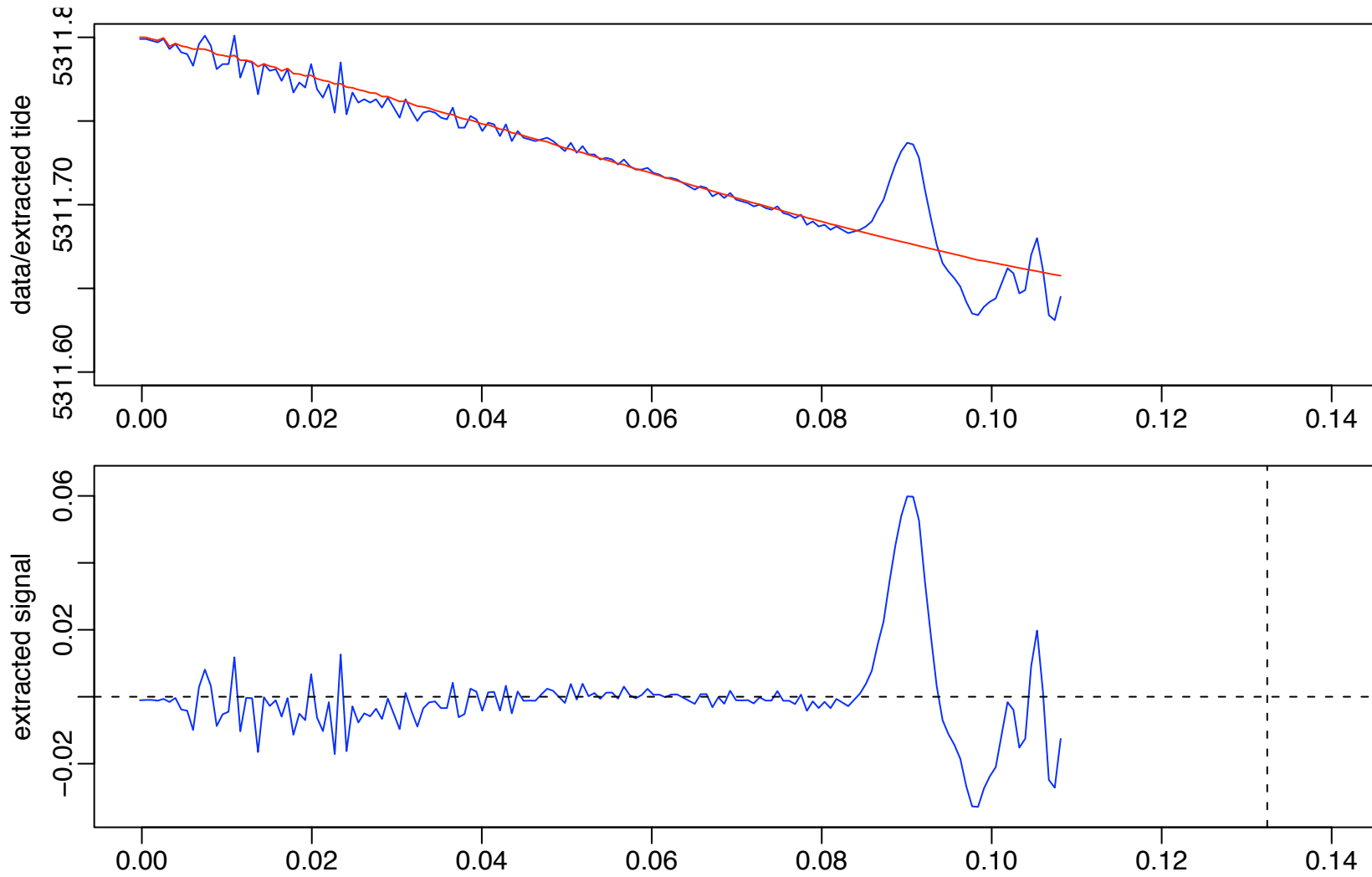
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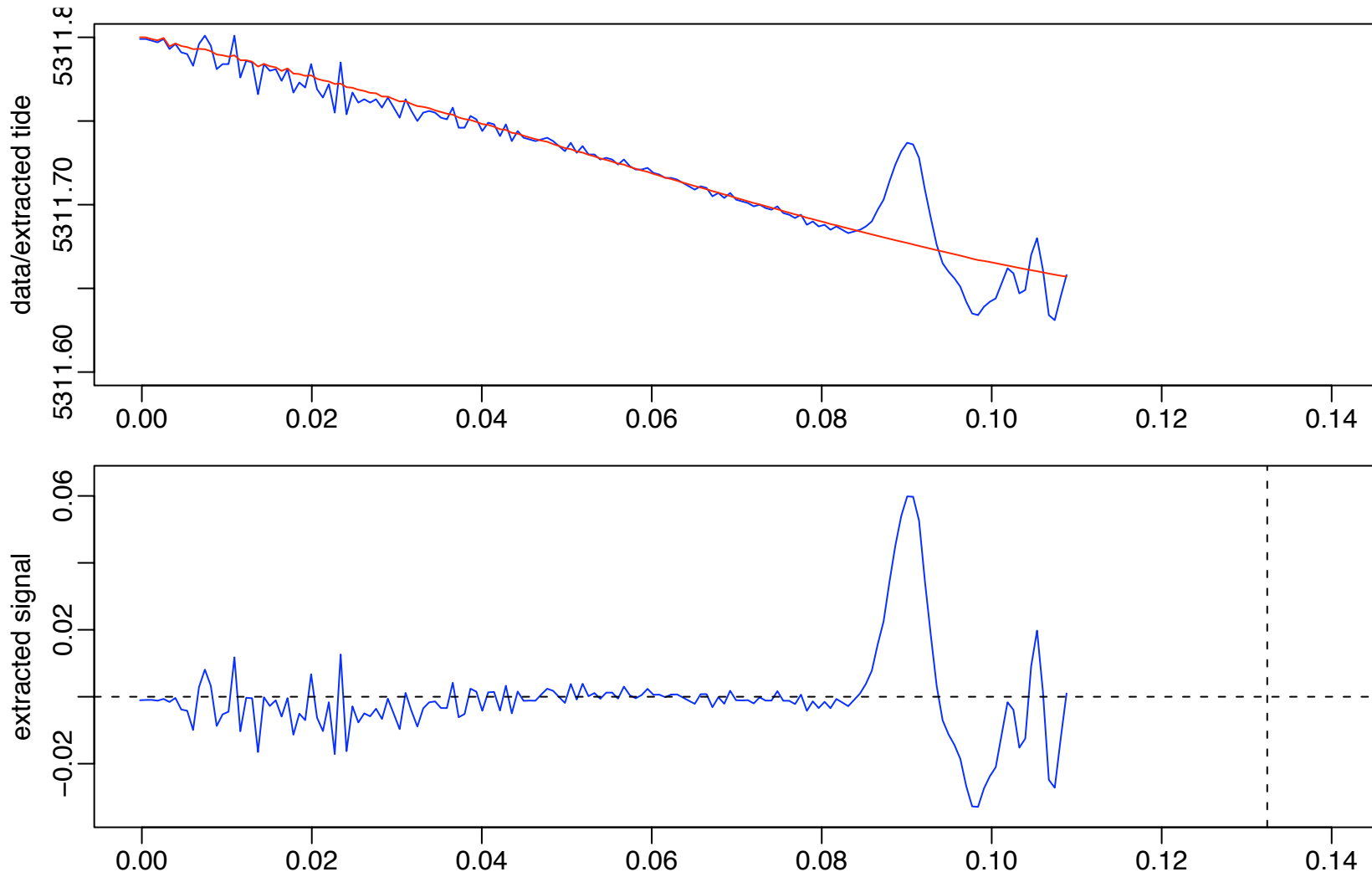
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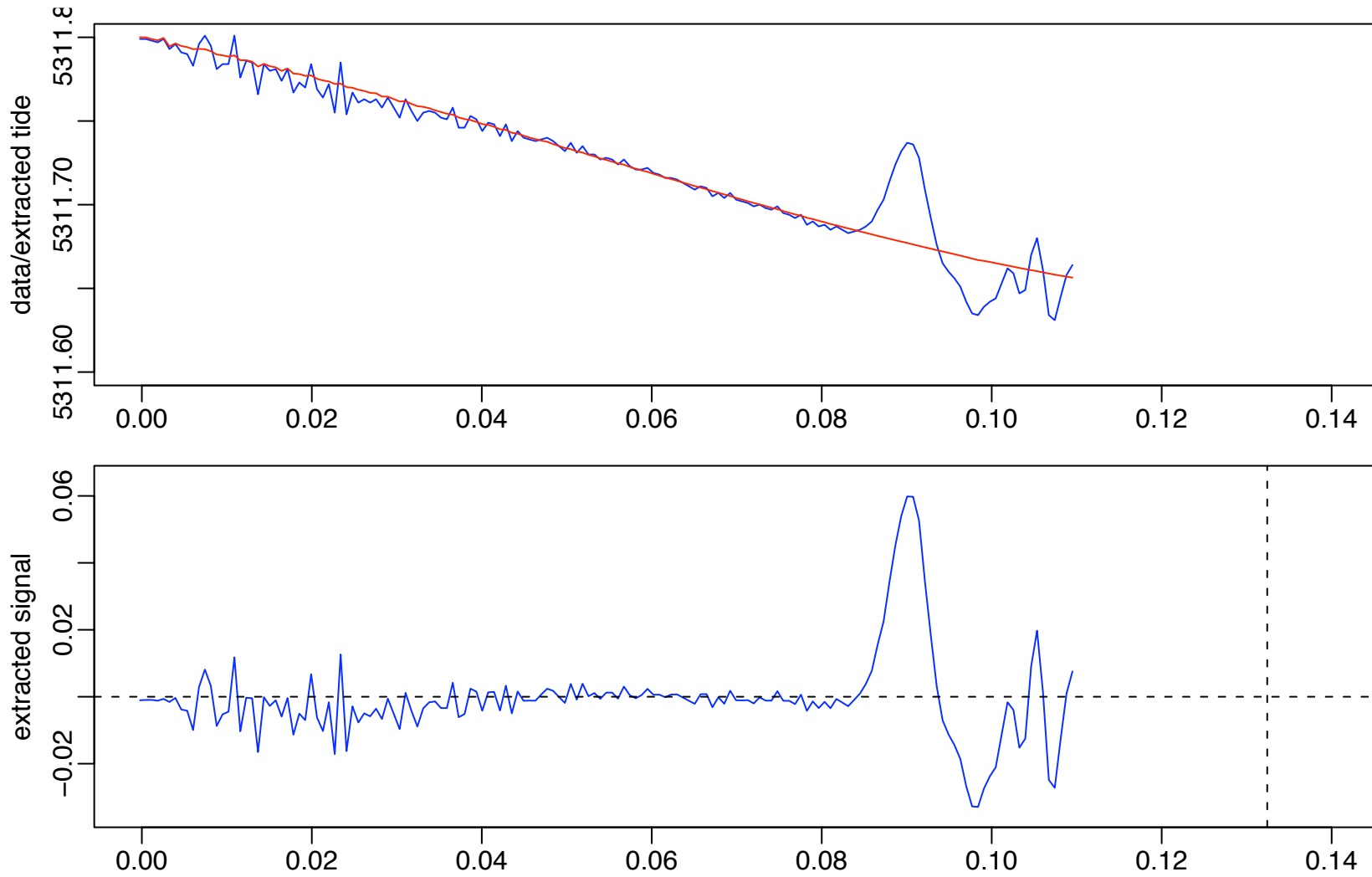
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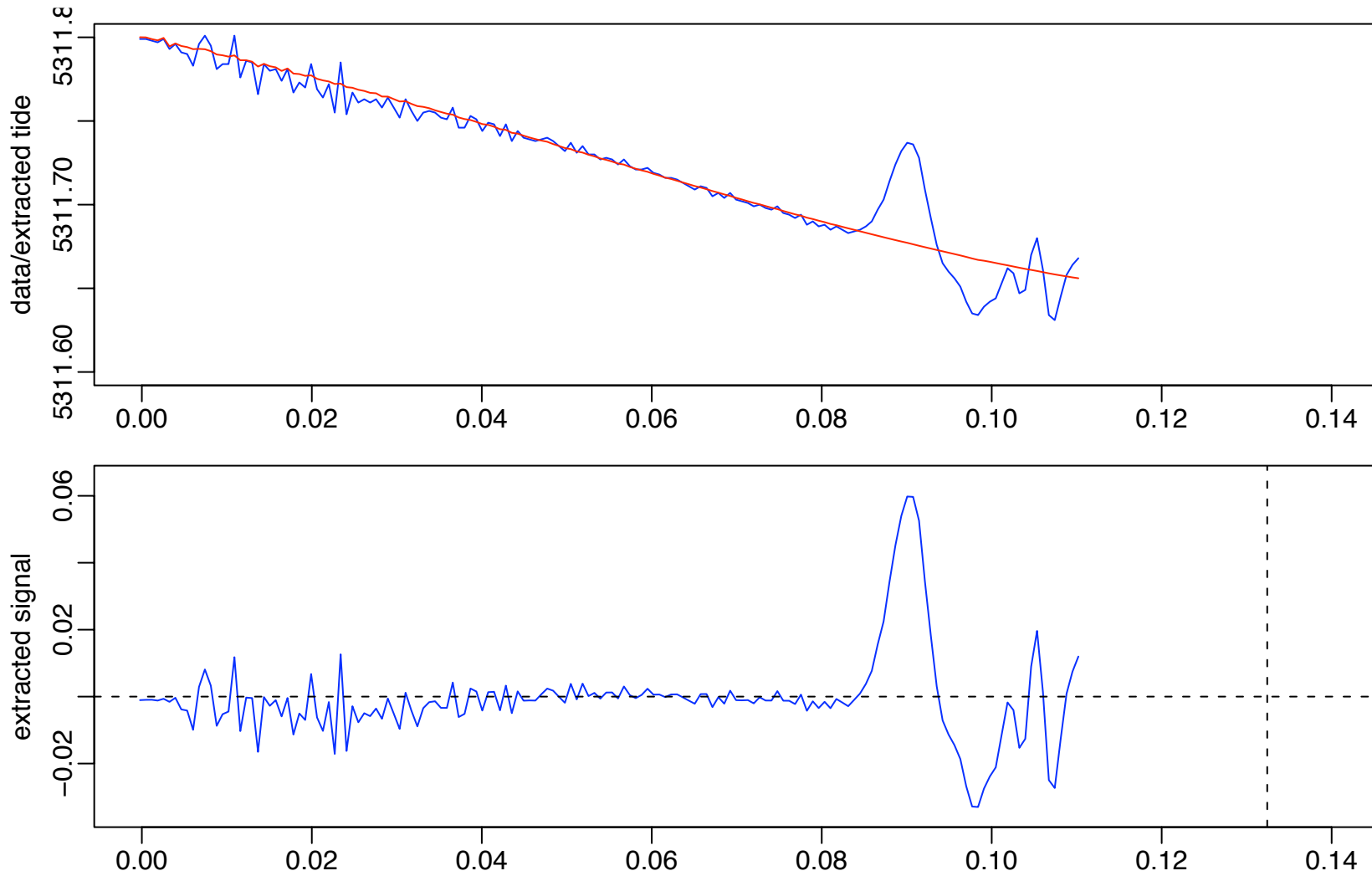


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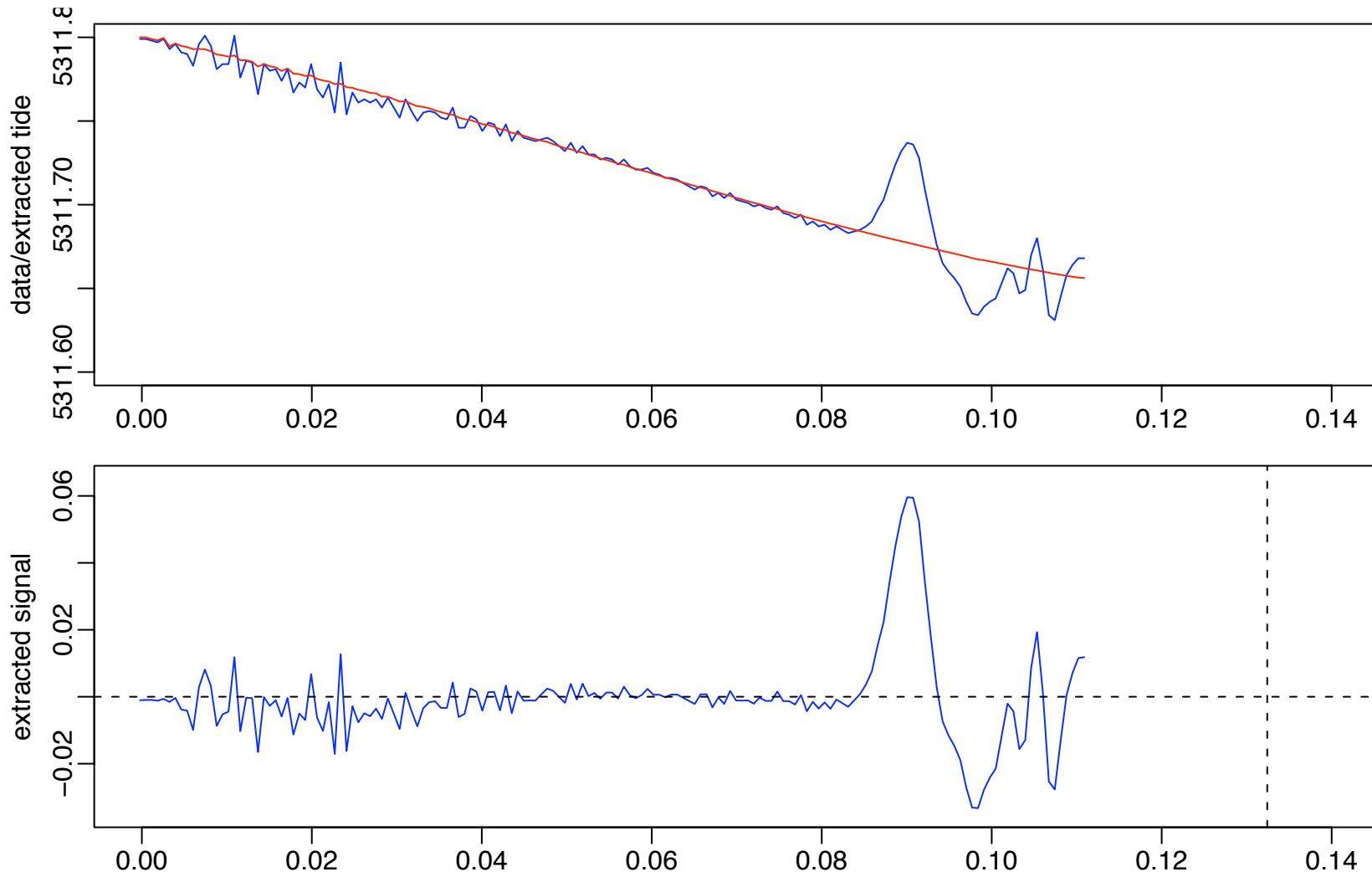




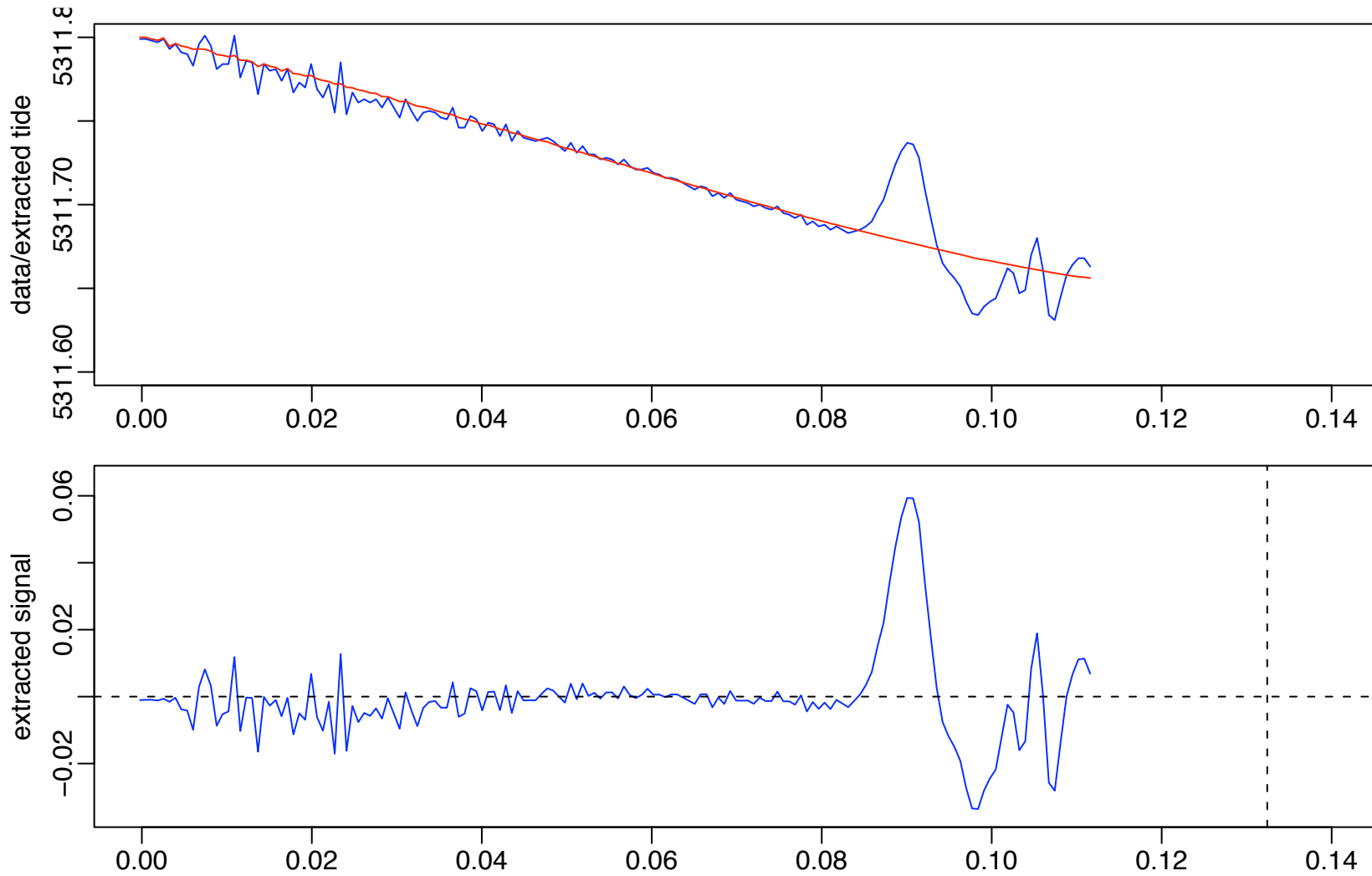
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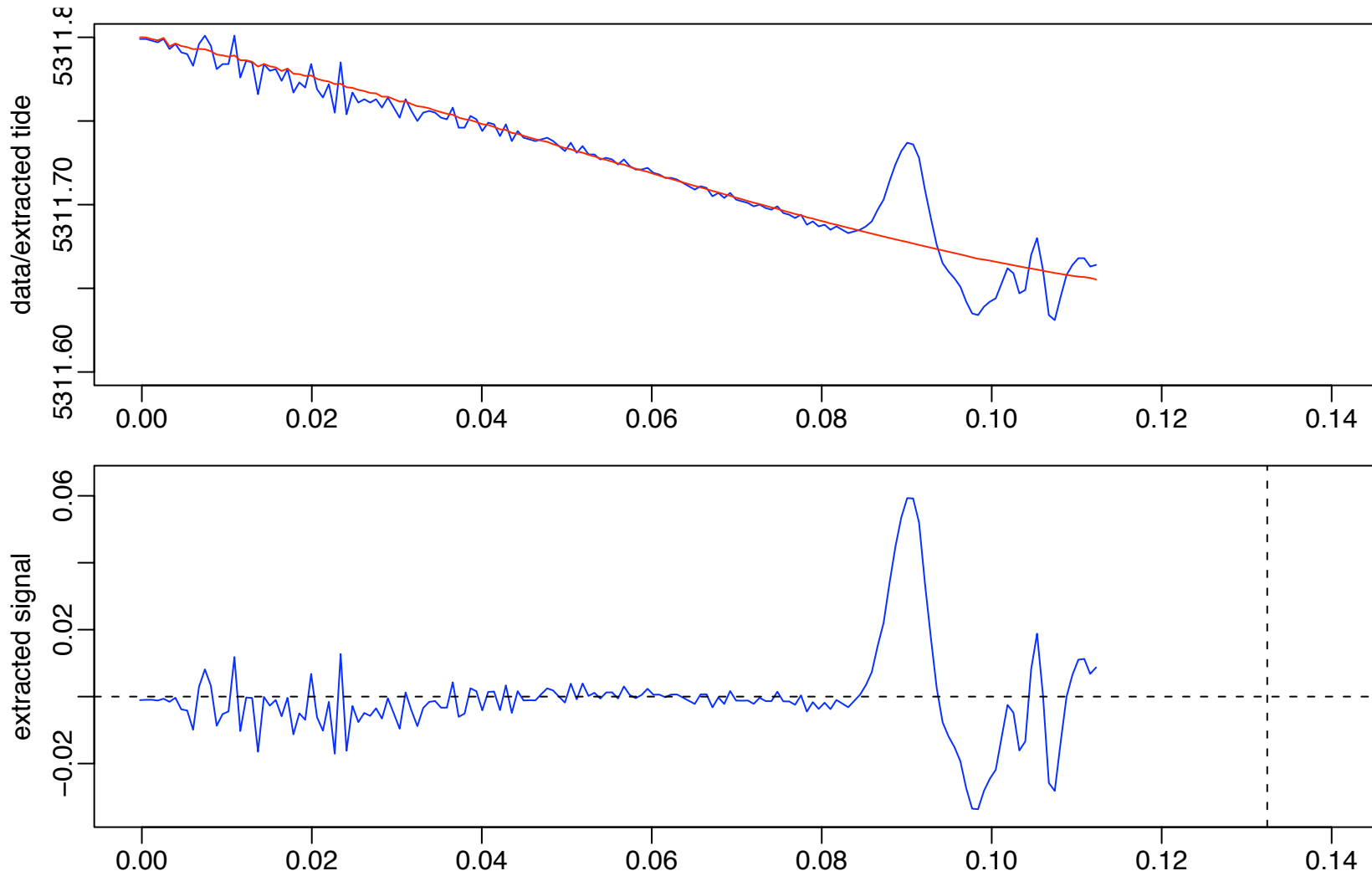
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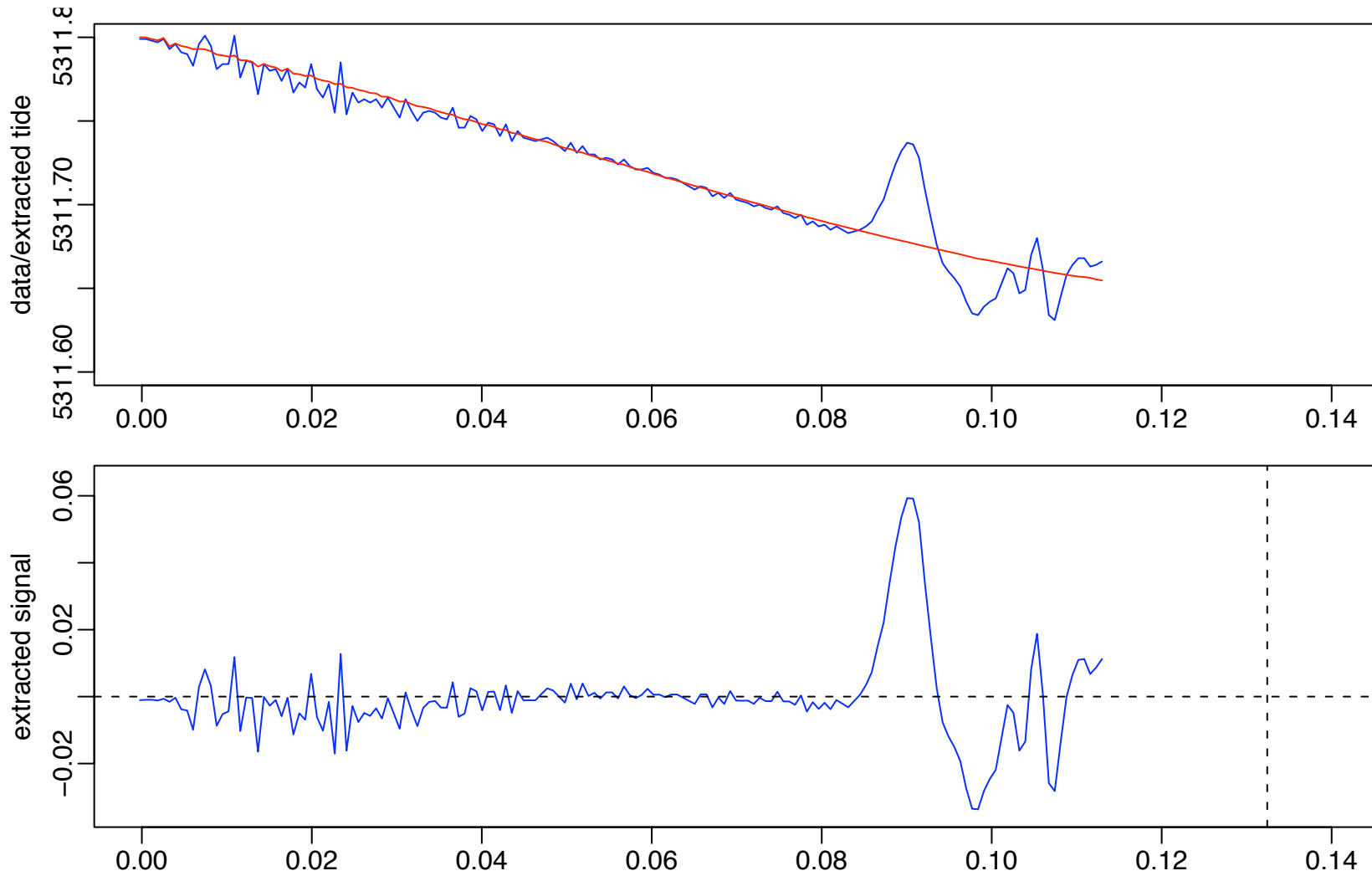
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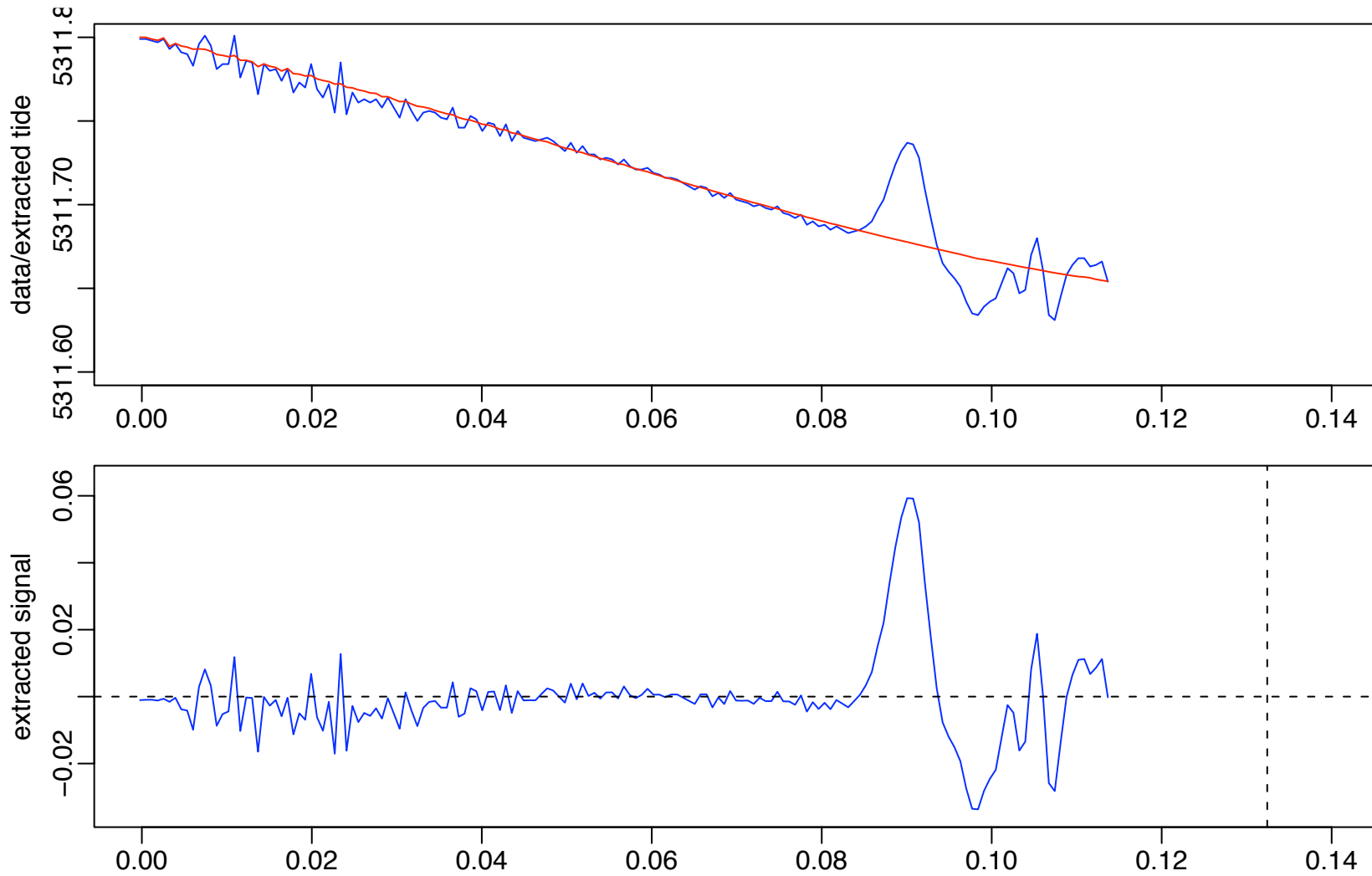
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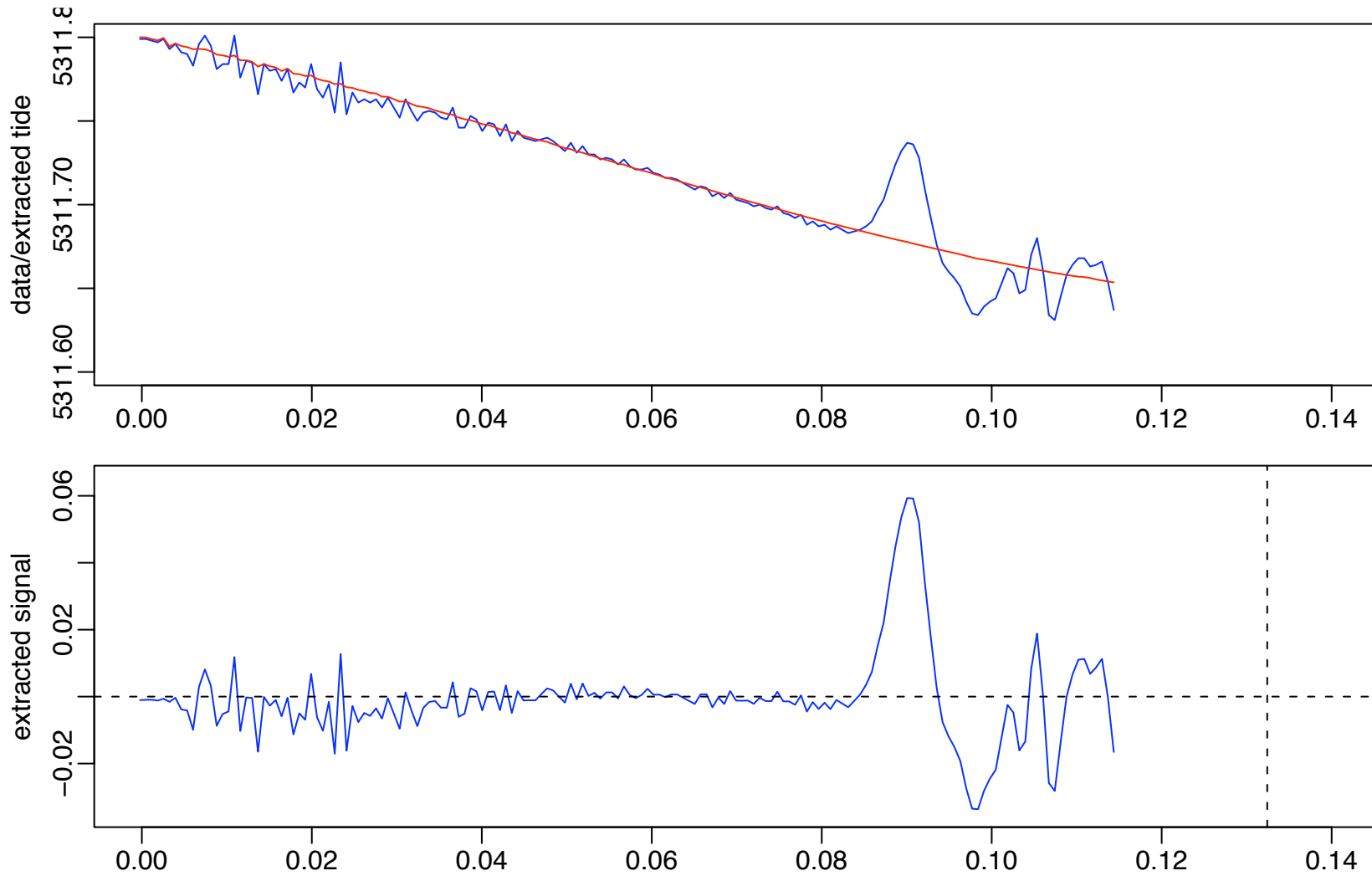
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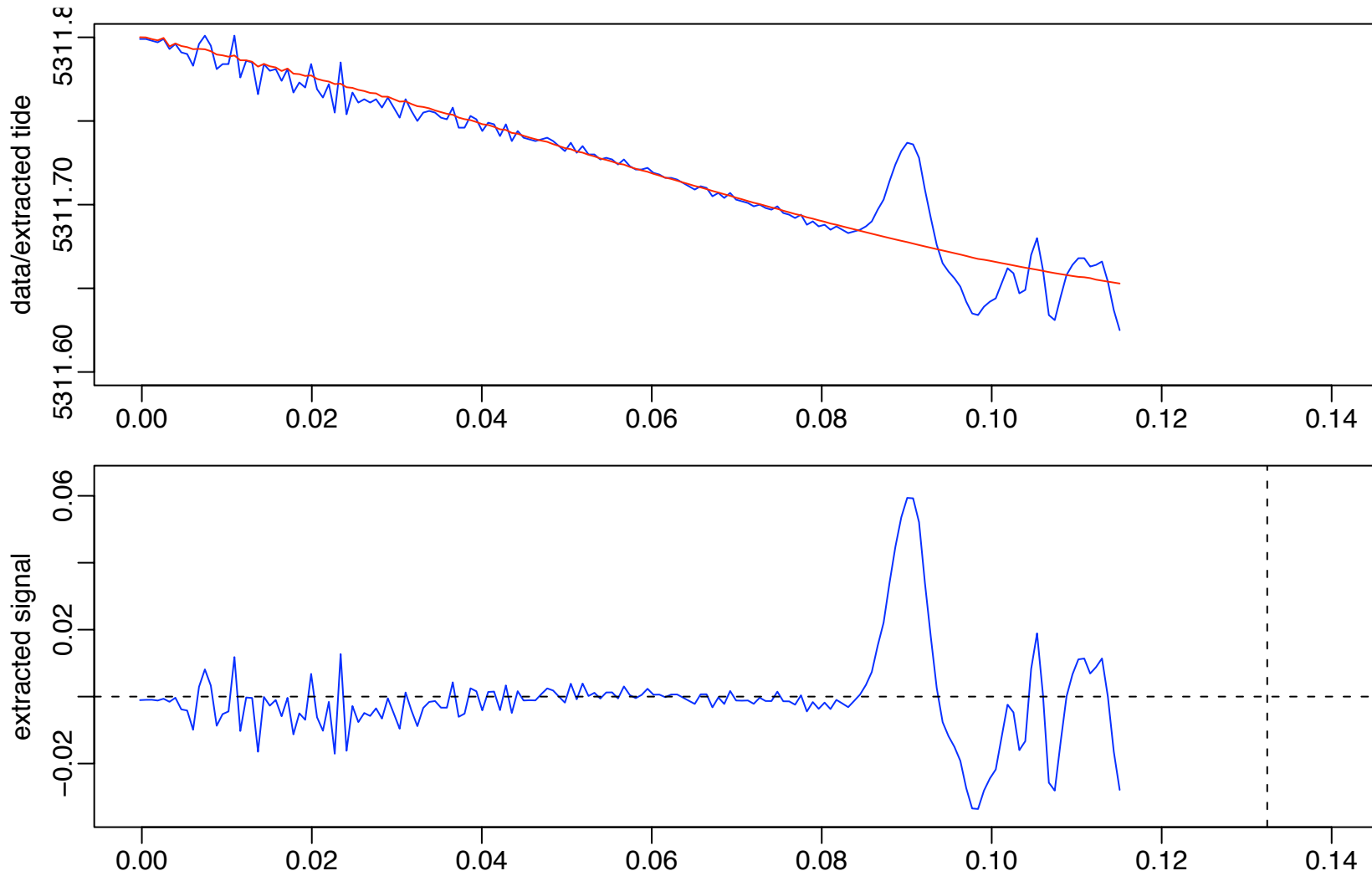
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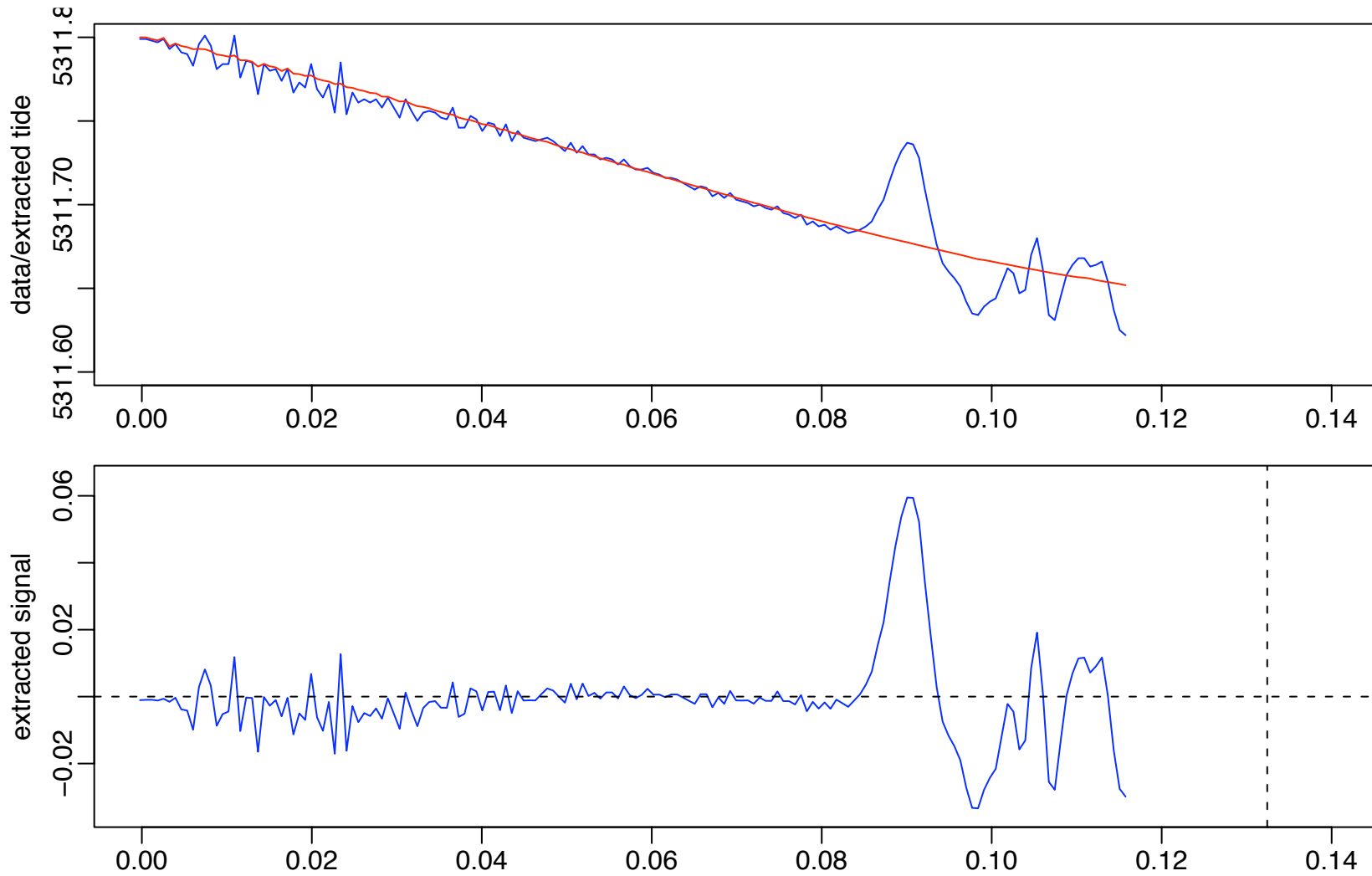


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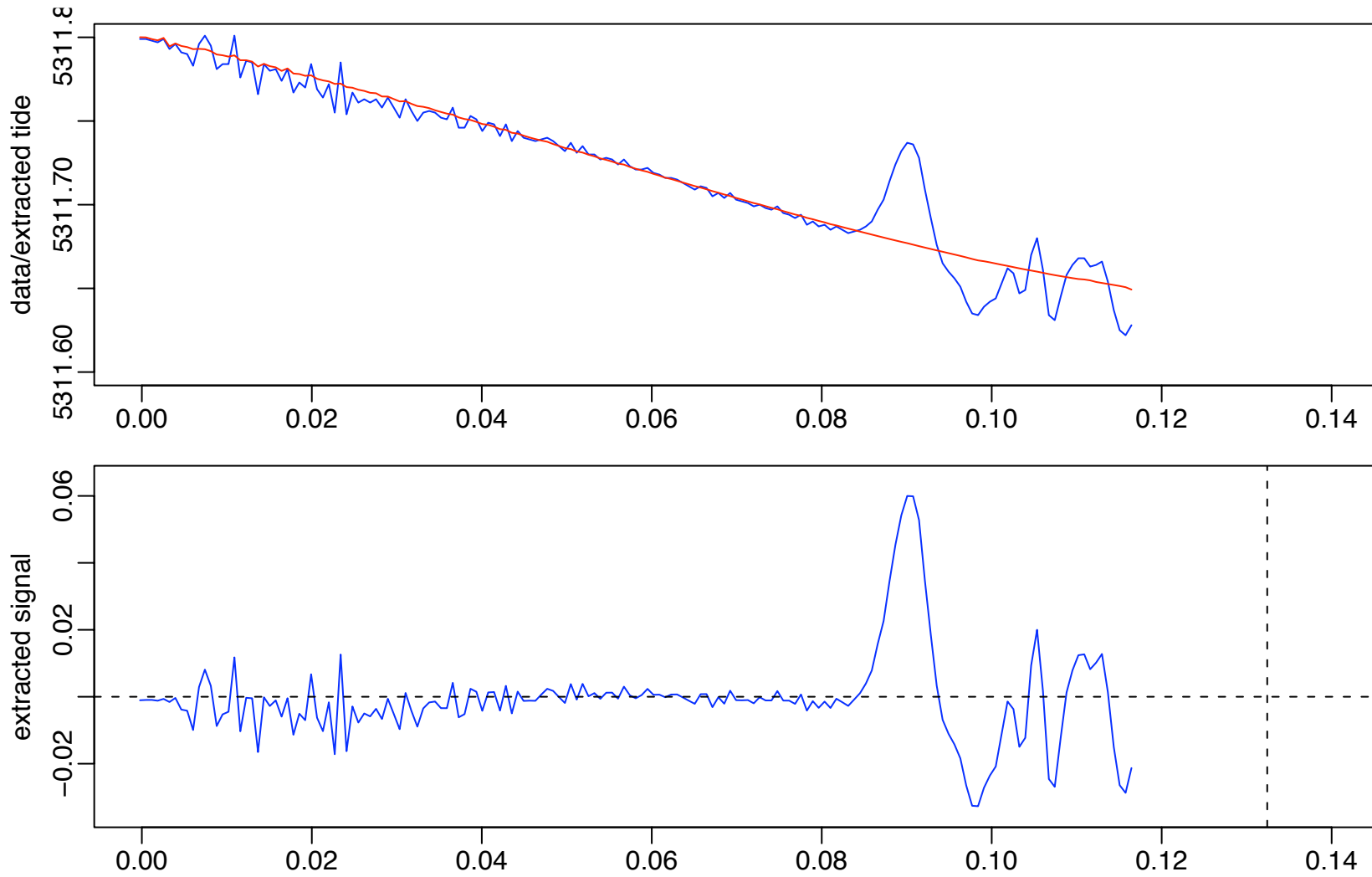




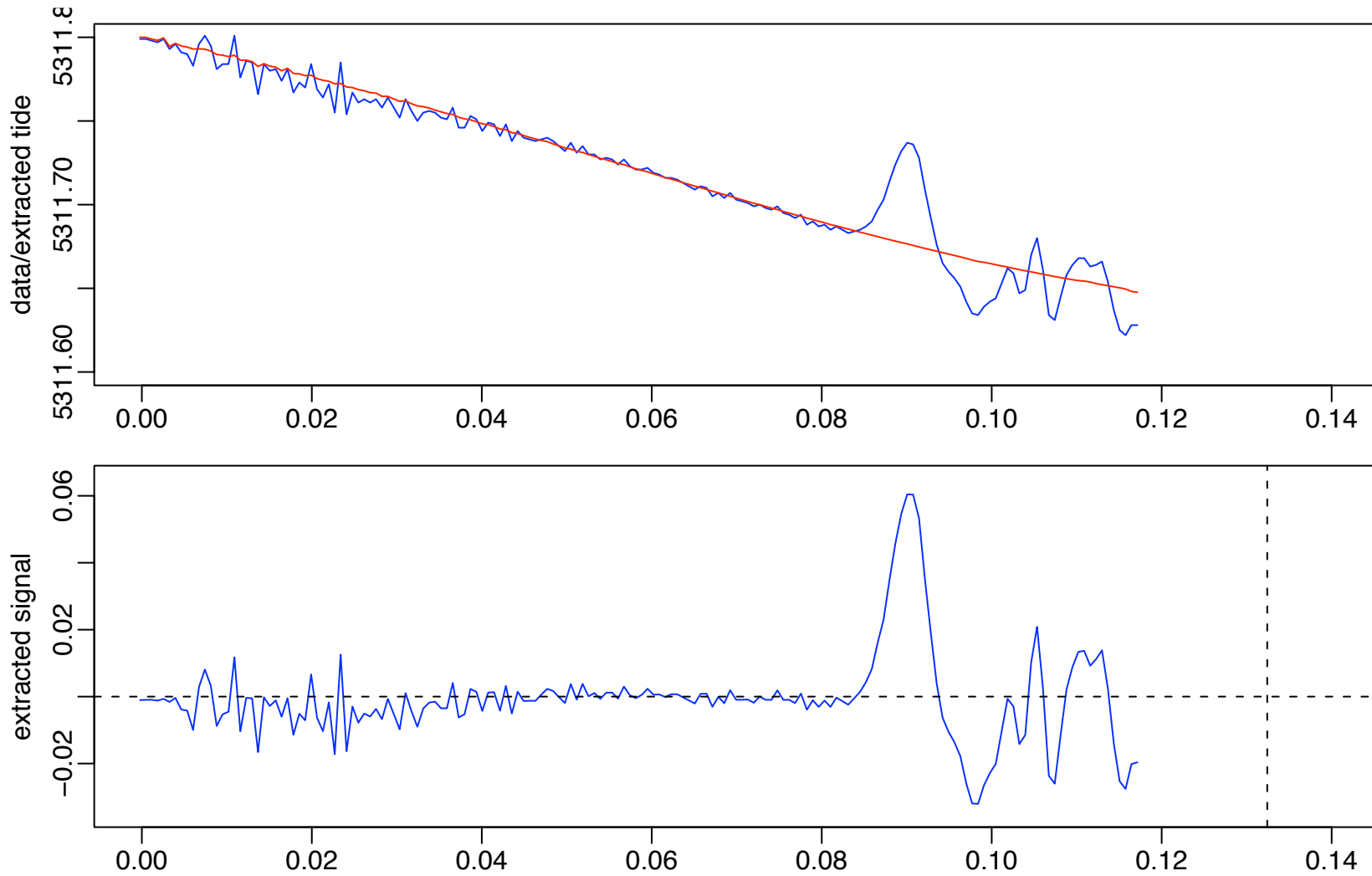
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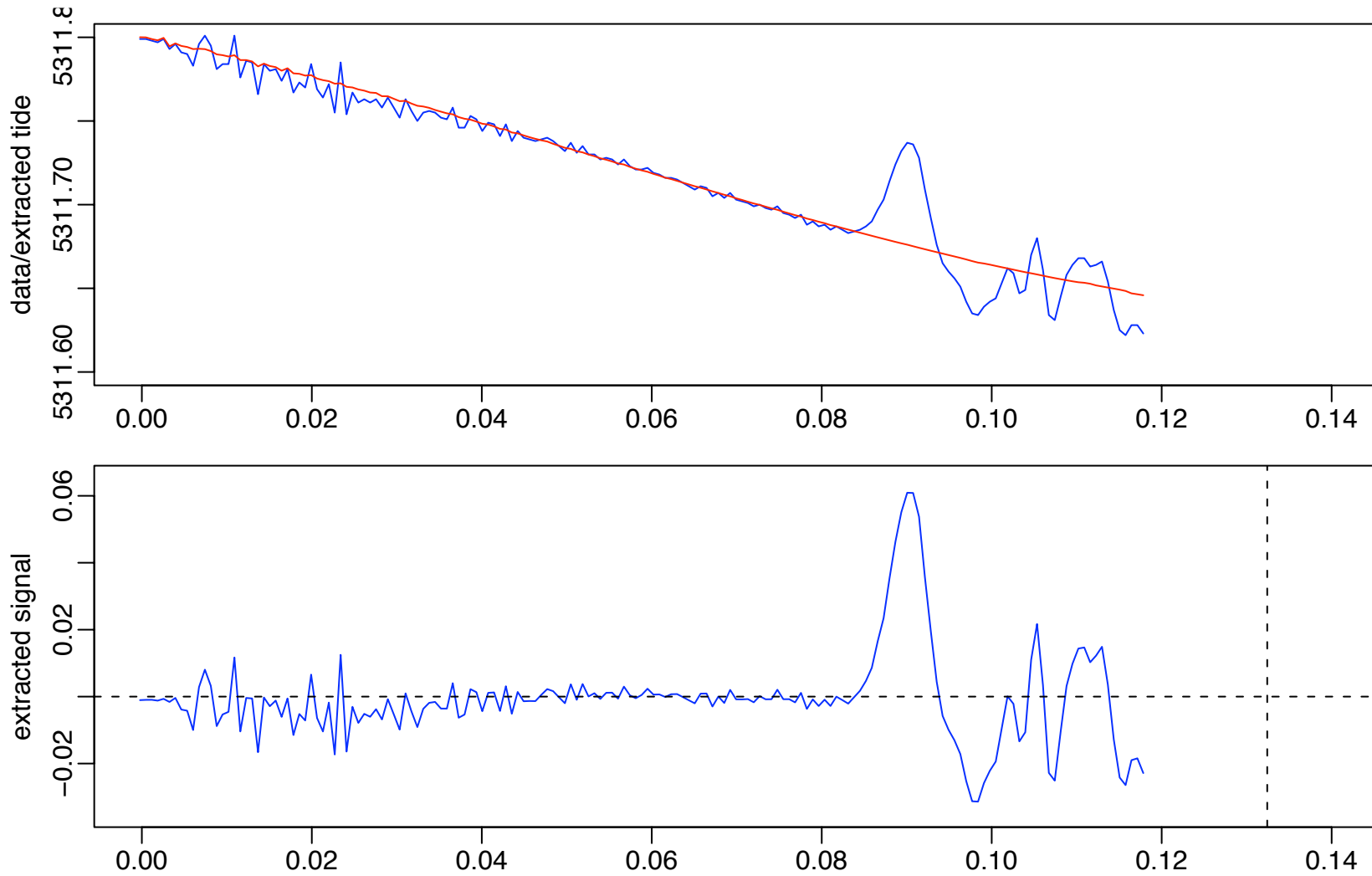
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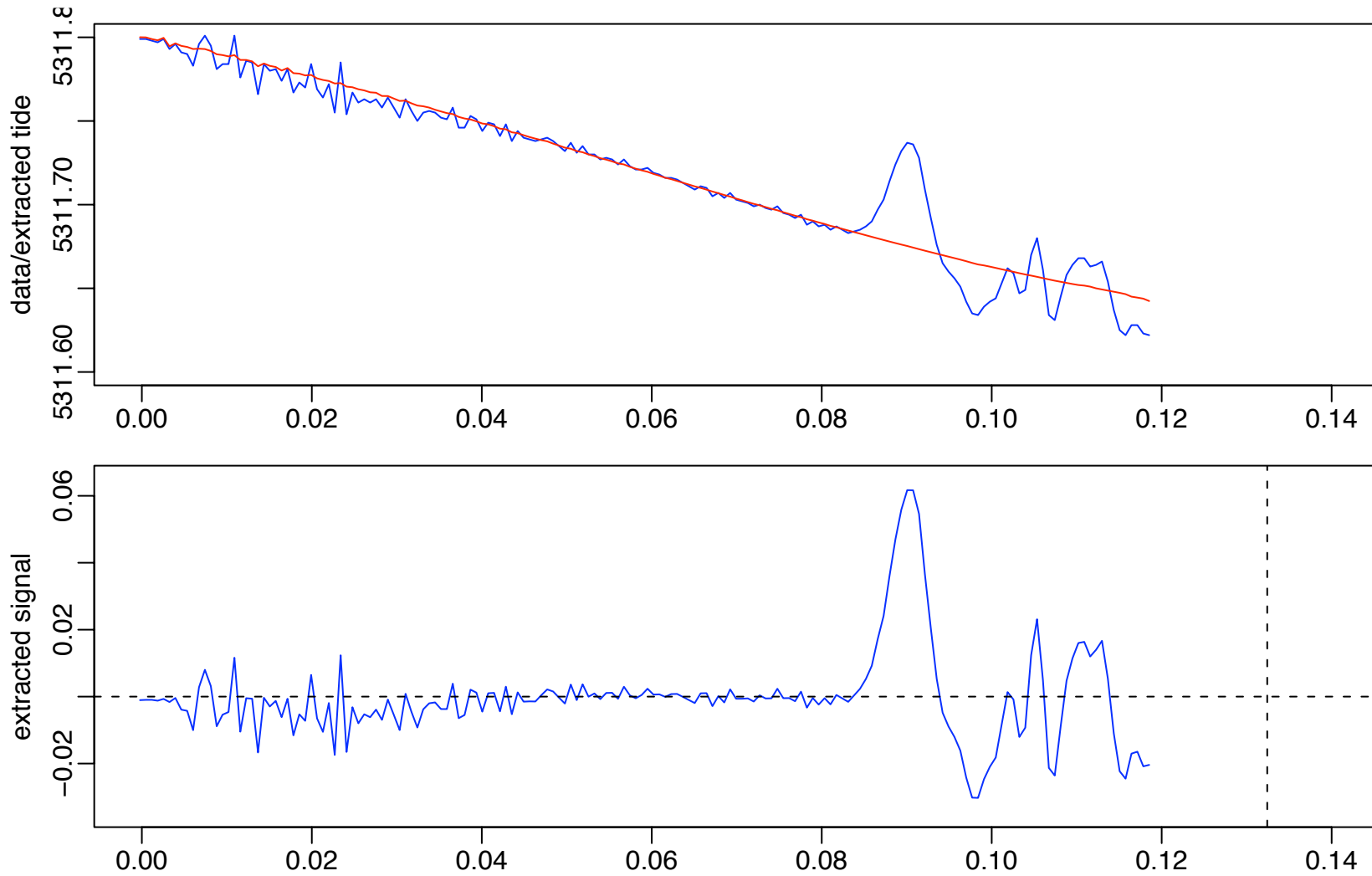
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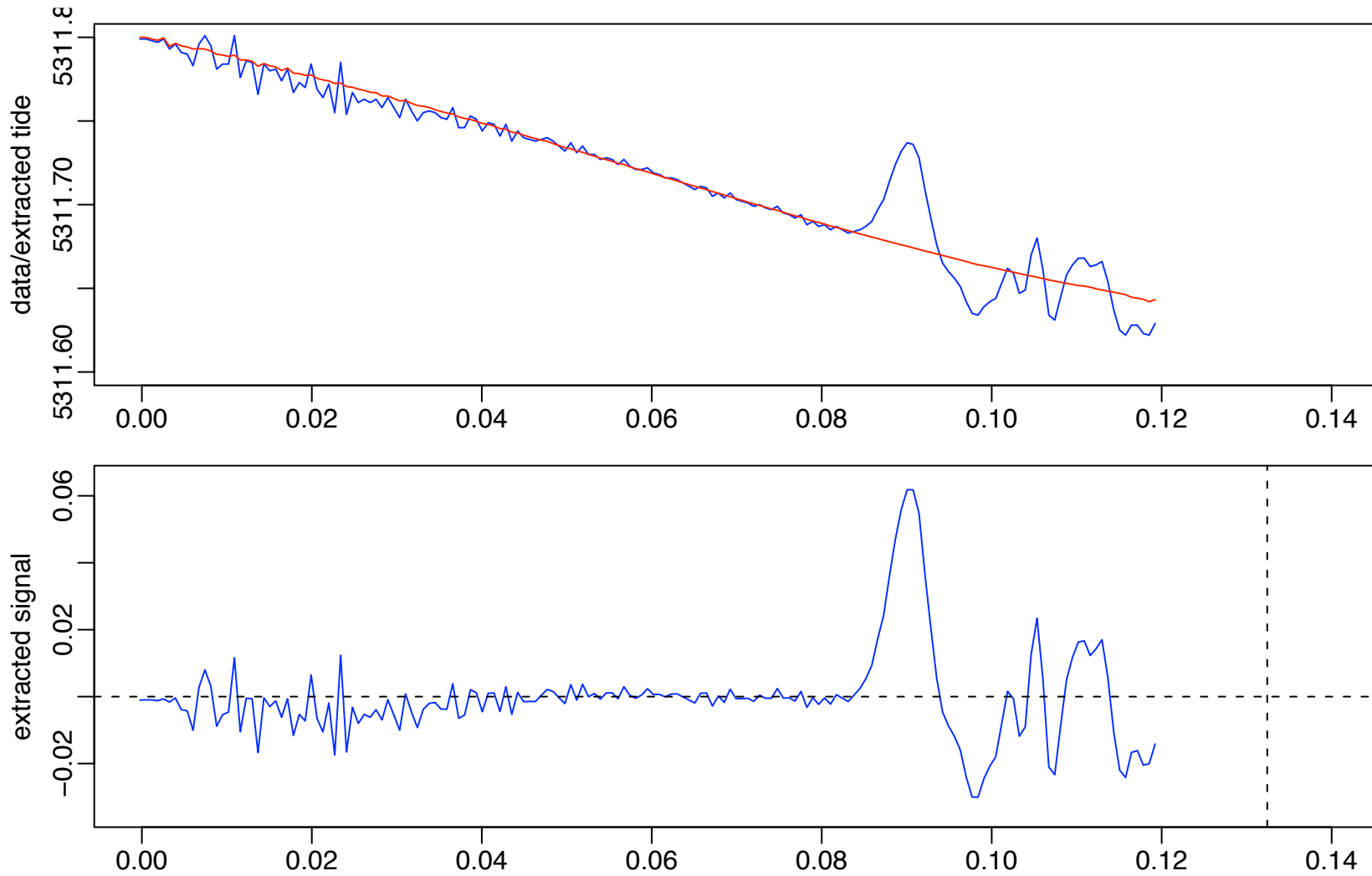
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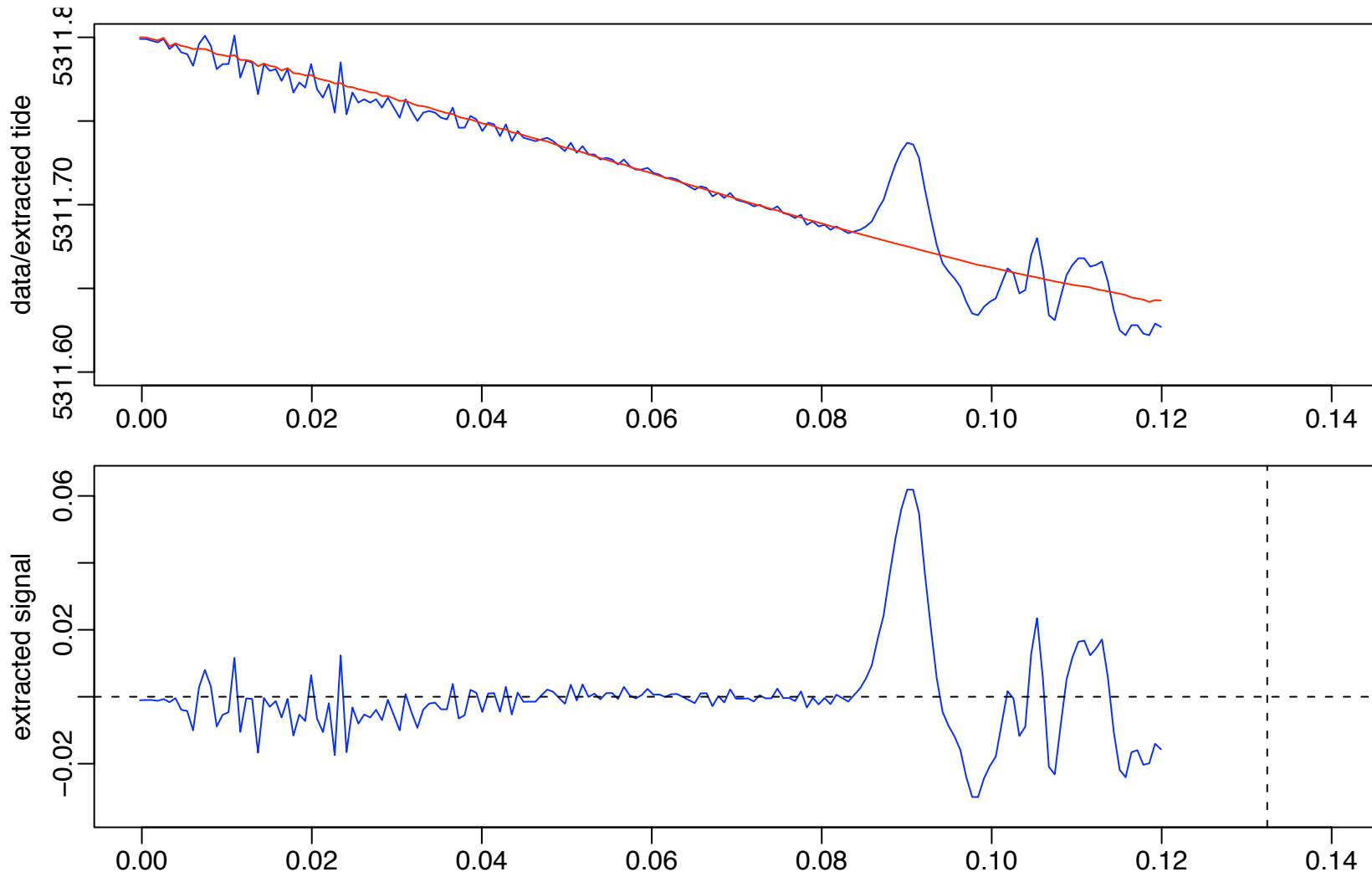
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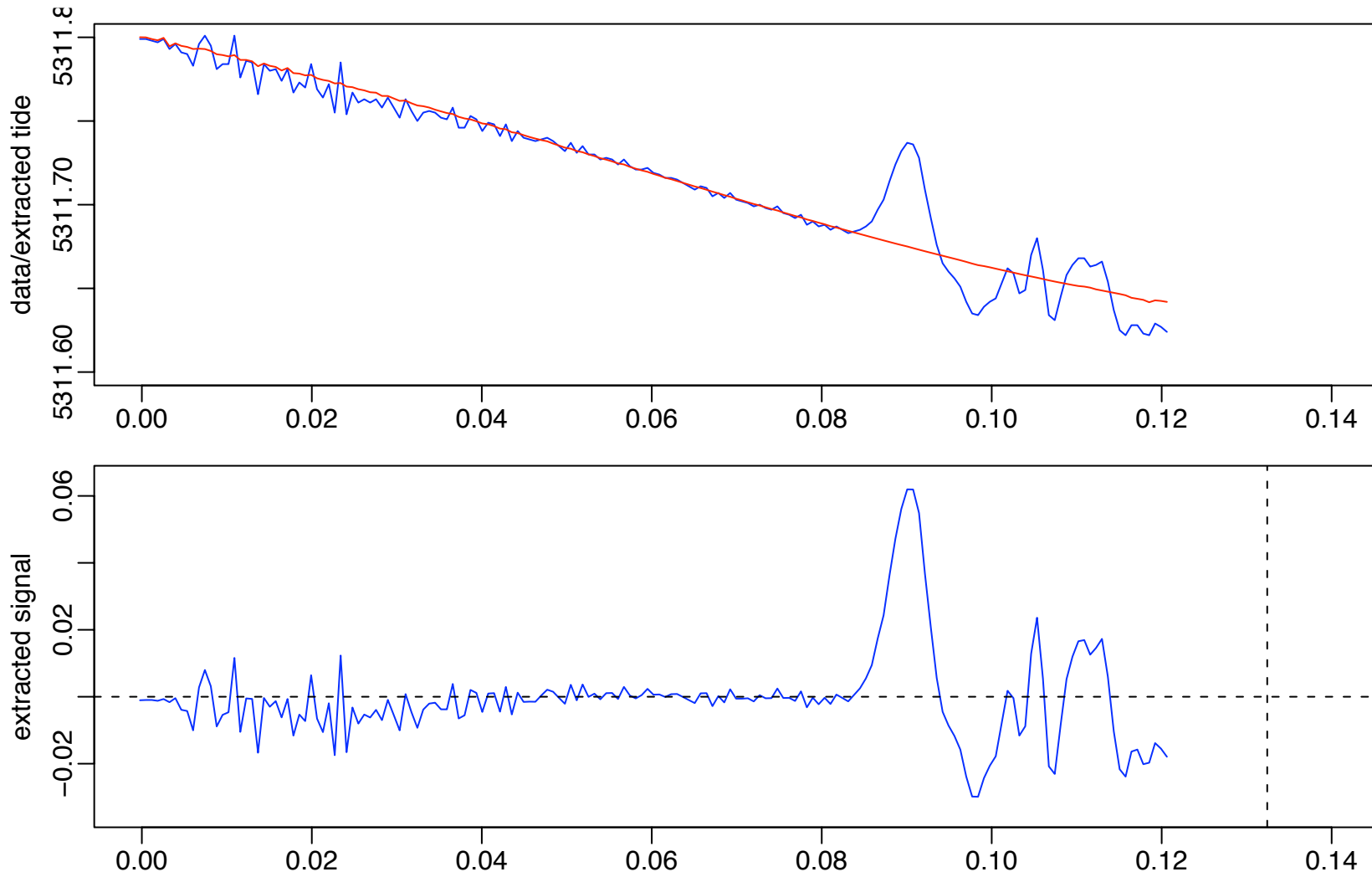
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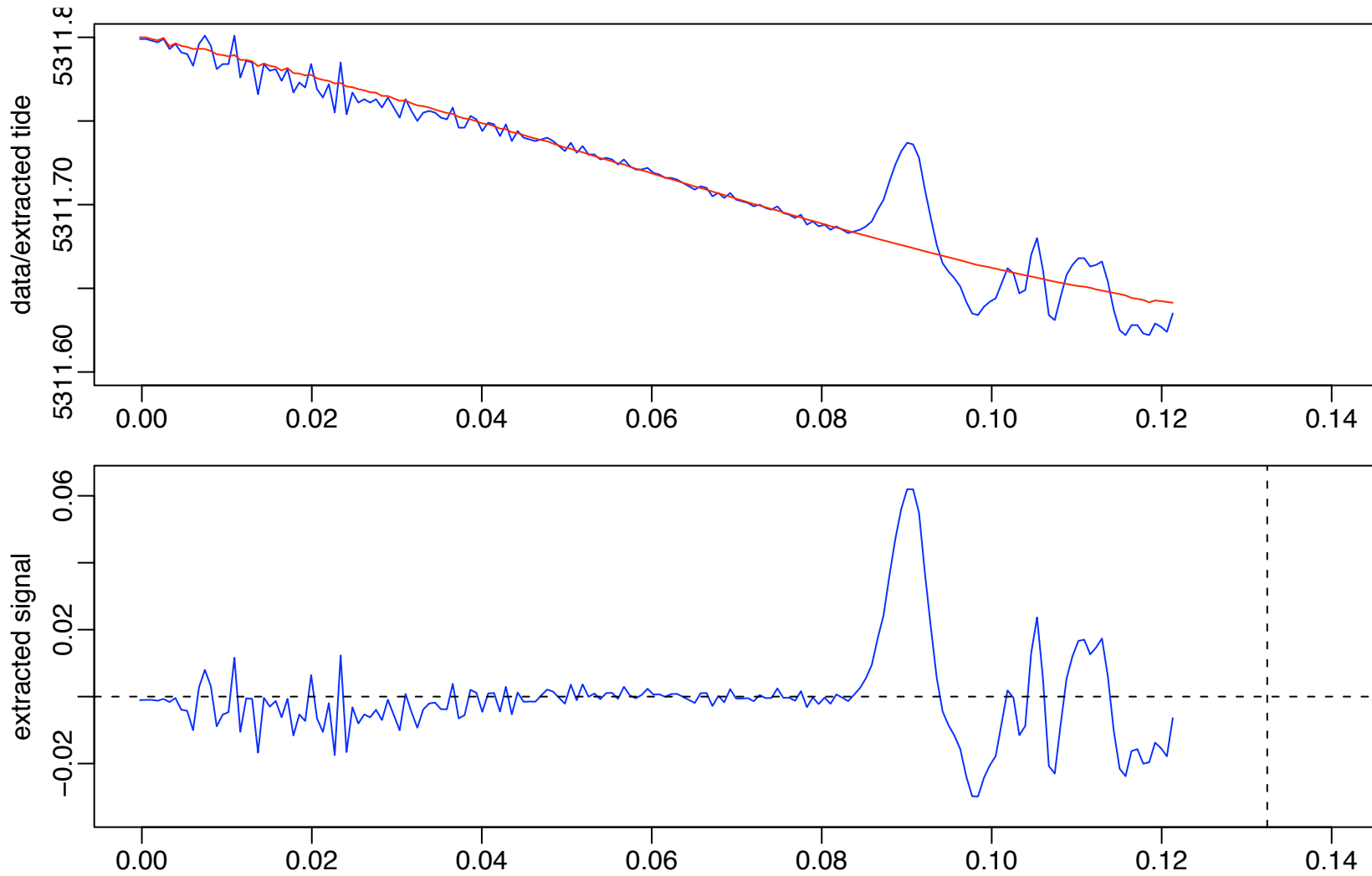


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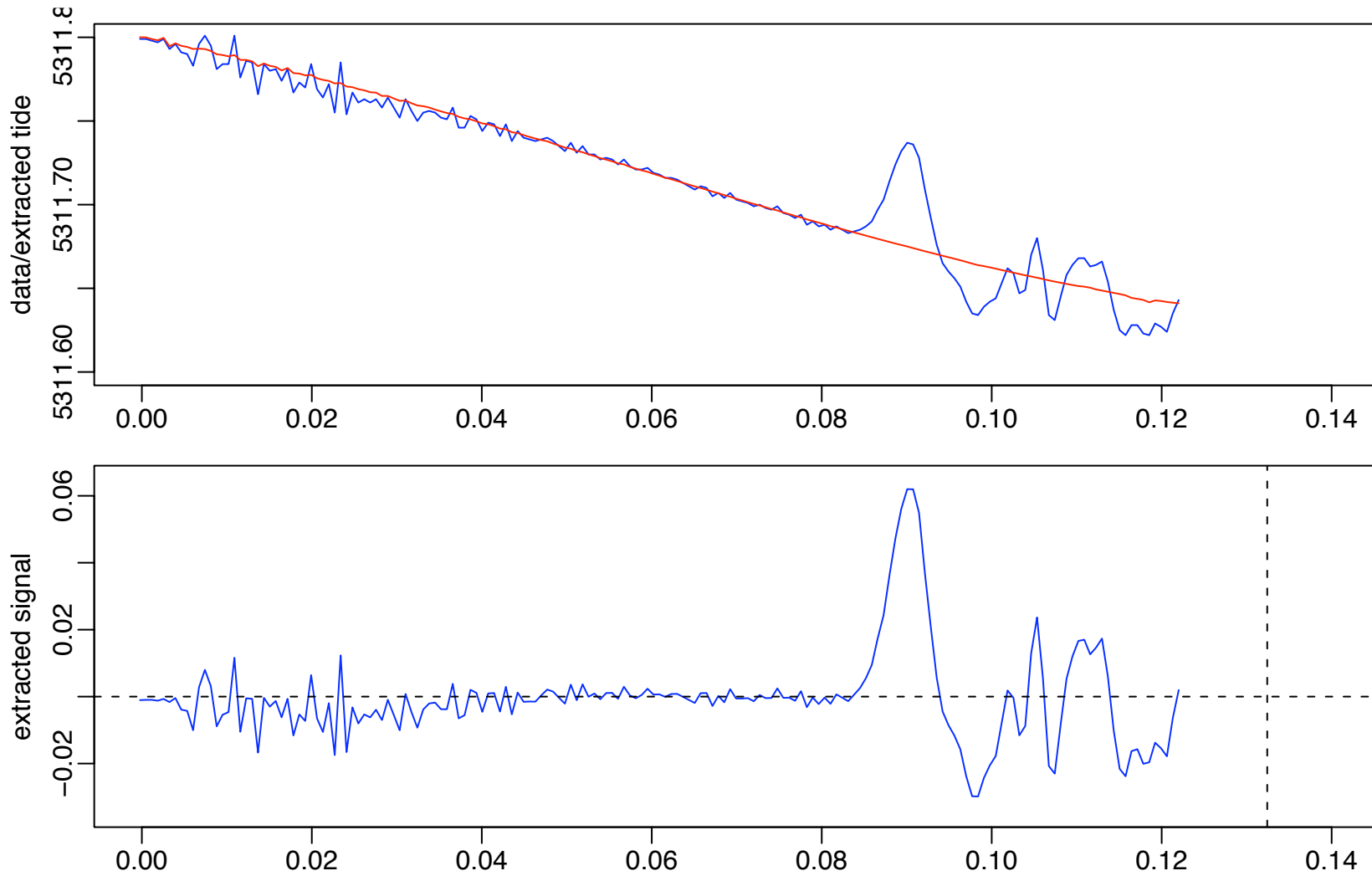




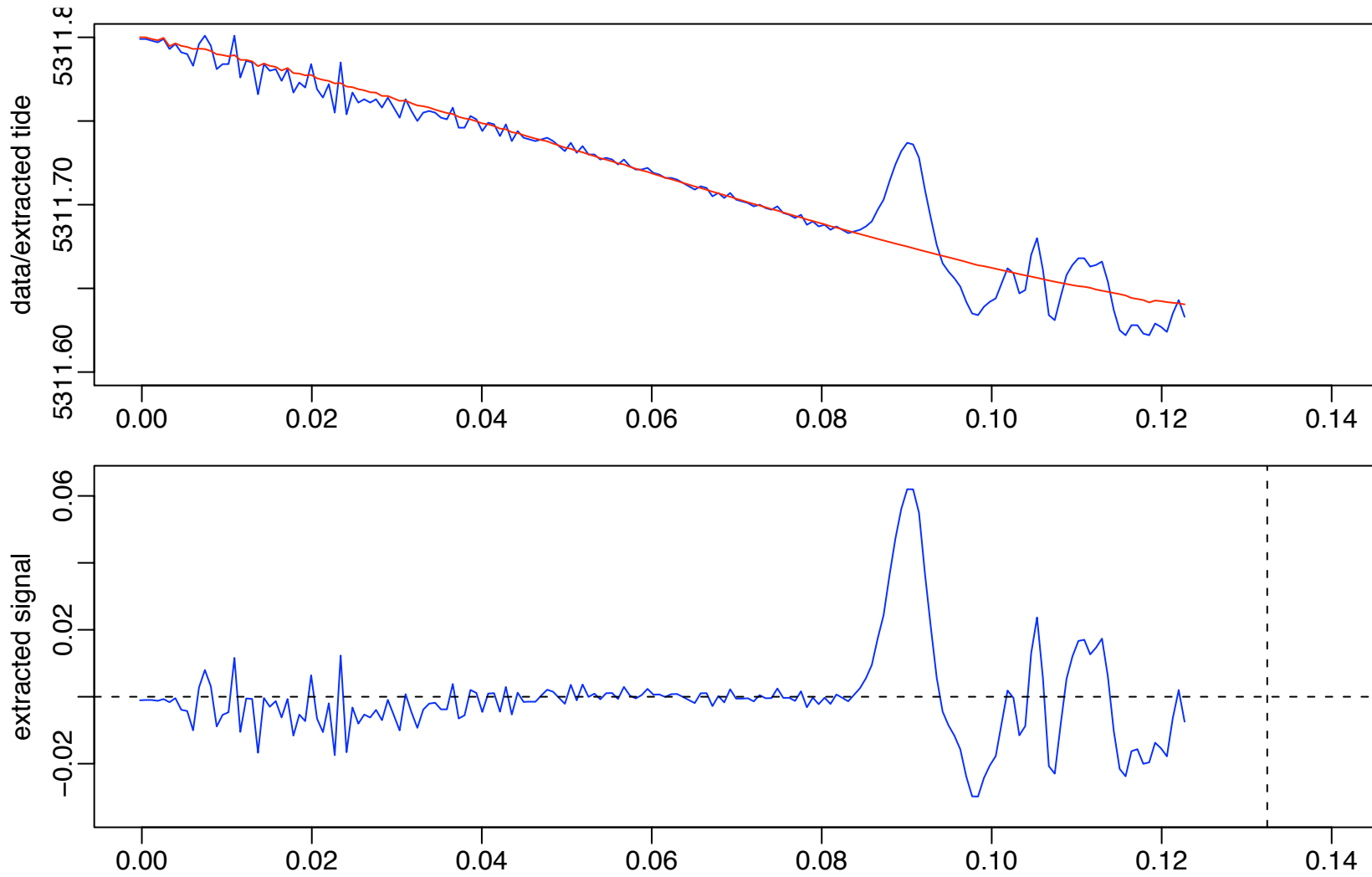
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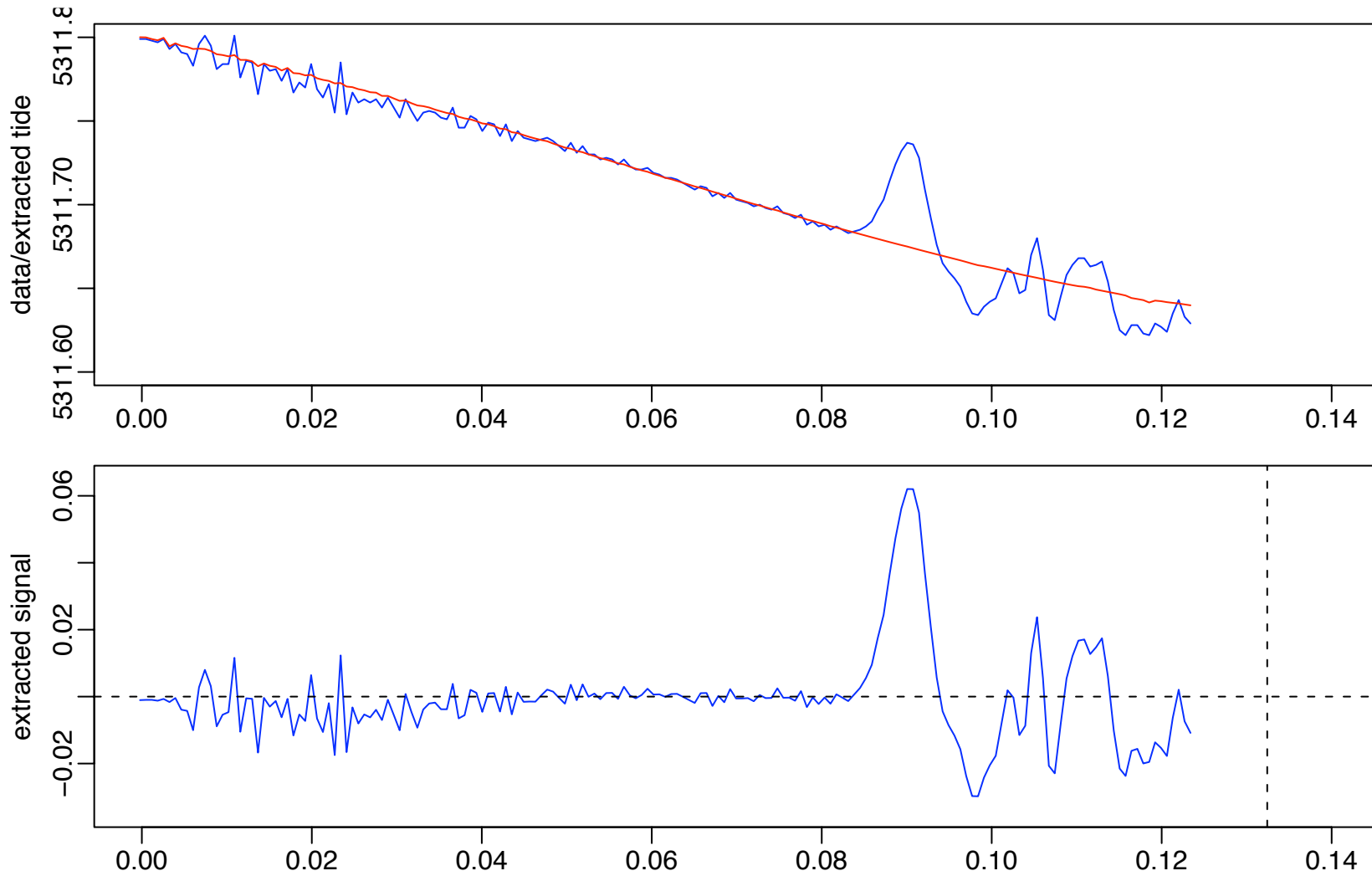
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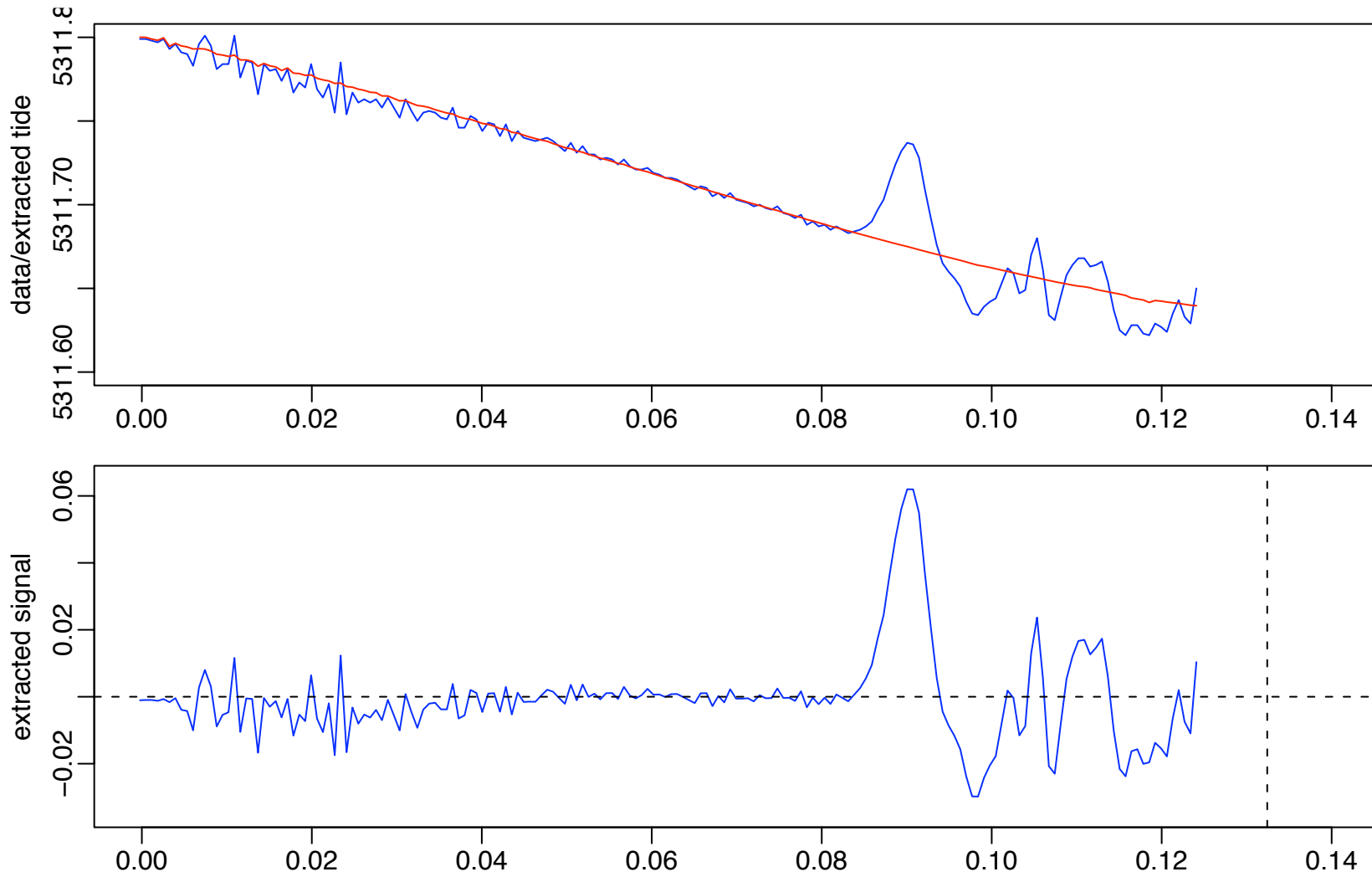
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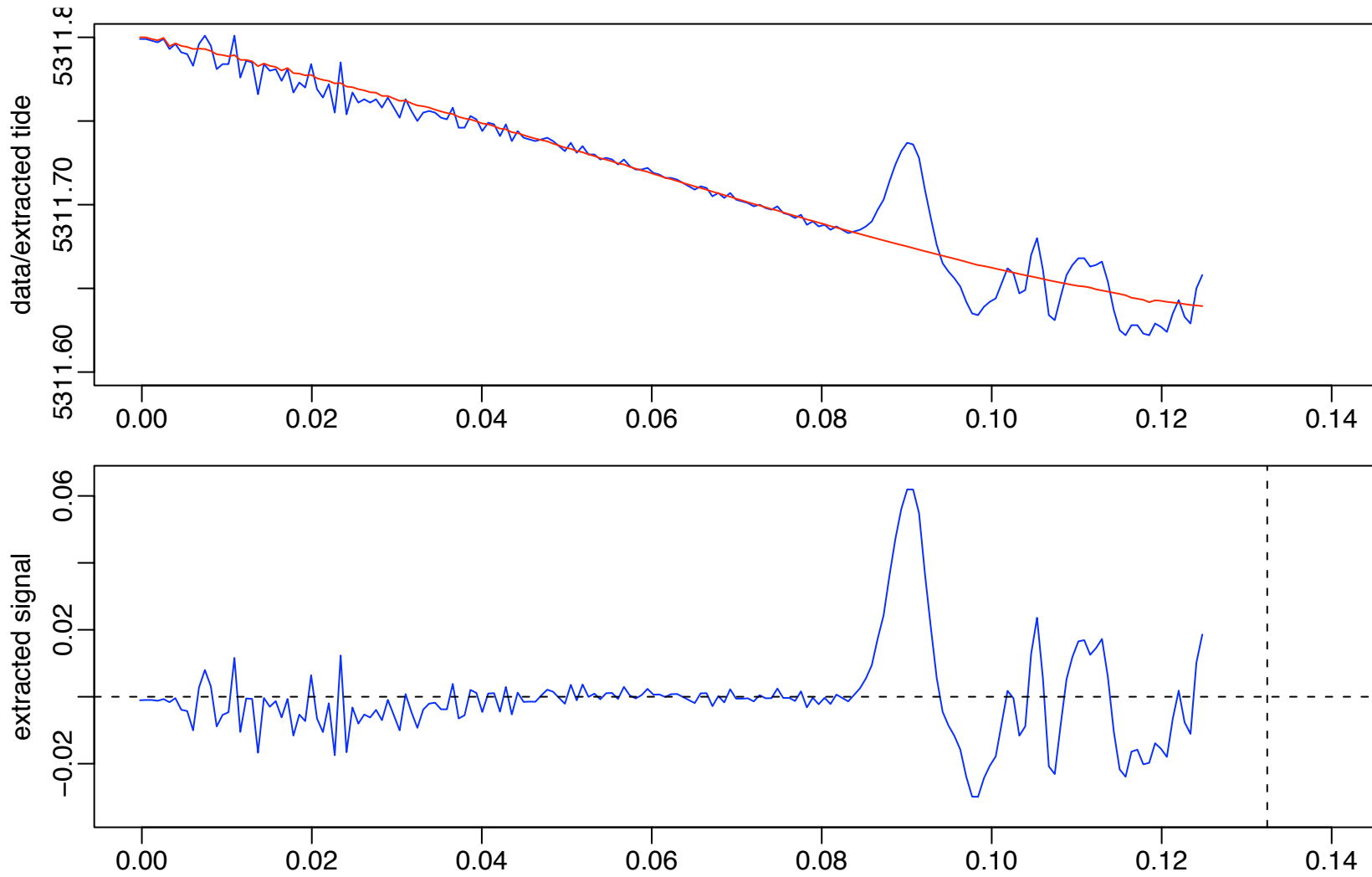
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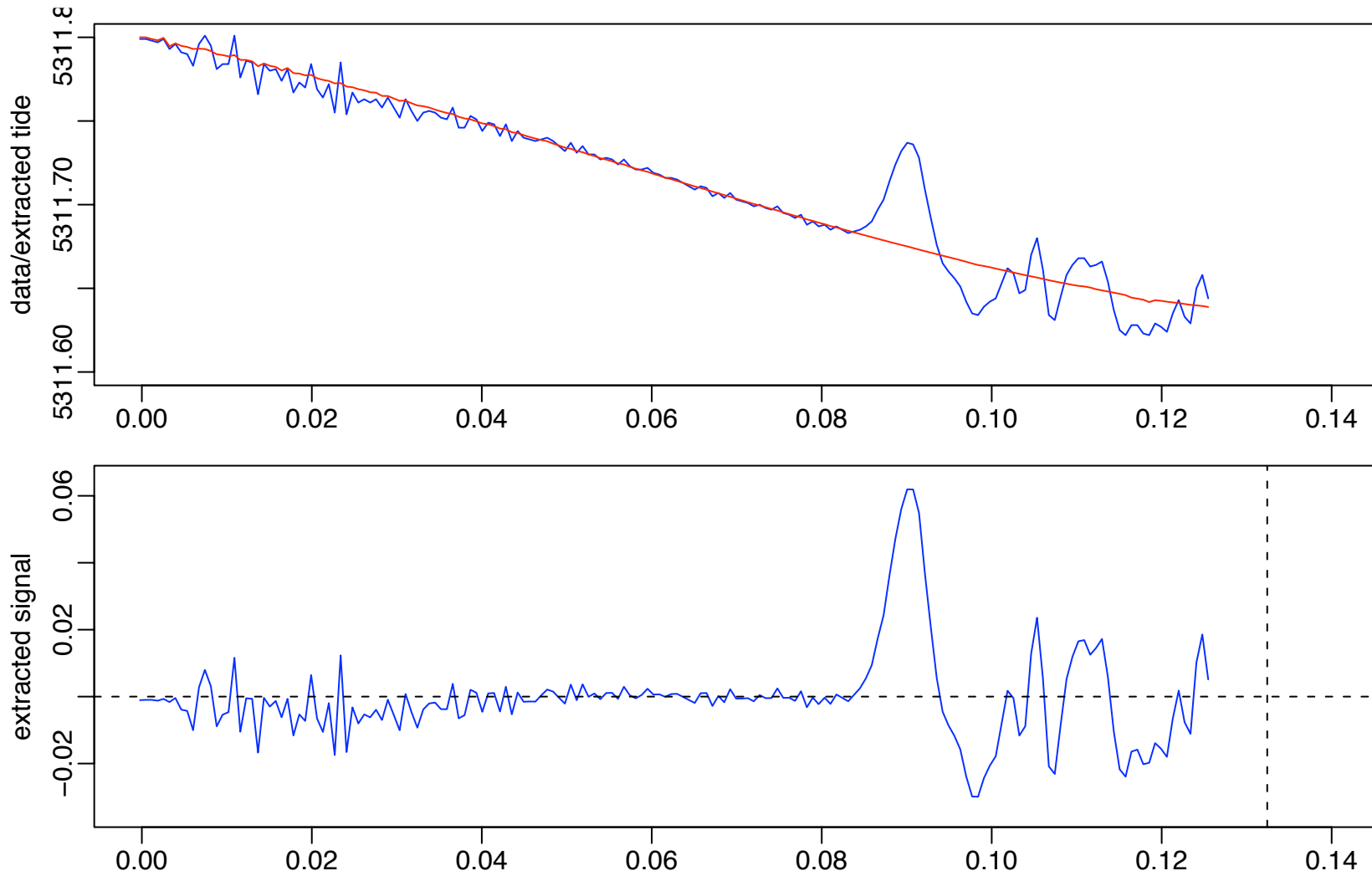
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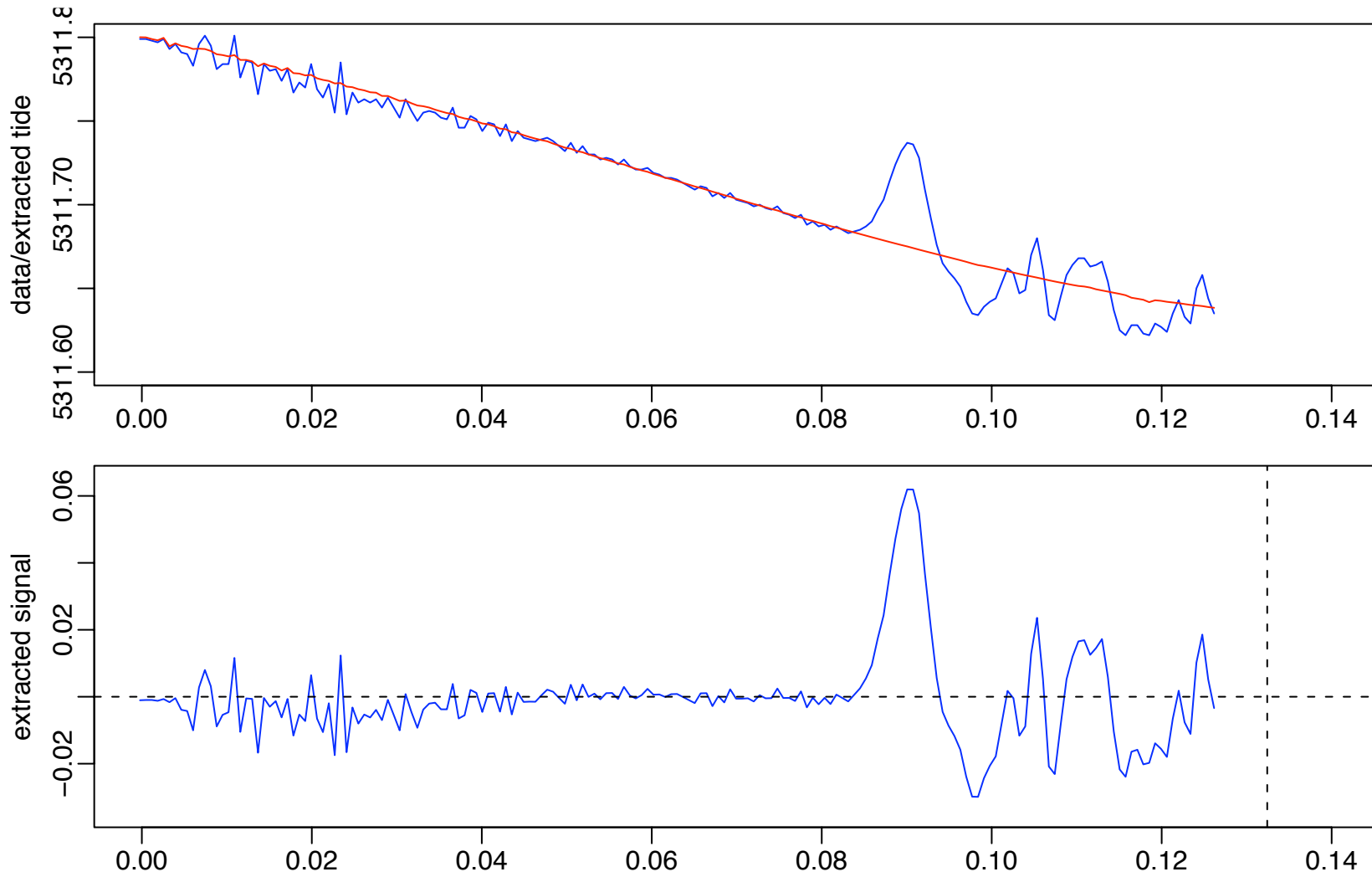
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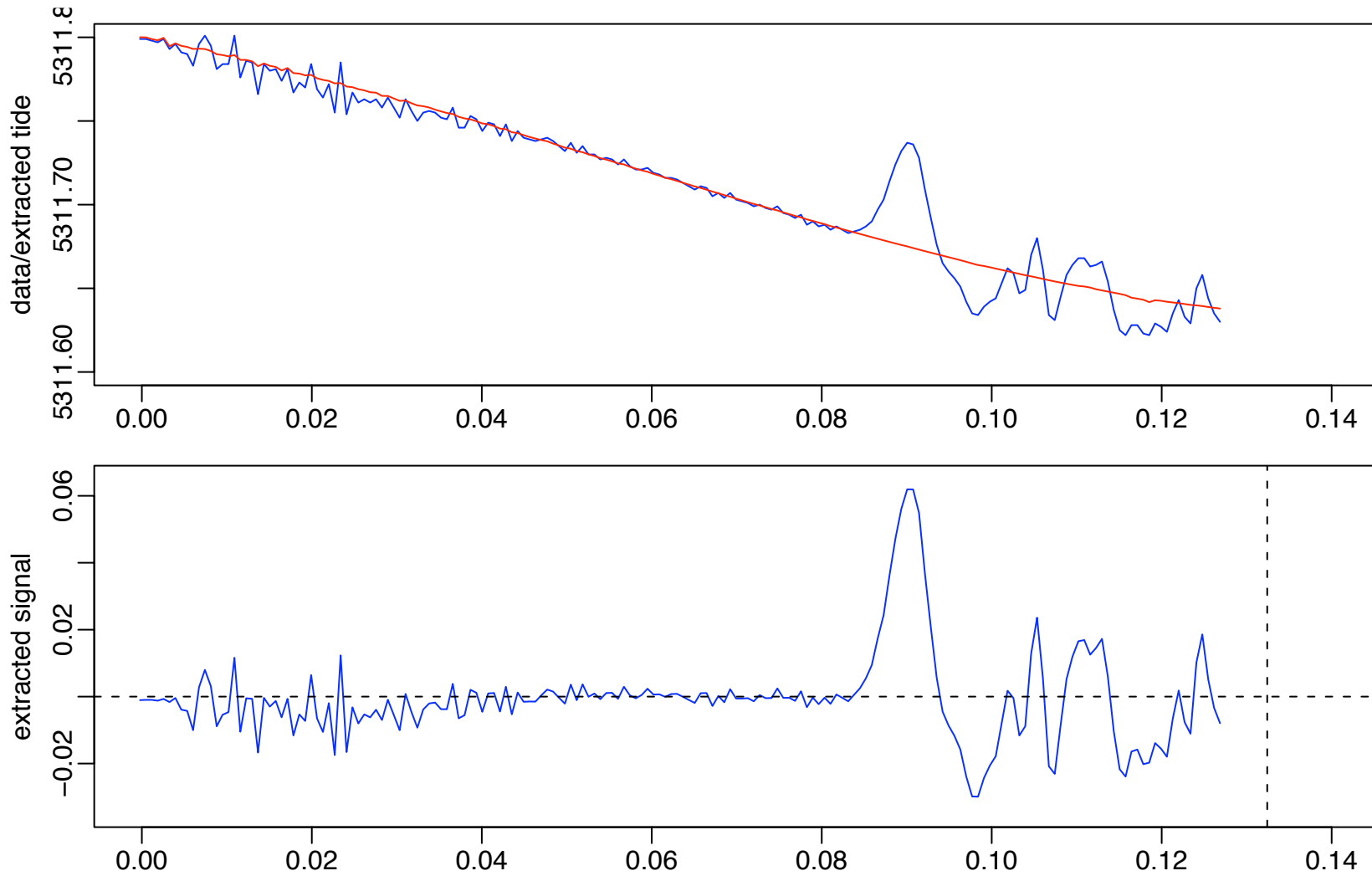


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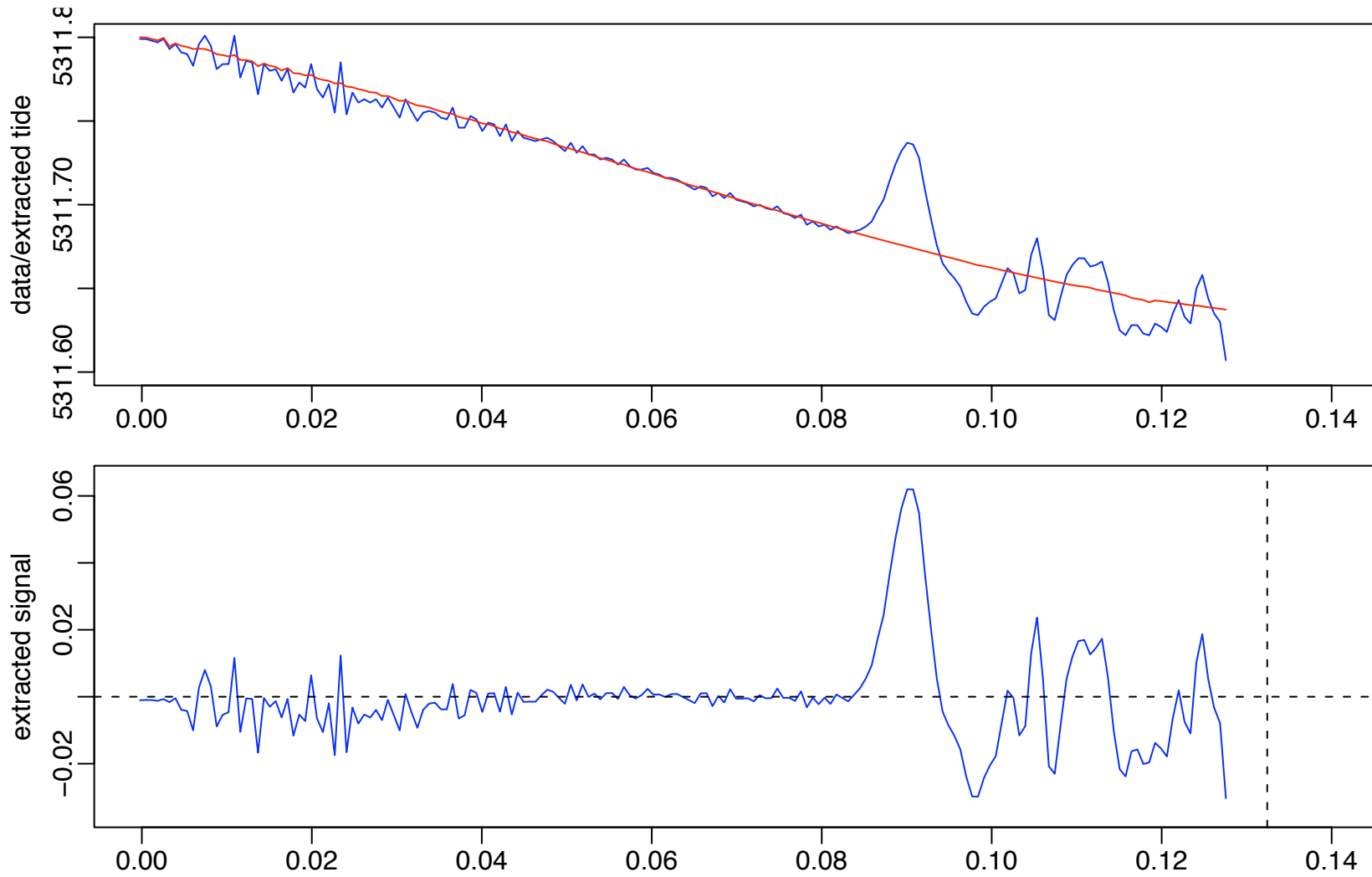




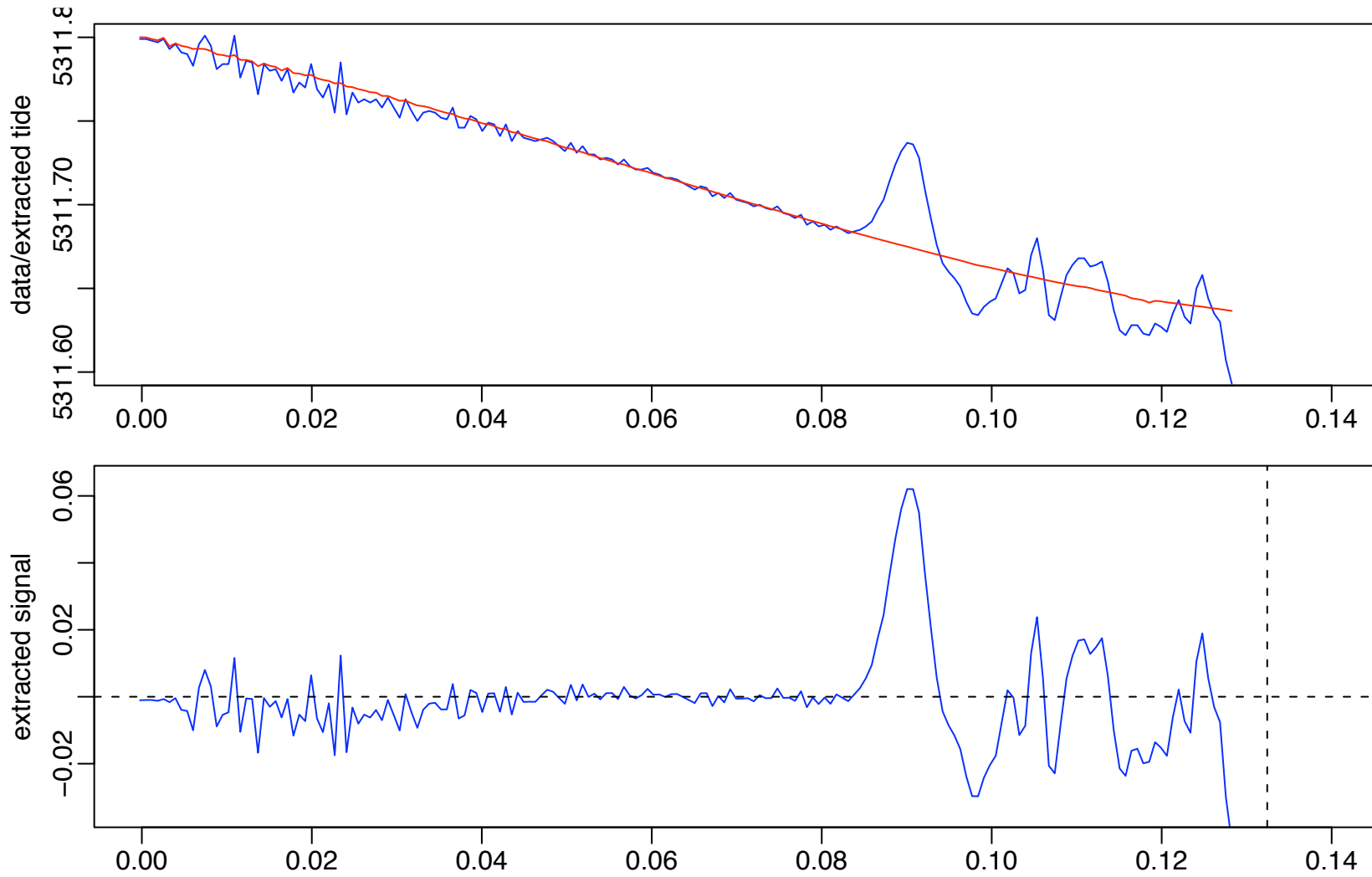
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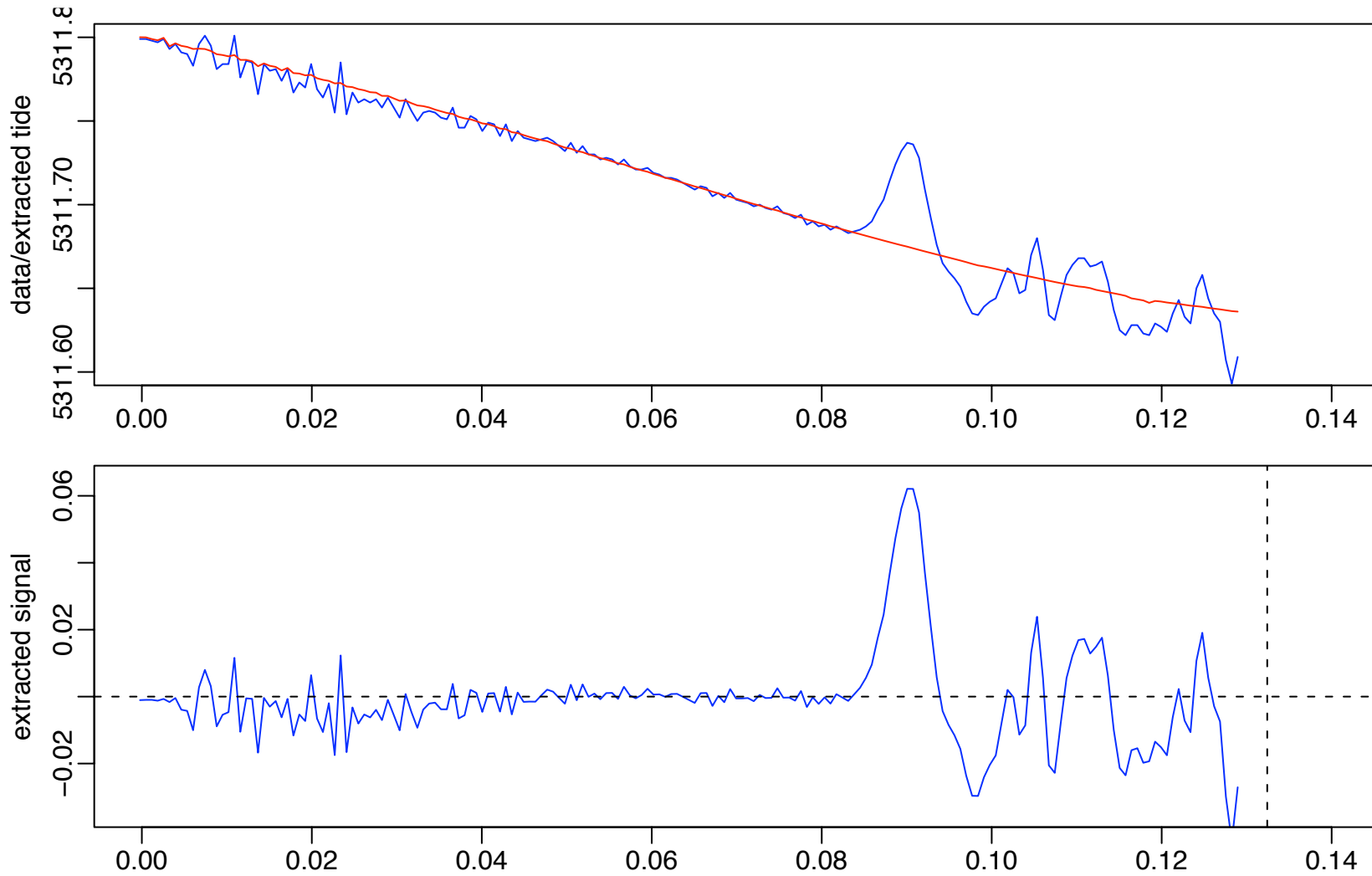
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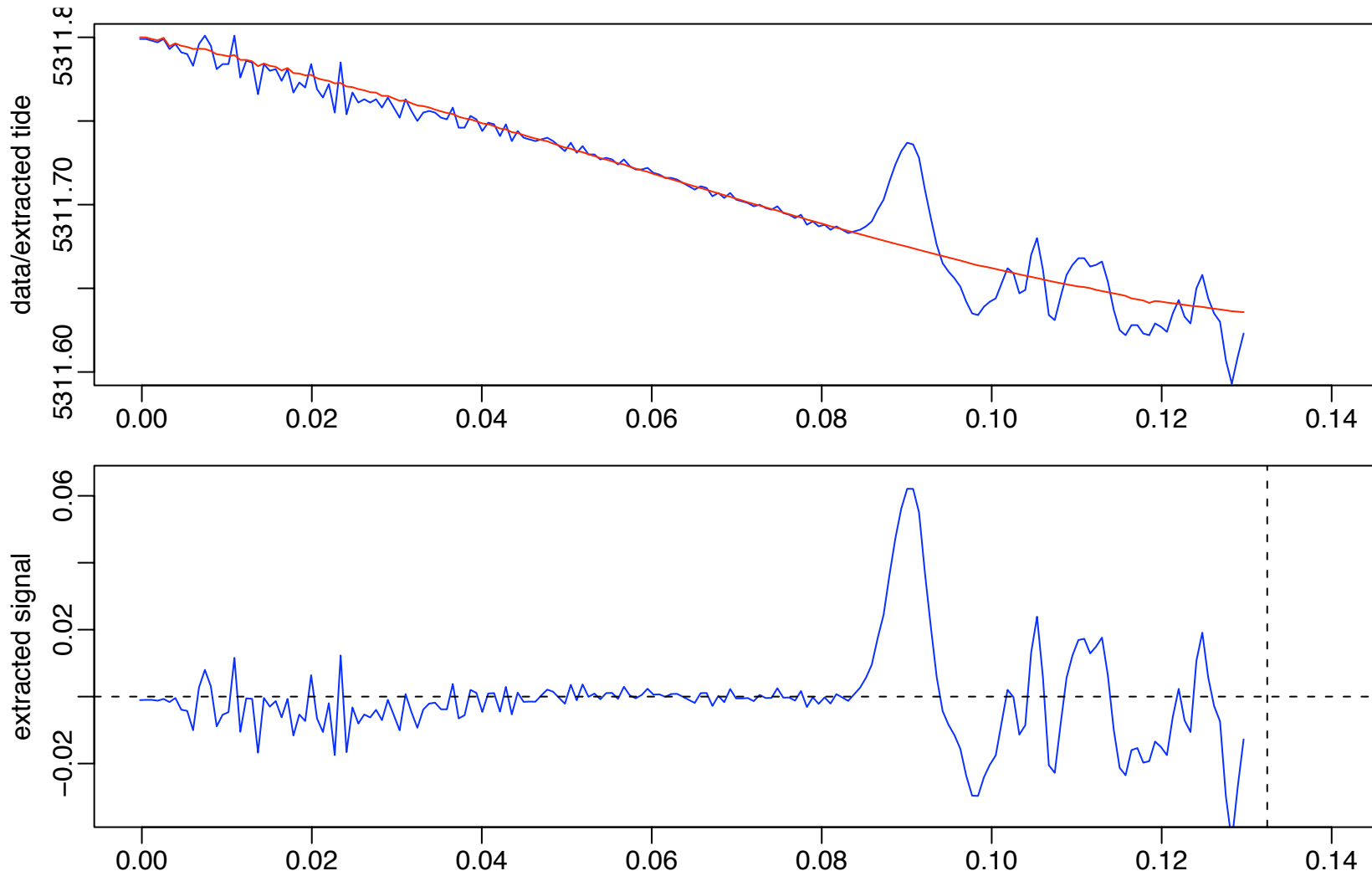
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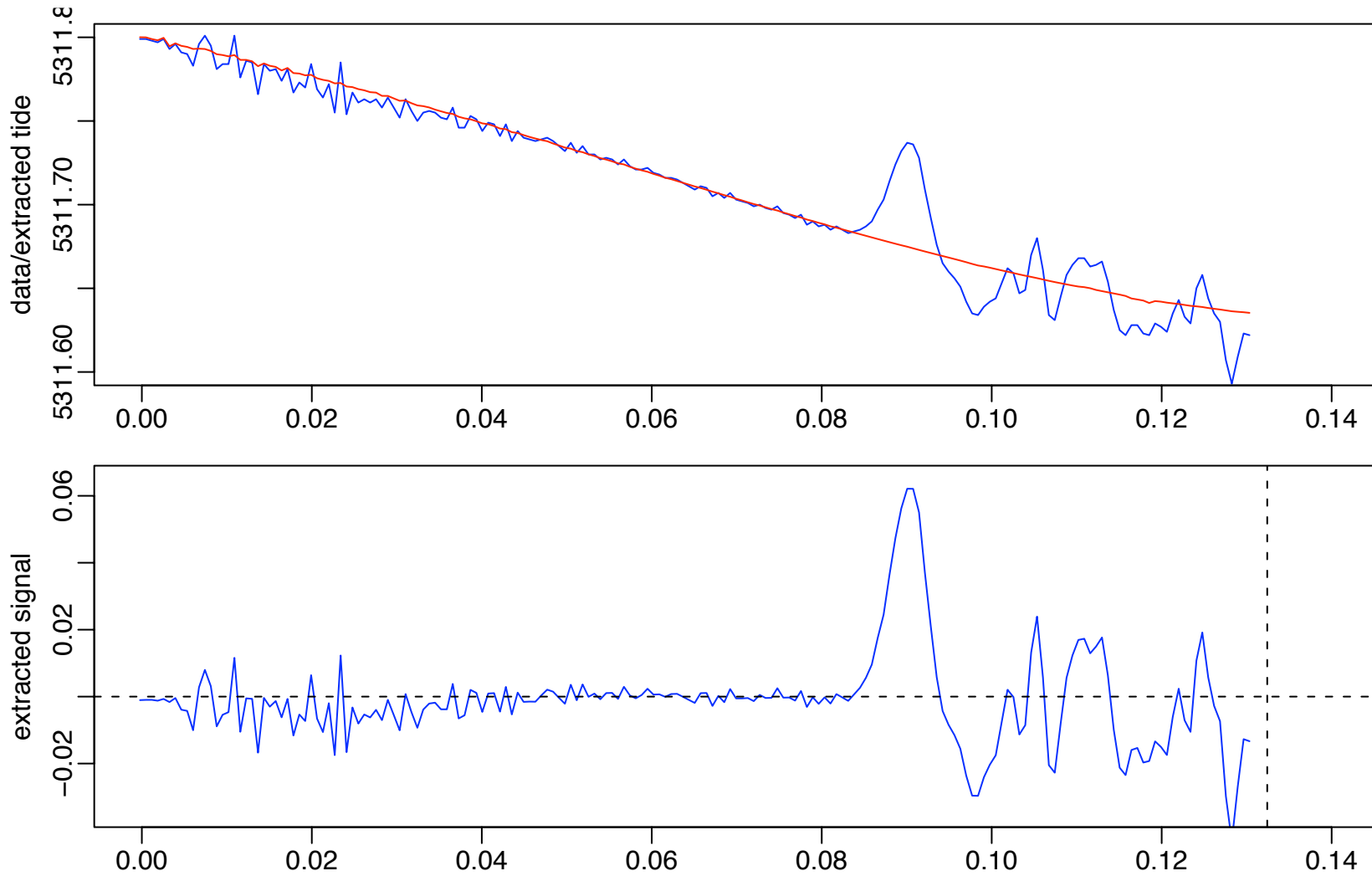
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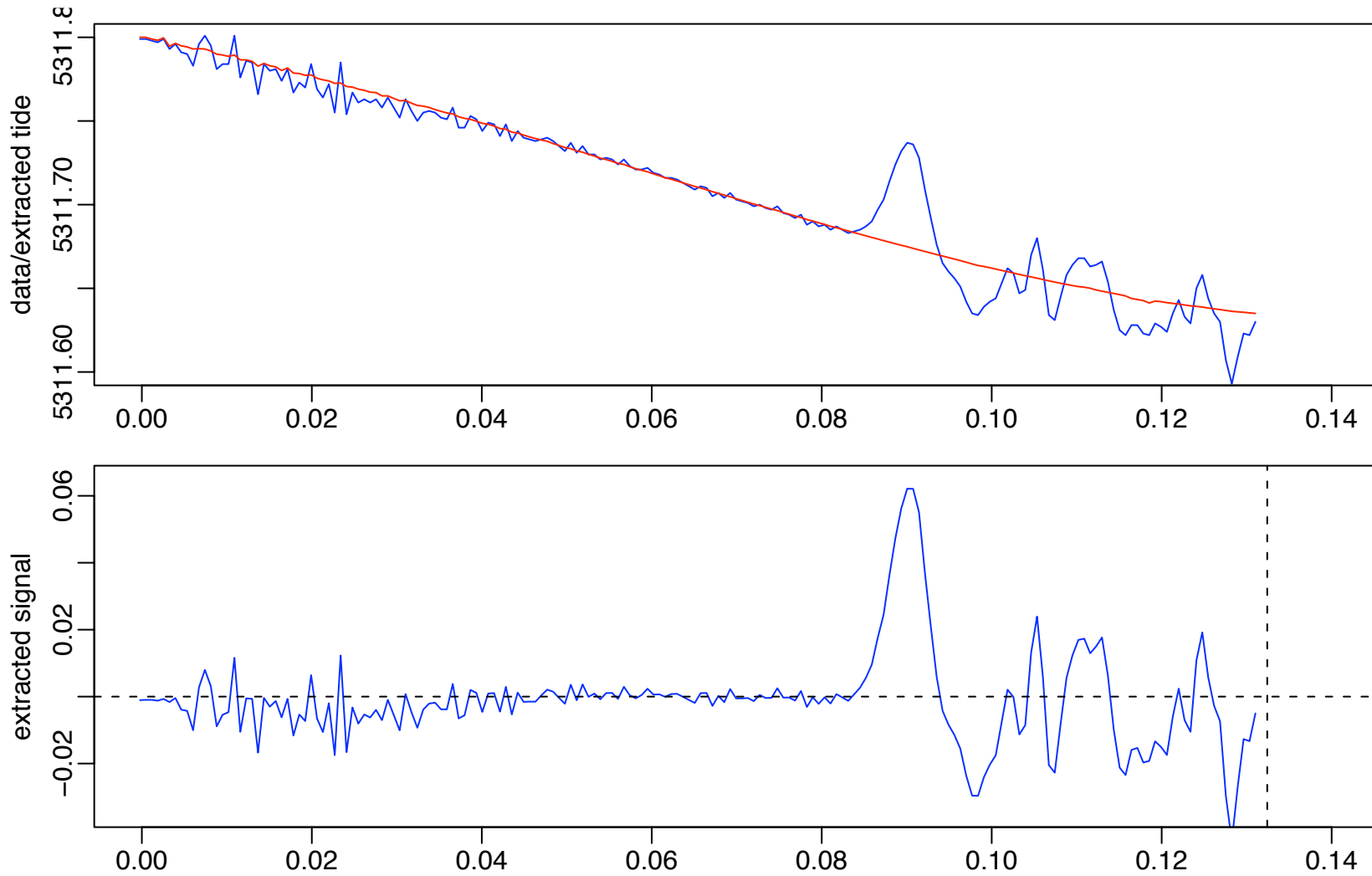
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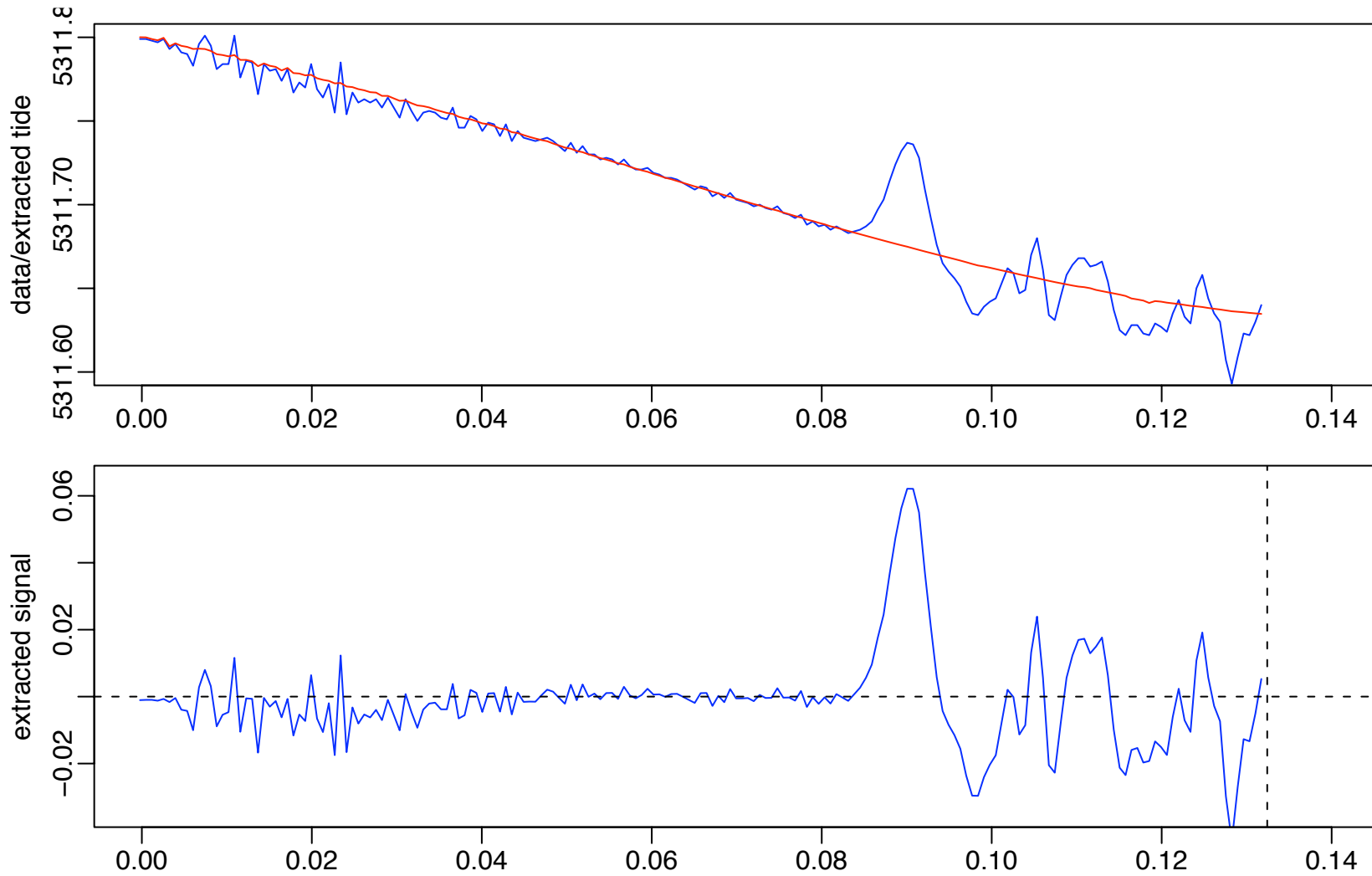
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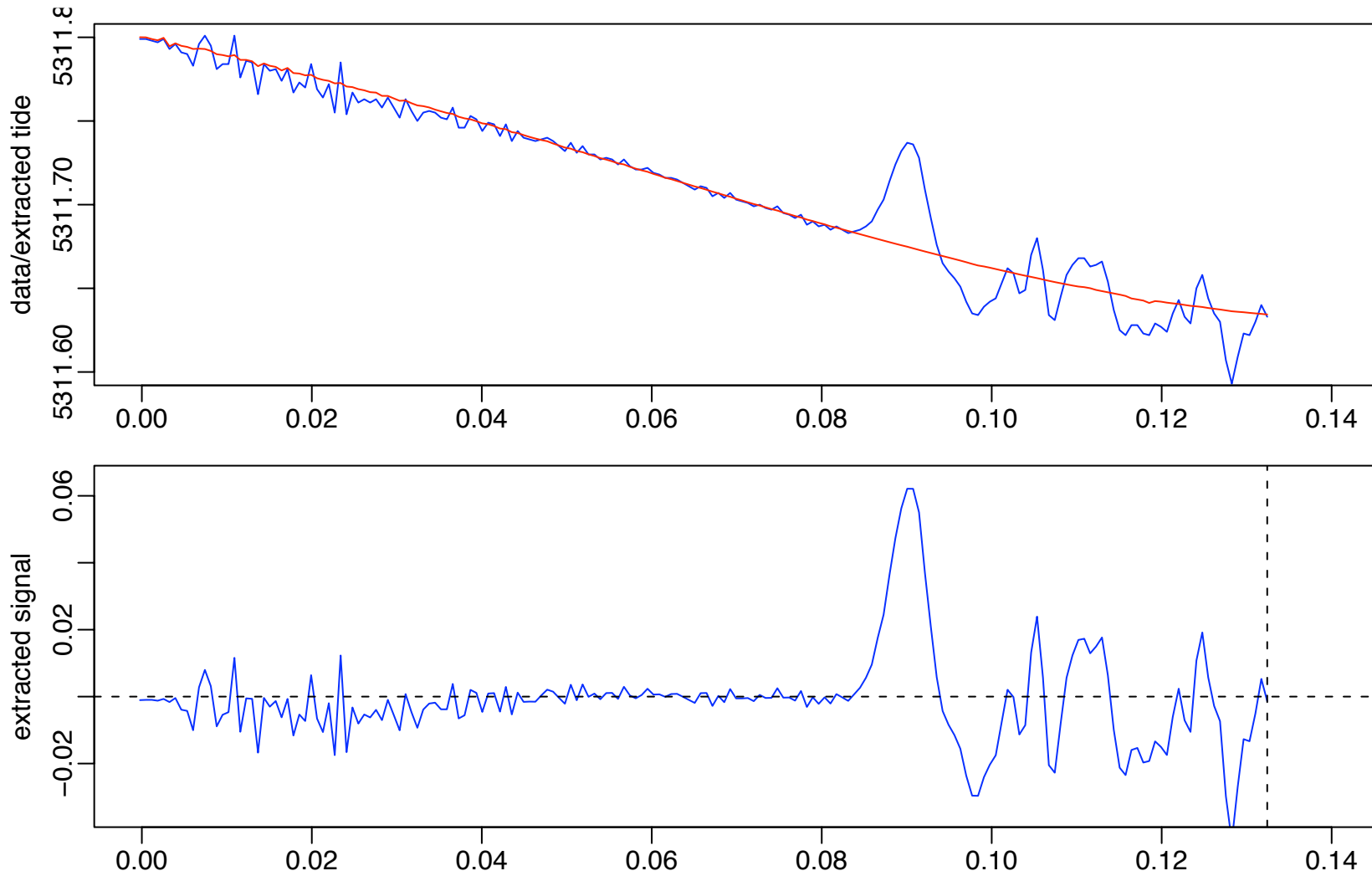


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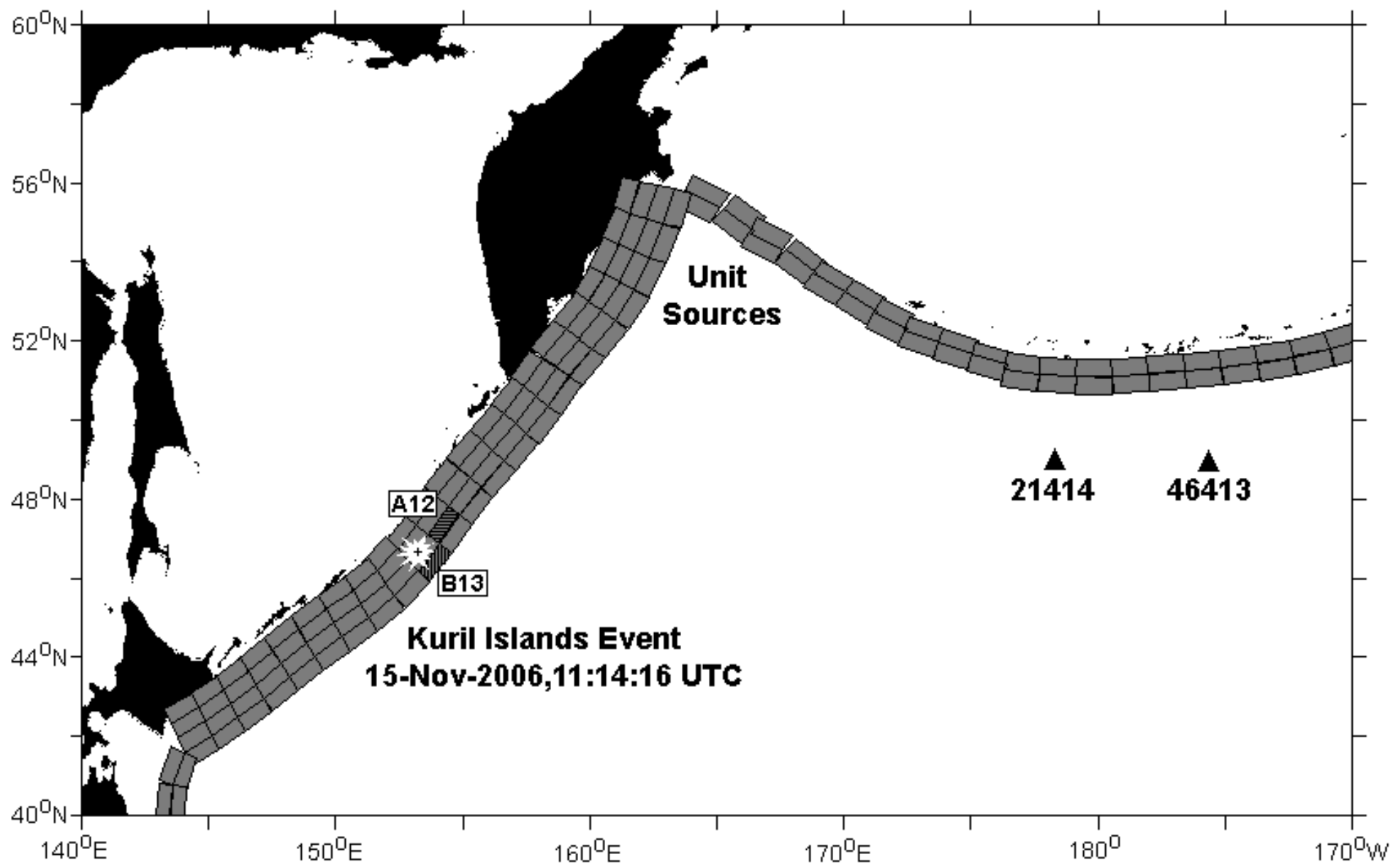


# Kalman Filtering/Smoothing – 14 Day Model



## Modelling of Detided DART<sup>®</sup> Buoy Data: I

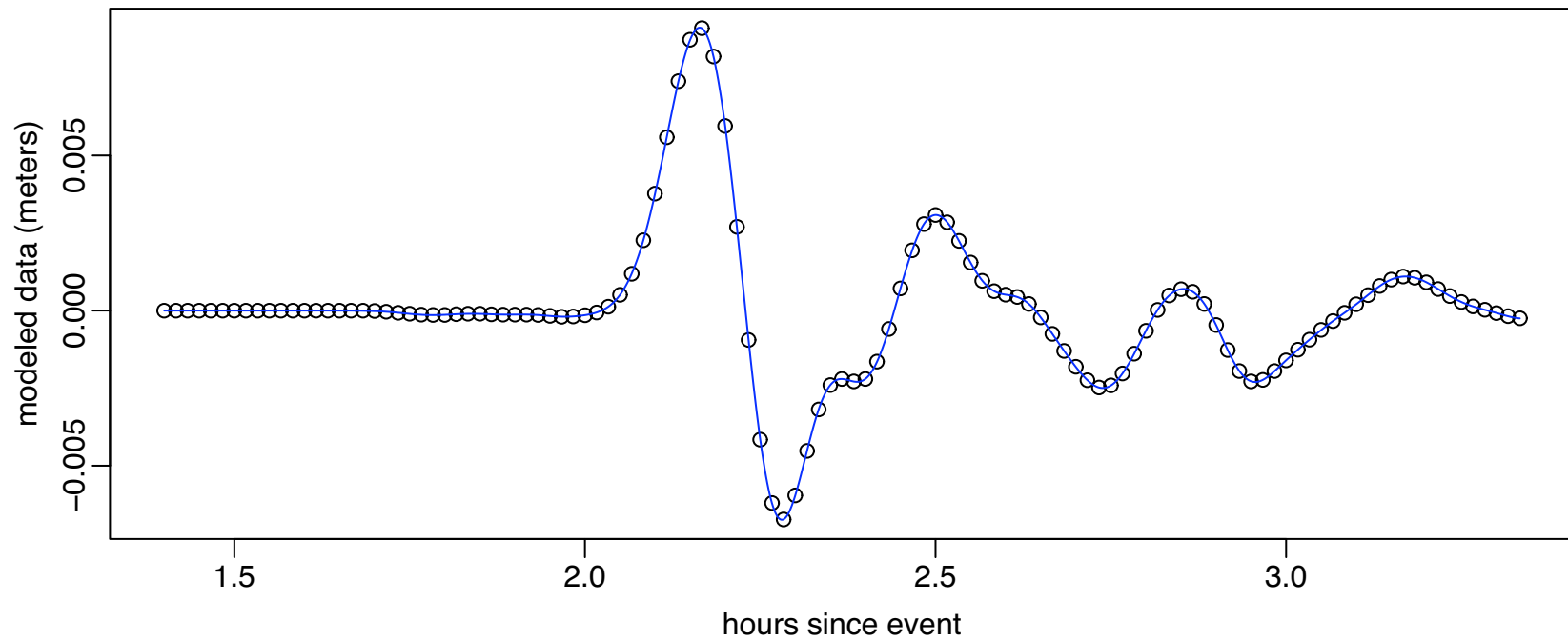
- geophysically-based propagation predictions are based on a simplification provided by so-called ‘unit sources’
- unit sources are rectangles of area  $100 \times 50 \text{ km}^2$  covering portions of globe from which tsunami-generating earthquakes can occur



## Modelling of Detided DART<sup>®</sup> Buoy Data: II

- propagation predictions must be precomputed (not enough time to compute them once a tsunami is in progress)
- database has been established with precomputed predictions for each pairing of particular unit source and particular buoy
- predictions assume earthquake is located in center of unit source and is of magnitude 7.5
- entry in database predicts what will be observed over time at a particular buoy given earthquake from a particular unit source

# Model for Buoy 21414 and Unit Source a12



## Modelling of Detided DART<sup>®</sup> Buoy Data: III

- will use buoy data to adjust predictions to handle earthquakes greater or less than 7.5 in magnitude
- consider case of 2 buoys (1, 2) and three unit sources ( $a, b, c$ )
- let  $\mathbf{x}_1$  and  $\mathbf{x}_2$  be relevant detided data from buoys 1 and 2
- let  $\mathbf{g}_{1,a}$  etc. be prediction of what buoy 1 should see from earthquake at unit source  $a$
- leads to following model for buoy data:

$$\mathbf{x} \equiv \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{1,a} & \mathbf{g}_{1,b} & \mathbf{g}_{1,c} \\ \mathbf{g}_{2,a} & \mathbf{g}_{2,b} & \mathbf{g}_{2,c} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} \boldsymbol{\epsilon}_1 \\ \boldsymbol{\epsilon}_2 \end{bmatrix} \equiv G\boldsymbol{\alpha} + \boldsymbol{\epsilon}$$

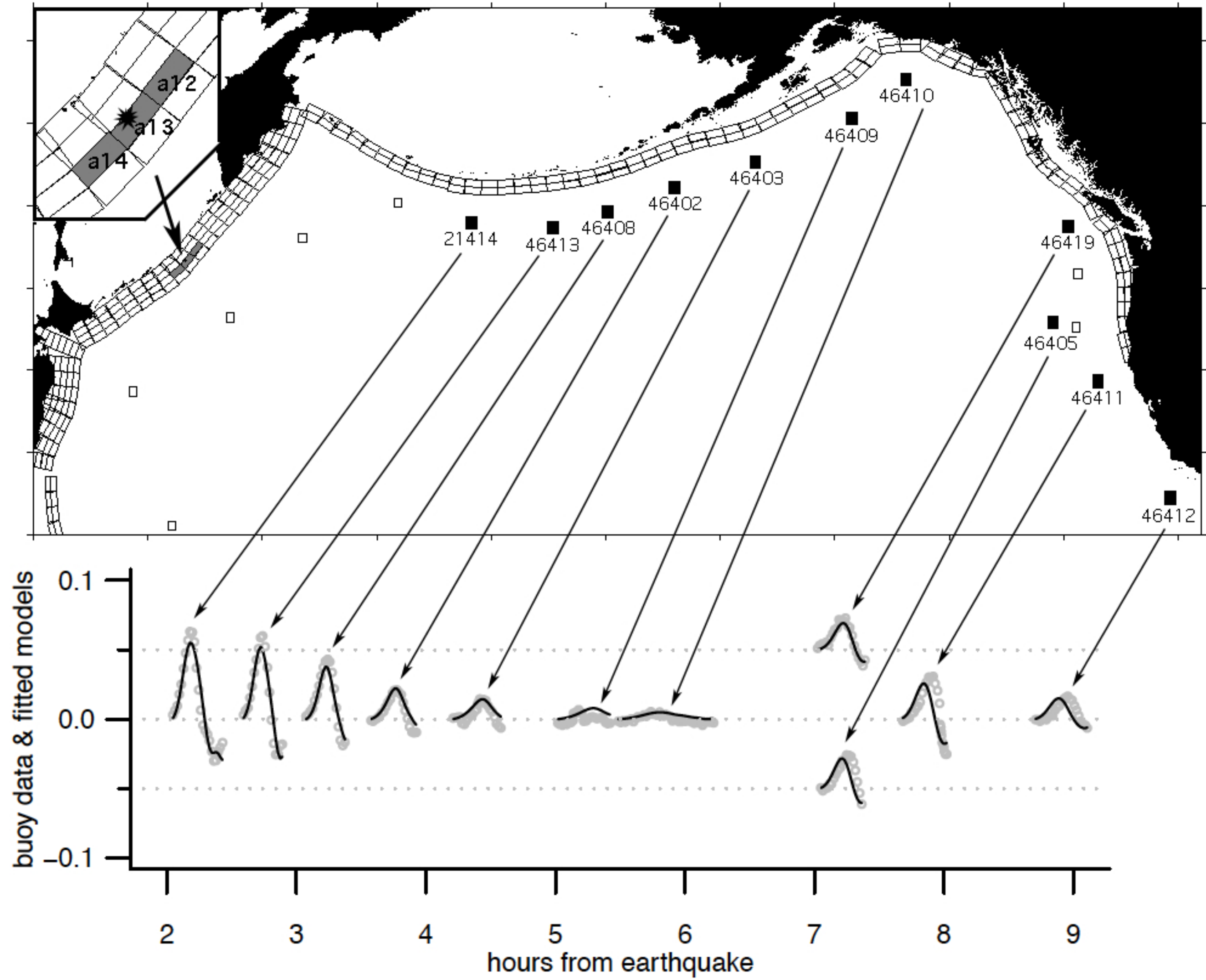
- error terms  $\boldsymbol{\epsilon}_j$  assumed to obey first-order autoregressive model

## Modelling of Detided DART<sup>®</sup> Buoy Data: IV

- solve for  $\boldsymbol{\alpha}$  using constrained least squares:

$$\text{minimize } \|\boldsymbol{\epsilon}\|^2 = \|\mathbf{x} - G\boldsymbol{\alpha}\|^2 \text{ subject to } \boldsymbol{\alpha} \geq \mathbf{0}$$

- constraint needed to get physically reasonable solution  $\hat{\boldsymbol{\alpha}}$
- note: if  $k$ th element of  $\hat{\boldsymbol{\alpha}}$  is set to zero,  $k$ th unit source effectively eliminated from model
- sum of elements of  $\hat{\boldsymbol{\alpha}}$  can be used to estimate tsunami magnitude  $T_M$
- uncertainty in  $T_M$  assessable using covariance matrix for  $\hat{\boldsymbol{\alpha}}$
- following example involves 11 buoys and 3 units sources





## Concluding Comments

- SIFT application currently relies on experienced operators to select appropriate unit sources
- desirable for future versions of SIFT to include statistically-oriented selection of unit sources (related to variable selection in linear regression)
- selection of data from buoy also currently done manually by operators – need for statistical guidance here also

## Thanks to . . .

- Ross Darnell for invitation to speak
- colleagues at NOAA Center for Tsunami Research in Seattle
  - Don Denbo
  - Marie Eblé
  - Edison Gica
  - Hal Mofjeld
  - Mick Spillane
  - Elena Tolkova
  - Vasily Titov
- Crime Scene for 2nd place finish (got \$13 in \$2 sweep!!!)