

**Homework 2 – Due in class on Mon., July 10**

Please do your own work

1) Consider this line of proof:      1      (1)       $\neg(P \vee Q)$       A

**What is the wff on line 1 – negation or disjunction?** (Circle the correct answer)

State for each of the following lines of proof whether the move is legal or illegal according to Lemmon's rules.

a)	2	(2)	$\neg P \vee \neg Q$	1 DN
b)	1	(2)	$\neg\neg(P \vee Q)$	1 DN
c)	2	(2)	$\neg\neg(P \vee Q)$	1 DN
d)	1	(2)	P	1 vE
e)	2	(2)	$\neg P \ \& \ \neg Q$	1 &I
f)	1	(2)	$\neg P \ \& \ \neg Q$	1 &I
g)	2	(2)	$P \vee Q$	1 RAA
h)	1	(2)	$P \vee Q$	1 RAA
i)	1	(2)	$\neg(P \vee Q) \vee (P \vee Q)$	1 vI
j)	2	(2)	$\neg(P \vee Q) \vee (P \vee Q)$	1 vI
k)	1	(2)	$P \rightarrow \neg Q$	1 CP
l)	1	(2)	$\neg(P \vee Q) \leftrightarrow \neg(P \vee Q)$	1 Df. $\leftrightarrow$
m)	1	(2)	$\neg(P \vee Q) \vee (R \leftrightarrow S)$	1 vI
n)	2	(2)	$\neg(P \vee Q) \vee (R \leftrightarrow S)$	1 vI

2) Consider the following lines of proof:

1	(1)	$\neg R \rightarrow \neg S$	A
2	(2)	$\neg R$	A
3	(3)	S	A

**What is the wff on line 1 – negation or conditional?** (Circle the correct answer)

State for each of the following lines of proof whether the move is legal or illegal according to Lemmon's rules.

a)	4	(4)	$\neg S$	1,2 MTT
b)	1,2	(4)	$\neg S$	1,2 MTT
c)	1,2	(4)	$\neg S$	1,2 MPP
d)	1,2	(4)	R	1,3 MTT
e)	1,3	(4)	R	1,3 MTT
f)	1,3	(4)	$\neg\neg R$	1,3 MTT
g)	2	(4)	$\neg\neg R$	2 DN
h)	1,2	(4)	$\neg\neg S$	1,2 MPP
i)	4	(4)	$\neg R \vee S$	2,3 vI
j)	2,3	(4)	$\neg R \vee S$	2,3 vI
k)	2,3	(4)	$\neg R \rightarrow S$	2,3 CP
l)	2,3	(4)	$\neg R \ \& \ S$	2,3 &I
m)	1,2	(4)	$\neg R \vee (\neg R \rightarrow \neg S)$	1,2 vI
n)	1,3	(4)	$S \ \& \ (\neg R \rightarrow \neg S)$	1,3 &I

3) Consider the following valid sequent:  $(P \ \& \ Q) \rightarrow R \vdash P \rightarrow (Q \rightarrow R)$

**What is the assumption - conjunction or a conditional?** (Circle the correct answer)

Suppose the proof begins like this:

1	(1)	$(P \ \& \ Q) \rightarrow R$	A	
2	(2)	P	A / CP	$\vdash Q \rightarrow R$

Now let us skip to line 17. Assuming that no errors have been made prior to this point, state for each of the following lines of proof whether or not it correctly completes the proof.

- |    |       |      |   |            |
|----|-------|------|---|------------|
| a) | 1,2   | (17) | $P \rightarrow (Q \rightarrow R)$       | 2,6 CP     |
| b) | 1,2   | (17) | $P \rightarrow (Q \rightarrow R)$       | 2,6 CP     |
| c) | 1     | (17) | $(P \ \& \ Q) \rightarrow R$            | 1 A        |
| d) | 1,2   | (17) | $P \rightarrow (Q \rightarrow R)$       | 1 A        |
| e) | 1,2   | (17) | $P \rightarrow (Q \rightarrow R)$       | 15, 16 &E  |
| f) | 1     | (17) | $P \rightarrow (Q \rightarrow R)$       | 15 &E      |
| g) | 1,2   | (17) | $P \rightarrow (Q \rightarrow R)$       | 4, 16 RAA  |
| h) | 1,2,3 | (17) | $P \rightarrow (Q \rightarrow R)$       | 2,6 CP     |
| i) | 1     | (17) | $P \rightarrow (Q \rightarrow R)$       | 16 DN      |
| j) | 1     | (17) | $P \rightarrow (Q \rightarrow R)$       | 14, 15 MPP |
| k) | 1     | (17) | $P \rightarrow (Q \rightarrow R)$       | 14, 15 MTT |
| l) | 1,2   | (17) | $--(P \rightarrow (Q \rightarrow R))$   | 3, 16 RAA  |
| m) | 1,2   | (17) | $--(P \rightarrow (Q \rightarrow R))$   | 3, 16 RAA  |
| n) | 1,2   | (17) | $((P \ \& \ Q) \rightarrow R) \ \& \ P$ | 1,2 &I     |

4) Consider this line of proof:      1      (1)       $(- \neg P \leftrightarrow Q) \ \& \ --R$       A

**What is the wff on line 1 – negation, biconditional or conjunction?** (Circle correct answer)

State for each of the following lines of proof whether the move is legal or illegal according to Lemmon's rules.

- |    |   |     |   |                             |
|----|---|-----|---|-----------------------------|
| a) | 2 | (2) | $--P \leftrightarrow Q$   | 1 &E                        |
| b) | 1 | (2) | $--P \leftrightarrow Q$   | 1 &E                        |
| c) | 1 | (2) | $P \leftrightarrow Q$   | 1 &E, DN                    |
| d) | 2 | (2) | $(- \neg P \leftrightarrow Q) \ \& \ R$                                 | 1 DN                        |
| e) | 1 | (2) | $(- \neg P \leftrightarrow Q) \ \& \ R$                                 | 1 DN                        |
| f) | 1 | (2) | R   | 1 DN, &E                    |
| g) | 1 | (2) | $--R$   | 1 &E                        |
| h) | 2 | (2) | $Q \vee ((- \neg P \leftrightarrow Q) \ \& \ --R)$                      | 1 vI                        |
| i) | 1 | (2) | $Q \vee ((- \neg P \leftrightarrow Q) \ \& \ --R)$                      | 1 vI                        |
| j) | 1 | (2) | $--((- \neg P \leftrightarrow Q) \ \& \ --R)$                           | 1 DN                        |
| k) | 2 | (2) | $(P \leftrightarrow Q) \ \& \ --R$                                      | 1 DN                        |
| l) | 1 | (2) | $((- \neg P \rightarrow Q) \ \& \ (Q \rightarrow --P)) \ \& \ - \neg R$ | 1 Df. $\leftrightarrow$     |
| m) | 1 | (2) | $((P \rightarrow Q) \ \& \ (Q \rightarrow P)) \ \& \ R$                 | 1 DN, Df. $\leftrightarrow$ |
| n) | 1 | (2) | $((- \neg P \leftrightarrow Q) \ \& \ --R) \vee (P \ \& \ - \ R)$       | 1 vI                        |

5) Complete the following two-part proof:

$P \vee (Q \& R) \vdash \vdash (P \vee Q) \& (P \vee R)$

(The first part may be done in as few as 12 lines, and the second part may be done in 11 or 13 lines. You may do the proof correctly with a different number of lines.)

<p>(a) <math>P \vee (Q \&amp; R) \vdash (P \vee Q) \&amp; (P \vee R)</math></p>	<p>(b) <math>(P \vee Q) \&amp; (P \vee R) \vdash P \vee (Q \&amp; R)</math></p>
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