$$a * b_0 = \sum_{u=-\infty}^{\infty} a_u b_{-u} \qquad\qquad a * b_2 = \sum_{u=-\infty}^{\infty} a_u b_{2-u}$$

Figure 24. Graphical illustration of convolution of the infinite sequences $\{a_t\}$ and $\{b_t\}$. The left-hand plot shows two lines. The upper line is labeled at equal intervals with elements of the infinite sequence $\{a_t\}$. The lower line is likewise labeled, but now with the *reverse* of $\{b_t\}$, i.e., $\{b_{-t}\}$ The zeroth element $a * b_0$ of the convolution of $\{a_t\}$ and $\{b_t\}$ is obtained by multiplying the a_t's and b_t's facing each other and then summing. In general, the tth element $a * b_t$ is obtained in a similar fashion *after* the lower line has been shifted to the right by t divisions – for example, the right-hand plot shows the alignment of the lines that yields the second element $a * b_2$ of the convolution.

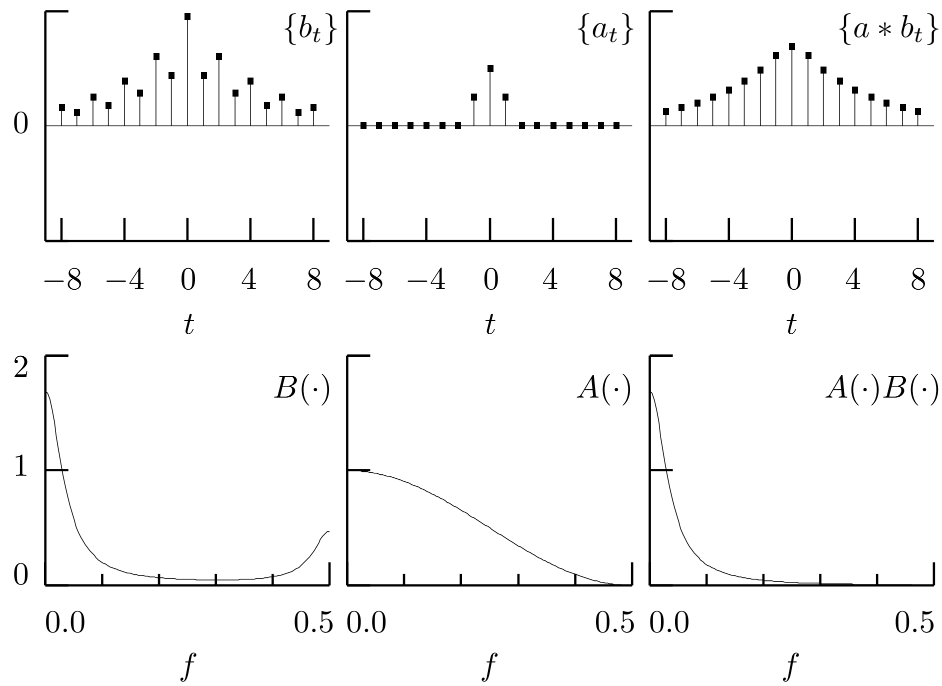


Figure 26. Example of filtering using a low-pass filter.

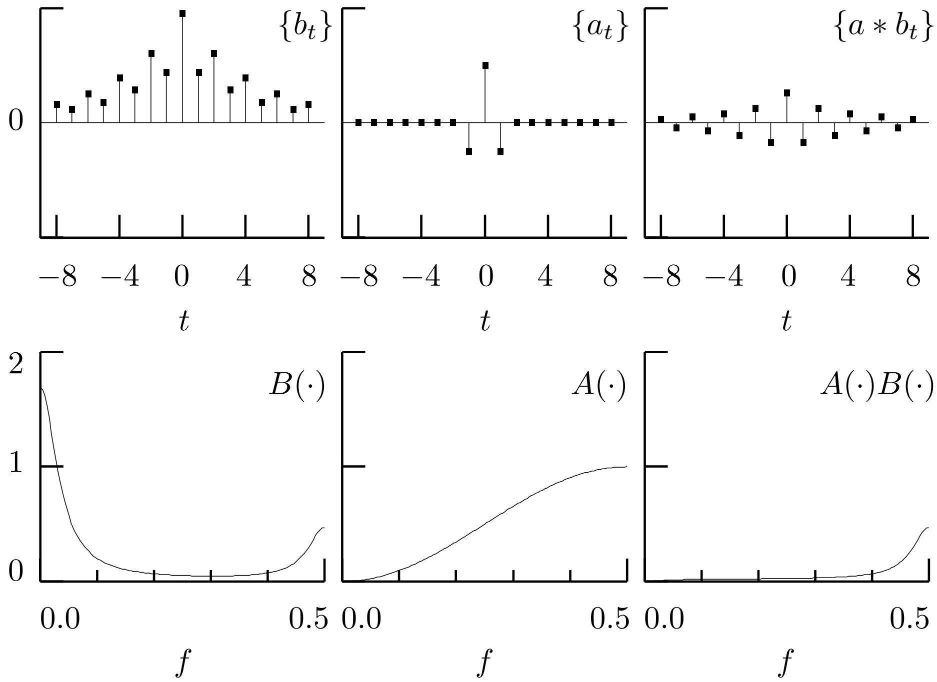
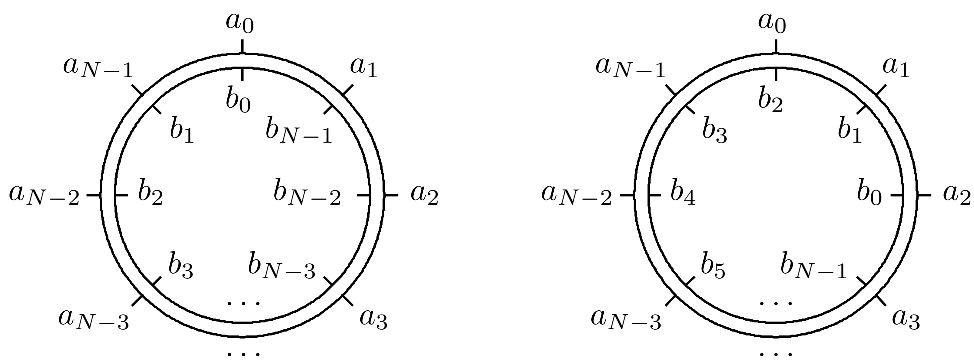
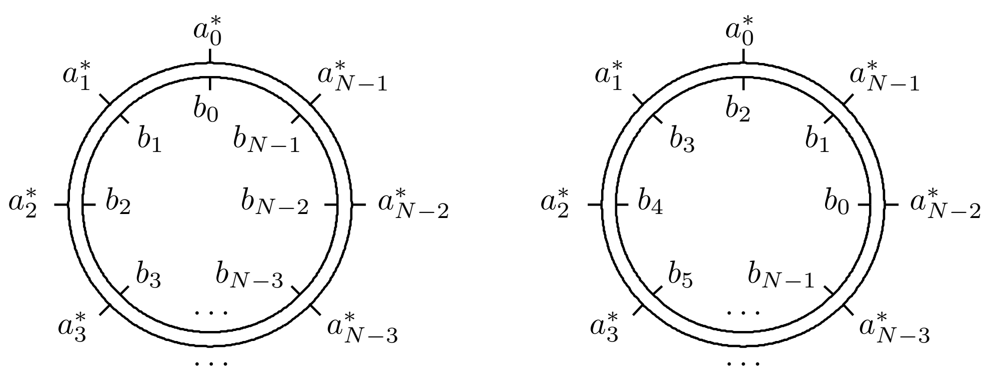


Figure 27. Example of filtering using a high-pass filter.



$$a * b_0 = \sum_{u=0}^{N-1} a_u b_{-u \bmod N} \qquad\qquad a * b_2 = \sum_{u=0}^{N-1} a_u b_{2-u \bmod N}$$

Figure 31a. Graphical illustration of circular convolution of the finite sequences $\{a_t\}$ and $\{b_t\}$. The left-hand plot shows two concentric circles. The outer circle is divided into N equal arcs, and the boundaries between the arcs are labeled clockwise from the top with $a_0, a_1, \ldots, a_{N-1}$. The inner circle is likewise divided, but now the boundaries are labeled counter-clockwise with $b_0, b_1, \ldots, b_{N-1}$. The zeroth element $a * b_0$ of the convolution of $\{a_t\}$ and $\{b_t\}$ is obtained by multiplying the a_t's and b_t's facing each other and then summing. In general, the tth element $a * b_t$ is obtained in a similar fashion *after* the inner circle has been rotated clockwise by t divisions – for example, the right-hand plot shows the alignment of the concentric circles that yields the second element $a * b_2$ of the convolution.



$$a^* \star b_0 = \sum_{u=0}^{N-1} a_u^* b_u \qquad\qquad a^* \star b_2 = \sum_{u=0}^{N-1} a_u^* b_{u+2 \bmod N}$$

Figure 31b. Graphical illustration of circular complex cross-correlation of $\{a_t\}$ and $\{b_t\}$. The layout is similar to Figure 31a.