

**Homework 2 – Due in class on Mon., July 10 (84 points possible)**

Please do your own work

1) Consider this line of proof:      1      (1)      - (P  $\vee$  Q)      A

**What is the wff on line 1 – negation or disjunction?** (Circle the correct answer)

2) State for each of the following lines of proof whether the move is legal or illegal according to Lemmon's rules. **(1 point each – 15 points total for this problem)**

- |    |   |     |   |                         |   |
|----|---|-----|---|-------------------------|---|
| a) | 2 | (2) | -P $\vee$ -Q                                  | 1 DN                    | <i>illegal</i><br>(assumption number wrong, rule misused) |
| b) | 1 | (2) | --- (P $\vee$ Q)                              | 1 DN                    | <i>legal</i>  |
| c) | 2 | (2) | --- (P $\vee$ Q)                              | 1 DN                    | <i>illegal</i><br>(assumption number wrong)               |
| d) | 1 | (2) | P   | 1 $\vee$ E              | <i>illegal</i> (rule misused)                             |
| e) | 2 | (2) | -P & -Q                                       | 1 &I                    | <i>illegal</i><br>(assumption number wrong, rule misused) |
| f) | 1 | (2) | -P & -Q                                       | 1 &I                    | <i>illegal</i> (rule misused)                             |
| g) | 2 | (2) | P $\vee$ Q                                    | 1 RAA                   | <i>illegal</i><br>(assumption number wrong, rule misused) |
| h) | 1 | (2) | P $\vee$ Q                                    | 1 RAA                   | <i>illegal</i> (rule misused)                             |
| i) | 1 | (2) | -(P $\vee$ Q) $\vee$ (P $\vee$ Q)             | 1 $\vee$ I              | <i>legal</i>  |
| j) | 2 | (2) | -(P $\vee$ Q) $\vee$ (P $\vee$ Q)             | 1 $\vee$ I              | <i>illegal</i><br>(assumption number wrong)               |
| k) | 1 | (2) | P $\rightarrow$ -Q                            | 1 CP                    | <i>illegal</i> (rule misused)                             |
| l) | 1 | (2) | -(P $\vee$ Q) $\leftrightarrow$ -(P $\vee$ Q) | 1 Df. $\leftrightarrow$ | <i>illegal</i> (rule misused)                             |
| m) | 1 | (2) | -(P $\vee$ Q) $\vee$ (R $\leftrightarrow$ S)  | 1 $\vee$ I              | <i>legal</i>  |
| n) | 2 | (2) | -(P $\vee$ Q) $\vee$ (R $\leftrightarrow$ S)  | 1 $\vee$ I              | <i>illegal</i><br>(assumption number wrong)               |

3) Consider the following lines of proof:

- |   |     |                     |   |
|---|-----|---------------------|---|
| 1 | (1) | -R $\rightarrow$ -S | A |
| 2 | (2) | -R                  | A |
| 3 | (3) | S                   | A |

**What is the wff on line 1 – negation or conditional?** (Circle the correct answer)

State for each of the following lines of proof whether the move is legal or illegal according to Lemmon's rules. **(1 point each – 15 points total for this problem)**

- |    |     |     |      |         |  |
|----|-----|-----|------|---------|--|
| a) | 4   | (4) | -S   | 1,2 MTT | <i>illegal</i><br>(assumption number wrong, wrong rule)    |
| b) | 1,2 | (4) | -S   | 1,2 MTT | <i>illegal</i> (wrong rule)                                |
| c) | 1,2 | (4) | -S   | 1,2 MPP | <i>legal</i>   |
| d) | 1,2 | (4) | R    | 1,3 MTT | <i>illegal</i><br>(assumption numbers wrong, rule misused) |
| e) | 1,3 | (4) | R    | 1,3 MTT | <i>illegal</i> (rule misused)                              |
| f) | 1,3 | (4) | ---R | 1,3 MTT | <i>illegal</i> (rule misused)                              |
| g) | 2   | (4) | ---R | 2 DN    | <i>legal</i>   |

h)	1,2	(4)	---S	1,2 MPP	<i>illegal</i> (rule misused)
i)	4	(4)	-R v S	2,3 vI	<i>illegal</i>
			(assumption number wrong, rule misused – supposed to have one line cited)		
j)	2,3	(4)	-R v S	2,3 vI	<i>illegal</i>
			(rule misused – supposed to have one line cited)		
k)	<del>2</del> ,3	(4)	-R → S	2,3 CP	<i>illegal</i>
			(rule misused – 2 is not an assumption for line 3)		
l)	2,3	(4)	-R & S	2,3 &I	<i>legal</i>
m)	1,2	(4)	-R v (-R → -S)	1,2 vI	<i>illegal</i>
			(rule misused – supposed to have one line cited)		
n)	1,3	(4)	S & (-R → -S)	1,3 &I	<i>legal</i>

3) Consider the following valid sequent:  $(P \& Q) \rightarrow R \vdash P \rightarrow (Q \rightarrow R)$

**What is the assumption - conjunction or a conditional?** (Circle the correct answer)

Suppose the proof begins like this:

1	(1)	$(P \& Q) \rightarrow R$	A
2	(2)	P	A / CP $\vdash Q \rightarrow R$

Now let us skip to line 17. Assuming that no errors have been made prior to this point, state for each of the following lines of proof whether or not it correctly completes the proof. **(1 point each – 15 points total for this problem)**

a)	1,2	(17)	$P \rightarrow (Q \rightarrow R)$	2,6 CP	<i>No</i> (assumption 2 should be discharged)
b)	1, <del>2</del>	(17)	$P \rightarrow (Q \rightarrow R)$	2,6 CP	<i>Yes</i>
c)	1	(17)	$(P \& Q) \rightarrow R$	1 A	<i>No</i> (wrong conclusion, wrong assumption number, rule misused – should cite zero line numbers)
d)	1,2	(17)	$P \rightarrow (Q \rightarrow R)$	1 A	<i>No</i> (wrong assumption numbers, rule misused – should cite zero line numbers)
e)	1,2	(17)	$P \rightarrow (Q \rightarrow R)$	15,16 &E	<i>No</i> (wrong assumption numbers for final line; rule misused – should cite one line number)
f)	1	(17)	$P \rightarrow (Q \rightarrow R)$	15 &E	<i>Yes</i>
g)	1, <del>2</del>	(17)	$P \rightarrow (Q \rightarrow R)$	4,16 RAA	<i>No</i> (assumption 4 must be the one that is discharged; wff must have at least one negation sign from RAA)
h)	1,2, <del>3</del>	(17)	$P \rightarrow (Q \rightarrow R)$	2,6 CP	<i>No</i> (wrong assumption numbers for final line; assumption 2 must be the one that is discharged)
i)	1	(17)	$P \rightarrow (Q \rightarrow R)$	16 DN	<i>Yes</i>
j)	1	(17)	$P \rightarrow (Q \rightarrow R)$	14,15 MPP	<i>Yes</i>
k)	1	(17)	$P \rightarrow (Q \rightarrow R)$	14,15 MTT	<i>No</i> (wff must have at least one negation sign from MTT)

l)	1,2	(17)	-- (P → (Q → R))	3,16 RAA	<i>No</i> (wrong conclusion, also problems with discharging assumptions)
m)	1,2	(17)	-- (P → (Q → R))	3,16 RAA	<i>No</i> (ditto)
n)	1,2	(17)	((P & Q) → R) & P	1,2 &I	<i>No</i> (wrong conclusion etc.)

4) Consider this line of proof:      1      (1)      (- -P ↔ Q) & --R      A

**What is the wff on line 1 – negation, biconditional or conjunction?** (Circle correct answer)

State for each of the following lines of proof whether the move is legal or illegal according to Lemmon's rules. (1 point each – 15 points total for this problem)

a)	2	(2)	--P ↔ Q	1 &E	<i>illegal</i> (wrong assumption number)
b)	1	(2)	--P ↔ Q	1 &E	<i>legal</i>
c)	1	(2)	P ↔ Q	1 &E, DN	<i>illegal</i> (illegal to cite or use two rules on the same line)
d)	2	(2)	(- -P ↔ Q) & R	1 DN	<i>illegal</i> (wrong assumption number; illegal to use DN on part of a line)
e)	1	(2)	(- -P ↔ Q) & R	1 DN	<i>illegal</i> (illegal to use DN on part of a line)
f)	1	(2)	R	1 DN, &E	<i>illegal</i> (illegal to cite or use two rules on the same line)
g)	1	(2)	--R	1 &E	<i>legal</i>
h)	2	(2)	Q ∨ ((- -P ↔ Q) & --R)	1 ∨I	<i>illegal</i> (wrong assumption number)
i)	1	(2)	Q ∨ ((- -P ↔ Q) & --R)	1 ∨I	<i>legal</i>
j)	1	(2)	--((- -P ↔ Q) & --R)	1 DN	<i>legal</i>
k)	2	(2)	(P ↔ Q) & --R	1 DN	<i>illegal</i> (wrong assumption number; illegal to use DN on part of a line)
l)	1	(2)	((- -P → Q) & (Q → - -P)) & - -R	1 Df. ↔	<i>legal</i>
m)	1	(2)	((P → Q) & (Q → P)) & R	1 DN, Df. ↔	<i>illegal</i> (illegal to cite or use two justifications on same line)
n)	1	(2)	((- -P ↔ Q) & --R) ∨ (P & - R)	1 ∨I	<i>legal</i>

5) Complete the following two-part proof:

$P \vee (Q \& R) \vdash \vdash (P \vee Q) \& (P \vee R)$

(The first part may be done in as few as 12 lines, and the second part may be done in 11 or 13 lines. You may do the proof correctly with a different number of lines.)

**(1 point for each line of proof – 24 points total for this problem)**

(a) $P \vee (Q \& R) \vdash (P \vee Q) \& (P \vee R)$				(b) $(P \vee Q) \& (P \vee R) \vdash P \vee (Q \& R)$			
1	(1)	$P \vee (Q \& R)$	A	1	(1)	$(P \vee Q) \& (P \vee R)$	A
2	(2)	P	A / vE	1	(2)	$P \vee Q$	1 &E
2	(3)	$P \vee Q$	2 vI	1	(3)	$P \vee R$	1 &E
2	(4)	$P \vee R$	2 vI	4	(4)	P	A / vE
2	(5)	$(P \vee Q) \& (P \vee R)$	3,4 &I	4	(5)	$P \vee (Q \& R)$	4 vI
6	(6)	$Q \& R$	A / vE	6	(6)	Q	A / vE
6	(7)	Q	6 &E	7	(7)	R	A / vE
6	(8)	R	6 &E	6,7	(8)	$Q \& R$	6,7 &I
6	(9)	$P \vee Q$	7 vI	6,7	(9)	$P \vee (Q \& R)$	8 vI
6	(10)	$P \vee R$	8 vI	1,6, <del>4</del> ,7	(10)	$P \vee (Q \& R)$	3,4,5,7,9 vE
6	(11)	$(P \vee Q) \& (P \vee R)$	9,10 &I	1, <del>4</del> ,6	(11)	$P \vee (Q \& R)$	2,4,5,6,10 vE
1, <del>2</del> ,6	(12)	$(P \vee Q) \& (P \vee R)$	1,2,5,6,11 vE				
Assumptions 2 and 6 are discharged on line 12				Assumptions 4 and 7 are discharged on line 10 Assumptions 4 and 6 are discharged on line 11			
				Also correct:			
				1,7, <del>4</del> ,6 (10) $P \vee (Q \& R)$ 2,4,5,6,9 vE			
				1, <del>4</del> ,7 (11) $P \vee (Q \& R)$ 3,4,5,7,10 vE			
				Assumptions 4 and 6 are discharged on line 10 Assumptions 4 and 7 are discharged on line 11			