

**Homework 1 – Due in class on Wed., June 28**

Please do your own work

- 1) State whether or not each of the following is a wff of the propositional calculus, as defined in class.
- a)  $\neg(P \rightarrow Q)$
  - b)  $(\neg P \rightarrow Q)$
  - c)  $(\neg(P \rightarrow Q))$
  - d)  $\neg(P \rightarrow Q) \ \& \ R \vee S$
  - e)  $A_1 \vee \neg B$
  - f)  $\neg \neg R \rightarrow S$
  - g)  $(R \ \& \ R)$
  - h)  $(R) \rightarrow (S \vee P)$
  - i)  $\neg \neg P$
  - j)  $(\neg \neg P)$
  - k)  $\neg Q, P \rightarrow Q \vdash \neg P$
  - l)  $\neg(P \ \& \ Q \ \& \ R \leftrightarrow S)$
  - m)  $\neg Q \rightarrow (P \vee R) \vee (S \leftrightarrow P)$
  - n)  $\neg B \leftrightarrow A$
  - o)  $(P \vee R) \ \& \ (S \leftrightarrow P)$
  - p)  $\neg \neg \neg (\neg \neg P \rightarrow \neg Q)$
  - q)  $\neg(P \vee \neg R)$
- 2) Consider the following assignments of statements to propositional variables:  
P: April Fool's Day is a paid holiday (*false*)  
Q: Christmas is a paid holiday (*true*)  
R: Halloween is a paid holiday (*false*)  
S: Thanksgiving is a paid holiday (*true*)  
Circle the main connective of each of the following wffs and state the truth-value of the wff (T or F) based on these assignments.
- a)  $\neg(P \rightarrow R) \ \& \ Q$
  - b)  $P \leftrightarrow ((P \vee S) \ \& \ R)$
  - c)  $(Q \vee \neg S) \rightarrow \neg P$
  - d)  $\neg(Q \vee \neg P) \vee (R \leftrightarrow \neg P)$
  - e)  $\neg(P \ \& \ (S \rightarrow Q))$
- 3) Using the same assignments of statements to propositional variables, translate the following English sentences into wffs of the propositional calculus.
- a) Halloween is not a paid holiday only if April Fool's Day is a paid holiday.
  - b) Either Christmas is a paid holiday, or Halloween is a paid holiday just in case Thanksgiving is not a paid holiday.
  - c) It is not the case that both Halloween is a paid holiday and Christmas is a paid holiday.
  - d) If Halloween is not a paid holiday, then both Thanksgiving is a paid holiday and Christmas is a paid holiday.
  - e) April Fool's Day is a paid holiday unless Thanksgiving is a paid holiday.
  - f) It is not the case that Thanksgiving is not a paid holiday.

- g) If April Fool's Day is a paid holiday and Thanksgiving is a paid holiday, then Christmas is not a paid holiday.
- h) Christmas is a paid holiday if neither Halloween is a paid holiday nor Thanksgiving is a paid holiday.
- 4) Consider the following assignments of wffs to metalogical variables:  
 $A_1 : (P \rightarrow Q) \rightarrow P$   
 $A_2 : P \rightarrow (Q \rightarrow P)$   
 $A_3 : \neg(P \vee \neg Q)$   
 $A_4 : \neg(\neg P \rightarrow Q)$   
 $A_5 : \neg P \ \& \ Q$   
 $A_6 : (\neg P \vee Q) \rightarrow (\neg P \ \& \ Q)$   
Classify the relations between the first and second wff of the following pairs as contrary, subcontrary, implies, is implied by, equivalent, contradictory, or independent (may be more than one):
- a)  $A_3$  and  $A_5$   
b)  $A_1$  and  $A_2$   
c)  $A_4$  and  $A_6$   
d)  $A_4$  and  $A_5$   
e)  $A_6$  and  $A_1$   
f)  $A_2$  and  $A_3$
- 5) Using the same assignments of wffs to metalogical variables as in the previous problem, classify each of the following as tautologous, contingent, consistent, inconsistent, valid, invalid, complete, safe, or undetermined (may be more than one):
- a)  $A_3 \vee \neg A_5$   
b)  $A_3 \ \& \ \neg A_5$   
c)  $A_1 \rightarrow A_2$   
d)  $A_2 \rightarrow A_1$   
e)  $A_5 \rightarrow \neg A_6$   
f)  $\neg A_5 \rightarrow A_6$   
g)  $A_2 \leftrightarrow A_3$   
h)  $A_4 \vee A_5$   
i)  $\neg(A_4 \vee A_5)$   
j)  $(A_3 \ \& \ \neg A_5) \rightarrow A_4$   
k)  $A_4 \rightarrow (A_3 \ \& \ \neg A_5)$   
l)  $(A_3 \vee \neg A_5) \rightarrow A_4$   
m)  $A_4 \rightarrow (A_3 \vee \neg A_5)$
- 6) For each of the following sequent-expressions, state whether it represents an argument that is valid, invalid, true, false, complete, incomplete, safe, unsafe, or undetermined (may be more than one).
- a)  $\vdash \neg(\neg P \rightarrow P)$   
b)  $\vdash (Q \rightarrow P) \rightarrow (P \vee \neg Q)$   
c)  $P \vdash Q \rightarrow P$   
d)  $P \leftrightarrow (P \vee Q), \neg P \vdash Q$

- e)  $P \rightarrow -Q, Q \vdash -P$   
 f)  $P \& -Q \vdash -(-P \vee Q)$

- 7) State whether or not each of the following arguments is valid, invalid, true, false, complete, incomplete, safe, unsafe, or undetermined (may be more than one).
- If Elton marries Madonna, then Oprah marries Rosie. Oprah does not marry Rosie. Therefore, Elton does not marry Madonna.
  - Either Tom Cruise marries Brad Pitt, or Barry Bonds does not use steroids. Barry Bonds uses steroids. Therefore, Tom Cruise marries Brad Pitt.
  - Tom Hanks died in 2003. Therefore, if Madonna marries Britney, then Madonna marries Britney.
  - Tom Hanks did not die in 2003. Therefore, if Bill Clinton did not divorce Hillary, then Tom Hanks died in 2003.
  - Tom Hanks did not die in 2003. Therefore, if Tom Hanks died in 2003, then Bill Clinton divorced Hillary.
  - Clinton died in 2003. Therefore, if Oprah married Rosie, then Clinton died in 2003.
  - It is not the case both that Madonna married Britney and Elton married Oprah. Therefore, either Madonna did not marry Britney or Elton did not marry Oprah.
- 8) Suppose that wff  $A_7$  is tautologous,  $A_8$  is inconsistent, and  $A_9$  is contingent. What (if anything) can you conclude about the validity of the following sequents?
- $A_8 \vdash A_9$
  - $\vdash A_7 \& A_9$
  - $A_9 \rightarrow A_7 \vdash A_9$
  - $A_7 \rightarrow A_9 \vdash A_8$
  - $A_8 \& A_7 \vdash A_9$
  - $A_8 \vdash A_7 \leftrightarrow A_8$

9) Going across each row, circle all and only correct answers.

(P&Q) & R	Implies	<input type="checkbox"/>	P	Q	R	-P	-Q	-R
	Is Implied By	<input type="checkbox"/>	P	Q	R	-P	-Q	-R
(P&Q) v R	Implies	<input type="checkbox"/>	P	Q	R	-P	-Q	-R
	Is Implied By	<input type="checkbox"/>	P	Q	R	-P	-Q	-R
(P&Q) → R	Implies	<input type="checkbox"/>	P	Q	R	-P	-Q	-R
	Is Implied By	<input type="checkbox"/>	P	Q	R	-P	-Q	-R
¬((P&Q)&R)	Implies	<input type="checkbox"/>	P	Q	R	-P	-Q	-R
	Is Implied By	<input type="checkbox"/>	P	Q	R	-P	-Q	-R
¬((P&Q) v R)	Implies	<input type="checkbox"/>	P	Q	R	-P	-Q	-R
	Is Implied By	<input type="checkbox"/>	P	Q	R	-P	-Q	-R
¬((P&Q)→R)	Implies	<input type="checkbox"/>	P	Q	R	-P	-Q	-R
	Is Implied By	<input type="checkbox"/>	P	Q	R	-P	-Q	-R

10) Give an example of an invalid sequent, and a valid sequent, *using exactly two different propositional variable-types, and at least one connective, in each sequent* (not necessarily in each wff). Then fill in the table below.

Is this sequent:	Example of an Invalid Sequent:	Example of a Valid Sequent:
↓		
Derivable in a safe system of rules?		
Derivable in a complete system of rules?		
Derivable in a safe and complete system of rules?		
If this sequent is derivable in some system of rules, then that system is:		
If this sequent is underivable in some system of rules, then that system is:		

11) For each of the following sequents, construct its corresponding conditional and state whether that wff is tautologous, contingent, consistent, inconsistent, valid, invalid, complete, safe, or undetermined (may be more than one).

- a)  $\neg(P \ \& \ R), R \rightarrow P \vdash \neg R$
- b)  $(P \ \& \ Q) \rightarrow R \vdash P \rightarrow (Q \rightarrow R)$
- c)  $P \rightarrow Q, \neg P \vdash \neg Q$
- d)  $\neg P \vdash P \rightarrow Q$
- e)  $P \leftrightarrow S, \neg P \ \& \ S \vdash Q \vee R$
- f)  $\neg\neg P, Q \vee \neg P \vdash \neg Q$
- g)  $\neg(P \ \& \ Q), \neg P \vdash \neg Q$