

**Figure 3.** Three wavelets. From left to right, we have one version of the Haar wavelet; a wavelet that is related to the first derivative of the Gaussian probability density function (PDF); and the Mexican hat wavelet, which is related to the second derivative of the Gaussian PDF.



**Figure 5.** Three Morlet wavelets  $\psi_{\omega_0}^{(M)}(\cdot)$ . These wavelets are complex-valued, so their real and imaginary parts are plotted using, respectively, thick and thin curves. The parameter  $\omega_0$  controls the frequency of the complex exponential that is then modulated by a function whose shape is dictated by the standard Gaussian PDF. As  $\omega_0$  increases from 3 to 7, the number of oscillations within the effective width of the Gaussian PDF increases.



**Figure 6.** Step function  $x(\cdot)$  successively taking on the values  $x_0, x_1, \ldots, x_{15}$  over a partitioning of the interval [a, b] into 16 equal subintervals. As defined by Equation (5), the average value of  $x(\cdot)$  over [a, b] is just the sample mean of all 16  $x_j$ 's (the dashed line indicates this average).



**Figure 8.** Average daily fractional frequency deviates for cesium beam atomic clock 571 (bottom plot) and its Mexican hat CWT. The fractional frequency deviates are recorded in parts in  $10^{13}$  (a deviation of -15 parts in  $10^{13}$  means that clock 571 lost about 129.6 billionths of a second in one day with respect to the US Naval Observatory master clock to which it was being compared). The vertical axis of the CWT plot is scale (ranging from 1 to 64 days), while the horizontal axis is the same as on the lower plot. The CWT plot is grey-scaled coded so that large magnitudes correspond to bright spots (regions where the plot is dark indicate scales and days at which the clock performed well).



**Figure 10.** Shifted and rescaled versions of the Haar wavelet  $\psi^{(H)}(\cdot)$ . The plots above show  $\psi_{\lambda,t}^{(H)}(\cdot)$ , which can be used to measure how much adjacent averages of a signal  $x(\cdot)$  over a scale of length  $\lambda$  change at time t. The top row of plots shows the effect of keeping  $\lambda$  fixed at unity while t is varied; in the bottom row t is fixed at zero while  $\lambda$  is varied.



Figure 14. Haar DWT coefficients for clock 571 and sample ACSs.



Figure 16. Energy analysis for clock 571 based on Haar DWT wavelet coefficients (curve) and Haar MODWT wavelet coefficients (o's).



Figure 18. Haar MODWT coefficients for clock 571.