

Homework 5 – Due in class on Fri., Aug. 11 – 80 points possible

Please do your own work

1) Consider the following English sentence:

Some grumpy fowls hate every iceberg.

Using the following assignments of predicate-letters, state whether or not each of the following wffs is a plausible translation of that English sentence.

F: is a fowl

G: is grumpy

H: hates (2-place predicate)

I: is an iceberg

(1 point each – 12 points total for this problem)

- a) $(x)((Gx \ \& \ Fx) \rightarrow (\exists y)(Iy \ \& \ Hxy))$ *No*
- b) $(\exists x)((Gx \ \& \ Fx) \ \& \ (y)(Iy \rightarrow Hxy))$ *Yes*
- c) $(x)((Gx \rightarrow Fx) \rightarrow (\exists y)(Iy \ \& \ Hxy))$ *No*
- d) $(\exists x)((Gx \ \& \ Fx) \rightarrow (y)(Iy \rightarrow Hxy))$ *No*
- e) $(x)((Gx \ \& \ Fx) \ \& \ (\exists y)(Iy \ \& \ Hxy))$ *No*
- f) $(\exists x)((Gx \rightarrow Fx) \ \& \ (y)(Iy \rightarrow Hxy))$ *No*
- g) $(\exists z)((Gz \ \& \ Fz) \ \& \ (x)(Ix \rightarrow Hxz))$ *Yes*
- h) $(x)(Ix \rightarrow (\exists y)((Gy \ \& \ Fy) \ \& \ Hyx))$ *Yes*
- i) $(x)(Ix \ \& \ (\exists y)((Gy \ \& \ Fy) \ \& \ Hyx))$ *No*
- j) $(x)(Ix \rightarrow (\exists y)((Gy \ \& \ Fy) \rightarrow Hyx))$ *No*
- k) $(x)(Ix \rightarrow (\exists y)((Gy \rightarrow Fy) \ \& \ Hyx))$ *No*
- l) $(y)(Iy \rightarrow (\exists x)((Gx \rightarrow Fx) \ \& \ Hyx))$ *No*

2) Consider the following English sentence and its translation:

If anyone hurts someone, then she hurts herself.

H: hurts (2-place predicate)

$(x)((\exists y)Hxy \rightarrow Hxx)$

State whether or not each of the following wffs is equivalent to that one.

(1 point each – 13 points total for this problem)

- a) $(x)(y)(Hxy \ \& \ Hxx)$ *No*
- b) $(x)(y)(\neg Hxx \rightarrow \neg Hxy)$ *Yes* (via sequents 118 and 9)
- c) $(x)(\neg Hxx \rightarrow (y)\neg Hxy)$ *Yes* (from b, via 3.4.3 a)
- d) $(\exists x)(y)(Hxy \ \& \ Hxx)$ *No*
- e) $(x)((y)Hxy \rightarrow Hxx)$ *No*
- f) $(x)(y)(Hxy \rightarrow Hxx)$ *Yes* (via sequent 118)
- g) $(x)(\exists y)(Hxy \rightarrow Hxx)$ *No*
- h) $(x)\neg(\exists y)\neg(Hxy \rightarrow Hxx)$ *Yes* (from f, via sequent 114)
- i) $(x)\neg(\exists y)(Hxy \ \& \ \neg Hxx)$ *Yes* (from h, via sequent 35g)
- j) $(x)(y)(\neg Hxy \vee Hxx)$ *Yes* (from f, via sequent 49.1)
- k) $(x)(y)(Hxy \vee \neg Hxx)$ *No*
- l) $(\exists y)Hxy \rightarrow (x)Hxx$ *No* (not a wff – the first ‘x’ is unbound)
- m) $(\exists y)(Hxy \rightarrow (x)Hxx)$ *No* (ditto)

3) Prove each of the following two-part sequents, using SI or TI when you can. *If you use a substitution-instance at any point, state what has been substituted for what.* (1 point per line – 55 points total for this problem)

a) $(x)(Fx \rightarrow -Gx) \vdash \vdash -(\exists x)(Fx \& Gx)$

The first part may be done in 12 lines, and the second part in 9 lines, although you may do the proof correctly with a different number of lines.

$(x)(Fx \rightarrow -Gx) \vdash -(\exists x)(Fx \& Gx)$		
1	(1)	$(x)(Fx \rightarrow -Gx)$ A
2	(2)	$(\exists x)(Fx \& Gx)$ A / RAA
3	(3)	$Fa \& Ga$ A / EE
1	(4)	$Fa \rightarrow -Ga$ 1 UE
3	(5)	Fa 3 &E
3	(6)	Ga 3 &E
1,3	(7)	$-Ga$ 4,5 MPP
1,3	(8)	$Ga \& -Ga$ 6,7 &I
1 ,3	(9)	$-(x)(Fx \rightarrow -Gx)$ 1,8 RAA
2, 3	(10)	$-(x)(Fx \rightarrow -Gx)$ 2,3,9 EE
1,2	(11)	$(x)(Fx \rightarrow -Gx) \& -(x)(Fx \rightarrow -Gx)$ 1,10 &I
1, 2	(12)	$-(\exists x)(Fx \& Gx)$ 2,11 RAA
(Assumption 1 is discharged on line 9, assumption 3 is discharged on line 10, and assumption 2 is discharged on line 12)		
$-(\exists x)(Fx \& Gx) \vdash (x)(Fx \rightarrow -Gx)$		
1	(1)	$-(\exists x)(Fx \& Gx)$ A
2	(2)	Fa A / CP $\vdash -Ga$
3	(3)	$Fa \& Ga$ A / RAA
3	(4)	$(\exists x)(Fx \& Gx)$ 3 EI
1,3	(5)	$(\exists x)(Fx \& Gx) \& -(\exists x)(Fx \& Gx)$ 1,4 &I
1, 3	(6)	$-(Fa \& Ga)$ 3,5 RAA
1,2	(7)	$-Ga$ 2,6 SI (S) 34
1, 2	(8)	$Fa \rightarrow -Ga$ 2,7 CP
1	(9)	$(x)(Fx \rightarrow -Gx)$ 8 UI
<i>On line 7, Fa is substituted for P, and Ga for Q</i>		
(Assumption 3 is discharged on line 6, and assumption 2 is discharged on line 8)		

b) $(\exists x)(Fx \rightarrow P) \vdash \vdash (x)Fx \rightarrow P$

The first part may be done in 7 lines, and the second part in 14 lines, although you may do the proof correctly with a different number of lines.

$(\exists x)(Fx \rightarrow P) \vdash (x)Fx \rightarrow P$			
1	(1)	$(\exists x)(Fx \rightarrow P)$	A
2	(2)	$Fa \rightarrow P$	A / EE
3	(3)	$(x)Fx$	A / CP $\vdash P$
3	(4)	Fa	3 UE
2,3	(5)	P	2,4 MPP
1,2,3	(6)	P	1,2,5 EE
1,3	(7)	$(x)Fx \rightarrow P$	3,6 CP

(Assumption 2 is discharged on line 6, and assumption 3 is discharged on line 7)

$(x)Fx \rightarrow P \vdash (\exists x)(Fx \rightarrow P)$			
1	(1)	$(x)Fx \rightarrow P$	A
2	(2)	$\neg(\exists x)(Fx \rightarrow P)$	A / RAA
3	(3)	$Fa \rightarrow P$	A / RAA
3	(4)	$(\exists x)(Fx \rightarrow P)$	3 EI
2,3	(5)	$(\exists x)(Fx \rightarrow P) \ \& \ \neg(\exists x)(Fx \rightarrow P)$	2,4 &I
2,3	(6)	$\neg(Fa \rightarrow P)$	3,5 RAA
2	(7)	$Fa \ \& \ \neg P$	6 SI (S) 35g
2	(8)	Fa	7 &E
2	(9)	$(x)Fx$	8 UI
1,2	(10)	P	1,9 MPP
2	(11)	$\neg P$	7 &E
1,2	(12)	$P \ \& \ \neg P$	10,11 &I
1,2	(13)	$\neg\neg(\exists x)(Fx \rightarrow P)$	2,12 RAA
1	(14)	$(\exists x)(Fx \rightarrow P)$	13 DN

On line 7, Fa is substituted for P, and P for Q
 (Assumption 3 is discharged on line 6, and assumption 2 is discharged on line 13)

c) $Fa \vdash \vdash (x)(x=a \rightarrow Fx)$

The first part may be done in 7 lines, and the second part in 4 lines, although you may do the proof correctly with a different number of lines.

$Fa \vdash \vdash (x)(x=a \rightarrow Fx)$			
1	(1)	Fa	A
2	(2)	$b=a$	A / CP $\vdash Fb$
	(3)	$b=b$	=I
2	(4)	$a=b$	2,3 =E
1,2	(5)	Fb	1,4 =E
1,2	(6)	$b=a \rightarrow Fb$	2,5 CP
1	(7)	$(x)(x=a \rightarrow Fx)$	6 UI
(Assumption 2 is discharged on line 6)			

$(x)(x=a \rightarrow Fx) \vdash Fa$			
1	(1)	$(x)(x=a \rightarrow Fx)$	A
1	(2)	$a=a \rightarrow Fa$	1 UE
	(3)	$a=a$	=I
1	(4)	Fa	2,3 MPP