

2) Complete the following two-part proof:

$\neg(-P \ \& \ Q) \vdash P \vee \neg Q$

(The first part may be done in as few as 15 lines, and the second part may be done in 11 lines. You may do the proof correctly with a different number of lines.)

(1 point for each line of proof – 26 points total for this problem)

(a) $\neg(-P \ \& \ Q) \vdash P \vee \neg Q$				(b) $P \vee \neg Q \vdash \neg(-P \ \& \ Q)$			
1	(1)	$\neg(-P \ \& \ Q)$	A	1	(1)	$P \vee \neg Q$	A
2	(2)	$\neg(P \vee \neg Q)$	A / RAA	2	(2)	$\neg P \ \& \ Q$	A / RAA
3	(3)	P	A / RAA	2	(3)	$\neg P$	2 &E
3	(4)	$P \vee \neg Q$	3 vI	2	(4)	Q	2 &E
2,3	(5)	$(P \vee \neg Q) \ \& \ \neg(P \vee \neg Q)$	2,4 &I	5	(5)	P	A / vE
2,3	(6)	$\neg P$	3,5 RAA	2,5	(6)	$P \ \& \ \neg P$	3,5 &I
7	(7)	$\neg Q$	A / RAA	2,5	(7)	$\neg(-P \ \& \ Q)$	2,6 RAA
7	(8)	$P \vee \neg Q$	7 vI	8	(8)	$\neg Q$	A / vE
2,7	(9)	$(P \vee \neg Q) \ \& \ \neg(P \vee \neg Q)$	2,8 &I	2,8	(9)	$Q \ \& \ \neg Q$	4,8 &I
2,7	(10)	$\neg \neg Q$	7,9 RAA	2,8	(10)	$\neg(-P \ \& \ Q)$	2,9 RAA
2	(11)	Q	10 DN	1,5,8	(11)	$\neg(-P \ \& \ Q)$	1,5,7,8,10 vE
2	(12)	$\neg P \ \& \ Q$	6,11 &I	Assumption 2 is discharged on lines 7 and 10			
1,2	(13)	$(\neg P \ \& \ Q) \ \& \ \neg(\neg P \ \& \ Q)$	1,12 &I	Assumptions 5 and 8 are discharged on line 11			
1,2	(14)	$\neg \neg(P \vee \neg Q)$	2,13 RAA				
1	(15)	$P \vee \neg Q$	14 DN				
Assumption 3 is discharged on line 6							
Assumption 7 is discharged on line 10							
Assumption 2 is discharged on line 14							

3) Consider the following sequent-expression:

$$-P \rightarrow Q \vdash P \vee Q$$

Are these two wffs really interderivable (in other words, is the sequent valid in both directions)?
 (Yes or no) *yes*

State, for each of the following, whether or not it is a substitution-instance of the sequent-expression above. **(1 point each – 15 points total for this problem)**

- | | |
|--|---|
| a) $P \rightarrow Q \vdash -P \vee Q$ | <i>no</i> |
| b) $-R \rightarrow Q \vdash R \vee Q$ | <i>yes (R substituted for P)</i> |
| c) $--P \rightarrow Q \vdash -P \vee Q$ | <i>yes (-P for P)</i> |
| d) $-P \rightarrow -Q \vdash P \vee -Q$ | <i>yes (-Q for Q)</i> |
| e) $-A_1 \rightarrow A_2 \vdash A_1 \vee A_2$ | <i>no (a step along the way)</i> |
| f) $-Q \rightarrow P \vdash Q \vee P$ | <i>yes (Q for P and P for Q)</i> |
| g) $-(P \vee S) \rightarrow Q \vdash (P \vee S) \vee Q$ | <i>yes ((P ∨ S) for P)</i> |
| h) $-P \rightarrow (S \vee Q) \vdash P \vee -(S \vee Q)$ | <i>no</i> |
| i) $Q \rightarrow Q \vdash -Q \vee Q$ | <i>no</i> |
| j) $R \rightarrow P \vdash -R \vee P$ | <i>no</i> |
| k) $-P \rightarrow Q \vdash P \vee Q$ | <i>yes (P for P and Q for Q)</i> |
| l) $--Q \rightarrow Q \vdash -Q \vee Q$ | <i>yes (-Q for P)</i> |
| m) $-(R \& S) \rightarrow -Q \vdash (R \& S) \vee -Q$ | <i>yes ((R & S) for P and -Q for Q)</i> |
| n) $(-R \vee -S) \rightarrow Q \vdash (R \& S) \vee Q$ | <i>no</i> |

4) On the front page of your first exam, you received a list, written in red on the left side of the page, of zero or more problems or parts of problems that you are to do or redo now. The answers are available online (see <http://staff.washington.edu/angelic1/logic/exam1answers.pdf>). Your job is to show, using truth-tables, why the online answers for those problems are correct. You should not miss any of the answers, since those are given to you. The points for this problem will be awarded for showing with truth-tables, and explaining briefly if needed, *why* those answers are correct.
(Points vary as stated on your own test)