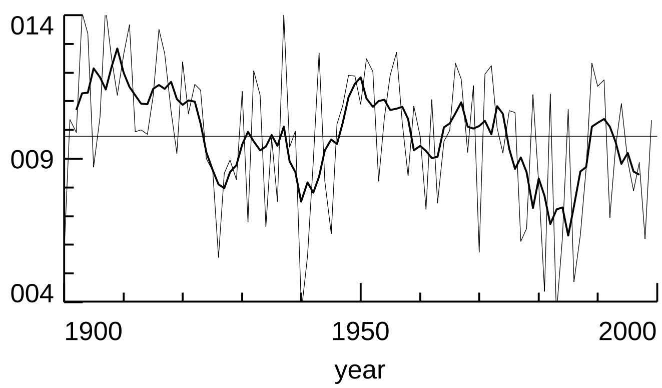

Figure 1: Plot of the NP index (thin curve) and a five year running average of the index (thick). The thin horizontal line depicts the sample mean (1009.8) for the index.

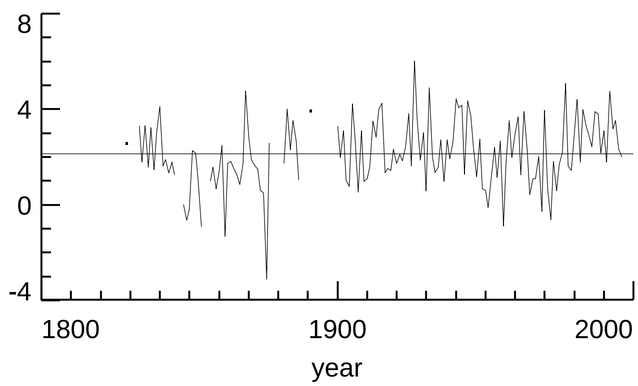

Figure 2: Plot of Sitka winter air temperatures (broken curve). The thin horizontal line depicts the sample mean (2.13) for the series.

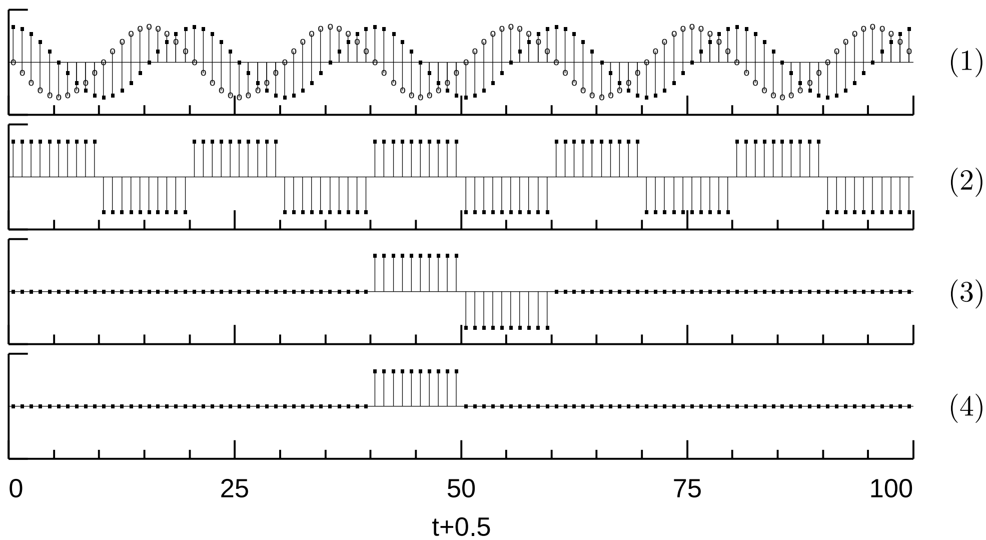

Figure 3: Examples of dictionary vectors $\mathbf{D}_k$ used in various matching pursuits of the NP index. The elements $D_{k,t}$, $t = 0, \ldots, 99$, for each $\mathbf{D}_k$ are plotted versus $t + 0.5$. The vector in (1) is a complex-valued vector from an orthonormal discrete Fourier transform (the real and imaginary parts are indicated by, respectively, solid dots and open circles). The period associated with this vector is twenty. In (2), the vector contains a square wave oscillation, also with a period of twenty. In (3), the vector is created from a discretized Haar wavelet function associated with changes on a scale of ten, while (4) shows one from a corresponding Haar scaling function.

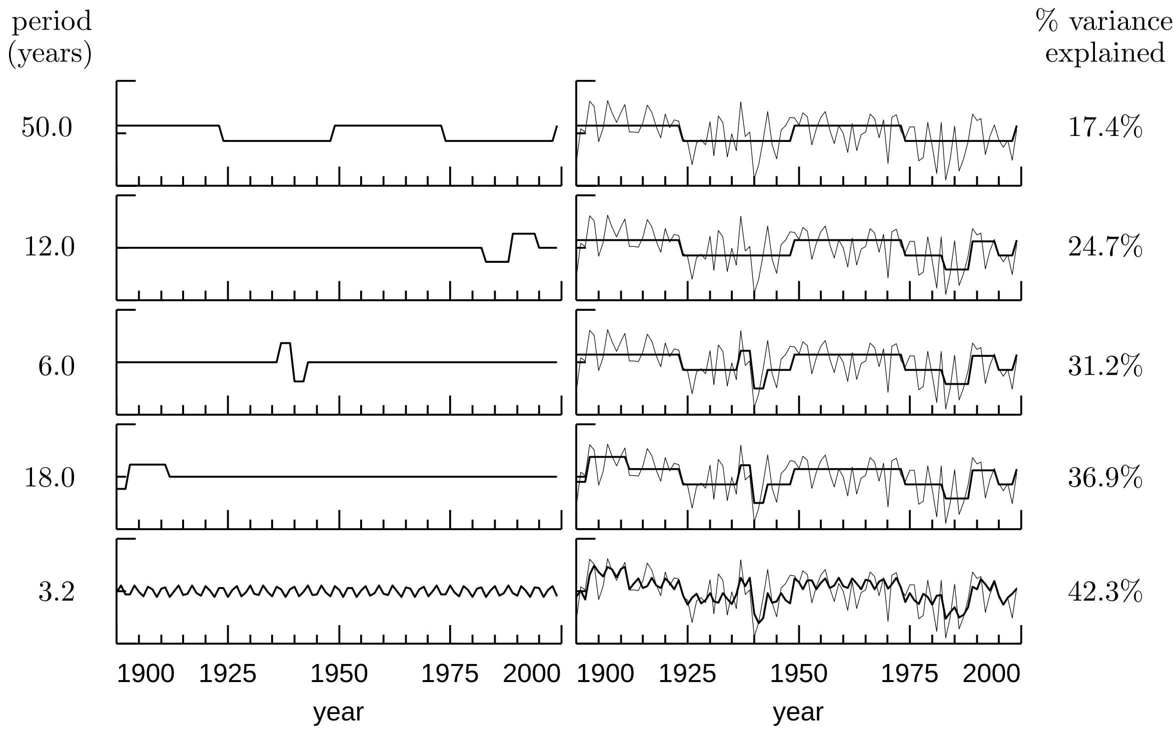

Figure 4: Matching pursuit of NP index using dictionary consisting of sinusoids, square wave oscillations, Haar wavelet vectors and Haar scaling vectors. The thin jagged curve in each right-hand plot shows the NP index $\mathbf{Z}$. The thick curves in the left-hand plots depict the vector that was selected in steps $m = 1, \dots, 5$ (top to bottom, respectively). The thick curves in the right-hand plots show the corresponding approximation $\widehat{\mathbf{Z}}^{(m)}$. The period associated with each vector is stated in the left-hand margin, while the right-hand margin lists the percentage of the variance that is explained by $\widehat{\mathbf{Z}}^{(m)}$ (by definition, this is $(\|\mathbf{Z}\|^2 - \|\mathbf{R}^{(m)}\|^2)/\|\mathbf{Z}\|^2 \times 100\%$, where $\mathbf{R}^{(m)} = \mathbf{Z} - \widehat{\mathbf{Z}}^{(m)}$).

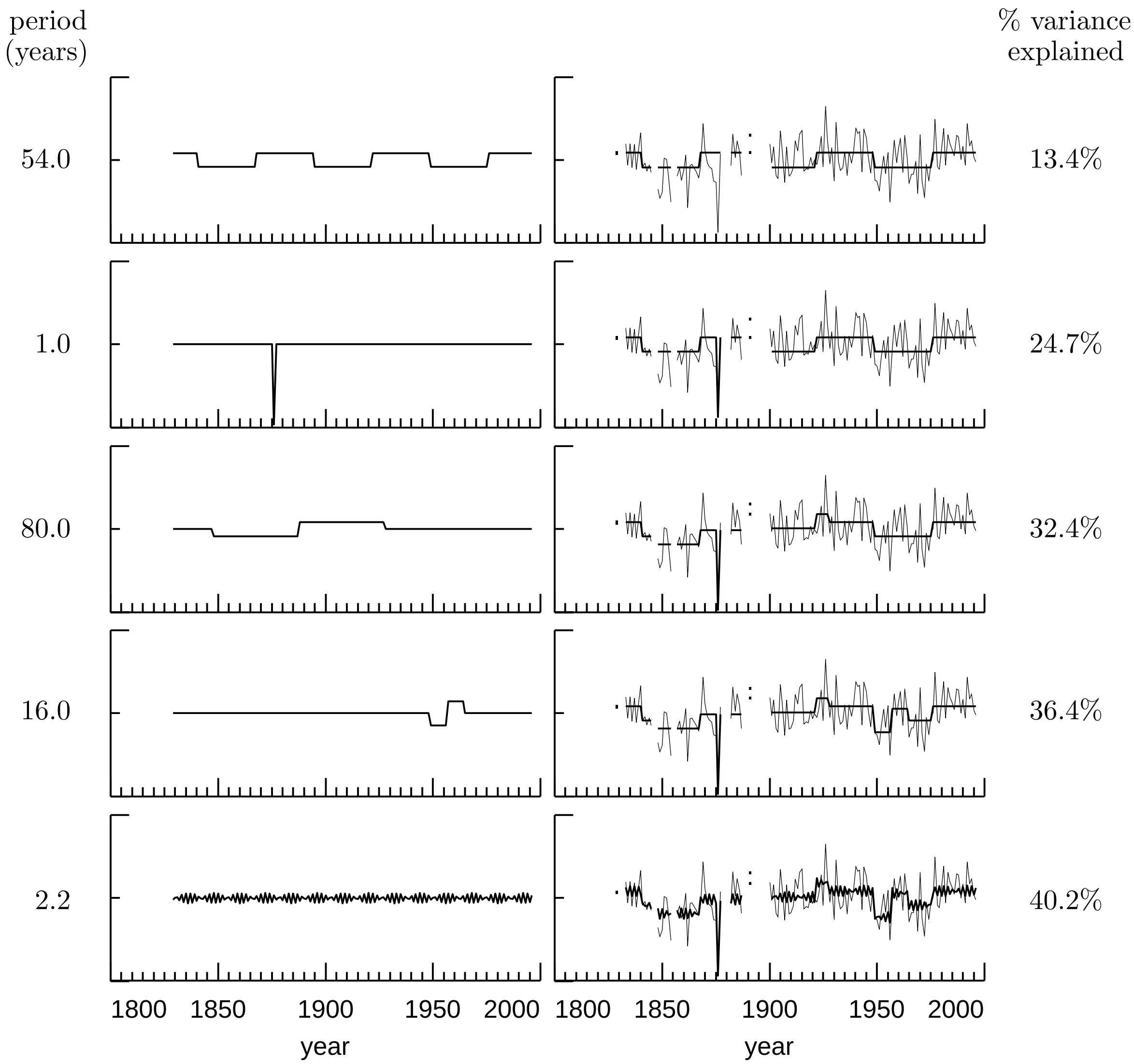

Figure 5: As in Figure 4, but now using the Sitka air temperatures.

model	parameter	95% CI	σ	95% CI
AR	$\hat{\phi} = 0.21$	$[0.02, 0.40]$	$\hat{\sigma}_\epsilon = 2.37$	$[2.01, 2.67]$
FD	$\hat{\delta} = 0.17$	$[0.02, 0.32]$	$\hat{\sigma}_\varepsilon = 2.35$	$[2.00, 2.66]$
SWO	$\hat{\beta} = -10.09$	$[-14.51, -5.67]$	$\hat{\sigma}_e = 2.21$	$[1.88, 2.50]$

Table 1: Autoregressive (AR), fractionally differenced (FD) and square wave oscillator (SWO) process parameter estimates for the NP index.

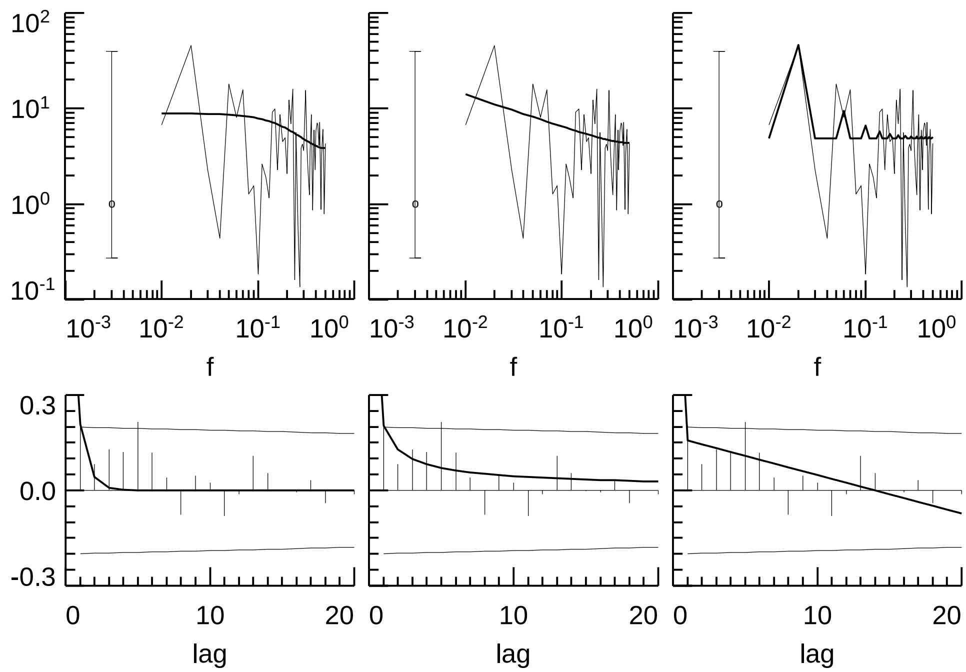

Figure 6: Sample autocorrelation sequence (ACS) and periodogram for the NP index, along with theoretical ACSs and spectral density functions (SDFs) for fitted AR, FD and SWO models (left, middle and right plots, respectively).

j	model	T_j	$Q_j(0.90)$	$Q_j(0.95)$	$Q_j(0.99)$	$\alpha = 0.05$ test result	$\hat{\alpha}$
1	AR	0.30	0.38	0.39	0.42	fail to reject	0.67
	FD	0.28	"	"	"	fail to reject	0.78
	WN	0.39	"	"	"	reject	0.05
2	AR	0.10	0.17	0.19	0.23	fail to reject	$\gg 0.1$
	FD	0.07	"	"	"	fail to reject	$\gg 0.1$
	SWO	0.10	"	"	"	fail to reject	$\gg 0.1$
	WN	0.21	"	"	"	reject	≈ 0.03
3	AR	4.65	7.74	9.45	13.31	fail to reject	0.32
	FD	3.12	"	"	"	fail to reject	0.54
	SWO	2.83	"	"	"	fail to reject	0.59
	WN	12.63	"	"	"	reject	0.01

Table 2: Model goodness of fit tests for the NP index.

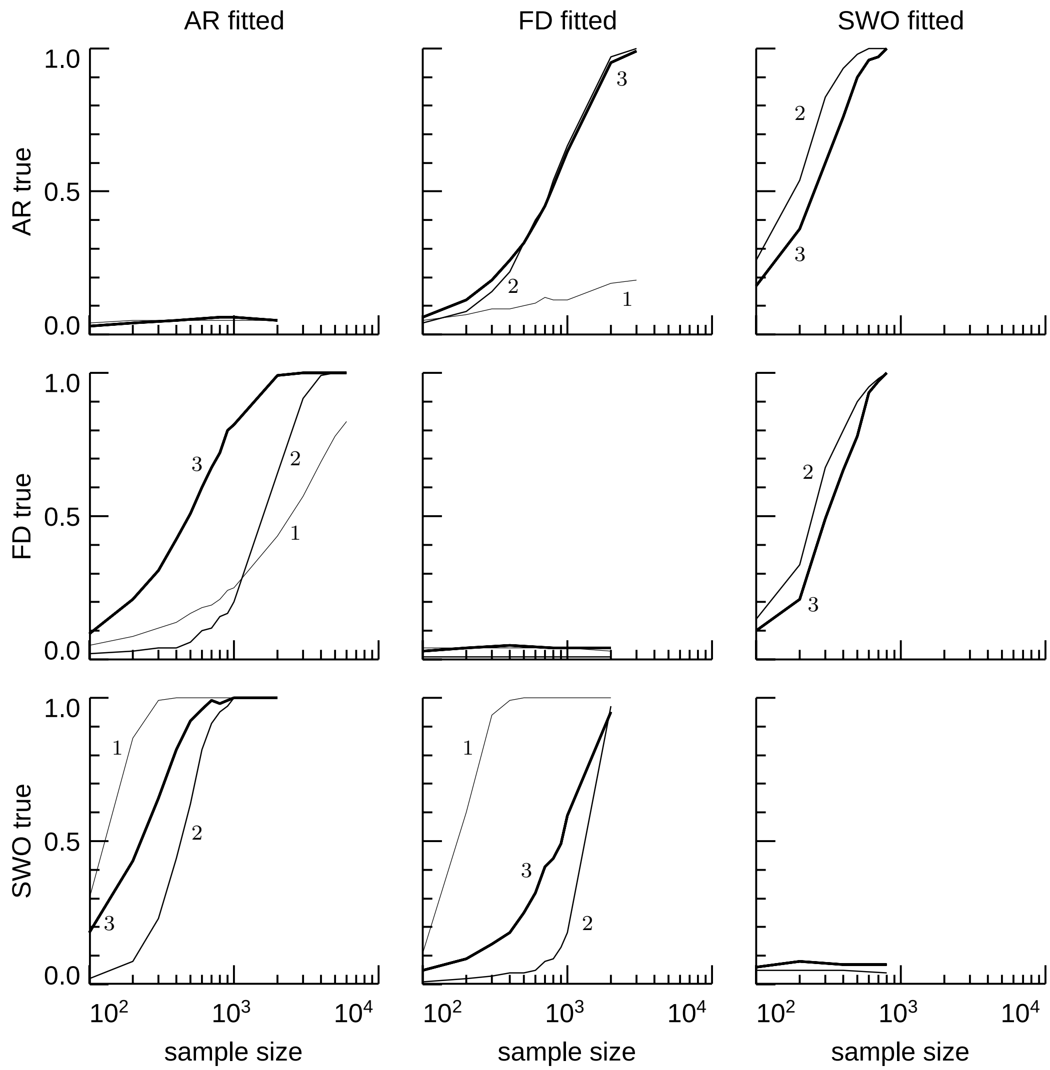

Figure 7: Probability (as a function of sample size) of rejecting the null hypothesis at a 0.05 level of significance that a fitted model A is adequate for a realization of a process B when using the test statistics T_1, T_2 and T_3. For the plots in the left- to right-hand columns, the fitted models A is, respectively, an FD, AR(1) and SWO model. The same ordering is used for the process B for the plots in the top to bottom rows.

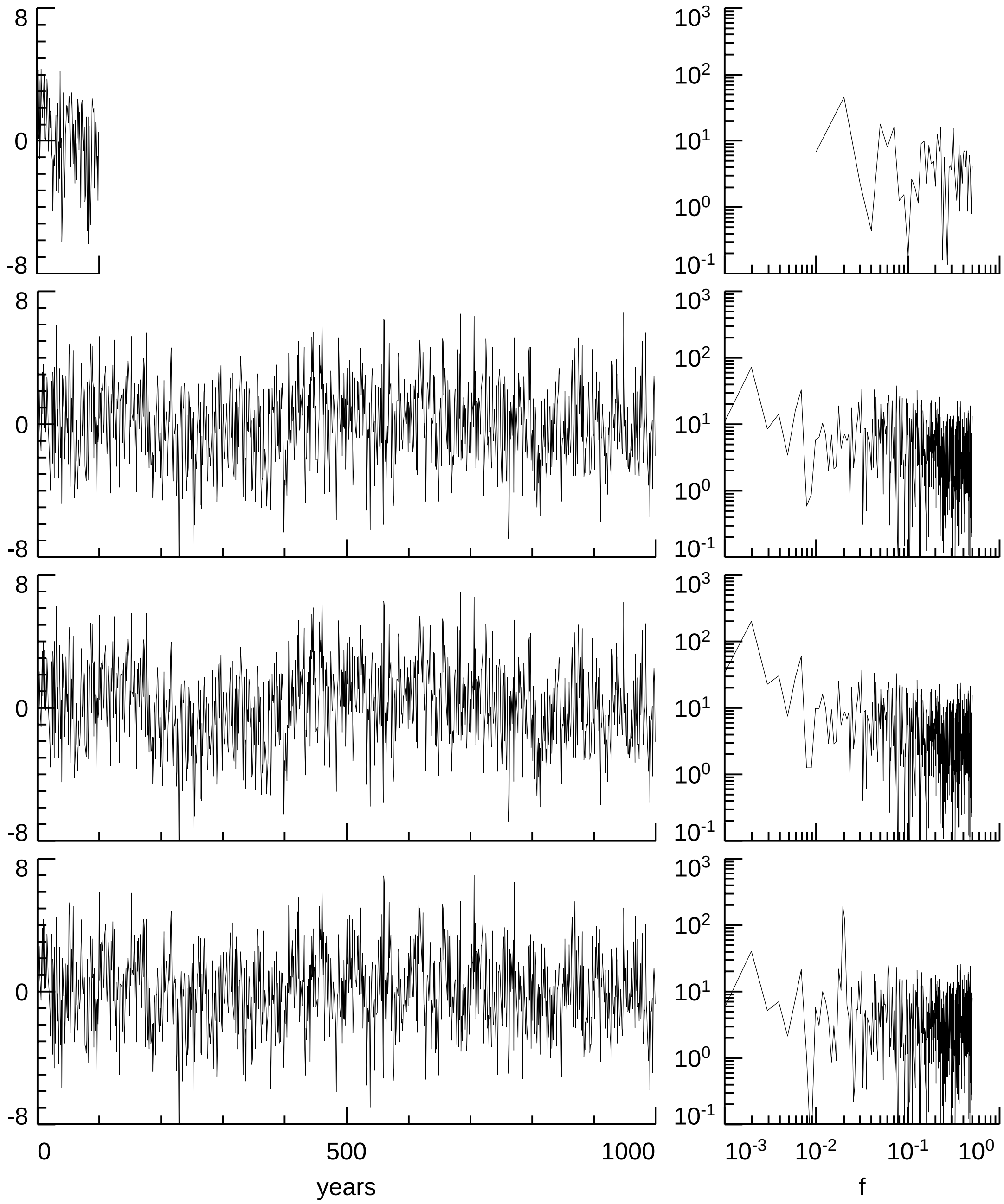

Figure 8: Simulated realizations (left-hand column, bottom three rows) of AR(1), FD and SWO processes (second, third and fourth rows, respectively) with model parameters set to values estimated for the NP index, along with associated periodograms (right-hand column). The actual NP index and its periodogram are shown in the top row. Each realization was created using an exact circulant embedding method. This method converts $2N$ uncorrelated standard Gaussian deviates into the desired realization of length N. For each series, the same $2N$ deviates were used to make it easier to compare realizations from the different models.

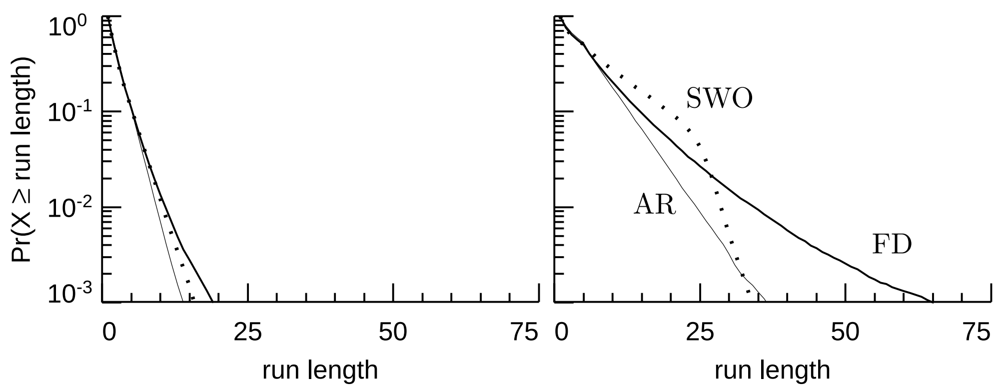

Figure 9: Probability of observing a run that is greater than or equal to a specified run length. The thin, thick and dotted curves denote the AR, FD and SWO processes. The left-hand plot is for processes without smoothing, whereas the right-hand plot is for processes subjected to a five year running average.