Homework 4 – Due in class on Fri., Aug. 4

Please do your own work

1) Using Lemmon's ten primitive rules (plus one definition), but not SI or TI, do or redo each proof from Exam 2 for which you received less than full points. This includes problems 1-4, plus extra credit, but not problems 5-6. On your test, written in red near your name, are the problems you are supposed to do.

2) Using Lemmon's SI and TI rules and the online sequent sheet, do the following proofs from Exam 2. For each proof, a likely number of lines is given, but you may do the proof correctly with a different number of lines. *If you use a substitution-instance at any point, state what has been substituted for what.*

   a) P & -Q ⊢ - (-P v Q) (6 lines)
   b) P ↔ - Q ⊢ Q → -P (4 lines)
   c) ⊢ -P → (P → Q) (3 lines)
   d) P v - Q , Q ⊢ P (4 lines)
   e) - (P → -Q) ⊢ P & Q (6 lines)
   f) - (P → -P) ⊢ P (3 lines)

3) State whether or not each of the following is a wff of the predicate calculus, as defined in class.
   a) Pt_1t_2
   b) Fabm
   c) Hxcy
   d) Gax → Fxa
   e) Gan ↔ -Fan
   f) R & R
   g) -(x)(Gxa)
   h) (x)-Gxa
   i) (x)Gan
   j) -(∃y)-(x)-(-Gx & Fxy)
   k) (∃y)-(x)((-Gcx & Fxy) v (z)Hz)
   l) (x)((-Gx & Fxy) v (y)Hy)
   m) (∃z) – (P ↔ Hz)
   n) (∃y)(Fy → (x)Fx)
   o) (∃m)(Fm → Gm)
   p) (Q → (P v R)) v ((∃z)Fz ↔ P)
   q) (y) – Gmya → - Gmna
   r) - Gmna → (∃z) - Gzna
   s) -(x)Fx ⊣ ⊢ (∃x)-Fx
   t) Q & $(v_1)Gv_1ab$
   u) (x)(Fx v ((∃x)Gx & (y)Gxy))
   v) (x)(Fxa v P) v – (∃z)(x)(P → Hxbz)
   w) (x)(Fx v P) v – (∃z)(y)(P → Hxyz)
   x) (x)((Fx v P) v – (∃z)(y)(P → Hxyz))

y) (x)((Fx v P) v – (∃z)(y)(P → Hxy))
z) (x)-(y)(-Gxyab)
aa) (x)-(y)Gxab
bb) (x)Fx → (∃x)Fx
cc) (x)Fxa → Fma

4) Circle the main quantifier or main connective of each of the following wffs.
a) -(∃x)(Fx & Gax)
b) (x) - (Fx → Gxb) v (Q & (∃x)Hx)
c) (x)(Fx → (∃y)(Gy & Hyx))
d) (∃x)(Gx & (y)(Fyx → Hy))
e) (x)(Fx → Hbcx) → (∃x)(Fx & Hbcx)
f) (x)(∃y)(z)Hbamxczy
g) (x)(∃y)-(z)Hbamxczy
h) -(x)(∃y)-(z)Hbamxczy
i) (∃z)Hz ↔ -(x)-(y)-Fxy
j) (∃x)(y)(Hxab & Gxcy)

5) Consider the following lines of proof:

1	(1)	(x)Fx → (x)Gx	A
2	(2)	-Gm	A

What is the wff on line 1 – universal quantification or conditional? (Circle the correct answer)

State for each of the following lines of proof whether the move is legal or illegal according to Lemmon's rules.

a)	3	(3)	Fm → Gm	1 UE
b)	1	(3)	Fm → Gm	1 UE
c)	3	(3)	(x)Gx	A / RAA
d)	2	(3)	(x)-Gx	2 UI
e)	2	(3)	-(x)Gx	2 UI
f)	1,2	(3)	-(x)Fx	1,2 MTT
g)	1,2	(3)	(x)-Fx	1,2 MTT
h)	2	(3)	(∃x)-Gx	2 EI
i)	2	(3)	-(∃x)Gx	2 EI

6) Consider the following line of proof:
   1 (1) - (∃x)(Fx & Gx) A
   What is the wff on line 1 – existential quantification or negation? (Circle the correct answer)

State for each of the following lines of proof whether the move is legal or illegal according to Lemmon's rules.

a)	1	(2)	-(Fa & Ga)	1 EE
b)	2	(2)	Fa & Ga	A / RAA
c)	1	(2)	(x) – (Fx & Gx)	1 UI
d)	1	(2)	(x) – (Fx & Gx)	1 DN
e)	1	(2)	-Fa & -Ga	1 EE
f)	1	(2)	Fa	A / RAA

7) Consider the following line of proof:
   1 (1) (∃x)-(Fx & Gx) A
   What is the wff on line 1 – existential quantification or negation? (Circle the correct answer)

State for each of the following lines of proof whether the move is legal or illegal according to Lemmon's rules.

a)	1	(2)	-(Fa & Ga)	1 EE
b)	1	(2)	-(Fa & Ga)	A / EE
c)	2	(2)	-(Fa & Ga)	A / EE
d)	1	(2)	-Fa v –Ga	1 DN
e)	1	(2)	-(x)(Fx & Gx)	1 UI
f)	1	(2)	-(x)(Fx & Gx)	1 DN

8) Consider the following lines of proof:
   1 (1) (x)(Fx → Gx) A
   1 (2) Fa → Ga 1 UE
   What is the wff on line 1 – universal quantification or conditional? (Circle the correct answer)
   State for each of the following lines of proof whether the move is legal or illegal according to Lemmon's rules.

a)	1	(3)	(y)(Fy → Gy)	1 UI
b)	1	(3)	(y)(Fy → Gy)	2 UI
c)	1	(3)	(∃x)(Fx → Gx)	2 EI
d)	1	(3)	(∃x)(Fx → Ga)	2 EI
e)	2	(3)	(∃x)(Fx → Ga)	2 EI
f)	3	(3)	Fb → Gb	1 UE
g)	1	(3)	(x)(Fx → Ga)	1,2 MPP
h)	1	(3)	(x)(Fa → Gx)	1,2 MPP

9) Complete the following two-part proof:
(∃x)-Fx ⊣ ⊢ - (x)Fx
(The first part may be done in as few as 7 lines, and the second part may be done in 11 lines. You may do the proof correctly with a different number of lines.)

(a) (∃x)-Fx ⊢ - (x)Fx	(b) - (x)Fx ⊢ (∃x)-Fx