Wavelets and ambient noise

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Example 1

- Geoacoustic inversion for seabed properties (attenuation and sound speed) using reverberation signal.
- Signal irregularities handled by using zoom-in capability of wavelets.
- Our task processes the signal and removes unmodeled local features (instead of detecting them), and then hands the processed signal off to the inversion team.

Translation invariant time-frequency frames

Windowed Fourier transform

$$F(\omega,t) = \int f(t')g(t'-t)e^{i\omega t'}dt'$$

Finite version (FFT)

$$F(k,n) = \sum_{m=0}^{N-1} f_m g(m-n) e^{2\pi i mk/N}$$

- Speed: O(N lg(N)) for one window
- Spectral content—fixed resolution
- Essential for analytic work:
  - convolution → product
  - translation → phase shift
  - non. rel. hydrogen → integrated $\frac{1}{r} \to \frac{1}{\vec{q}^2} \to \sum Y_{nlm}(\Omega)Y^*_{nlm}(\Omega')$

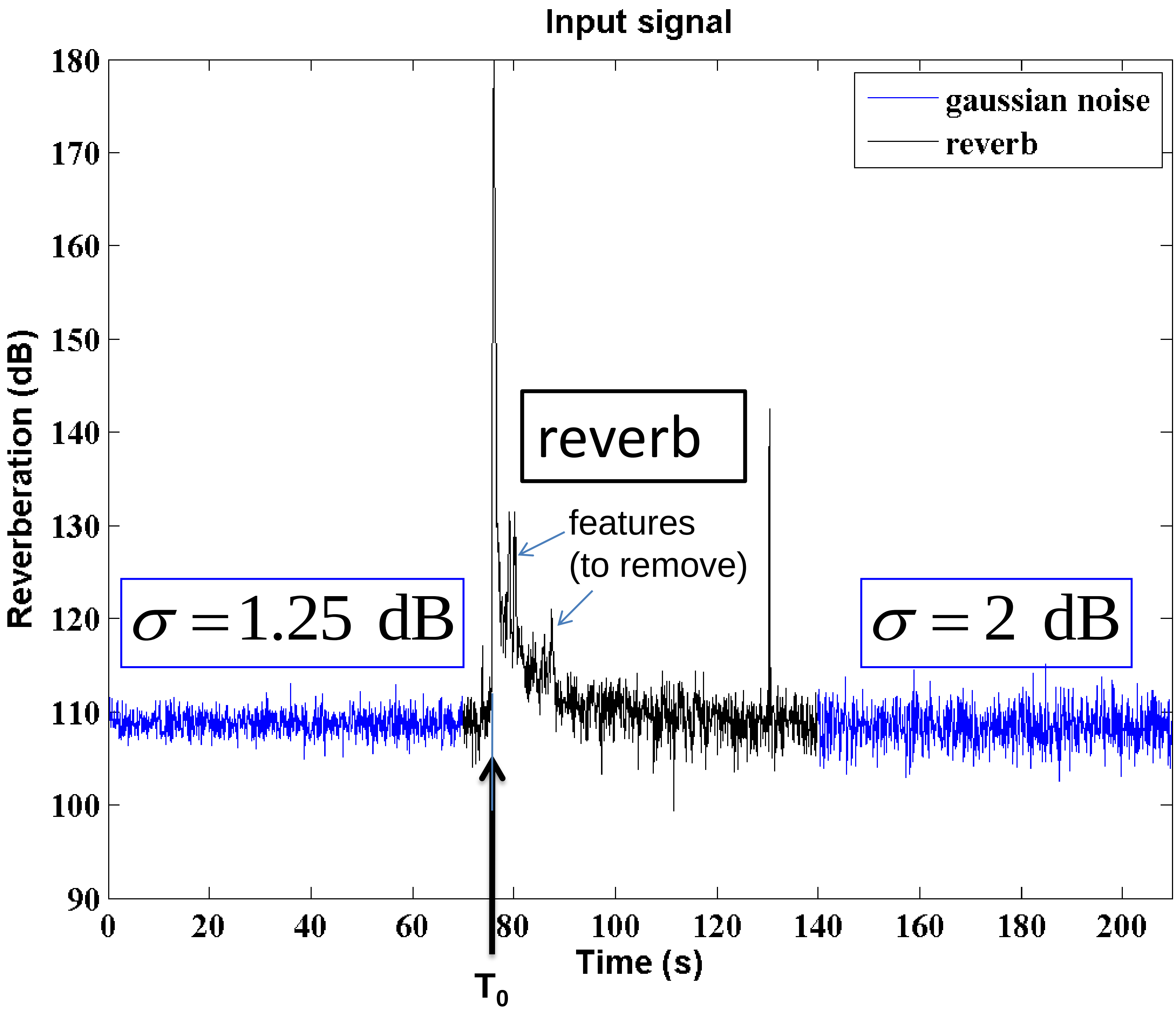
Wavelet transform

$$W(s,t) = \int f(t')\frac{1}{\sqrt{s}}\psi\left(\frac{t'-t}{s}\right)dt'$$

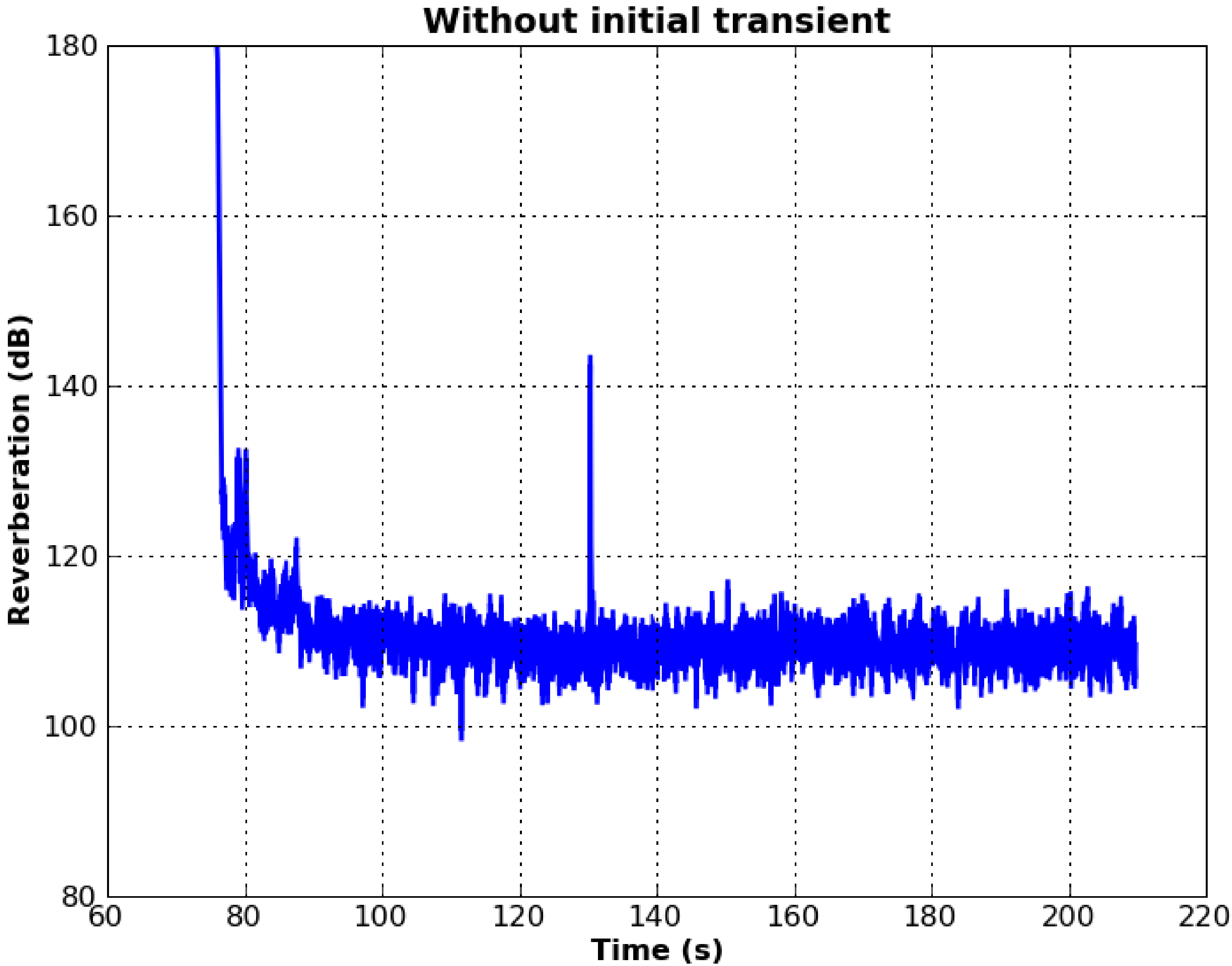
Finite version (MODWT)

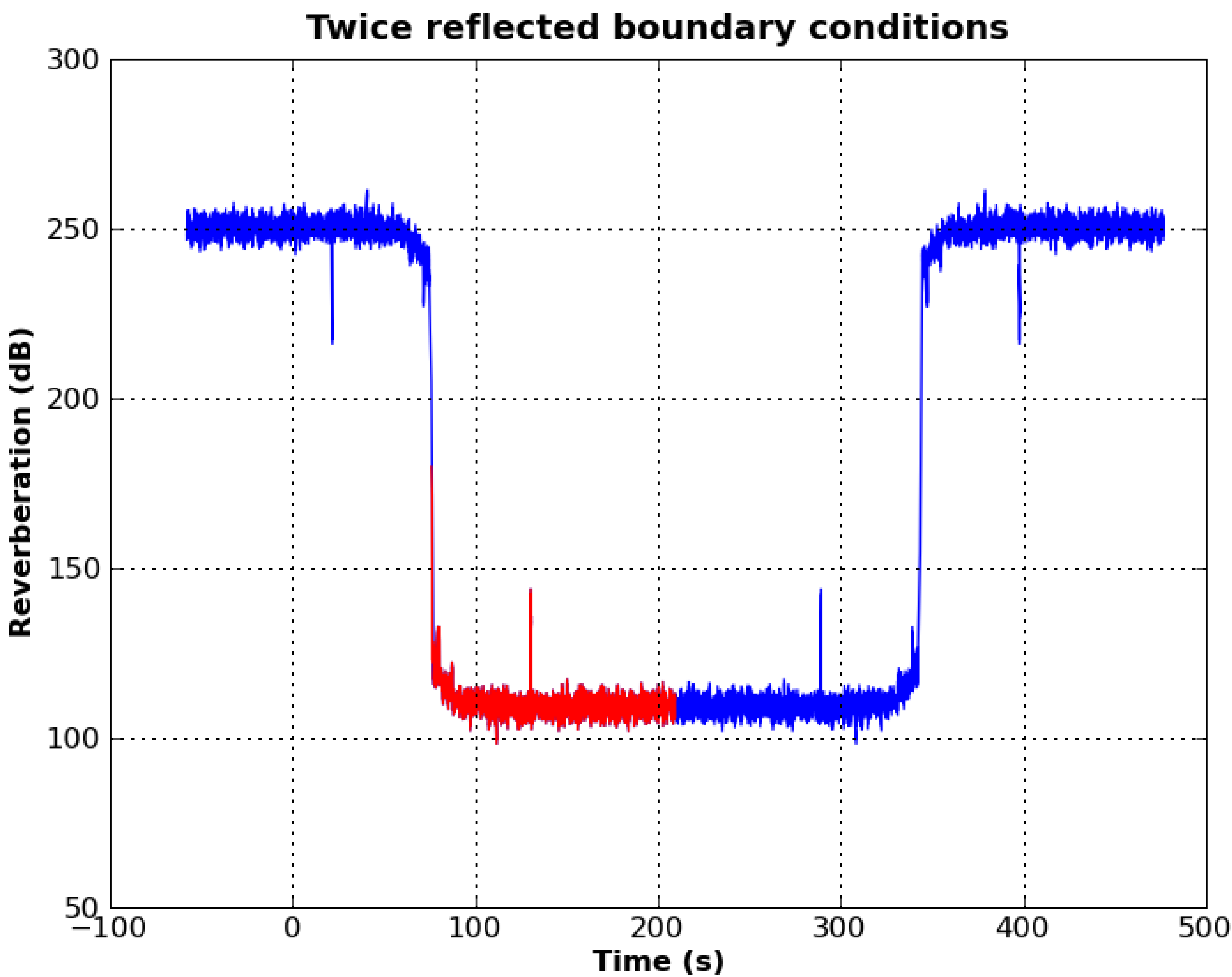
$$W(2^j,n) = \sum_{m=0}^{N-1} f_m \frac{1}{\sqrt{2^j}}\psi\left(\frac{m-n}{2^j}\right)$$

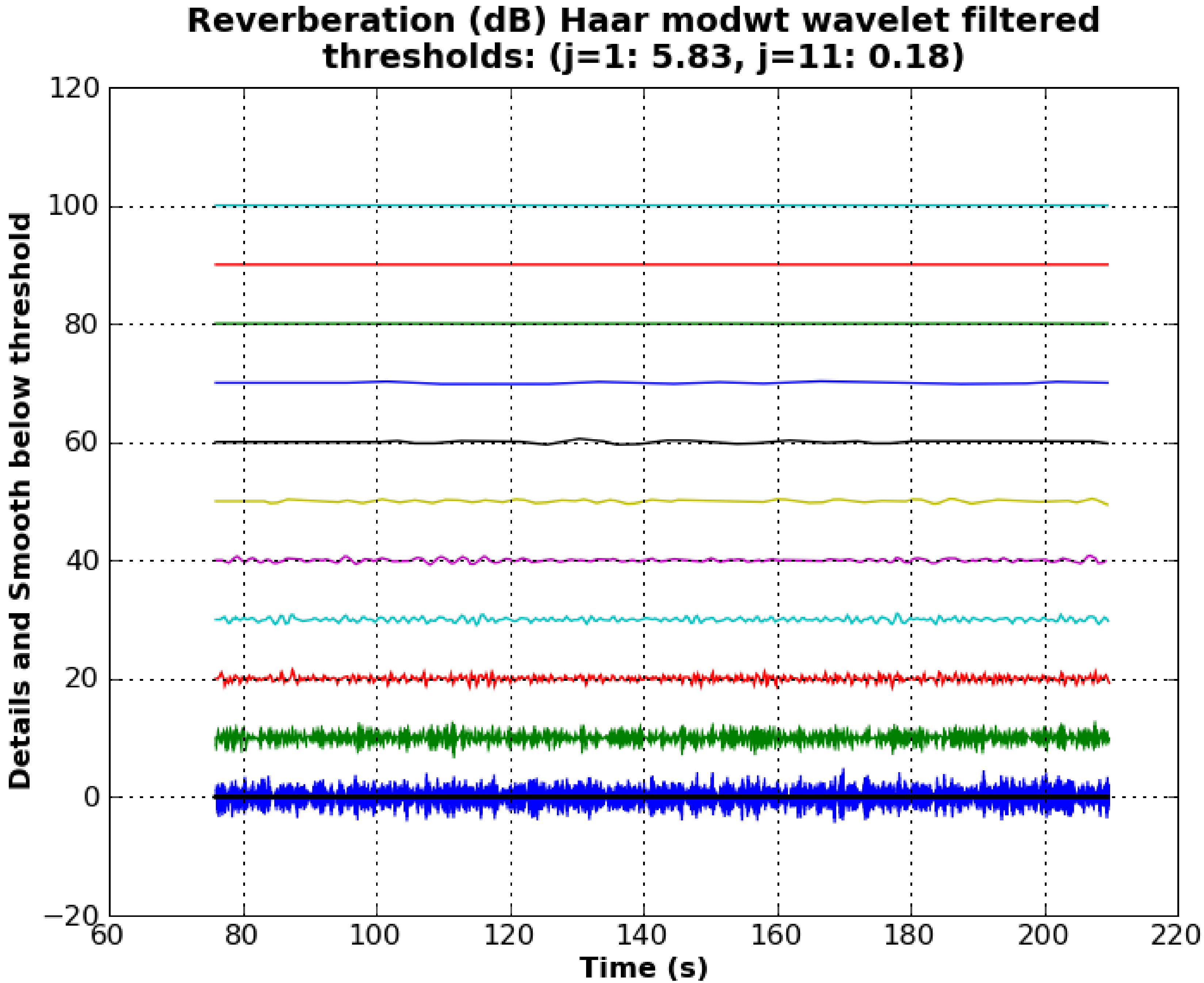
- Speed: O(N lg(N)) with pyramid algorithm
- Spectral content—variable resolution
  - high freq/short times,
  - low freq/long times
- Good for detecting edges, transients
- Denoising, compression, sparse representation applications



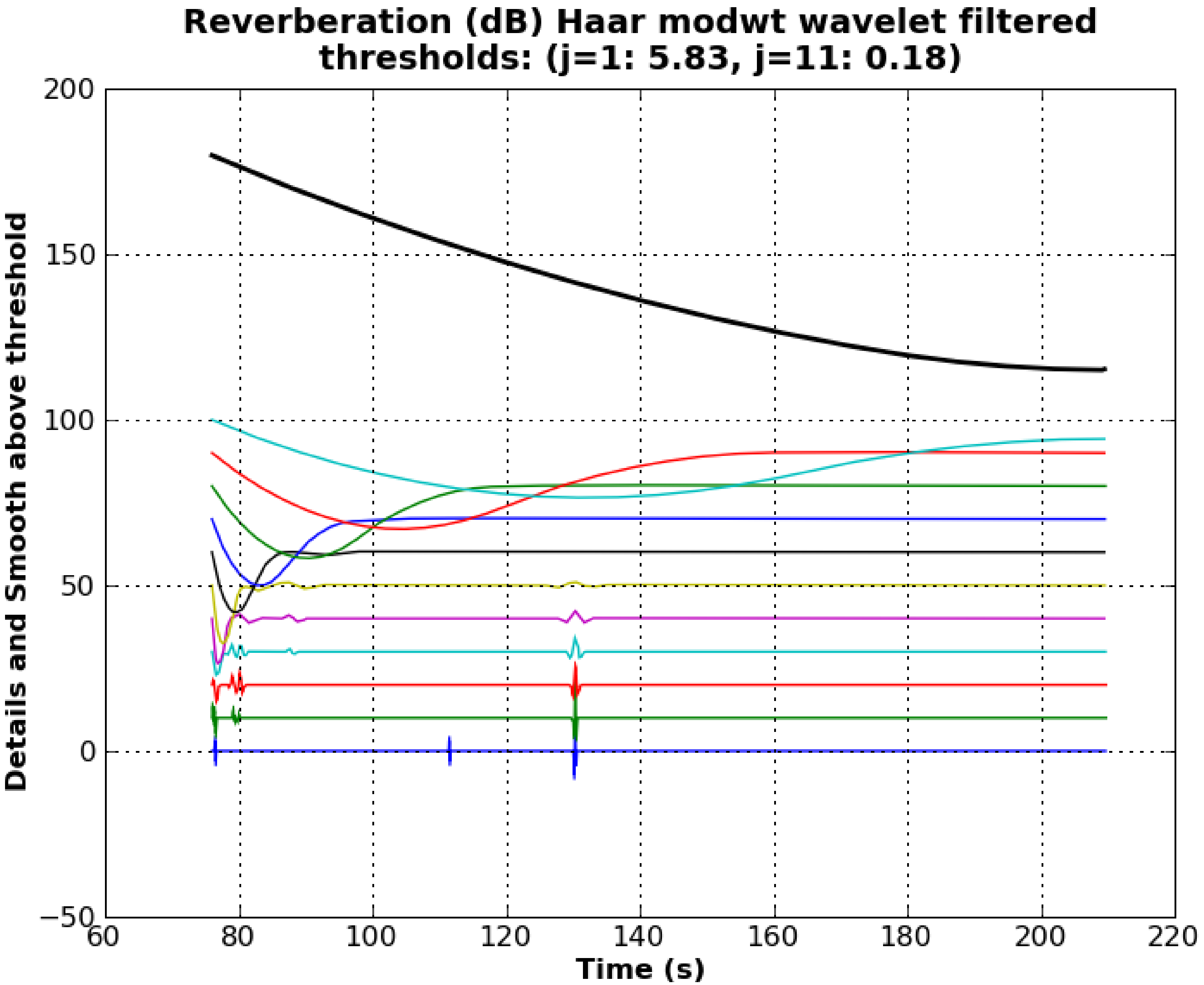
Input signal
180
170
160
150
140
130
120
110
100
90
Reverberation (dB)
0
20
40
60
80
100
120
140
160
180
200
Time (s)
gaussian noise
reverb
reverb
features
(to remove)
$\sigma = 1.25$ dB
$\sigma = 2$ dB
T_0

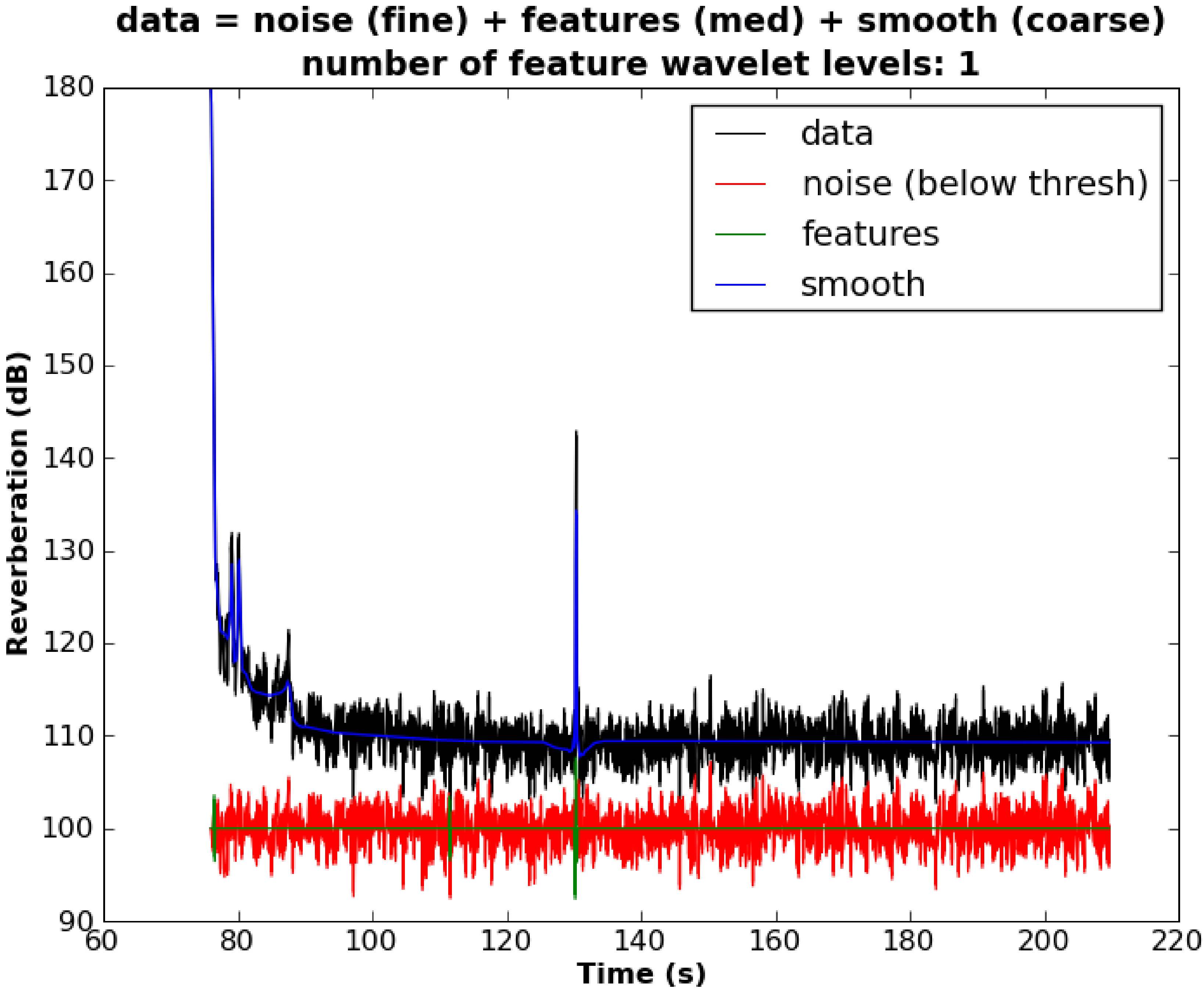

Without initial transient
Reverberation (dB)
180
160
140
120
100
80
60
80
100
120
140
160
180
200
220
Time (s)

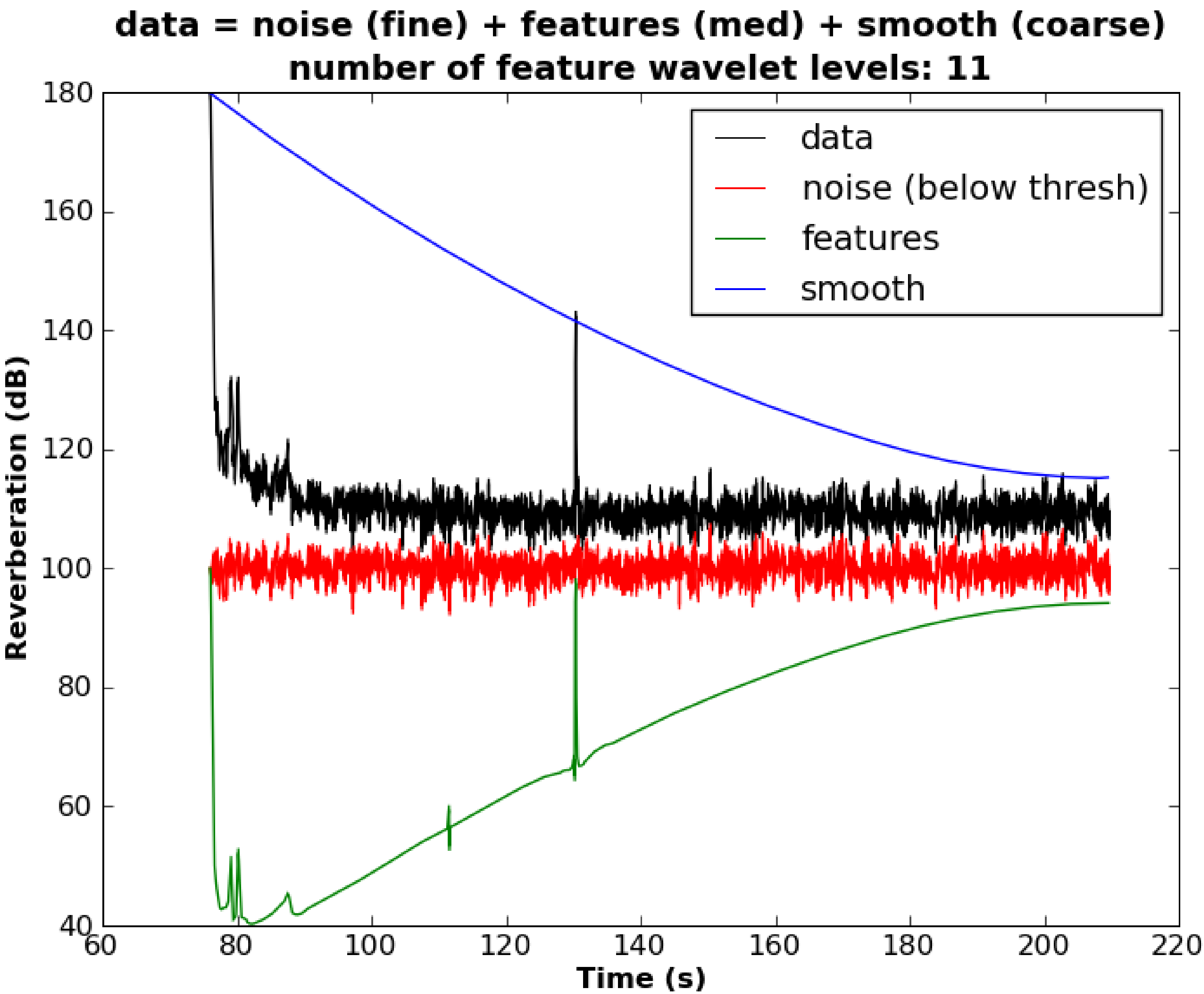

Twice reflected boundary conditions
Reverberation (dB)
Time (s)
300
250
200
150
100
50
−100
0
100
200
300
400
500

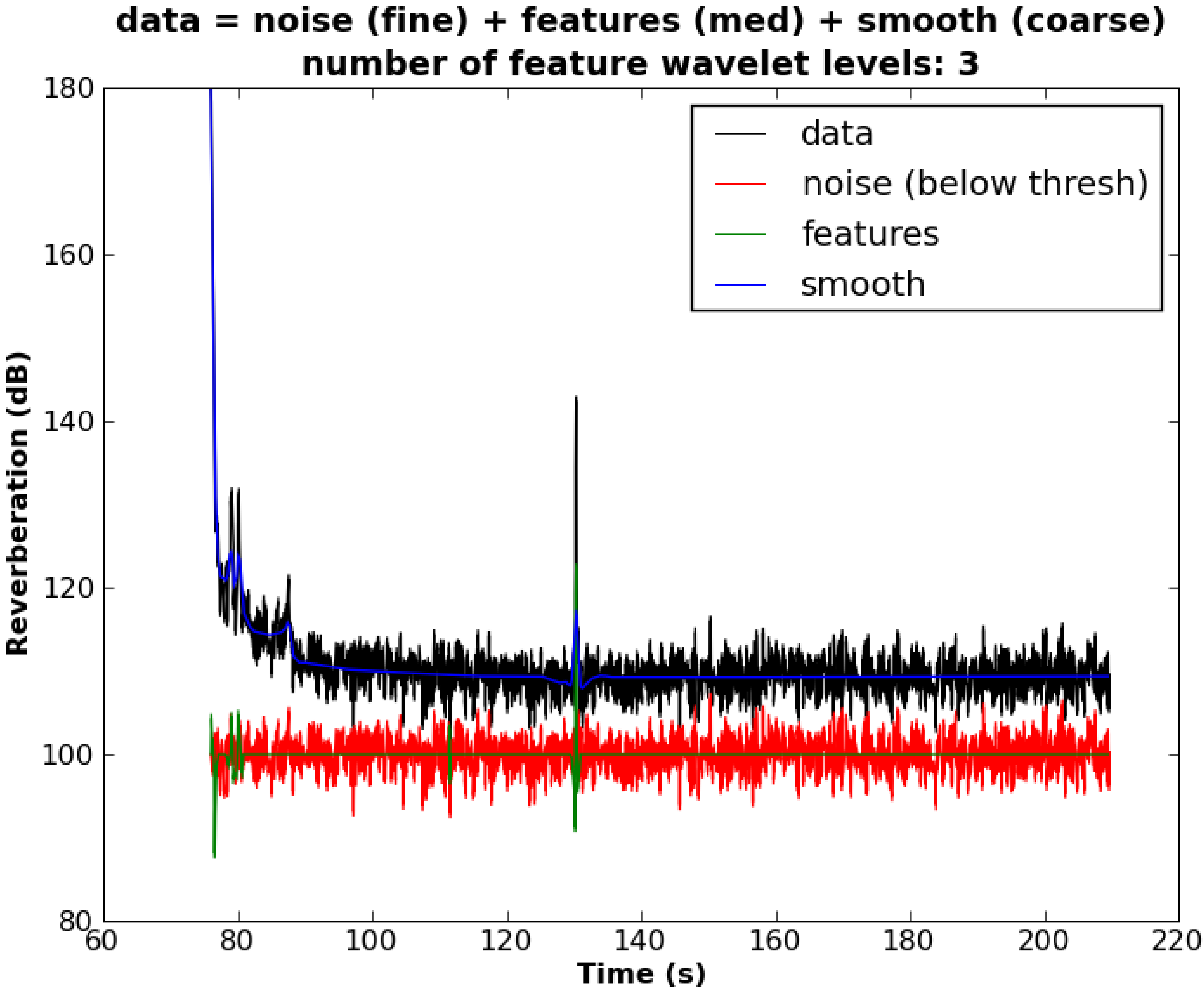


Reverberation (dB) Haar modwt wavelet filtered
thresholds: (j=1: 5.83, j=11: 0.18)
Details and Smooth below threshold
Time (s)
120
100
80
60
40
20
0
−20
60
80
100
120
140
160
180
200
220

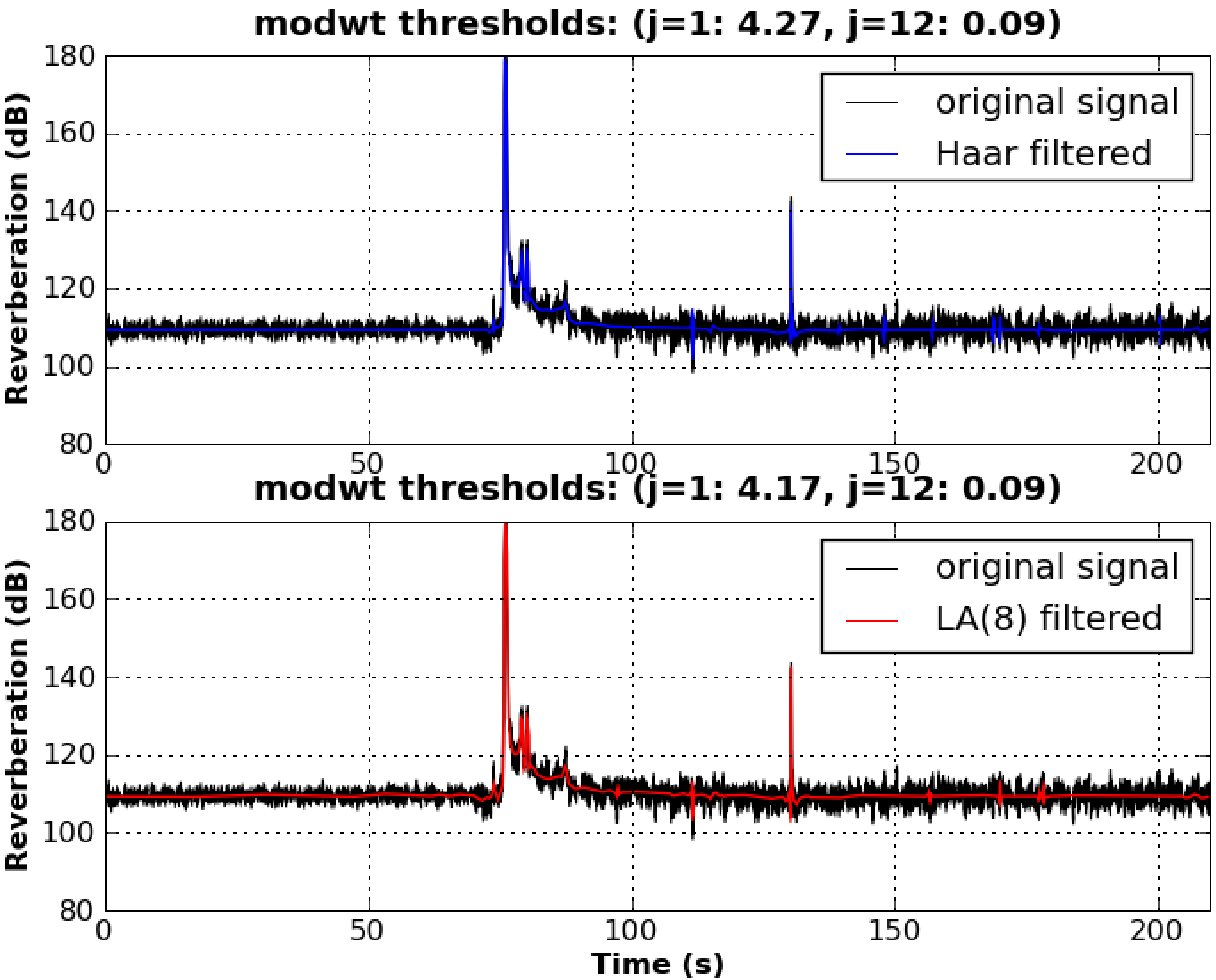

Reverberation (dB) Haar modwt wavelet filtered
thresholds: (j=1: 5.83, j=11: 0.18)
200
150
100
50
0
−50
60
80
100
120
140
160
180
200
220
Time (s)
Details and Smooth above threshold


data = noise (fine) + features (med) + smooth (coarse)
number of feature wavelet levels: 1
Reverberation (dB)
Time (s)
180
170
160
150
140
130
120
110
100
90
60
80
100
120
140
160
180
200
220
data
noise (below thresh)
features
smooth


data = noise (fine) + features (med) + smooth (coarse)
number of feature wavelet levels: 11
data
noise (below thresh)
features
smooth
Reverberation (dB)
Time (s)
180
160
140
120
100
80
60
40
60
80
100
120
140
160
180
200
220



data = noise (fine) + features (med) + smooth (coarse)
number of feature wavelet levels: 3
data
noise (below thresh)
features
smooth
Reverberation (dB)
Time (s)
180
160
140
120
100
80
60
80
100
120
140
160
180
200
220


modwt thresholds: (j=1: 4.27, j=12: 0.09)
Reverberation (dB)
180
160
140
120
100
80
0
50
100
150
200
original signal
Haar filtered
modwt thresholds: (j=1: 4.17, j=12: 0.09)
Reverberation (dB)
180
160
140
120
100
80
0
50
100
150
200
original signal
LA(8) filtered
Time (s)

Example 2

- Geoacoustic inversion with a natural source, in this case ambient noise from ocean surface waves.
- Presented following paper at recent Acoustical Society meeting in Miami (select slides shown; full version online as a POMA paper).
- Basic idea was to place receiver (moored underwater glider) near bottom in shallow water and attempt measurement of upward and downward beams to obtain bottom loss.

Ambient noise inversion for bottom loss with a glider

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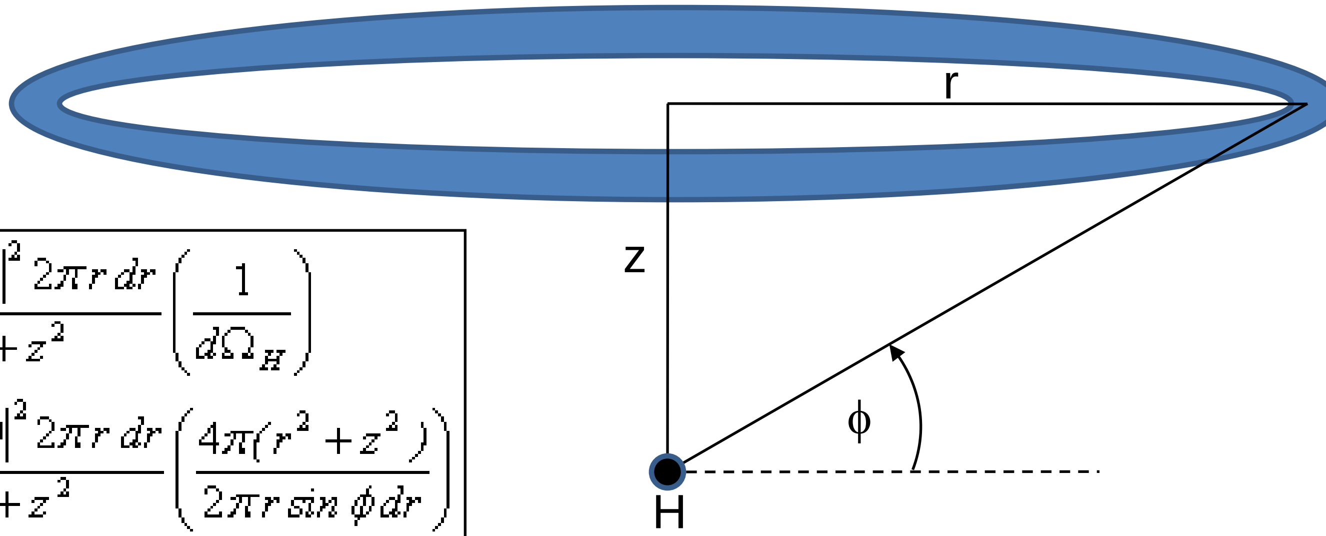
Physical surface noise model



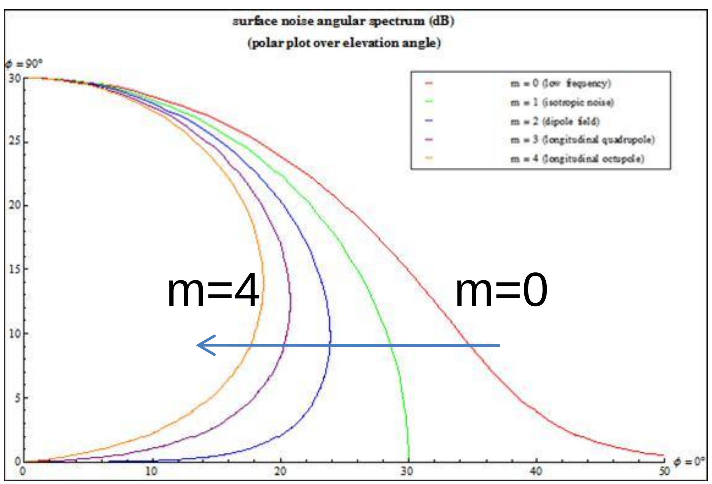
[Burdic]

$$|N(\phi)|^2 = \frac{dI_H}{d\Omega_H} = \frac{I_0 |g(\phi)|^2 2\pi r\,dr}{r^2+z^2}\left(\frac{1}{d\Omega_H}\right)$$

$$= \frac{I_0 |g(\phi)|^2 2\pi r\,dr}{r^2+z^2}\left(\frac{4\pi(r^2+z^2)}{2\pi r \sin\phi\, dr}\right)$$

$$= \frac{4\pi I_0 |g(\phi)|^2}{\sin\phi}$$

impulse: $|N_{\phi_0}(\phi)|^2 = 4\pi I_0\, \delta(\phi - \phi_0)$



The surface noise angular spectrum is modeled by [Cron and Sherman, 1962]

multipole: $|g(\phi)|^2 = \sin^m(\phi)$

where

- m = 0: impulsive field at ϕ = 0 (distant shipping, low frequency)
- m = 1: monopole field (isotropic noise, low-med freq. transition region)
- m = 2: dipole field (surface noise, med frequency)
- m = 3: longitudinal quadrupole (fine structure)
- m = 4: longitudinal octupole (hyperfine structure)

Ambient noise inversion: A simple method with a glider [Arvelo, Harrison, and many others]

Array noise power output

$$P_M^{v} = 2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{-\alpha s_{DP}} + |R_b|^2 e^{-\alpha s_{BB}}}{1 - |R_s|^2 |R_b|^2 e^{-\alpha(s_{DP}+s_{BB})}} \left|N(\phi)\right|^2 \left|G_M(\phi)\right|^2 \cos\phi \, d\phi$$

Surface noise angular spectrum (previous slide)

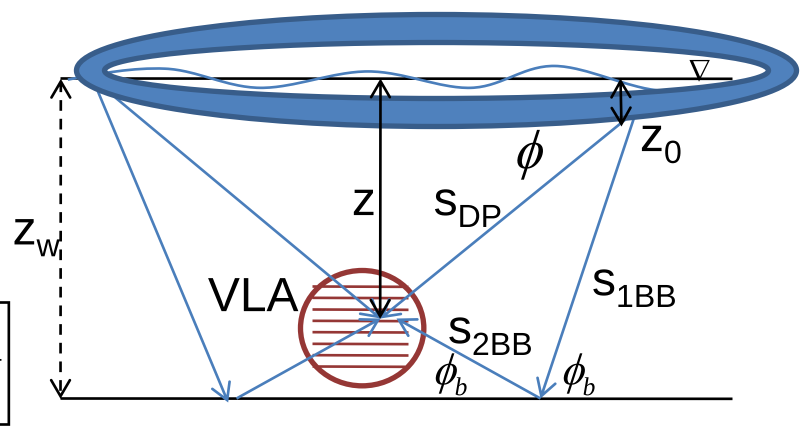
Array pattern function

$$G_M(\phi) = \int g_M(z) \exp\left[-\frac{2\pi i z}{\lambda}(\sin\phi - \sin\phi_0)\right] dz$$

$$g_M(z) \equiv \frac{1}{M} \sum_{n=-\frac{M}{2}}^{n=\frac{M}{2}-1} \delta(z - n d_z)$$

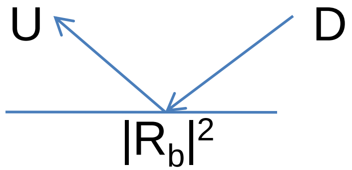
$$\int g_M(z) dz \equiv 1$$

$$s_{DP} = \frac{z - z_0}{\sin\phi} \quad \text{and} \quad s_{BB} = \frac{2z_w - (z + z_0)}{\sin\phi_b}$$



Measurement model

“ U = D $|R_b|^2$ ”



BL ~ difference between the upward and downward looking vertical beam powers:

$$\Delta VN \equiv 10\log\left[P_M^{\mathrm{v}}\left(+\phi_0\right)\right] - 10\log\left[P_M^{\mathrm{v}}\left(-\phi_0\right)\right]$$

$$= -10\log\left[|R_b|^2\, e^{-\alpha(s_{BB}-s_{DP})}\right]$$

$$\equiv BL + \frac{\alpha}{2}(s_{BB} - s_{DP})20\log e$$

$$= BL + \frac{s_{BB} - s_{DP}}{2}\,\alpha_{water}(dB/m)$$

Sources/further reading

- Wavelets
  - Percival and Walden, *WMTSA*, 2006
  - Mallat, *A wavelet tour…The sparse way*, 2009
  - Daubechies, *Ten lectures on wavelets*, 1992
  - Kaiser, *…Complex spacetime—towards a new synthesis*, 1990
- Ambient noise
  - Kuperman and Ingenito, *Spatial correlation…surface noise…*, 1980
  - Cox, *Spatial Correlation…arbitrary fields…ambient noise*, 1973
  - Cron and Sherman, *Spatial-correlation…noise models*, 1962
  - Eckart, *Theory of noise in continuous media*, 1953
  - Many others [textbooks: Burdic, Frisk, Jensen et al., Jackson (EM)…]